Supplementary Material for "Reflective amplification without population inversion from a strongly driven superconducting qubit"

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DETAILS FOR THE NUMERICAL SIMULATIONS

In panels (b) and (d) of Fig. 3, we show numerical simulations corresponding to the experimental results in panels (a) and (c), respectively. These theoretical plots are done using the parameters in Table 1. Even though no free fitting parameters are used, we see excellent agreement between theory and experiment. To model the two-tone spectroscopy results, we solve the master equation

$$\begin{aligned}\dot{\rho} = &-\frac{i}{\hbar}\left[\sum_{m=0}^{N-1}\hbar\Delta_m |m\rangle\langle m| - i\frac{\Omega_{\text{pump}}}{2}\left(\Sigma_+ - \Sigma_-\right), \rho\right] \\ &+\sum_{m=1}^{N-1} m\Gamma_1 \mathcal{D}\left[|m-1\rangle\langle m|\right]\rho \\ &+\Gamma_\phi \mathcal{D}\left[\sum_{m=1}^{N-1} m |m\rangle\langle m|\right]\rho, \end{aligned} \tag{S1}$$

where ρ is the density matrix for the qubit with energy levels Δ_m in the rotating frame of the pump, $\Sigma_- = \sum_{m=1}^{N-1}\sqrt{m}\,|m-1\rangle\langle m|$, $\Sigma_+ = \Sigma_-^\dagger$, $\mathcal{D}[X]\rho = X\rho X^\dagger - \frac{1}{2}\left\{X^\dagger X, \rho\right\}$, and N is the number of qubit levels. We then use quantum linear-response theory [S1] to calculate the susceptibility

$$\chi(\omega_\text{p}) = i\int_0^\infty \mathrm{d}t\, e^{i\omega_\text{p}t}\left\langle\left[\Sigma_-(t), \Sigma_p(0)\right]\right\rangle, \tag{S2}$$

where $\Sigma_p = -i\left(\Sigma_+ - \Sigma_-\right)$, average over pump phases, and extract the reflection coefficient

$$r = 1 + \Gamma_1\chi(\omega_\text{p}), \tag{S3}$$

similar to Refs. [S2, S3]. The numerical simulations use methods from Ref. [S4] and are implemented in QuTiP [S5, S6].

ADDED NOISE IN OUR AMPLIFIER

Using input-output theory [S7], we compute the added noise that is introduced by the gain of the amplifier. Given the peak gain of 7 % for $\omega_\text{p}/2\pi \approx 4.49\,\text{GHz}$ at $P_\text{pump} = -114\,\text{dBm}$, the added noise as a function of frequency, i.e.,

$$A(\omega) = \left[\frac{\hbar\omega}{e^{\hbar\omega/kT}-1} + \hbar\omega\right]\left(1 - |G(\omega)|^{-2}\right), \tag{S4}$$

is shown in Fig. S1. Given the experimental measurements shown in the main text, we model the gain distribution $|G(\omega)^2|$ on a Lorentzian centered at 4.49 GHz with full width at half maximum (FWHM) of 45 MHz. The resulting plot coincides with a typical noise figure from the inverted-oscillator model for a quantum amplifier. Following the calculation of this inversion model, at the peak 7 % gain point, we calculate the equivalent noise temperature to the added noise term to be 102 mK for this model.

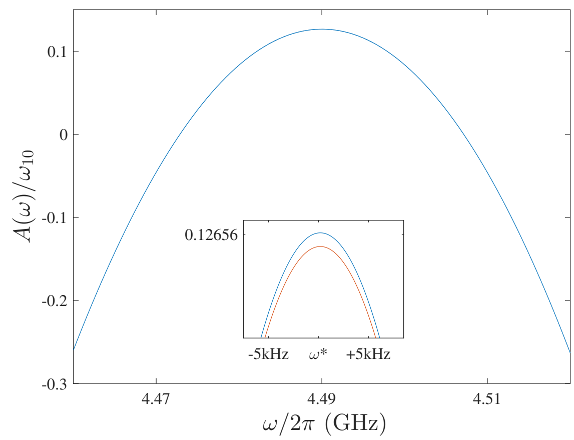

Figure S1. Added noise distribution of the quantum amplifier, in dimensionless unit scaled to the resonance photon number at 4.49 GHz. The inset shows a magnified area about the noise peak, comparing the performance of the amplifier (the upper curve) with reference to the vacuum-fluctuation floor (the lower curve). The maximum noise added above the floor peaks at the shifted $\omega^*/2\pi = 4.49003\,\text{GHz}$, is less than 10^{-8} photons, showing good performance of the amplifier at the low environmental temperature.

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