Amplified Optomechanical Transduction of Virtual Radiation Pressure

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Here we describe how, utilizing a time-dependent optomechanical interaction, a mechanical probe can provide an amplified measurement of the virtual photons dressing the quantum ground state of an ultrastrongly coupled light-matter system. We calculate the thermal noise tolerated by this measurement scheme and discuss an experimental setup in which it could be realized.

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Introduction.—When probing a physical system, a competition can emerge between the intensity of the *response* induced and the *information* gained. This is especially true in quantum systems, where internal coherences are very fragile against backaction noise [1,2]. Strongly coupled composite systems, such as light and matter in a cavity quantum electrodynamic (QED) device [3,4], introduce additional challenges in this regard. Such systems have been traditionally realized with atoms in highly reflective optical cavities [5,6], semiconductors coupled to microcavities [7–9], and artificial atoms coupled to microwave cavities in circuit QED [10–14]. Very recently, the semiconductor and superconductor examples of such devices have reached the ultrastrong and deep-strong coupling regimes [7–12,14–19] characterized by interaction strengths comparable to the bare light-matter resonant energies and by states in which matter is dressed by "virtual photons."

The term "virtual" implies that specific actions must be taken to make such excitations observable. For example, highly invasive schemes have been designed to cause virtual photons to be emitted as extracavity radiation by nonadiabatic modulation of the interaction between light and matter [20–22] or by inducing transitions outside the system's interacting Hilbert space [22–26]. By doing this, all internal coherence between light and matter present in the initial state is destroyed. Very recently, it became clear that one can potentially observe the presence of these ground-state excitations while inducing only a minimal backaction (i.e., transitions between eigenstates of the system one is trying to measure [1,27,28]) on the system. In the language of quantum optics, such measurements are called nondemolition [1]. For example, in the seminal work of Lolli *et al.* [27], an ancillary qubit is used to investigate the ground state of an ultrastrongly coupled light-matter system with minimal disturbance. However, there, to achieve a large signal-to-noise ratio, they require many atoms in the cavity. In addition, they rely on an ultrastrong coupling between the light-matter system and the probe, which induces backaction effects. In turn, non-negligible loss in the light-matter system is needed to return it to the ground state after an accidental disturbance.

Here, we propose a method in which the interaction with the probe is weak and commutes with the system's Hamiltonian. The consequent evasion of quantum backaction noise removes the need for loss and leads to a high signal-to-noise ratio for realistic parameters, even in the presence of only a single atom in the cavity. In addition, this method combines two existing technologies in an unique way. In particular, we employ a hybrid matter-cavity-mechanical device [29–43] where a mechanical mode, acting as the probe, couples via radiation pressure to a cavity-QED system (in which resonant matter ultrastrongly interacts with the confined light). While the photons dressing the ground state of the strongly coupled cavity-QED system can displace the mechanical mode through a "virtual radiation-pressure" [4,44] effect (akin to variations of the Casimir force experiment [45–49]), such a force is typically extremely weak. Here we show that, even with a relatively weak optomechanical probe interaction strength, a modulation of the cavity-mechanical probe (i.e., optomechanical) interaction at the probe frequency can amplify the transduction of these virtual excitations into an observable displacement of the mechanical probe.

We begin with a description of the composite system, part by part, and intuitively derive the requirements for the detection of virtual radiation-pressure effects with such a mechanical probe at zero temperature. We then give an analytical quantitative analysis, which includes thermal noise affecting both light-matter and mechanical systems. As a result, we estimate the strength of the optomechanical coupling, and the bounds on the thermal noise, needed to resolve the effect within the standard quantum limit. Finally, we outline several explicit physical systems in which our proposal could be realized.

Ultrastrong coupling of light and matter.—The interaction between (a mode of) light confined in a cavity and a matter degree of freedom (modeled as a two-level system) is described by the quantum Rabi model ($\hbar = 1$)

$$H_R = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(\sigma_+ + \sigma_-)(a + a^\dagger), \quad (1)$$

where the fundamental mode of the cavity, with frequency ω, is described by the annihilation operator a and the two-level system (assumed resonant with the cavity) is described by the Pauli operator σ_z. In this model, the light-matter interaction is fully characterized by the normalized coupling $\eta \equiv \Omega/\omega$. In the weak-coupling regime ($\eta \ll 1$), terms which do not conserve the total free excitation number can be neglected, leading to the Jaynes-Cummings interaction. Therefore, the ground state $|G\rangle$ of the system does not contain photons; i.e., $\bar{n}_{\mathrm{GS}} = \langle G|a^\dagger a|G\rangle = 0$. However, in the ultrastrong coupling regime ($\eta > 0.1$), hybridization effects play an important role, and these qualitatively change the nature of the ground state (GS) which becomes dressed by virtual photons; e.g., the second-order perturbation theory in η implies

$$\bar{n}_{\mathrm{GS}} = \langle G|a^\dagger a|G\rangle \approx \frac{\eta^2}{4}. \quad (2)$$

Importantly, when weakly coupled to a low-temperature environment, the system relaxes to the hybridized ground state $|G\rangle$, out of which photons cannot escape [15,50,51]. To observe such virtual excitations, we now introduce a mechanical probe. We show that active modulation of the probe's interaction with the above system allows for an amplified measurement of the ground-state photons.

Optomechanical interaction.—The optomechanical interaction of a mechanical probe with the light-matter system described above can be most easily understood through the picture of a Fabry-Perot cavity with a mechanically compliant mirror coupled to a spring with frequency ω_m. This frequency is usually much smaller than the cavity frequency ω. The interaction between photons inside the cavity and the mirror displacement is essentially radiation pressure; i.e., momentum kicks on the mechanical spring due to the bouncing of photons off the mirror. It can be described, to lowest order in the displacement of the mirror, as

$$H = H_R + \omega_m b^\dagger b + g_0 a^\dagger a(b + b^\dagger), \quad (3)$$

where b is the annihilation operator of the mechanical mode and g_0 is the vacuum optomechanical coupling strength. Note that, when matter is within the cavity, a third-order interaction term can arise because of modulation of the light-matter coupling strength Ω as the cavity length varies in time [35]. Here we neglect that interaction, as it can be made negligible (while still maintaining a strong light-matter dipole coupling) by moving the position of the matter inside the cavity slightly away from the maximum of the electric field. Thus, here we focus on the standard optomechanical interaction term, for which the coupling amplitude g_0 corresponds to the frequency shift of the cavity when the mechanical displacement is equal to its zero-point motion x_{zp} [30]. Because of this interaction, in the absence of matter, an average of n photons in the cavity exerts a radiation-pressure force $P_n = ng_0/x_{\mathrm{zp}}$ on the mirror, inducing a displacement

$$|\langle x\rangle_n| = 2n\eta_m x_{\mathrm{zp}}, \quad (4)$$

as a function of the normalized optomechanical coupling $\eta_m \equiv g_0/\omega_m$. Let us now provide some intuition on how the situation changes when an atom interacts with the cavity field. At sufficiently low temperatures, the cavity-QED composite system is in its ground state which still exerts a (virtual) radiation pressure on the mirror, readily found by setting $n = \bar{n}_{\mathrm{GS}}$, giving

$$|\langle x\rangle_{\mathrm{GS}}| = \frac{\eta^2}{2}\eta_m x_{\mathrm{zp}}. \quad (5)$$

To resolve the effect within the standard quantum limit, we need to impose $|\langle x\rangle_{\mathrm{GS}}| > x_{\mathrm{zp}}$, which leads to

$$\eta_m > \frac{2}{\eta^2}. \quad (6)$$

While it is now possible for many different cavity-QED systems to reach the ultrastrong coupling regime $\eta \sim 0.1$, most realizations of optomechanical systems tend to be in the weak coupling regime $\eta_m \ll 1$, limiting the practicality of Eq. (6) (although proposals to achieve stronger couplings do exist [34,52–59]).

However, we can overcome this limitation by modulating the optomechanical coupling $g_0 \mapsto g_0(t)$, akin to recent proposals to enhance effective Kerr nonlinearities with a modulated optomechanical coupling [60], to enhance the readout of qubits with a modulated longitudinal coupling [61], or by modulating other parameters of the atom-cavity system [62–64]. Intuitively, this modulation effectively turns radiation pressure into a built-in (photon-number-dependent) resonant driving force. With this interpretation in mind, by considering a modulation at the mechanical frequency

$$g_0 \mapsto g_0 \cos \omega_m t, \quad (7)$$

we immediately find [65] that the mechanical displacement is enhanced by the factor $|\chi(\omega_m)|/|\chi(0)| = \omega_m/\Gamma_m$ in terms of the frequency-dependent mechanical susceptibility $\chi(\omega)$ and the mechanical decay rate Γ_m. This effectively corresponds to the substitution $\eta_m \mapsto \bar{\eta}_m$ (with $\bar{\eta}_m = g_0/\Gamma_m$) in Eq. (6), obtaining the much more realistic requirement

$$\bar{\eta}_m > \frac{2}{\eta^2}. \quad (8)$$

This suggests that the amplified observation of ground-state excitations is feasible and constitutes our first main result.

While this result holds for zero temperature, at small but finite temperatures, correlations between the system and the mechanical probe arise, which can complicate the problem of distinguishing the small thermal occupation of the light-matter system from virtual excitations.

To understand in detail the competition between ground-state occupation and unwanted environmental influence, we perform a detailed analysis, based on an analytical low-energy effective model. This allows us to estimate temperature-dependent bounds for the observation of the virtual excitations. In addition, we will show that the protocol presented here does not amplify the intrinsic mechanical thermal noise, which we expect to be the most relevant in realistic implementations (wherein the mechanical probe frequency is much smaller than the strongly coupled light-matter parameters).

Effective model.—With the modulation of the optomechanical coupling described in Eq. (7), and in a frame rotating at the mechanical frequency ω_m, the Hamiltonian in Eq. (3) becomes

$$H = H_R + \frac{g_0}{2} a^\dagger a (b + b^\dagger), \tag{9}$$

where we performed a rotating wave approximation (see the Supplemental Material [66] for the nonresonant driving case).

A Born-Markov perturbative master-equation treatment of the interaction with the environment for the system in Eq. (9) can be written as $\dot{\rho} = -i[H, \rho] + \mathcal{L}_R(\rho) + \mathcal{L}_m(\rho)$ [50,76], where the term $\mathcal{L}_m = \Gamma_m\{\bar{n}_m \mathcal{D}[b^\dagger](\rho) + (1 + \bar{n}_m)\mathcal{D}[b](\rho)\}$ is the Liouvillian, accounting for the bath of the mechanical degree of freedom, as a function of its thermal occupation number $\bar{n}_m$ and where $\mathcal{D}[O](\rho) = \frac{1}{2}(2O\rho O^\dagger - \rho O^\dagger O - O^\dagger O \rho)$. The Liouvillian $\mathcal{L}_R$ depends on the environments coupled to the photonic and matter systems and, importantly, in the ultrastrong coupling regime, causes transitions between dressed states which diagonalize the light-matter Hamiltonian [15,50]. We now assume a regime where the population of the light-matter system is restricted to its lowest (dressed) energy states, i.e., the ground $|G\rangle$ and first two excited states $|\pm\rangle$. Under this approximation, we can project H to this low-energy subspace. Not surprisingly, in this limit, the model can be given a bosonic representation under the replacement $|G\rangle\langle\pm| \mapsto a_\pm$. In this way, it is possible to provide an analytical treatment of the model, including a self-consistent quantification of the low-temperature effects. Under these assumptions, to second order in η, the Hamiltonian reads

$$H = \omega_+ a_+^\dagger a_+ + \omega_- a_-^\dagger a_- + \frac{g_0}{2} \hat{\alpha}(b + b^\dagger), \tag{10}$$

where $\omega_\pm = \omega(1 \pm \eta)$ and $\hat{\alpha} = (\alpha_+ a_+^\dagger a_+ + \alpha_- a_-^\dagger a_- + \xi)$, with $\alpha_\pm = \frac{1}{2} \mp \eta/4$, $\xi = \eta^2/4$, and where we neglected terms rotating at frequencies 2ω and $2\eta\omega$ in the optomechanical interaction term ([66], Sec. I). Interestingly, this allows for an effective decoupling of the modes a_+ and a_-.

In addition, this result enlarges the domain of our analysis ([66], Sec. III) to physical systems with *a priori* bosonized matter degrees of freedom (as is typical for many-particle systems like quantum wells). The Hamiltonian in Eq. (10) shows a critical feature of our scheme: the interaction between the mechanical probe and the system's Hamiltonian commute, allowing for quantum nondemolition measurements. Without this condition, one would need to rely on further dissipation processes to reduce backaction noise [27].

In the linearized approximation we are considering, a completely equivalent master equation for the coupled system can naturally be written ([66], Sec. II) in terms of three independent baths as

$$\dot{\rho} = -i[H, \rho] + \mathcal{L}_+(\rho) + \mathcal{L}_-(\rho) + \mathcal{L}_m(\rho), \tag{11}$$

where $\mathcal{L}_\pm(\rho) = \kappa_\pm\{\bar{n}_\pm \mathcal{D}[a_\pm^\dagger](\rho) + (1 + \bar{n}_\pm)\mathcal{D}[a_\pm](\rho)\}$ and $\kappa_\pm$ are linear combinations of the decay rates of the light-matter subsystems calculated at the frequencies $\omega_\pm$. For simplicity, in the following, we will assume that the occupation numbers are equal: $\bar{n}_\pm = \bar{n}$ (see [66] for details and a more general analysis).

Enhanced readout.—From Eq. (10), note that the force acting on the mechanical mode $P = g_0\hat{\alpha}/2x_{\mathrm{zp}}$ has two contributions: the usual radiation pressure (dependent on the number of normal excitations in the light-matter system) and virtual radiation pressure (proportional to ξ, accounting for ground-state effects). Following Eq. (11), the Heisenberg equation of motion for the dimensionless quadrature of the mechanical mode $|\langle\tilde{X}\rangle| = |\langle i(b^\dagger - b)\rangle|/\sqrt{2}$ in the steady state leads to

$$|\langle\tilde{X}\rangle| = \sqrt{2}\bar{\eta}_m(\alpha_+\bar{n}_+ + \alpha_-\bar{n}_- + \xi), \tag{12}$$

which is the expected result from our intuitive analysis in the introduction: The modulation of the coupling induces a displacement of the mechanical probe with an amplified amplitude proportional to $\bar{\eta}_m = g_0/\Gamma_m$. As implicitly done throughout this Letter, we omitted zero-point energy contributions [77]. As shown by this expression, the total displacement has two physically different contributions, i.e., $|\langle\tilde{X}\rangle_{\mathrm{GS}}| = (\xi/\alpha)|\langle\tilde{X}\rangle|$ (accounting for virtual radiation-pressure effects) and $|\langle\tilde{X}\rangle_{\bar{n}}| = (1 - \xi/\alpha)|\langle\tilde{X}\rangle|$ (accounting for finite temperature effects), where $\alpha = \langle\hat{\alpha}\rangle = \alpha_+\bar{n}_+ + \alpha_-\bar{n}_- + \xi$.

Signal-to-noise ratio.—To analyse the interplay between the two different contributions to the displacement and to what degree they can be resolved, both from one another and from the mechanical system's own vacuum fluctuations (the standard quantum limit), we use the ratio $F \equiv |\langle\tilde{X}\rangle|/\delta\tilde{X}$, where $(\delta\tilde{X})^2 = \langle\tilde{X}^2\rangle - \langle\tilde{X}\rangle^2$, a general analytical expression of which is shown in [66] (Sec. III B). At finite temperatures, the mechanical probe and the light-matter system become correlated, leading to a nontrivial expression for this variance. Using Eq. (12), we can define the analogous ratio for the ground-state signal contribution alone as

$$F_{\mathrm{GS}} \equiv \frac{|\langle \tilde{X} \rangle_{\mathrm{GS}}|}{\delta \tilde{X}}, \tag{13}$$

which quantifies our ability to resolve virtual radiation-pressure effects. We plot [78,79] these quantities as a function of the thermal occupation of the light-matter system in Fig. 1. Close to the ground state, $F \to 0$ in the absence of matter (black curve), while $F \to F_{\mathrm{GS}} \neq 0$ when matter is present in the cavity in the ultrastrong coupling regime (dashed blue curve).

For a more quantitative analysis, we now consider two minimal conditions to observe the influence of virtual radiation pressure on the mechanical displacement, i.e., the conditions

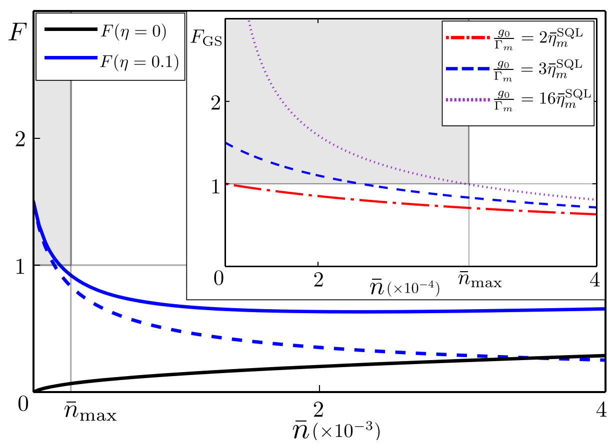

FIG. 1. Total displacement visibility F in the presence (full blue curve, $\eta = 0.1$) and absence (full black curve, $\eta = 0$) of matter in the cavity as a function of the number of thermal light-matter excitations $\bar{n}$ (for an optomechanical coupling $g_0/\Gamma_m = 3\bar{\eta}_0^{\mathrm{SQL}}$, for $\bar{\eta}_0^{\mathrm{SQL}} = 2/\eta^2$). For high values of $\bar{n}$, the two curves asymptotically converge to a parallel behavior ([66], Sec. III B). In the absence of matter, when $\bar{n} \to 0$ a zero photon population implies no displacement (black curve). However, in the presence of matter, virtual photons can displace the mechanical oscillator even for $\bar{n} \to 0$ (blue curve). The relative displacement contribution purely due to virtual radiation-pressure effects F_{GS} is represented by the blue dashed curve showing that, for $\bar{n} \to 0$, the displacement is mainly due to the dressed structure of the ground state. The gray vertical line represents the theoretical upper bound $\bar{n}_{\max}$. Below this critical value, it is possible to tune g_0/Γ_m to resolve the ground-state signal. This is shown in the inset, which magnifies the main plot around $\bar{n} = \bar{n}_{\max}$. The blue curve corresponds to the same color-coded ones in the main figure. The dotted purple and dot-dashed red curves are plotted for different values of g_0/Γ_m ($16\bar{\eta}_0^{\mathrm{SQL}}$ and $2\bar{\eta}_0^{\mathrm{SQL}}$, respectively). For $\bar{n} < \bar{n}_{\max}$, it is always possible to find optomechanical couplings which, in principle, allow one to resolve the ground-state signal (i.e., $F_{\mathrm{GS}} > 1$).

$$|\langle \tilde{X} \rangle_{\mathrm{GS}}| > |\langle \tilde{X} \rangle_{\bar{n}}|, \qquad F_{\mathrm{GS}} > 1. \tag{14}$$

The first condition requires the observed total displacement to be mainly due to ground-state effects. The second condition requires the signal to be resolved with respect to the standard-quantum-limit noise [1,2] (see the threshold in Fig. 1).

From the analysis following Eq. (12), the first condition translates to an upper bound $\bar{n}_{\mathrm{GS}}$ on the allowed thermal occupation of the light-matter system for the ground-state effects to dominate. Complementarily, the second condition implies the ability to resolve the ground-state contribution to the signal in Eq. (12) with respect to its total uncertainty $\delta\tilde{X}$. It translates into both a lower bound $\bar{\eta}_m^{\mathrm{SQL}}$ on the normalized optomechanical coupling and another upper bound $\bar{n}^{\mathrm{SQL}}$ on the thermal light-matter occupation. By solving the Heisenberg equation of motion using Eq. (11), we find [66] the following explicit conditions:

$$\bar{n} < \bar{n}_{\max}, \qquad \bar{\eta}_m > \bar{\eta}_m^{\mathrm{SQL}}. \tag{15}$$

This is the second main result of our work, generalizing Eq. (8) to finite temperatures. Here, $\bar{n}_{\max} = \min(\bar{n}^{\mathrm{GS}}, \bar{n}^{\mathrm{SQL}})$ (with $4\bar{n}^{\mathrm{GS}} = \eta^2$, $8\bar{n}^{\mathrm{SQL}} = \beta\eta^4$ at lowest significant order in η where the expression for $n_{\max}$ does not depend on the bosonic or spin nature of the model), and $\bar{\eta}_m^{\mathrm{SQL}} = 4[(1+2n_b)/(\eta^4 - 16R)]^{1/2}$ [with $R = \bar{n}(1+\bar{n})(\alpha_+^2/\beta_+ + \alpha_-^2/\beta_-)$, $\beta_\pm = 1 + 2\kappa_\pm/\Gamma_m$].

Consistent with our initial intuitive reasoning, when n_b, $R \to 0$, the second expression in Eq. (15) is equivalent to the zero-temperature result given in Eq. (8). Moreover, we note that mechanical thermal occupation is not amplified by this protocol, and its influence can be understood as a weak renormalization of the optomechanical coupling $g_0 \mapsto g_0/(1+n_b)^{1/2}$.

In summary, one can observe the amplified ground-state occupation when the temperature is low enough such that ground-state effects both dominate the displacement ($\bar{n} < \bar{n}_{\mathrm{GS}}$) *and* can be resolved from thermal and vacuum fluctuations (which requires $\bar{n} < \bar{n}^{\mathrm{SQL}}$ and sufficiently large optomechanical coupling $\bar{\eta}_m > \bar{\eta}_m^{\mathrm{SQL}}$); see Fig. 1.

Experimental feasibility.—Experimentally, optomechanical devices operating at microwave frequencies can achieve both strong electromechanical couplings g_0 and ultrastrong light-matter interaction ($\eta > 0.1$ [12]). For concreteness, we consider a microwave cavity capacitively coupled to a micromechanical membrane [60,80] whose motion modulates the frequency of the cavity. The consequent optomechanical interaction can then be modulated by using an additional tunable capacitor and/or inductance (for example, using a SQUID threaded by an external magnetic field [60,66]). The experimental parameters (explicitly referring to Ref. [80]) realized in these systems are very promising, with a thermal occupation of the cavity being $\bar{n}^{\rm exp} \sim 10^{-11}$ and a renormalized optomechanical coupling $\bar{\eta}_m^{\rm exp} \sim 6$. By using the other relevant experimental parameters in Ref. [80], we find $\bar{n}_{\max} \sim 0.1$, so that $\bar{n}^{\rm exp} < \bar{n}_{\max}$, i.e., the first condition in Eq. (8), can be satisfied by a large margin. In order to fulfill the second constraint in Eq. (8), i.e., $\bar{\eta}_m < \bar{\eta}_m^{\rm exp}$, we find that the normalized Rabi frequency has to be $\eta > 0.8$. This is compatible with the other main requirement for the observation of the effect given in Eq. (8), i.e., $\eta > (\bar{\eta}_m^{\rm exp}/2)^{1/2} \sim 0.6$. These conditions are, in principle, possible in circuit-QED devices [12].

Conclusions.—We presented a method to probe the structure of the dressed ground state by introducing an optomechanical coupling between the cavity mode and a mechanical measurement device. Compared to other proposals [27], our method is effectively quantum nondemolition, exhibits higher sensitivity, and requires only weak optomechanical coupling. Critically, we showed that a time-dependent modulation of the optomechanical coupling leads to an effective amplification of the measurement strength, allowing one to peer into the dressed ground state. We expect that this technique could also be applied to other measurement problems based on the same optomechanical interaction.

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[65] Intuitively, the mechanical displacement induced by the (Fourier transformed) radiation-pressure force $P_n(\omega)$ can be written as $\Delta x = \int_{-\infty}^{\infty} d\omega e^{i\omega t}\chi(\omega)P_n(\omega)$ in terms of the mechanical susceptibility $\chi(\omega)=\omega_m/(\omega_m^2-\omega^2-i\omega\Gamma_m)$. For constant optomechanical couplings, we have $P_n \propto \delta(0)$. By modulating the coupling g_0 at the mechanical frequency, the radiation pressure effectively becomes a driving force with $P_n \propto [\delta(\omega-\omega_m)+\delta(\omega+\omega_m)]/2$. By inserting these expression in the one for Δx, we then find (in a rotating frame) the enhancement factor reported in the main text.

[66] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.053601 for further details. In Section I, we derive the effective Hamiltonian describing the low-energy physics of hybrid opto-mechanical systems where the cavity mode interacts ultrastrongly with a two-level atom. In Section II, we model the interaction of the hybrid opto-mechanical system and its environment which leads to the Lindblad master equation reported in the main text. In section III, we derive an expression for the quadrature averages and variances which define the regime where virtual radiation pressure effects are in principle observable. In section IV, we present details about the experimental feasibility (in electro-mechanical systems) of the amplification protocol. The Supplemental Material includes Refs. [67-75].

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