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Enhancement of the critical current in quasiperiodic pinning arrays: One-dimensional chains and Penrose lattices

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Abstract

Here we summarize results from our study of the critical depinning current J_c versus the applied magnetic flux Φ, for: (i) quasiperiodic (QP) one-dimensional (1D) chains and (ii) 2D arrays of pinning centers placed on the nodes of a five-fold Penrose lattice. In 1D QP chains, the peaks in $J_c(\Phi)$ are determined by a sequence of harmonics of the long and short segments of the chain. The critical current $J_c(\Phi)$ has a remarkable self-similarity. In 2D QP pinning arrays, we predict analytically and numerically the main features of $J_c(\Phi)$, and demonstrate that the Penrose lattice of pinning sites provides an enormous enhancement of $J_c(\Phi)$, even compared to triangular and random pinning site arrays. This huge increase in $J_c(\Phi)$ could be useful for applications.

Keywords: Vortex lattice; Critical current; Quasicrystals

1. Introduction

Recent progress in the fabrication of nanostructures has provided a wide variety of well-controlled vortex-confinement topologies, including different regular pinning arrays [1–5]. A main fundamental question in this field is how to drastically increase vortex pinning, and thus the critical current J_c, using artificially-produced periodic arrays of pinning sites (APS). The increase and, more generally, control of the critical current J_c in superconductors by its patterning (perforation) can be of practical importance for applications in microelectronic devices. A peak in the critical current $J_c(\Phi)$, for a given value of the magnetic flux per unit cell, say Φ_1, can be engineered using a superconducting sample with a periodic APS with a matching field $H_1 = \Phi_1/A$ (where A is the area of the pinning cell), corresponding to one trapped vortex per pinning site. However, this peak in $J_c(\Phi)$, while useful to obtain, *decreases very quickly for fluxes away from* Φ_1. Thus, the desired peak in $J_c(\Phi)$ is *too narrow* and not very robust against changes in Φ. It would be greatly desirable to have samples with APS with *many* periods. This multiple-period APS sample would provide either very many peaks or an extremely broad peak in $J_c(\Phi)$, as opposed to just one (narrow) main peak (and its harmonics). We achieve this goal (*a very broad* $J_c(\Phi)$) here by studying samples with *many built-in periods*. Here, we study vortex pinning by 1D quasiperiodic (QP) chains and by 2D APS located on the nodes of QP lattices (e.g., a five-fold Penrose lattice) [6]. We show that the use of the 2D QP (Penrose) lattice of pinning sites results in *a remarkable enhancement of* $J_c(\Phi)$, as compared to other APS, including triangular and random APS. In contrast to superconducting networks, for which only the areas of the network plaquettes play a role [7], for vortex pinning by QP pinning arrays, the specific geometry of the elements which form the QP lattice and their arrangement are important, making the problem far more complex.

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2. Simulation

We model a three-dimensional (3D) slab, infinitely long in the z-direction, by a 2D (in the xy-plane) simulation cell with periodic boundary conditions. The external magnetic field is applied along the z-direction. We perform simulated annealing simulations by numerically integrating the overdamped equations of motion (see, e.g., [8,9]):

$$\eta \mathbf{v}_i = \mathbf{f}_i = \mathbf{f}_i^{\mathrm{vv}} + \mathbf{f}_i^{\mathrm{vp}} + \mathbf{f}_i^{\mathrm{T}} + \mathbf{f}_i^{\mathrm{d}}. \quad (1)$$

Here $\mathbf{f}_i$ is the total force per unit length acting on vortex i, $\mathbf{f}_i^{\mathrm{vv}}$ and $\mathbf{f}_i^{\mathrm{vp}}$ are the forces due to vortex–vortex and vortex–pin interactions, respectively, $\mathbf{f}_i^{\mathrm{T}}$ is the thermal stochastic force, and $\mathbf{f}_i^{\mathrm{d}}$ is the driving force; η is the viscosity, which is set to unity. The force due to the vortex–vortex interaction is $\mathbf{f}_i^{\mathrm{vv}} = \Sigma_j^{N_{\mathrm{v}}} f_0 K_1(|\mathbf{r}_i - \mathbf{r}_j|/\lambda)\mathbf{r}_{ij}$, where N_{v} is the number of vortices, K_1 a modified Bessel function, λ the penetration depth, $\mathbf{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j|$, and $f_0 = \Phi_0^2/8\pi^2\lambda^3$. Here $\Phi_0 = hc/2e$. The pinning force is $\mathbf{f}_i^{\mathrm{vp}} = \Sigma_k^{N_{\mathrm{p}}} f_{\mathrm{p}} \cdot (|\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})}|/r_{\mathrm{p}})\Theta \times [(r_{\mathrm{p}} - |\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})}|)/\lambda]\mathbf{r}_{ik}^{(\mathrm{p})}$, where N_{p} is the number of pinning sites, f_{p} the maximum pinning force of each short-range parabolic potential well located at $\mathbf{r}_k^{(\mathrm{p})}$, r_{p} is the range of the pinning potential, Θ is the Heaviside step function, and $\mathbf{r}_{ik}^{(\mathrm{p})} = (\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})})/|\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})}|$. All the lengths (fields) are expressed in units of λ (Φ_0/λ^2). The ground state of a system of moving vortices is obtained by simulating the field-cooled experiments. For deep short-range (δ-like) potential wells, the energy required to depin vortices trapped by pinning sites is proportional to the number of pinned vortices, $N_{\mathrm{v}}^{(\mathrm{p})}$. Therefore, in this approximation, we can define the critical current as follows: $j_{\mathrm{c}}(\Phi) = j_0 N_{\mathrm{v}}^{(\mathrm{p})}(\Phi)/N_{\mathrm{v}}(\Phi)$, where j_0 is a constant, and study the dimensionless value $J_{\mathrm{c}} = j_{\mathrm{c}}/j_0$. We use narrow potential wells as pinning sites, with $r_{\mathrm{p}} = 0.04$–0.1λ.

3. Critical current in quasiperiodic arrays of pinning sites

3.1. 1D quasicrystal

A 1D QP chain [6] can be constructed by iteratively applying the Fibonacci rule ($L \rightarrow LS$, $S \rightarrow L$), which generates an infinite sequence of two line segments, long L and short S. For an infinite QP sequence, the ratio of the numbers of long to short elements is the golden mean $\tau = (1 + \sqrt{5})/2$ [6]. The position of the nth point where a new segment, either L or S, begins is determined (e.g., for $a_S = 1$ and $a_L = \tau$) by: $x_n = n + [n/\tau]/\tau$, where $[x]$ denotes the integer part of x. To study the critical depinning current J_{c} in 1D QP pinning chains, we place pinning sites to the points where the L or S elements of the QP sequence link to each other. The results of calculating $J_{\mathrm{c}}(N_{\mathrm{v}})$ for different chains and the same $\gamma = 1/\tau$ ($\gamma = a_S/a_L$ is the ratio of the length of the short segment, a_S, to the length of the long segment, a_L) are shown in Fig. 1. For sufficiently long chains, the positions of the main peaks in J_{c}, to a significant extent, do not depend on the length of the chain. The set of peaks in J_{c} includes a Fibonacci

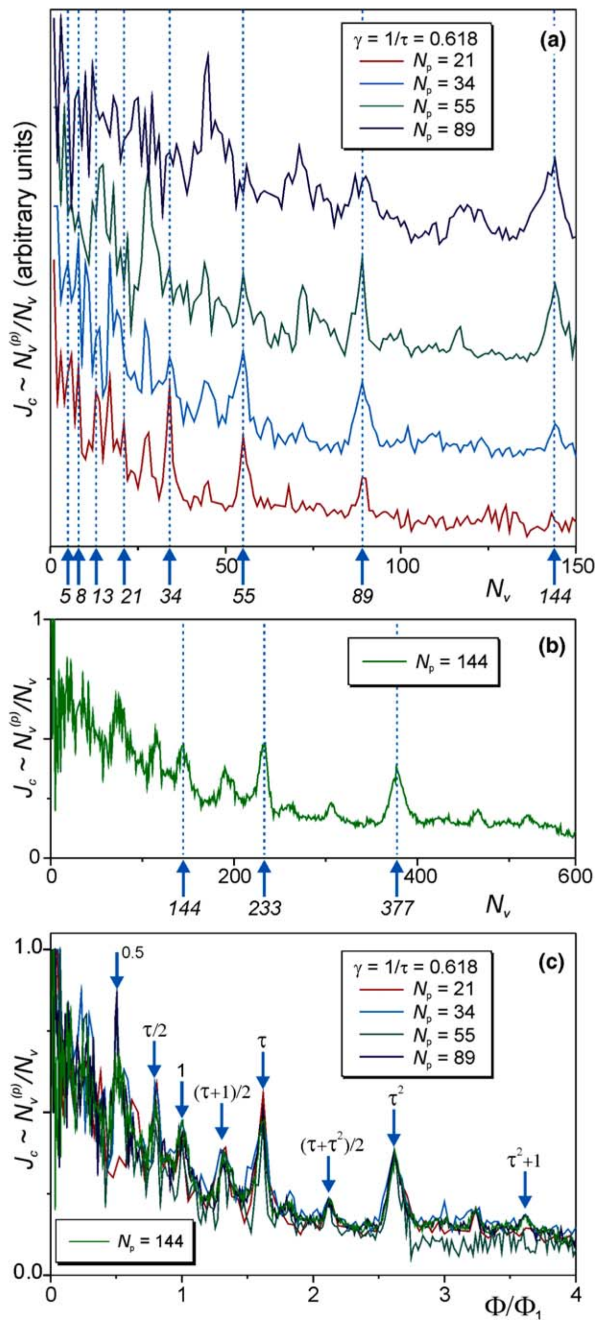

Fig. 1. (a) The critical depinning current J_{c}, versus the number of vortices, $N_{\mathrm{v}} \sim \Phi$, for 1D QP chains, $N_{\mathrm{p}} = 21$ (red bottom line), $N_{\mathrm{p}} = 34$ (blue line), $N_{\mathrm{p}} = 55$ (green line), and $N_{\mathrm{p}} = 89$ (dark blue top line), for $\gamma = a_S/a_L = 1/\tau$. Here we use: $f_{\mathrm{p}}/f_0 = 1.0$ and $r_{\mathrm{p}} = 0.1\lambda$. Independently of the length of the chain, the peaks include the sequence of successive Fibonacci numbers and their subharmonics. (b) $J_{\mathrm{c}}(N_{\mathrm{v}})$ for a long chain, $N_{\mathrm{p}} = 144$, and the same $\gamma = 1/\tau$. (c) The function $J_{\mathrm{c}}(\Phi/\Phi_1)$ for the same 1D chains (using same colors). The curves for different chains display the same set of peaks, namely, at $\Phi/\Phi_1 = 1$ (first matching field) and $\Phi/\Phi_1 = 0.5$, as well as at the golden-mean-related values: $\Phi/\Phi_1 = \tau$, $\tau/2$, $(\tau+1)/2 = \tau^2/2$, $(\tau^2+\tau)/2 = \tau^3/2$, $\tau^2 = \tau + 1$, $\tau^2 + 1$. This behavior demonstrates the self-similarity of $J_{\mathrm{c}}(\Phi)$. (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

sequence: $N_{\mathrm{v}} = 13$, 21, 34, 55, 89, 144, as well as the peaks at first matching field (different for each chain) and other "harmonics". Being rescaled, normalized by the numbers of pins in each chain, the J_c curves reproduce each other and have many pronounced peaks for golden-mean-related values of Φ/Φ_1 (Φ_1 is the flux corresponding to the first matching field, H_1, when $N_v = N_p$). The same peaks of $J_c(\Phi)$, for different chains, are revealed before and after rescaling because of the *self-similarity of* $J_c(\Phi)$. The self-similarity of $J_c(\Phi)$ has also been studied in reciprocal k-space and will be presented elsewhere [10].

3.2. 2D quasiperiodic array of pinning sites: Penrose lattice

Consider now a 2D QP APS, namely, an APS located at the nodes of a five-fold Penrose lattice. This lattice is a 2D QP structure, or quasicrystal, also referred to as Penrose tiling [6]. These structures possess a perfect local rotational (five- or ten-fold) symmetry, but do not have translational long-range order. Being constructed of a series of building blocks of certain simple shapes combined according to specific local rules, these structures can extend to infinity without any defects [6]. The unusual self-similar diffraction pattern of a Penrose lattice exhibits a dense set of ''Bragg'' peaks because the lattice contains *an infinite number of periods in it* [6]. *It is precisely this unusual property that is responsible for the striking* $J_c(\Phi)$*'s obtained here.*

The structure of a five-fold Penrose lattice is presented in the inset to Fig. 2. As an illustration only, a small five-fold symmetric fragment with 46 points is shown. The elemental building blocks are rhombuses with equal sides, a, and angles which are multiples of $\theta = 36°$. There are two kinds of rhombuses: (i) those having angles 2θ and 3θ (''thick''), and (ii) rhombuses with angles θ and 4θ (''thin'').

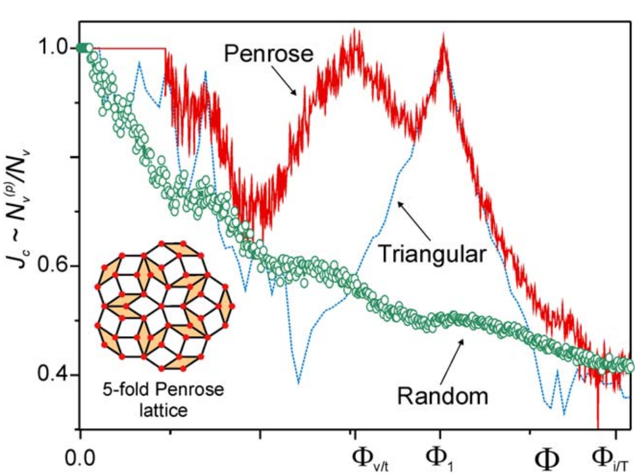

Fig. 2. The critical current $J_c(\Phi)$ for a triangular (blue dashed line), random (green open circles) and QP (301-sites Penrose-lattice) APS (red solid line). The Penrose lattice provides a remarkable enhancement of $J_c(\Phi)$ over a very wide range of values of Φ because it contains many periods in it. The inset shows an example (a 46-sites sample) of a five-fold Penrose lattice, consisting of ''thick'' (empty) and ''thin'' (orange-filled) rhombuses. The pinning parameters are: $f_p/f_0 = 2.0$ and $r_p = 0.1\lambda$. (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

Let us analyze whether any specific matching effects can exist between the Penrose pinning lattice and the interacting vortices, which affect the magnetic-field dependence of the critical depinning current $J_c(\Phi)$. Quasicrystalline patterns are intrinsically incommensurate with the flux lattice for any value of the magnetic field [7], therefore, in contrast to periodic APS, one might a priori assume a lack of sharp peaks in $J_c(\Phi)$ for QP APS. However, the existence of many periods in the Penrose lattice can lead to a hierarchy of matching effects for certain values of the applied magnetic field, resulting in strikingly-broad shapes for $J_c(\Phi)$. These could be valuable for applications demanding unusually broad $J_c(\Phi)$'s.

First, there is a ''first matching field'' (the corresponding flux is denoted as Φ_1) when each pinning site is occupied by a vortex. This matching effect is accompanied by a broad maximum involving three kinds of local ''commensurability'' effects of the flux lattice: with the rhombus side a; with the short diagonal of a thick rhombus, $1.176a$, which is close to a; and with the short diagonal of a thin rhombus, which is $a/\tau \approx 0.618a$.

Another matching is related with the filling of all the pinning sites on the vertices of the thick rhombuses and only three out of four of the pinning sites on the vertices of thin rhombuses. For this value of the flux, matching conditions are fulfilled for two close distances, a (the side of a rhombus) and $1.176a$ (the short diagonal of a thick rhombus), but are not fulfilled for the short diagonal, a/τ, of the thin rhombus. Therefore, this 2D QP feature is related to τ, although not in such a direct way as in the case of a 1D QP pinning array. This 2D QP matching results in a very wide maximum of the function $J_c(\Phi)$ at $\Phi = \Phi_{\text{vacancy/thin}} \equiv \Phi_{v/t} = 0.757\Phi_1$ (Fig. 2).

For higher vortex densities ($\Phi = \Phi_{\text{interstitial/thick}} \equiv \Phi_{i/T} = 1.482\Phi_1$) a single interstitial vortex is inside each thick rhombus. These interstitial vortices can easily move; thus J_c has no peak at $\Phi_{i/T}$. The position of this feature is determined by the number of vortices at $\Phi = \Phi_1$, which is $N_v(\Phi) = N_p$, plus the number of thick rhombuses, $N_{\text{rh}}^{\text{thick}} = N_{\text{rh}}/\tau$.

In order to better understand the structure of $J_c(\Phi)$ for the Penrose pinning lattice, we compare the elastic E_{el} and pinning E_{pin} energies of the vortex lattice at H_1 and at (the lower field) $H_{v/t}$, corresponding to the two maxima of J_c. Vortices can be pinned if the gain $E_{\text{pin}} = U_{\text{pin}}\beta n_{\text{pin}}$ of the pinning energy is larger than the increase of the elastic energy [11] related to local compressions: $E_{\text{el}} = C_{11}[(a_{\text{eq}} - b)/a_{\text{eq}}]^2$. Here, $U_{\text{pin}} \sim f_p r_p$, n_{pin} is the density of pinning centers, $\beta(H \leqslant H_1) = H/H_1 = B/(\Phi_0 n_{\text{pin}})$, and $\beta(H > H_1) = 1$ is the fraction of occupied pinning sites ($\beta = 1$ for $H = H_1$, and $\beta = 0.757$ for $H = H_{v/t}$), $a_{\text{eq}} = [2/3^{1/2}\beta n_{\text{pin}}]^{1/2}$ is the equilibrium distance between vortices in the triangular lattice, b is the minimum distance between vortices in the distorted pinned vortex lattice ($b = a/\tau$ for $H = H_1$ and $b = a$ for $H = H_{v/t}$), and $C_{11} = B^2/[4\pi(1 + \lambda^2 k^2)]$ is the compressibility modulus for short-range deformations [8] with characteristic spatial scale

$k \approx (n_{\rm pin})^{1/2}$. The dimensionless difference of the pinning and elastic energies is $E_{\rm pin} - E_{\rm el} = \beta f_{\rm diff} n_{\rm pin} \Phi_0^2/(4\pi\lambda^2)$, where

$$f_{\rm diff} = 4\pi\lambda^2 U_{\rm pin}/\Phi_0^2 - \beta \times [1 - b(3^{1/2}\beta n_{\rm pin}/2)^{1/2}]^2. \quad (2)$$

Near matching fields, J_c has a peak only if $f_{\rm diff} > 0$. Since only two matching fields provide $f_{\rm diff} > 0$, then our analysis explains the two-peak structure observed in J_c shown in Fig. 2. Note that for weaker pinning, the two-peak structure gradually turns into one very broad peak, and eventually zero peaks for weak enough pinning [10].

For comparison, we show the $J_c(\Phi)$ for Penrose–lattice, triangular and random pinning arrays (Fig. 2). The latter is an average over five realizations of disorder. Notice that the QP lattice leads to a very broad and potentially useful enhancement of the critical current $J_c(\Phi)$, even compared to the triangular or random APS. The remarkably broad maximum in $J_c(\Phi)$ is due to the fact that the Penrose lattice has many (infinite, in the thermodynamic limit) periodicities built in it [6]. In principle, each one of these periods provides a peak in $J_c(\Phi)$. In practice, like in quasicrystalline difraction patterns, only few peaks are strong. This is also consistent with our study.

4. Conclusions

The critical depinning current $J_c(\Phi)$ was studied in 1D QP chains and in 2D QP arrays (the five-fold Penrose lattice) of pinning sites. A hierarchical and self-similar $J_c(\Phi)$ was obtained. We physically analyzed all the main features of $J_c(\Phi)$. Our analysis shows that the QP lattice provides an unusually broad critical current $J_c(\Phi)$, that could be useful for practical applications demanding high J_c's over a wide range of fields.

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References

[1] M. Baert et al., Phys. Rev. Lett. 74 (1995) 3269; V.V. Moshchalkov et al., Phys. Rev. B 54 (1996) 7385.
[2] J.E. Villegas et al., Science 302 (2003) 1188; J.E. Villegas et al., Phys. Rev. B 68 (2003) 224504; M.I. Montero et al., Europhys. Lett. 63 (2003) 118.
[3] A.M. Castellanos et al., Appl. Phys. Lett. 71 (1997) 962; R. Wördenweber et al., Phys. Rev. B 69 (2004) 184504.
[4] L. Van Look et al., Phys. Rev. B 66 (2002) 214511.
[5] A.V. Silhanek et al., Phys. Rev. B 67 (2003) 064502.
[6] J.-B. Suck, M. Schreiber, P. Häussler (Eds.), Quasicrystals, Springer, Berlin, 2002.
[7] A. Behrooz et al., Phys. Rev. B 35 (1987) 8396; F. Nori et al., Phys. Rev. B 36 (1987) 8338; F. Nori, Q. Niu, Phys. Rev. B 37 (1988) R2364; Q. Niu, F. Nori, Phys. Rev. B 39 (1989) 2134.
[8] C. Reichhardt et al., Phys. Rev. B 57 (1988) 7937.
[9] B.Y. Zhu et al., Phys. Rev. Lett. 92 (2004) 180602; B.Y. Zhu et al., Physica E18 (2003) 318; B.Y. Zhu et al., Phys. Rev. B 68 (2003) 014514; B.Y. Zhu et al., Physica C 665 (2003) 388; B.Y. Zhu et al., Physica C 404 (2004) 260.
[10] V. Misko, S. Savel'ev, F. Nori, Phys. Rev. Lett. 95 (2005) 177007; V.R. Misko, S. Savel'ev, F. Nori, cond-mat/0502480; cond-mat/0511098; unpublished.
[11] E.H. Brandt, Rep. Prog. Phys. 58 (1995) 1465.