Frustration Elimination and Excited State Search in Coherent Ising Machines

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Frustration, that is, the impossibility of satisfying the energetic preferences between all spin pairs simultaneously, underlies the complexity of many fundamental properties in spin systems, including the computational difficulty in determining their ground states. Coherent Ising machines (CIMs) have been proposed as a promising analog computational approach to efficiently find different degenerate ground states of large and complex Ising models. However, CIMs also face challenges in solving frustrated Ising models: frustration not only reduces the probability of finding good solutions, but it also prohibits the leveraging of quantum effects in doing so. To circumvent these detrimental effects of frustration, we show how frustrated Ising models can be mapped to frustration-free CIM configurations by including ancillary modes and modifying the coupling protocol used in current CIM designs. Such frustration elimination may empower current CIMs to benefit from quantum effects in dealing with frustrated Ising models. In addition, these ancillary modes can also enable error detection and searching for excited states.

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Introduction—Ising models [1–5] have been widely studied because these models exhibit, despite their seemingly simple Hamiltonians, a rich variety of interesting properties, exemplified, e.g., by the theory of spin glasses [6–10]. The price to pay is that the properties of large Ising models with random all-to-all couplings are notoriously difficult to study both experimentally and numerically [11–16]. For instance, finding their ground states is known to be NP-hard.

Coherent Ising machines (CIMs) [17–30] have been developed as optical analog computers with the potential to simulate Ising models more efficiently. These machines feature flexible optical couplings that can, in principle, be adjusted to realize any desired Ising interaction, overcoming the nearest-neighbor restrictions of other hardware approaches. In particular, this allows them to enter deeply into, and explore, the realm of frustrated Ising problems. Frustrated couplings, which are omnipresent in generic Ising models, lie at the heart of many intriguing properties of Ising models, including the computational hardness of finding ground states [31–38]. Therefore, dealing with frustrated Ising models is a core objective of CIMs.

However, frustrated couplings represent challenges as well for CIMs. Most Ising models with frustrated couplings correspond to CIM configurations where loss acts inhomogeneously on different effective optical spins [39–45], thus breaking the homogeneous-amplitude requirement of CIMs [17]. Moreover, frustration in CIMs results in intrinsic single-photon loss, which decoheres quantum states [46–52] and thus impedes the advancement of quantum CIMs. Therefore, finding improved ways to deal with frustrated couplings can significantly affect the performance of CIMs.

Although frustration is an intrinsic property of generic Ising models, frustrated optical couplings can be circumvented in CIMs while preserving the to-be-found ground states. Since the energy difference between different spin configurations is mapped to the total-loss difference between optical modes in CIMs [17,19,53], frustration is absent in CIMs whenever ground states of Ising models are mapped to loss-free optical configurations. Unfortunately, these "zero points," i.e., the Ising energies corresponding to vanishing optical loss, cannot be adjusted freely in current CIMs.

To solve this obstacle in CIMs, we provide a method to shift the lossless point of CIMs with the help of ancillary modes. In our proposal, the signal degenerate-optical-parametric-oscillator (DOPO) modes express the spin configurations in the same way as in the current CIMs, while the additional ancillas shift the correspondence between Ising energy and optical loss, so that Ising ground

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states are mapped to lossless optical configurations. Eliminating frustration this way promotes current CIMs mainly in three aspects: First, it allows for the presence of stronger quantum effects in CIMs, e.g., steady states can emerge as superpositions of all the degenerate ground states of the underlying Ising models despite their frustration. Enabling these quantum effects is essential for developing CIMs operating in the quantum regime. Second, inhomogeneities caused by frustration are absent, thus improving the performance of CIMs in the semiclassical regime. Finally, by shifting the lossless point of CIMs to excited energies, it is possible to search for excited states.

Mapping Ising models to CIMs—In a CIM, the spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ are expressed by coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, respectively, which are steady states of the uncoupled DOPOs described by the Hamiltonian $H = \sum_k S(a_k^2 + a_k^{\dagger 2})$ and the two-photon loss $\mathcal{L}_d(\rho) = \sum_k(\Gamma_{\mathrm{tp}}/2)(2a_k^2\rho a_k^{\dagger 2} - a_k^{\dagger 2}a_k^2\rho - \rho a_k^{\dagger 2}a_k^2)$. At the same time, Ising coupling terms,

$$J_{n,m}\sigma_z^{(n)}\sigma_z^{(m)}, \tag{1}$$

are represented by collective loss channels $\mathcal{L}_{m,n}(\rho) = (\Gamma_c/2)(2L_{n,m}\rho L_{n,m}^\dagger - L_{n,m}^\dagger L_{n,m}\rho - \rho L_{n,m}^\dagger L_{n,m})$, with the Lindblad operator,

$$L_{n,m} = a_n + \mathrm{sign}(J_{m,n})a_m, \tag{2}$$

effectively mapping the energies of different spin configurations to the total loss in the CIM. Here, $\sigma_z^{(n)}$ is the Pauli matrix of the nth spin, $J_{n,m} = \pm J$ with $J > 0$, a_n is the annihilation operator of the nth optical mode in the CIM, and ρ is the density matrix [54].

For $J_{n,m} > 0$, the Ising coupling term (1) contributes a positive energy if the two spins are aligned, and negative energy if they are antialigned. In the corresponding optical coupling (2), the Lindblad term causes loss if the two optical modes have the same phase α, while two optical modes with opposite phases describe a dark state of the loss term. Similarly, if $J_{n,m} < 0$, two optical modes with the same phases describe a dark state of the corresponding loss term.

Therefore, the total energy of a spin configuration, i.e., the summed-up contributions of all the Ising coupling terms (1), is proportional to the total optical loss in the CIM. In particular, the ground state of the Ising model corresponds to the optical configuration with minimum loss. Note that an optical configuration without loss in a CIM corresponds to a spin configuration with minimum possible energy $E_{\mathrm{MPE}} = -JN_{\mathrm{c}}$, where N_{c} is the total number of coupling terms (1). In such cases, the steady states of CIMs are catlike superposition states in the frustration-free ground-state space [50,54,58]. As the phase transition point (threshold pump power) of a CIM depends on the loss, the optical configuration with minimum loss, which is located at the transition point, can be found by gradually increasing the pump [54].

Obstacles caused by frustrated couplings—For illustration, let us consider the simplest possible situation featuring frustration, realized by three DOPO modes, as shown in Fig. 1(a). In this example, all three optical couplings energetically prefer coupled modes with opposite phases, which cannot be simultaneously satisfied by any configuration. Therefore, the system unavoidably sustains loss from at least one coupling channel, thus prohibiting quantum superposition and entanglement. The resulting decoherence represents an intrinsic obstacle for developing quantum CIMs.

In addition to suppressing quantum effects, frustration hinders the original function of CIMs, that is, finding ground states of Ising models in the semiclassical regime. In a frustrated CIM configuration, the loss is distributed inhomogeneously among the modes, as illustrated in Fig. 1(b). Consequently, different modes assume different amplitudes, e.g., mode 1 is above threshold while modes 2 and 3 remain below threshold. Such inhomogeneous amplitudes render the mapping in Eqs. (1), (2) defective, thus preventing CIMs from finding correct solutions [54].

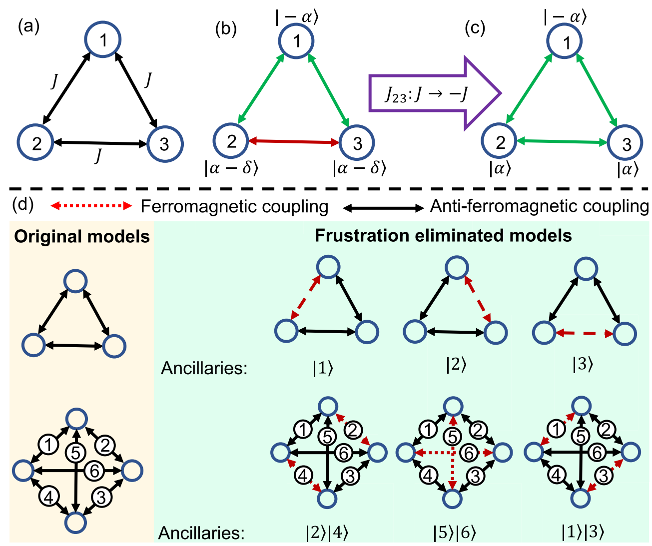

FIG. 1. Illustration of frustration elimination. (a) Paradigmatic three-mode example featuring frustration, reflected by the absence of a dark state. (b) One of the three degenerate ground states of the configuration in (a), sustaining loss due to the coupling between mode 2 and mode 3. The red double arrow represents the coupling contributing loss, and the green double arrows represent the loss-free couplings. (c) Flipping the coupling between modes 2 and 3 eliminates frustration for the ground state in (b). (d) Simultaneous frustration elimination for all degenerate ground states with the help of an ancilla. Although frustration in general prohibits dark states in CIMs, it is possible to map all the ground states of a frustrated Ising model to an optical dark state with the help of ancillary modes.

In general, Ising models with ground-state energies larger than E_{MPE} are, in the current CIM design, mapped according to Eqs. (1), (2) to such frustrated couplings.

Frustration elimination: General idea—For a specific ground state, e.g., $|-\alpha\rangle|\alpha\rangle|\alpha\rangle$, the loss in the frustrated configuration shown in Fig. 1(b) can be removed by flipping the sign of the lossy coupling term, as shown in Fig. 1(c). However, such direct manipulation cannot address all the degenerate ground states of the original Ising model, e.g., it fails for the solution $|\alpha\rangle|\alpha\rangle|-\alpha\rangle$.

Therefore, we introduce an ancilla that controls the flipping of the coupling terms. As the ancilla, by design, removes the loss, the steady state can then be a superposition of the different solutions, as illustrated in Fig. 1(d) and explicitly verified below. In the resulting state, the ancillary part mediates the superposition of three different, unfrustrated Ising models, the ground states of which are encoded in the signal modes. The set of these frustration-free ground states comprises all the degenerate ground states of the original frustrated model.

Figure 1(d) illustrates how to adapt frustration elimination to general frustrated Ising models (1). Flipping an appropriate coupling term, e.g., coupling 5 in Fig. 1(d), reduces the energy by at most $2J$ for all spin configurations, and the corresponding ground states of the original model maintain to carry the lowest energy. The lowest energy can be further reduced by flipping several coupling terms, each flip enabled by a separate ancilla as illustrated in Fig. 1(d), while preserving the ground states. Given a sufficient number of ancillary modes, the energy of the ground states E_{ground} can then be reduced to the minimum possible energy E_{MPE}, and the frustration is completely eliminated. Specifically, the number of necessary ancillas is determined by $(E_{\mathrm{ground}} - E_{\mathrm{MPE}})/(2J)$, which can be identified by either current CIMs or by gradually increasing the number of ancillas.

Ancilla realization in CIMs—The desired ancillas for frustration elimination can be realized in CIMs by additional optical modes. For a Lindblad operator $(L = a_1 + a_2)$ with the dark states $|\alpha\rangle|-\alpha\rangle$ and $|-\alpha\rangle|\alpha\rangle$, the dark states can be modified by including a third mode as

$$L = a_1 + a_2 + 2a_{\mathrm{an}}. \tag{3}$$

The collective loss (3) supports the dark states

$$|\alpha\rangle|\alpha\rangle|-\alpha\rangle_{\mathrm{an}}, \quad \text{and} \quad |-\alpha\rangle|-\alpha\rangle|\alpha\rangle_{\mathrm{an}}, \tag{4}$$

where the two signal modes now take the same sign.

To control multiple loss channels with a single ancillary mode, e.g., the case illustrated in Fig. 1(d), we introduce an effective hyperspin [59,60], which can take different directions, as the ancillary mode

$$L_{\mathrm{an}} = \sum_{n,m} e^{i\phi_{m,n}}[a_n + \mathrm{sign}(J_{m,n})a_m] + 2a_{\mathrm{an}}. \tag{5}$$

Different collective coupling terms are now associated with different phases $\phi_{n,m}$, corresponding to rotations of spin directions; while the ancillary mode (hyperspin) autonomously picks up one phase (direction) and flips the corresponding coupling term. An arbitrary phase can be imposed on DOPO modes by a phase and amplitude modulator [19].

Such ancillary states can be realized by nondegenerate optical parametric oscillators (NDOPOs) described by the Hamiltonian $H_{\mathrm{NDOPO}} = S(a_{\mathrm{ani}}a_{\mathrm{ans}} + a_{\mathrm{ani}}^\dagger a_{\mathrm{ans}}^\dagger)$ and the Lindblad operator $L_{\mathrm{NDOPO}} = a_{\mathrm{ani}}a_{\mathrm{ans}}$ with an ancillary idler mode a_{ani} and an ancillary signal mode a_{ans}. The steady states of an NDOPO are also coherent states $|\alpha_{\mathrm{ani}}\rangle|\alpha_{\mathrm{ans}}\rangle$ with $\alpha_{\mathrm{ani}}\alpha_{\mathrm{ans}} = 2S/\Gamma_{\mathrm{NDOPOtp}}$, where $\Gamma_{\mathrm{NDOPOtp}}$ is the two-photon loss rate of the NDOPO. Note that an additional loss channel with the same loss rate is required to fix the "length" of the hyperspin,

$$\begin{aligned} L_{\mathrm{ani}} &= \sum_{n,m} e^{i\phi_{m,n}}[a_n + \mathrm{sign}(J_{m,n})a_m] + 2a_{\mathrm{ani}}, \\ L_{\mathrm{ans}} &= \sum_{n,m} e^{-i\phi_{m,n}}[a_n + \mathrm{sign}(J_{m,n})a_m] + 2a_{\mathrm{ans}}, \end{aligned} \tag{6}$$

with $2S/\Gamma_{\mathrm{NDOPOtp}} = |a_n|^2$. To satisfy the two loss channels in Eq. (6) simultaneously, the two NDOPO modes must have amplitudes with the same absolute value, e.g., $a_{\mathrm{ani}} = \exp(i\phi_{m,n})a_n$ and $a_{\mathrm{ans}} = \exp(-i\phi_{m,n})a_n$.

Numerical demonstration of frustration elimination—We numerically simulate the evolution of the frustration eliminated CIM configuration that corresponds to the frustrated Ising configuration in Fig. 1(a). The DOPOs and NDOPOs share the same two-photon loss rate $\Gamma_{\mathrm{tp}} = \Gamma_{\mathrm{NDOPOtp}} = \Gamma$, and the frustration-eliminating loss channels (6) denote three coupling terms with the phases, $\phi_{1,2} = 0$, $\phi_{2,3} = (\pi/2)$, and $\phi_{3,1} = (\pi/4)$. The three frustration-eliminated configurations in Fig. 1(d) and the Z_2 symmetry result in a six-component dark state,

$$|\psi(\infty)\rangle = \frac{1}{\sqrt{6+\delta}} \sum_{k=1}^{6} |\phi\rangle_{\mathrm{s}}^{k} \otimes |\psi\rangle_{\mathrm{an}}^{k}, \tag{7}$$

where the signal mode part $|\phi\rangle_{\mathrm{s}}^{k}$ encodes the corresponding Ising ground state, while the ancillary part $|\psi\rangle_{\mathrm{an}}^{k}$ indicates the unsatisfied coupling.

The high fidelity (≈ 0.95) with respect to the pure dark state (7) in Fig. 2(a) implies that the optical effective spin system is not frustrated and resides in a loss-free ground mode.

Next, we discuss the realization of ancillary modes in general cases of multiple coupling flips and the advantages in the semiclassical regime. Consider the following coupling terms:

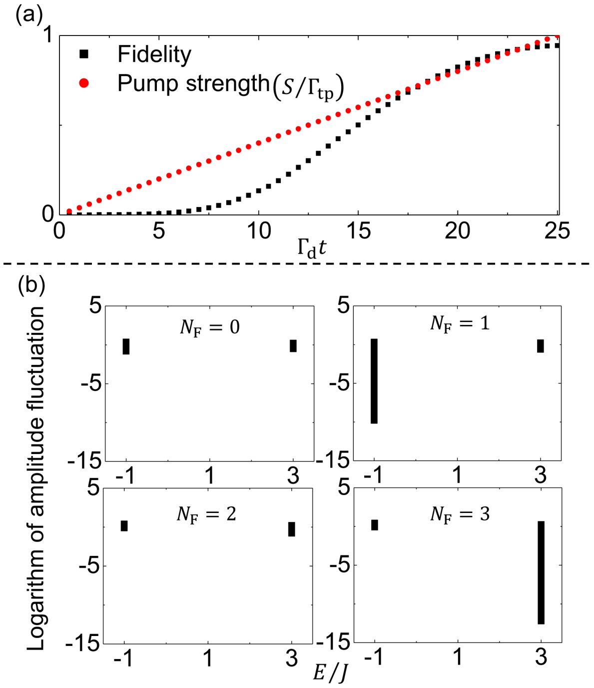

FIG. 2. Ground-state search under frustration elimination for the frustrated Ising configuration presented in Fig. 1. Shown is the time-resolved fidelity of the state, generated with a time-dependent pump, with respect to the pure dark state (7). The latter describes a superposition of the six possible ground states. The numerical demonstration of the close reproduction of (7) convincingly confirms the validity of the mapping of the frustrated Ising model to an optical effective spin system without frustration. The collective loss rate is $\Gamma_{\mathrm{c}}=3\Gamma$, and the two-photon loss rates for the DOPOs and NDOPOs have the same value ($\Gamma_{\mathrm{tp}}=\Gamma_{\mathrm{NDOPOtp}}=\Gamma$). (b) Self error detection and arbitrary state searching with frustration elimination method for the model presented in Fig. 1(a). The correct solutions are indicated by vanishing amplitude inhomogeneity. More active ancillary modes can shift the target state to excited states.

$$
\begin{aligned}
L_{\mathrm{ans}}^{n,m} &= L_{n,m} + i(b_{\mathrm{control}} + b_{n,m}) + 2a_{\mathrm{ans}}^{n,m},\\
L_{\mathrm{ani}}^{n,m} &= L_{n,m} - i(b_{\mathrm{control}} + b_{n,m}) + 2a_{\mathrm{ani}}^{n,m},\\
L_{\mathrm{control}} &= \sum b_{n,m} + (2N_{\mathrm{F}} - N_{\mathrm{c}})b_{\mathrm{control}}.
\end{aligned}
\tag{8}
$$

Here, b_{control} and $b_{n,m}$ are DOPO ancillary modes, $a_{\mathrm{ans}}^{n,m}$ and $a_{\mathrm{ani}}^{n,m}$ are ancillary NDOPO modes, and N_{F} is the number of coupling terms flipped. The first two lines in Eq. (8) imply that a flipped coupling term corresponds to an ancilla with phase 0, so that the corresponding DOPO ancillary mode $b_{n,m}$ forms a dark mode with the control mode b_{control}. The third line in Eq. (8) constrains the number of $b_{n,m}$ with negative phases according to b_{control} to be N_{flip}. Note that Eq. (6) is not a special case of Eq. (8). The scheme presented in (6) is more resource efficient but not suitable for flipping multiple couplings.

We apply Eq. (8) to the model in Fig. 1(a) and study its advantages in addition to quantum effects by simulating the corresponding semiclassical equations [54]. The relation between the logarithm of the amplitude inhomogeneity and the Ising energy of the solution candidates is calculated with 16^4 random initial conditions and shown in Fig. 2(b). The results with $N_{\mathrm{F}}=0,1$ reflect the frustration elimination process illustrated in Fig. 1(d). Amplitude inhomogeneity is large due to frustration for $N_{\mathrm{F}}=0$, and can be strongly suppressed for $N_{\mathrm{F}}=1$. Although we may get wrong solution candidates, these errors can be identified by the amplitude inhomogeneity. In addition to the error detection, the frustration elimination method can also be used to search for excited states, as shown with the results for $N_{\mathrm{F}}=2,3$. By flipping more coupling terms $N_{\mathrm{F}}=3$, the global minimum with strongly suppressed amplitude inhomogeneity captures the degenerated first excited state. More examples are provided in the Supplemental Material [54].

Realization of the frustration-elimination setup—The NDOPO modes required in Eq. (6) can be realized with an additional fiber loop [61–63], and the collective loss terms in Eq. (6) may be realized by delay lines with multiple ports [54], or feedback based on a field-programmable gate array (FPGA). According to Eq. (8), the necessary number of ancillary modes for a Ising model with N_{c} coupling terms is $(3N_{\mathrm{c}}+1)$.

The proposed method can also be extended to qubit-based Ising machines [2] leveraging the strong nonlinearity in superconducting circuits [64,65], e.g., in a coupling $J_{n,m}\sigma_z^{(n)}\sigma_z^{(m)}\sigma_{z,\mathrm{an}}^{(n,m)}$. The total number of flipped coupling terms can be constrained by a coupling among the ancillary qubits $H_{\mathrm{I}}=\sum g\sigma_{-,\mathrm{an}}^{(n,m)}\sigma_{+,\mathrm{an}}^{(n',m')}+\mathrm{H.c.}$, which conserves the total number of ancillary qubits with up direction. This extended method could also be applied to some optical Ising machines with strong nonlinearity [66]. In the semiclassical regime, our scheme can also be applied in oscillator-based Ising machines [67,68] and spatial CIMs [69], as the coherent couplings applied in these systems are equivalent to the feedback pulses calculated by a field-programmable gate array (FPGA).

Conclusions—We have developed a method for mapping frustrated Ising models to optical effective spin systems without frustration, while preserving the set of ground states. In our proposal, the DOPO modes perform, similar to current CIMs, the ground state search, while additional ancillary modes autonomously identify and flip the coupling terms causing frustration. The ancillary modes and switchable couplings can be realized by NDOPO modes and multiport optical couplings, respectively. As DOPOs and NDOPOs can share the same pump field, our modified CIM can be operated in line with the current protocol, i.e., by gradually increasing the pump intensity. Moreover, our numerical results confirm that the system produces a superposition of all the desired solutions when following the common CIM solution-searching protocol. Although researchers debate on whether CIMs with a mixed final state benefit from quantum effects, the quantum advantages of pure final states enabled by our frustration-elimination method have been widely studied in spin systems [54,55,70–73]. For CIMs operated in the semiclassical regime, our method can realize self-error detection and excited-state searching.

Our proposal makes it possible to avoid the intrinsic single-photon loss caused by frustration in the current CIM design, so that the exploitation of loss-vulnerable quantum effects becomes conceivable. Such quantum effects would not only be essential for developing quantum CIMs, but may also help in the solution searching. In addition, other severe problems related to frustration (in particular, detrimental amplitude inhomogeneity) are resolved by frustration elimination. Finally, our proposal is also compatible with the measurement-feedback coupling paradigm, and there, too, can enable error detection and excited-state searching.

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