Supplemental Material: Disorder-robust entanglement transport

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In this Supplemental Material we provide details of our numerical simulation of the Haldane model, which gives an independent validation of our analytical results. Similar to the calculations presented in the main text, we use a first quantization approach which encompasses either distinguishable or indistinguishable particles, depending on the choice of initial state. In the Haldane model, the time evolution of a single particle wavefunction $\Psi_{nm} = (a_{nm}, b_{nm})$ is governed by the Schrödinger equation

$$i\partial_t a_{nm} = \omega_{nm}^{(a)} a_{nm} + t_1 \sum_{\text{n.n}} b_{n'm'} + t_2 \sum_{\text{n.n.n}} a_{n'm'} e^{i\phi_{n'm'}}, \tag{S1a}$$

$$i\partial_t b_{nm} = \omega_{nm}^{(b)} b_{nm} + t_1 \sum_{\text{n.n}} a_{n'm'} + t_2 \sum_{\text{n.n.n}} b_{n'm'} e^{-i\phi_{n'm'}}, \tag{S1b}$$

where (n, m) index the unit cells of the lattice, a honeycomb lattice formed by two sublattices (a, b), t_1 is the nearest neighbor hopping strength, t_2 is the next-nearest neighbor hopping strength, signs of the fluxes $\phi_{n'm'} = \pm\phi$ alternate between adjacent next-nearest neighbors, and $\omega_{mn}^{(j)}$ describes uncorrelated on-site disorder uniformly distributed in the width $[-W/2, W/2]$. Fourier transforming yields the Bloch Hamiltonian,

$$H(\boldsymbol{k}) = 2t_2 \cos\phi \sum_i \cos(\boldsymbol{k}\cdot\boldsymbol{a}_i)\sigma_0 + t_1 \sum_i [\cos(\boldsymbol{k}\cdot\boldsymbol{\delta}_1)\sigma_x + \sin(\boldsymbol{k}\cdot\boldsymbol{\delta}_i)\sigma_y] - 2t_1 \sin\phi \sum_i \sin(\boldsymbol{k}\cdot\boldsymbol{a}_j)\sigma_z, \tag{S2}$$

where the lattice vectors are $\boldsymbol{a}_{1,2,3}$, $\boldsymbol{\delta}_{1,2,3}$ are displacements between neighboring lattice sites, and σ_j are Pauli matrices. The gap size is $6\sqrt{3}t_2 \sin\phi$ with band extrema $6t_2 \cos\phi \pm 3t_1$.

The evolution of a two (non-interacting) particle state $\mid \Psi\rangle = \sum_{x_1,x_2} \psi_{x_1,x_2} \mid x_1\rangle\otimes \mid x_2\rangle$ is governed by the symmetric Hamiltonian $\hat{H}_{\text{tot}} = \hat{H} \otimes \hat{1} + \hat{1} \otimes \hat{H}$. In this case, the Schrödinger equation reads

$$i\partial_t \mid \Psi\rangle = \sum_{x_1,x_2} \psi_{x_1,x_2} (\hat{H} \mid x_1\rangle\otimes \mid x_2\rangle + \mid x_1\rangle \otimes \hat{H} \mid x_2\rangle). \tag{S3}$$

The evolution equation for the wavefunction is obtained by multiplying both sides by $\langle r_1 \mid \otimes\langle r_2 \mid$,

$$i\partial_t \psi_{r_1,r_2} = \sum_x (\psi_{x,r_2}\langle r_1 \mid \hat{H} \mid x\rangle + \psi_{r_1,x}\langle r_2 \mid \hat{H} \mid x\rangle) = \sum_x (\psi_{x,r_2} H_{r_1,x} + \psi_{r_1,x} H_{r_2,x}), \tag{S4}$$

which is equivalent to the matrix equation

$$i\partial_t \psi = H\psi + (H\psi)^T, \tag{S5}$$

thus the solution is

$$\psi(t) = e^{-itH}\psi(0)e^{-itH^T}. \tag{S6}$$

We solve Eq. (S6) for 100 different realizations of the disorder and construct the disorder-averaged density matrix as $\bar{\rho}(t) = \sum_{i=1}^{100} \mid \psi_i(t)\rangle\langle\psi_i(t) \mid$, from which we obtain the correlation functions plotted in Fig. 5 of the main text.