SUPPLEMENTAL MATERIAL:

1. Calculations of spin, generalized Stokes parameters and degrees of polarization

Here we calculate the spin angular momentum, generalized Stokes parameters, and degree of polarization for each of the interfering waves and for the resulting knotted polarizations described by Eqs. (1)–(3) and Fig. 1 of the main text.

Lissajous-knotted polarizations. For the polarizations described by Eq. (1) in the main text, the complex field amplitudes are:

$$\mathbf{F}_1 = \left(A_1 e^{-i\phi_1}, 0, 0\right), \quad \mathbf{F}_2 = \left(0, A_2 e^{-i\phi_2}, 0\right), \quad \mathbf{F}_3 = \left(0, 0, A_3 e^{-i\phi_3}\right). \tag{S1}$$

Obviously, these linear polarizations do not possess any spin, and the resulting spin Eq. (4) also vanishes:

$$\mathrm{Im}\left(\mathbf{F}_n^* \times \mathbf{F}_n\right) = \mathbf{0}, \quad n = 1,2,3, \qquad \mathbf{S} = \mathbf{0}. \tag{S2}$$

The generalized Stokes parameters for the nth wave are defined as [11–14]:

$$\Lambda_0^{(n)} = \mathbf{F}_n^* \cdot \mathbf{F}_n, \quad \Lambda_3^{(n)} = \frac{3}{2}\left(\left|F_{nx}\right|^2 - \left|F_{ny}\right|^2\right), \quad \Lambda_8^{(n)} = \frac{\sqrt{3}}{2}\left(\left|F_{nx}\right|^2 + \left|F_{ny}\right|^2 - 2\left|F_{nz}\right|^2\right),$$

$$\Lambda_1^{(n)} = 3\,\mathrm{Re}\left(F_{nx}^* \cdot F_{ny}\right), \quad \Lambda_4^{(n)} = 3\,\mathrm{Re}\left(F_{nz}^* \cdot F_{nx}\right), \quad \Lambda_6^{(n)} = 3\,\mathrm{Re}\left(F_{ny}^* \cdot F_{nz}\right),$$

$$\Lambda_2^{(n)} = 3\,\mathrm{Im}\left(F_{ny}^* \cdot F_{nx}\right), \quad \Lambda_5^{(n)} = 3\,\mathrm{Im}\left(F_{nz}^* \cdot F_{nx}\right), \quad \Lambda_7^{(n)} = 3\,\mathrm{Im}\left(F_{nz}^* \cdot F_{ny}\right). \tag{S3}$$

For the Lissajous-knotted fields (S1), Eqs. (S3) yield:

$$\Lambda_1^{(1)} = \Lambda_2^{(1)} = \Lambda_4^{(1)} = \Lambda_5^{(1)} = \Lambda_6^{(1)} = \Lambda_7^{(1)} = 0, \quad \Lambda_0^{(1)} = A_1^2, \quad \Lambda_3^{(1)} = \frac{3}{2}A_1^2, \quad \Lambda_8^{(1)} = \frac{\sqrt{3}}{2}A_1^2,$$

$$\Lambda_1^{(2)} = \Lambda_2^{(2)} = \Lambda_4^{(2)} = \Lambda_5^{(2)} = \Lambda_6^{(2)} = \Lambda_7^{(2)} = 0, \quad \Lambda_0^{(2)} = A_2^2, \quad \Lambda_3^{(2)} = -\frac{3}{2}A_2^2, \quad \Lambda_8^{(2)} = \frac{\sqrt{3}}{2}A_2^2,$$

$$\Lambda_1^{(3)} = \Lambda_2^{(3)} = \Lambda_3^{(3)} = \Lambda_4^{(3)} = \Lambda_5^{(3)} = \Lambda_6^{(3)} = \Lambda_7^{(3)} = 0, \quad \Lambda_0^{(3)} = A_3^2, \quad \Lambda_8^{(3)} = -\sqrt{3}A_3^2. \tag{S4}$$

Obviously, each of the interfering plane waves is fully polarized, and their degrees of polarizations [12] are: $P^{(n)} = \sqrt{\sum_{l=1}^{8}\left(\Lambda_l^{(n)}\right)^2} \Big/ \sqrt{3}\,\Lambda_0^{(n)} = 1$.

The generalized Stokes parameters for the polychromatic interference field, Eq. (5) in the main text, are the sums of the parameters (S4), $\Lambda_l = \sum_n \Lambda_l^{(n)}$, which yields: $\Lambda_1 = \Lambda_2 = \Lambda_4 = \Lambda_5 = \Lambda_6 = \Lambda_7 = 0$,

$$\Lambda_0 = A_1^2 + A_2^2 + A_3^2, \quad \Lambda_3 = \frac{3}{2}\left(A_1^2 - A_2^2\right), \quad \Lambda_8 = \frac{\sqrt{3}}{2}\left(A_1^2 + A_2^2 - 2A_3^2\right). \tag{S5}$$

This results in the degree of polarization

$$P=\frac{\sqrt{\sum_{i=1}^{8}\Lambda_i^2}}{\sqrt{3}\Lambda_0}=\frac{\sqrt{A_1^4+A_2^4+A_3^4-A_1^2A_2^2-A_2^2A_3^2-A_1^2A_3^2}}{A_1^2+A_2^2+A_3^2}. \tag{S6}$$

For waves with equal amplitudes, $A_1=A_2=A_3$, as in Fig. 1(a), the wave becomes totally unpolarized: $P=0$. This is natural, because depolarized 3D light represents incoherent oscillations with equal amplitudes along the three axes. At the same time, 2D Lissajous polarizations with $A_1=A_2$ and $A_3=0$ yield $P=1/2$, which is the known value for a totally unpolarized 2D wave [12,52].

Torus-knotted polarizations. For the polarizations described by Eq. (2) in the main text, the complex field amplitudes are:

$$\mathbf{F}_1=A\left(1,0,-i\right),\quad \mathbf{F}_2=A\left(0,e^{-i\phi_2},0\right),\quad \mathbf{F}_3=A\left(0,0,e^{-i\phi_3}\right). \tag{S7}$$

Here, the first wave is circularly-polarized and carries nonzero spin:

$$\mathrm{Im}\left(\mathbf{F}_1^*\times\mathbf{F}_1\right)=\left(0,2A^2,0\right),\quad \mathrm{Im}\left(\mathbf{F}_2^*\times\mathbf{F}_2\right)=\mathrm{Im}\left(\mathbf{F}_3^*\times\mathbf{F}_3\right)=\mathbf{0}. \tag{S8}$$

Together with the amplitudes $\left|\mathbf{F}_1\right|^2=2A^2$, $\left|\mathbf{F}_2\right|^2=\left|\mathbf{F}_3\right|^2=A^2$, and frequencies $\omega_2/\omega_1=q/p$, $\omega_3/\omega_1=\left(q-p\right)/p$, this results in the normalized spin angular momentum (4) of the total field:

$$\mathbf{S}=\left(0,\frac{2}{2+p/q+p/\left(q-p\right)},0\right). \tag{S9}$$

In particular, for the trefoil knot in Fig. 2(b), $\left(p,q\right)=\left(2,3\right)$, the spin magnitude is $\left|\mathbf{S}\right|\simeq 0.43$.

The generalized Stokes parameters (S3) for each of the interfering fields (S7) yield:

$$\Lambda_1^{(1)}=\Lambda_2^{(1)}=\Lambda_4^{(1)}=\Lambda_6^{(1)}=\Lambda_7^{(1)}=0,\quad \Lambda_0^{(1)}=2A^2,\quad \Lambda_3^{(1)}=\frac{3}{2}A^2,\quad \Lambda_5^{(1)}=3A^2,\quad \Lambda_8^{(1)}=-\frac{\sqrt{3}}{2}A^2,$$

$$\Lambda_1^{(2)}=\Lambda_2^{(2)}=\Lambda_4^{(2)}=\Lambda_5^{(2)}=\Lambda_6^{(2)}=\Lambda_7^{(2)}=0,\quad \Lambda_0^{(2)}=A^2,\quad \Lambda_3^{(2)}=-\frac{3}{2}A^2,\quad \Lambda_8^{(2)}=\frac{\sqrt{3}}{2}A^2,$$

$$\Lambda_1^{(3)}=\Lambda_2^{(3)}=\Lambda_3^{(3)}=\Lambda_4^{(3)}=\Lambda_5^{(3)}=\Lambda_6^{(3)}=\Lambda_7^{(3)}=0,\quad \Lambda_0^{(3)}=A^2,\quad \Lambda_8^{(3)}=-\sqrt{3}A^2. \tag{S10}$$

As before, each of the interfering waves is fully polarized, and their degrees of polarizations $P^{(n)}=1$.

From Eqs. (S10), we obtain the generalized Stokes parameters for the total torus-knotted field:

$$\Lambda_1=\Lambda_2=\Lambda_3=\Lambda_4=\Lambda_6=\Lambda_7=0,\quad \Lambda_0=4A^2,\quad \Lambda_5=3A^2,\quad \Lambda_8=-\sqrt{3}A^2. \tag{S11}$$

This results in the degree of polarization

$$P=\frac{1}{2}. \tag{S12}$$

The nonzero degree of polarization is partially related to the presence of nonzero spin (S9) in torus knots [15]; total depolarization implies zero spin.

Figure-eight knotted polarization. For the polarization described by Eq. (3) and Fig. 1(c) in the main text, the complex field amplitudes are:

$$\mathbf{F}_1 = A\left(1,0,0.6i\right), \quad \mathbf{F}_2 = A\left(1,0.4i,i\right), \quad \mathbf{F}_3 = A\left(0,-i,0\right). \tag{S13}$$

The first and the second waves are elliptically-polarized and carry nonzero spins:

$$\mathrm{Im}\left(\mathbf{F}_1^* \times \mathbf{F}_1\right) = A^2\left(0,-1.2,0\right), \quad \mathrm{Im}\left(\mathbf{F}_2^* \times \mathbf{F}_2\right) = A^2\left(0,-2,0.8\right), \quad \mathrm{Im}\left(\mathbf{F}_3^* \times \mathbf{F}_3\right) = \mathbf{0}. \tag{S14}$$

Together with the amplitudes $\left|\mathbf{F}_1\right|^2 = 1.36A^2$, $\left|\mathbf{F}_2\right|^2 = 2.16A^2$, $\left|\mathbf{F}_3\right|^2 = A^2$, and frequencies $\omega_2/\omega_1 = 3$, $\omega_3/\omega_1 = 6$, this results in the normalized spin angular momentum (4) of the total field:

$$\mathbf{S} \simeq \left(0,-0.83,0.12\right), \quad \left|\mathbf{S}\right| \simeq 0.84. \tag{S15}$$

The generalized Stokes parameters (S3) for each of the interfering fields (S13) yield:

$$\Lambda_1^{(1)} = \Lambda_2^{(1)} = \Lambda_4^{(1)} = \Lambda_6^{(1)} = \Lambda_7^{(1)} = 0, \ \Lambda_0^{(1)} = 1.36A^2, \ \Lambda_3^{(1)} = \frac{3}{2}A^2, \ \Lambda_5^{(1)} = -1.8A^2, \ \Lambda_8^{(1)} = 0.14\sqrt{3}A^2,$$

$$\Lambda_1^{(2)} = \Lambda_4^{(2)} = \Lambda_7^{(2)} = 0, \ \Lambda_0^{(2)} = 2.16A^2, \ \Lambda_2^{(2)} = -1.2A^2,$$

$$\Lambda_3^{(2)} = 1.26A^2, \ \Lambda_5^{(2)} = -3A^2, \ \Lambda_6^{(2)} = 1.2A^2, \ \Lambda_8^{(2)} = -0.42\sqrt{3}A^2,$$

$$\Lambda_1^{(3)} = \Lambda_2^{(3)} = \Lambda_4^{(3)} = \Lambda_5^{(3)} = \Lambda_6^{(3)} = \Lambda_7^{(3)} = 0, \ \Lambda_0^{(3)} = A^2, \ \Lambda_3^{(3)} = -\frac{3}{2}A^2, \ \Lambda_8^{(3)} = \frac{\sqrt{3}}{2}A^2. \tag{S16}$$

As before, each of the interfering waves is fully polarized, and their degrees of polarizations $P^{(n)} = 1$.

From Eqs. (S16), we obtain the generalized Stokes parameters for the total figure-eight-knotted interference field: $\Lambda_1 = \Lambda_4 = \Lambda_7 = 0$,

$$\Lambda_0 = 4.52A^2, \ \Lambda_6 = -\Lambda_2 = 1.2A^2, \ \Lambda_3 = 1.26A^2, \ \Lambda_5 = -4.8A^2, \ \Lambda_8 = 0.22\sqrt{3}A^2. \tag{S17}$$

This results in the degree of polarization

$$P \simeq 0.67. \tag{S18}$$

The nonzero degree of polarization is partially related to the presence of nonzero spin (S15) in the figure-eight knot [15].

2. Equations for surface water (gravity) waves

Here we derive the main wave equation describing the motion of water particles in gravity wave fields. For this, we consider linear waves on a surface of an incompressible fluid. The fluid motion can be described by the Eulerian 3D velocity field $\mathbf{v}\left(x,y,z,t\right)$, the velocity potential ϕ: $\mathbf{v} = \nabla\phi$, and the local z-elevation of the fluid surface with respect to the equilibrium: $\mathcal{Z}\left(x,y,t\right)$. We will also use the 3D velocity field at the water surface: $\boldsymbol{\mathcal{V}}\left(\mathbf{r}_\perp,t\right) = \mathbf{v}\big|_{z=0}$, and the corresponding displacement field: $\boldsymbol{\mathcal{R}} = \left(\mathcal{X},\mathcal{Y},\mathcal{Z}\right)$, $\boldsymbol{\mathcal{V}}\left(\mathbf{r}_\perp,t\right) = \partial_t \boldsymbol{\mathcal{R}}\left(\mathbf{r}_\perp,t\right)$. Since the surface waves are effectively 2D modes, we will use the in-plane 2D quantities: $\mathbf{r}_\perp = \left(x,y\right)$, $\nabla_\perp = \left(\partial_x,\partial_y\right)$, $\mathbf{v}_\perp = \left(v_x,v_y\right)$, $\boldsymbol{\mathcal{V}}_\perp = \left(\mathcal{V}_x,\mathcal{V}_y\right)$, $\boldsymbol{\mathcal{R}}_\perp = \left(\mathcal{X},\mathcal{Y}\right)$.

Then, the equations of motion can be written as [53]:

$$\partial_t \phi\big|_{z=0} = -g\mathcal{Z}\,, \quad \nabla\cdot\mathbf{v} = \nabla_\perp\cdot\mathbf{v}_\perp + \partial_z v_z = 0\,, \tag{S19}$$

where g is the gravitational acceleration. Applying the operator $\nabla_\perp$ to the first equation (S19) and using the definitions above, we derive

$$\partial_t^2 \boldsymbol{\mathcal{R}}_\perp = -g\nabla_\perp \mathcal{Z}\,. \tag{S20}$$

Next, applying the ∂_t operator to the first Eq. (S19) and extending it for $z \neq 0$, we obtain $\partial_t^2\phi = -gv_z$. Applying here the ∂_z operator and using the second Eq. (S19), taken at $z = 0$, we arrive at $\partial_t^2 \mathcal{V}_z = g\nabla_\perp\cdot\boldsymbol{\mathcal{V}}_\perp$, or, equivalently for the displacement field:

$$\partial_t^2 \mathcal{Z} = g\nabla_\perp\cdot\boldsymbol{\mathcal{R}}_\perp\ . \tag{S21}$$

Equations (S20) and (S21) are the desired wave equations for the field $\boldsymbol{\mathcal{R}}\left(\mathbf{r}_\perp, t\right)$. Notably, these equations resemble the equations of acoustics for longitudinal sound waves in (2+1)D spacetime [4,9], where the 2D vector field $\boldsymbol{\mathcal{R}}_\perp$ and the scalar field $\mathcal{Z}$ play the role of the velocity and pressure fields, respectively. The only difference is that Eqs. (S20) and (S21) contain the second-order rather than first-order derivatives in time. It is easy to see that the wave solutions of these equations obey the dispersion relation $\omega^2 = gk$, which corresponds to deep-water (gravity) waves [53].

3. Knotted motion of water particles in interference of gravity waves

See the Supplemental Movie for the temporal evolution of the water surface and the motion of water particles described by Eq. (6) in the main text and shown in Fig. 2(a).