Hybrid Quantum Device with Nitrogen-Vacancy Centers in Diamond Coupled to Carbon Nanotubes

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We show that nitrogen-vacancy (NV) centers in diamond interfaced with a suspended carbon nanotube carrying a dc current can facilitate a spin-nanomechanical hybrid device. We demonstrate that strong magnetomechanical interactions between a single NV spin and the vibrational mode of the suspended nanotube can be engineered and dynamically tuned by external control over the system parameters. This spin-nanomechanical setup with strong, intrinsic, and tunable magnetomechanical couplings allows for the construction of hybrid quantum devices with NV centers and carbon-based nanostructures, as well as phonon-mediated quantum information processing with spin qubits.

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Carbon-based structures and devices are very commonly used in our everyday life and in state-of-the-art science and technology. In quantum information science, nitrogen-vacancy (NV) centers in diamond are outstanding solid state qubits due to their long coherence times and high controllability [1–5]. In nanomechanics, mechanical resonators made out of allotropes of carbon (such as nanotubes [6–8], diamond [9–11], and graphene [12]), are being extensively studied for fundamental research and practical applications [13–23].

Recently, much attention has been paid to coupling NV spins in diamond to mechanical resonators, which can be achieved extrinsically [24–34] or intrinsically [35–39]. In the first case, the interaction arises from the relative motion of the NV spin and a source of local magnetic-field gradients [24]. In such setups, a magnetic tip mounted on a vibrating cantilever [40] is often used to generate the magnetic coupling between an NV spin and the mechanical motion [24–34]. However, creating very strong, well-controlled, local gradients remains challenging for such setups, in particular when arrays of NV centers are placed in close proximity to the same cantilever. Thus far, experiments with the extrinsic coupling scheme have yet to reach the strong-coupling regime [26–29]. In the second case, the coupling of a diamond cantilever to the spin of an embedded NV center is induced by crystal strain during mechanical motion [35–39]. Unfortunately, the strain-induced interaction between a single NV spin and the cantilever quantized motion is inherently tiny [37,38], which makes the strong strain coupling at a single quantum level very challenging.

In this Letter, we propose that NV centers in diamond interfaced with carbon nanotubes can facilitate a spin-nanomechanical hybrid device. This hybrid structure takes advantage of the unprecedented mechanical and electrical characteristics of carbon nanotubes, as well as the exceptional coherence properties of NV centers in diamond. We demonstrate that the physics of an NV center in diamond placed near a carbon nanotube with a dc current flowing through it can be well mapped to cavity quantum electrodynamics (QED). In particular, going beyond earlier work in this field [24–39], we show that the magnetomechanical interaction can be engineered and dynamically tuned by external control of the driving microwave fields and electric current through the nanotube. The resulting coupling strength can be roughly 3 orders of magnitude stronger than that for cold atoms coupled to nanowires [16,17]. An inherent advantage of our NV-nanotube hybrid system is the intrinsic nature of the coupling. Thus, no additional components, such as external magnetic tips, are required to tune the coupling. Another distinct feature of this intrinsic coupling scheme is that it is scalable to arrays of NV centers in diamond. This spin-nanomechanical structure with strong intrinsic magnetomechanical couplings would open up new avenues towards the design of hybrid quantum devices [41] with NV centers and carbon-based nanostructures. It also allows for quantum information processing with NV spin qubits [42,43], and could serve as novel nanoscale sensors [44–48] in physical and life science.

Model.—We consider a setup as shown in Fig. 1(a), where the magnetic field of a current-carrying nanotube is coupled to an NV center spin embedded in a nanodiamond. The nanotube of length $L$ is suspended along the $x$ axis at a distance $d$ from the diamond nanocrystal. When it vibrates, $d$ varies by the nanotube’s effective transverse displacement. In the following, we assume the transverse displacement to be along the $y$ direction, and express it with the oscillator operator $\hat{a}$ of the fundamental oscillating mode,
i.e., \( \hat{u}_v = (\hbar/2m_\text{nt})^{1/2}(\hat{a} + \hat{a}^\dagger) \), where \( m \) is the effective mass of the nanotube, and \( \omega_\text{nt} \) is the mechanical vibration frequency [49]. The magnetic field \( \vec{B}_\text{nt} (\vec{r}) \) of the current-carrying nanotube at position \( \vec{r} \) can be calculated by the Biot-Savart law. For a long nanotube \( (L \gg d) \), the magnetic field has the form

\[ \vec{B}_\text{nt} (\vec{r}) = \mu_0 \vec{J}_x \times \vec{r}/2\pi |\vec{r}|^2, \]  

in a reference frame with axes as in Fig. 1. Here \( \vec{e}_x \) is the unit vector in the \( x \) direction, and \( I \) is the electric current in the nanotube.

NV centers in diamond consist of a substitutional nitrogen atom and an adjacent vacancy, which have a spin \( S = 1 \) ground state, with zero-field splitting \( D = 2\pi \times 2.87 \text{ GHz} \), between the \( |m_s = \pm 1\rangle \) and \( |m_s = 0\rangle \) states. For moderate applied magnetic fields, static and low frequency components of magnetic fields \( B_z \) cause Zeeman shifts of states \( |m_s = \pm 1\rangle \), while external microwave fields with magnetic field \( B_{\text{gs}} \) drive Rabi oscillations between \( |m_s = 0\rangle \) and the excited states \( |m_s = \pm 1\rangle \), as shown in Fig. 1(b). For convenience, we denote as the \( z \) axis the crystalline axis of the NV center.

The interaction of a single NV center located at \( \vec{r} \) with the total magnetic field (external driving and from the nanotube) can be written as

\[ \hat{H}_\text{NV} = \hbar D \hat{S}_z^2 + \mu_B g_z B_z \hat{S}_z + \mu_B g_s (\vec{B}_\text{nt}(\vec{r}) + \vec{B}_d) \cdot \hat{\vec{S}} \],

with \( g_s = 2 \) the Landé factor of the NV center, \( \mu_B \) the Bohr magneton, and \( \hat{\vec{S}} \) the spin operator of the NV center. We consider using a single microwave field polarized in the \( x \) direction, \( \vec{B}_d = B_0 \cos \omega_0 t \vec{e}_x \), and place the NV center in the position where the magnetic field of the nanotube is in the \( z \) direction, \( \vec{B}_\text{nt} = B_\text{nt} \hat{e}_z \). The magnetic field \( \vec{B}_\text{nt}(\vec{r}) \) felt by the NV center is modulated by the nanotube’s vibration. Expanding the magnetic field \( \vec{B}_\text{nt}(\vec{r}) \) up to first order in \( \hat{u}_y \), we have

\[ \hat{H}_\text{NV} = \hbar D \hat{S}_z^2 + \mu_B g_s (\vec{B}_d + \vec{B}_0) \cdot \hat{\vec{S}} + \mu_B g_s \vec{B}_d \cdot \hat{\vec{S}} + \mu_B g_s \hat{S}_z \vec{B}_\text{nt} \hat{u}_y. \]

We define \( \hat{h}_\Delta = \hbar D \pm \mu_B g_s (B_0 + B_\text{nt}) - \hbar \omega_0 \), \( \hbar \Omega = (\sqrt{2}/4)\mu_B g_s \hbar \omega_0 \), and restrict the following discussion to symmetric detunings, \( \Delta_+ = \Delta_- = \Delta \). When \( |\Delta| \gg \Omega \), we obtain the effective Hamiltonian [49]

\[ \hat{H}_\text{q} = \hbar \omega_0 \hat{a}^\dagger \hat{a} + \frac{\hbar}{2} \hbar \Delta \hat{S}_z + \hbar g (\hat{\sigma}_+ + \hat{\sigma}_-) (\hat{a}^\dagger + \hat{a}). \]

Here, \( \Lambda = 2\Omega^2/\Delta, \quad h\gamma = \mu_B g_s (\hbar/2m_\text{nt})^{1/2} \partial_\Omega \vec{B}_\text{nt}, \) and we switch to the new basis \( \{|\hat{B}\rangle = (1/\sqrt{2}) (|+1\rangle + |-1\rangle) \) and \( \{|\hat{D}\rangle = (1/\sqrt{2}) (|+1\rangle - |-1\rangle) \). Here \( \hat{\sigma}_+ = |\hat{B}\rangle \langle \hat{B} - |\hat{D}\rangle \langle \hat{D}|, \quad \hat{\sigma}_- = |\hat{B}\rangle \langle \hat{D}|, \) and \( \hat{\sigma}_z = |\hat{D}\rangle \langle \hat{D}|. \) The states \( |\hat{B}\rangle \) and \( |\hat{D}\rangle \) are often referred to as bright and dark states for NV spins [24,57]. If we choose \( \Delta = -\omega_\text{nt} \), then, under the rotating-wave approximation, we obtain the standard Jaynes-Cummings (JC) Hamiltonian

\[ \hat{H}_\text{JC} = \hbar \omega_0 \hat{a}^\dagger \hat{a} + \frac{\hbar}{2} \hbar \Lambda \hat{\sigma}_z + \hbar g (\hat{\sigma}_+ \hat{a}^\dagger + \hat{\sigma}_- \hat{a}). \]

However, if we choose \( \Lambda = -\omega_\text{nt} \), which can be controlled by the parameters \( \Delta \) and \( \Omega \), we obtain the anti-Jaynes-Cummings (AJC) Hamiltonian

\[ \hat{H}_\text{AJC} = \hbar \omega_0 \hat{a}^\dagger \hat{a} + \frac{\hbar}{2} \hbar \Lambda \hat{\sigma}_z + \hbar g (\hat{\sigma}_- \hat{a}^\dagger + \hat{\sigma}_+ \hat{a}). \]

Thus, our system mimics the standard model in cavity QED of a single atom coupled to a single cavity mode. The type of interactions can be designed by external control over the driving fields. This mapping allows the powerful toolbox of cavity QED to be transferred to these systems.

**Two proposed experimental setups.**—We consider two different designs for coupling an NV spin to a current-carrying carbon nanotube resonator. Figure 2(a) displays a nanotube, carrying a dc current, suspended above a bulk single crystal diamond sample. Individual, optically resolvable NV centers are implanted 5–10 nm below the surface of the diamond sample [27,58]. Figure 2(b) shows another feasible design, where a diamond nanoparticle hosting a single NV center is closely placed near the nanotube. Diamond nanoparticles can have a size of less than 10 nanometers, and only host one NV defect [26]. The spin states of NV centers can be controlled and manipulated by microwaves from external microwave antennas. A confocal microscope can be used to excite and polarize the NV spin, and detect photoluminescence to read out the NV spin polarization.

Carbon nanotubes can possess current-carrying capacities exceeding 10 \( \mu \text{A/nm}^2 \) [59–64], with lengths which can range from tens of nanometers to tens of micrometers. In experiments, the carbon nanotube could be actuated and deflected electrostatically over several nanometers with ac and dc voltages applied to the gate.
Thus, the distance between the nanotube and the NV center can be fine-tuned electrostatically. A recent experiment [23], has reported a device with a graphene membrane suspended some 10–50 nm above a single NV center.

To evaluate the single spin-phonon coupling strength $g$, we use the magnetic field generated by an infinite long tube, as given by the Biot-Savart law. In this case, it reads

$$g = \frac{\mu_0 g I}{2\pi \sqrt{2\hbar m_0 a d^2}}. \tag{4}$$

This magnetomechanical coupling strength depends on the dimensions of the nanotube and the distance $d$, as well as the current $I$ flowing through the nanotube. Thus, it can be easily tuned by control of the system parameters.

We consider a carbon nanotube of length $L \sim 2 \mu m$, radius $r \sim 1.5$ nm, and wall thickness $t \sim 0.353$ nm, suspended at a distance $d \sim 30$ nm from the NV center [49]. The tube carries a dc current $I \sim 60 \mu A$, and vibrates at a frequency $\omega_m/2\pi \sim 2$ MHz, with effective mass $m \sim 7 \times 10^{-21}$ kg [49]. With the above given parameters, one can obtain $g/2\pi \sim 10$ kHz. By changing the distance $d$ and the dimensions of the nanotube, as well as the current $I$ flowing through the nanotube, the coupling strength $g$ can be further adjusted (see Fig. 3). For a much closer distance $d \sim 10$ nm, the magnetomechanical coupling strength can even reach $g/2\pi \sim 100$ kHz. This coupling strength is comparable to that of a single NV spin coupled to a vibrating cantilever with a strong local magnet [24, 25] or a superconducting circuit [65–70], and is about a factor of 1000 larger than the coupling achieved with cold atoms [16, 17].

**Dephasing and dissipation.**—In realistic situations, we need to consider spin dephasing and mechanical dissipation. The full dynamics of our system that takes these incoherent processes into account is described by the master equation

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar} [\hat{H}_{JC}, \hat{\rho}] + \gamma_m D[\sigma_z] \hat{\rho} + n_{th}\gamma_m D[\hat{a}^\dagger] \hat{\rho} + (n_{th} + 1)\gamma_m D[\hat{a}] \hat{\rho}. \tag{5}$$

with $D(\hat{a})\hat{\rho} = \hat{a}\rho \hat{a}^\dagger - \frac{1}{2} \rho \hat{a}^\dagger \hat{a} - \frac{1}{2} \hat{a}^\dagger \hat{a} \rho$ for a given operator $\hat{a}$. The strong coupling regime can be reached if the coherent coupling strength $g$ exceeds both the electronic spin decay rate $\gamma_s$ and the intrinsic damping rate of the mechanical mode $\gamma_m$, i.e., $g > \{\gamma_s, n_{th}\gamma_m\}$, with $n_{th} = (e^{\hbar\omega_m/\hbar T} - 1)^{-1}$ the thermal phonon number at the environment temperature $T$. For a mechanical resonator with frequency $\omega_m$ and quality factor $Q$, the mechanical damping rate is $\gamma_m = \omega_m/Q$. The recent fabrication of carbon nanotube resonators can possess quality factors exceeding $10^5$ [8]. Together with the oscillator frequency $\omega_m/2\pi \sim 2$ MHz [49], this value of $Q$ implies an oscillator damping rate $\gamma_m/2\pi \sim 20$ Hz, and would translate into phonon mean free path $l_c \sim QL \sim 10$ cm [49]. Assuming an environmental temperature $T \sim 10$ mK in a dilution refrigerator, the thermal phonon number is about $n_{th} \sim 100$. Therefore, we obtain $g > n_{th}\gamma_m$. When it comes to the NV center, the dephasing time $T_2$ can be increased to several milliseconds in ultrapure diamond [71], leading to a dephasing rate $\gamma_s/2\pi \sim 1$ kHz. We can ignore single spin relaxation, as $T_1$ can be several minutes at low temperatures. Therefore, the strong-coupling regime can be reached in this setup, i.e., $g > \{\gamma_s, n_{th}\gamma_m\}$.

The strong-coupling regime described by the Hamiltonian (2) enables coherent quantum state transfer between the spin and the resonator. Moreover, in combination with optical pumping and detection techniques for spin qubits, this would provide the basic ingredients for detecting and manipulating quantum states of the nanotube resonator.

In Fig. 4, we show the numerical simulations of quantum dynamics of the coupled system through solving the master equation (5). As the initial state, we take the product state of the NV spin and the mechanical resonator with the occupation number $n_m = 0.2$, e.g., as a result of sideband cooling [72, 73]. In the time domain, vacuum Rabi oscillations are direct evidence of the coherent energy exchange between the spin qubit and the resonator phonon mode. We obtain numerical results for the time evolution of the mean phonon number and the probability for the NV center being in the excited state. We find that vacuum Rabi oscillations can occur for these parameters. In such a process, the spin state can be transferred from the NV center to the nanotube resonator, and vice versa.
Elementary quantum information.—The magnetomechanical interaction allows us to use the nanotube vibration mode as a quantum bus to perform more complex tasks. Controlled spin-spin couplings could be realized for two distant NV centers separated by micrometer distances. Based on these effective interactions, we now explore the possibility of implementing quantum information processing with spin qubits.

We consider two separated NV centers coupled to the same vibration mode of the nanotube in the dispersive regime $|\Lambda - \omega_{\text{nt}}| \gg g$. This will lead to an effective long range spin-spin interaction via the exchange of virtual phonons, $\hat{H}_{\text{eff}} = h\lambda_{\text{eff}}(\hat{\sigma}_z^1\hat{\sigma}_z^2 + \hat{\sigma}_z^1\hat{\sigma}_z^2)$, with the coupling strength $\lambda_{\text{eff}} = g^2/|\Lambda - \omega_{\text{nt}}|$. The coherence length of the phonon mediated NV spin coupling is about $l_c \sim QL$ [49], which can be much larger than the distance between two NV spins separated by a distance on the order of the nanotube’s length. If we choose $|\Lambda - \omega_{\text{nt}}|/2\pi \sim 1$ MHz, and $g/2\pi \sim 100$ kHz, then we can obtain the spin-spin coupling strength $\lambda_{\text{eff}}/2\pi \sim 10$ kHz. This strong spin-spin interaction allows for the implementation of a SWAP gate and quantum states transfer between two NV centers.

In the following, we encode the $j$th logical qubit in the two spin states of the $j$th NV center, i.e., $|0\rangle_j = |0\rangle_j$ and $|1\rangle_j = |D\rangle_j$. Such qubit encoding has proven to be more robust against low-frequency magnetic-field noise [57]. Our main task is to realize a SWAP gate and quantum information transfer between two qubits. To implement this protocol, we need a microwave to drive the transition between the qubit state $|0\rangle_j$ and the bright state $|B\rangle_j$ in each qubit with Rabi frequency $\Omega_j$ and frequency detuning $\delta_j$. The dynamics of the entire system is described by the Hamiltonian $\hat{H} = \sum_j h\delta_j|B\rangle_j\langle B| + \sum_j h\Omega_j|B\rangle_j\langle 0| + \text{H.c.} + \hat{H}_{\text{coupl}}$. The spin-spin interaction can be diagonalized with the states $|\pm\rangle_j = 1/\sqrt{2}(|0\rangle_j + |D\rangle_j)$. It is easy to show that in the subspace defined by $\{0,1\}_q \equiv |0\rangle_q^1|0\rangle_q^2, |+\rangle_q^1|+\rangle_q^2, |+\rangle_q^1|-\rangle_q^2, |0\rangle_q^1|0\rangle_q^2\}$, the system Hamiltonian has the form [49]

\[
\hat{H} = h\delta_+|+\rangle_q^1\langle +| + h\delta_-|-\rangle_q^1\langle -| + h\Omega_1|+\rangle_q^1\langle 0,1| + h\Omega_2|-\rangle_q^2\langle 0,1| + \text{H.c.},
\]

with $\delta_+ = \lambda_{\text{eff}} + (\delta_1 + \delta_2)/2$, $\delta_- = \lambda_{\text{eff}} - (\delta_1 + \delta_2)/2$, $\Omega_j = \Omega_j/\sqrt{2}$, $j = 1, 2$. Thus, we can find that if the two qubits are initially prepared in the state $|0\rangle_q^1|1\rangle_q^2$ or $|1\rangle_q^1|0\rangle_q^2$, then the dynamics of the system will be confined in the subspace governed by the Hamiltonian (6). In Fig. 5 we present detailed numerical simulations for the dynamics of the coupled system. It can be found that at the moment $T_{\text{sw}} = \pi/\lambda_{\text{eff}}$, the system evolves from the state $|0\rangle_q^1|1\rangle_q^2$ to the state $|1\rangle_q^1|0\rangle_q^2$ via the intermediate states $|\pm\rangle_q$ through microwave driving, and vice versa. The state $|1\rangle_q^1|1\rangle_q^2$ remains unchanged during this process, while the system will be brought from the state $|0\rangle_q^1|0\rangle_q^2$ to the state $|B\rangle_q^1|0\rangle_q^2$ or $|0\rangle_q^1|B\rangle_q^2$, and back again at the moment $T_{\text{sw}}$. Therefore, in the language of quantum information theory, this operation corresponds to a SWAP gate. This gate can be exploited to realize quantum state transfer between two NV spins, i.e., $(\alpha|0\rangle_q^1 + \beta|1\rangle_q^1)|1\rangle_q^2 \rightarrow |1\rangle_q^1(\alpha|0\rangle_q^2 + \beta|1\rangle_q^2)$.

The gate fidelity is limited by the following factors: (i) spin decoherence induced by phonon excitations with an effective decay rate $\Gamma = n_{\text{ph}}g^2/|\Lambda - \omega_{\text{nt}}|^2$; (ii) single spin dephasing due to low-frequency noise with a dephasing rate $\gamma$, which is assumed to be Markovian for simplicity. Recent work has shown that by coupling a single NV spin to another two-level system and encoding quantum information in the dark states $|0\rangle$ and $|D\rangle$, the coherent time $T_2$ can be prolonged [57]. For isotopically purified diamond, we can safely choose $T_2 \sim 1$ ms. Taking these factors together, we find the gate fidelity can be estimated as $F \sim (1 - T_{\text{sw}}\Gamma - T_{\text{sw}}/T_2) > 0.95$ for the given parameters, with an operating time $T_{\text{sw}} \sim 50$ μs.

Conclusions.—We have proposed a spin-nanomechanical hybrid device where a single NV center spin in diamond is coupled to the vibrational mode of a suspended current-carrying carbon nanotube. It makes the strong spin-mechanical coupling at a single quantum level feasible, and allows fast mechanical control of spin qubits and efficient phonon cooling by an NV center. Such a device can find applications in phonon-mediated quantum information processing with NV spin qubits, and could serve as novel nanoscale sensors for detecting tiny pressure, temperature, electric, and magnetic-field changes.

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