Effect of disorder on the transverse magnetoresistance of Weyl semimetals

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We study the effect of random potentials created by different types of impurities on the transverse magnetoresistance of Weyl semimetals. We show that the magnetic field and temperature dependence of the magnetoresistance is strongly affected by the type of impurity potential. We analyze in detail two limiting cases: (i) the ultraquantum limit, when the applied magnetic field is so high that only the zeroth and first Landau levels contribute to the magnetotransport, and (ii) the semiclassical situation, for which a large number of Landau levels come into play. A formal diagrammatic approach allowed us to obtain expressions for the components of the electrical conductivity tensor in both limits. In contrast to the oversimplified case of the δ -correlated disorder, the long-range impurity potential (including that of Coulomb impurities) introduces an additional length scale, which changes the geometry and physics of the problem. We show that the magnetoresistance can deviate from the linear behavior as a function of magnetic field for a certain class of impurity potentials.

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I. INTRODUCTION

In recent years, problems related to quantum transport in materials with the Dirac spectrum of charge carries, in particular in Weyl semimetals, have attracted considerable interest [1,2]. Much effort was focused on the longitudinal magnetoresistance, where the negative contribution associated with the so-called chiral anomaly arising due the transfer of charge carriers between Weyl points with opposite chiralities plays a dominant role [3-8].

At low magnetic fields, rather nontrivial manifestations of the weak localization and antilocalization effects have also been addressed [9,10]. No less interesting is the behavior of the transverse magnetoresistance, where a nonsaturating linear magnetic field dependence is observed at high fields [11-14]. The nature of such unusual behavior has been widely discussed.

The main physical mechanisms in the ultraquantum regime were revealed in the seminal work of Abrikosov [15]. He considered a gapless semiconductor with a linear dispersion law near the chemical potential. The electron density was chosen in such a way that only the zeroth Landau level took part in the charge transport and the charge carriers were scattered by impurities characterized by the screened Coulomb potential.

This problem was generalized in the detailed studies presented in Refs. [16,17], which were stimulated by numerous experimental observations of the linear magnetoresistance. In these papers, the transverse magnetoconductivity and Hall conductivity were obtained in both ultraquantum and socalled semiclassical limits (the latter is characterized by the contribution of many Landau levels into the charge transport). However, their main emphasis was on the case of pointlike impurities, i.e., the disorder correlation length was much smaller than the charge-carrier wavelength and the magnetic length. Even the case of Coulomb disorder was treated in the framework of modified δ -correlated disorder.

A rigorous study of long-range disorder in Weyl semimetals was undertaken in Ref. [18], where the perturbed Keldysh model was employed. The main focus of Ref. [18] was on the density of states of charge carriers, but the authors also addressed the magnetoconductivity with the Coulomb disorder. However, their usage of Drude-type expression for the conductivity was never derived diagrammatically, and, as a result, their answers were of a qualitative nature. A different approach to tackle the magnetoconductivity in metals with the long-range disorder was used in Ref. [19]. Although that approach lacks some rigor, it appeals to an intuitively transparent semiclassical picture involving the concept of guiding centers. It is also worth mentioning the numerical analysis undertaken in Ref. [20], where the magnetoconductivity was treated in the framework of the self-consistent Born approximation in the case of Coulomb impurities at an arbitrary position of the chemical potential. Finally, for the screened Coulomb potential of impurities, the electron transport was also analyzed in the case of a gapped Dirac spectrum [21,22].

This way, the analytical results for the magnetoconductivity with long-range impurity potentials have so far been either of qualitative nature or lacked rigor, which only a diagrammatic or kinetic-equation approach can provide.

The primary goal of this paper is obtaining analytical results for the magnetoconductivity for a general type of long-range disorder through the use of a consistent diagrammatic approach. We employ the Kubo-Středa formalism and use the diagrammatics to implement the averaging for obtaining the corresponding results for the transverse magnetoconductivity and Hall conductivity.

In this work, we consider a disorder potential of quite a general form and characterize it by its short-range asymptotics, which we assume to be of a power-law type:

$$u(r) \sim r^{-1-\gamma}, \quad r \ll r_0, \tag{1}$$

where r_0 is a characteristic radius of the disorder potential. The characteristic scale can be understood as the screening radius of the potential. We find that depending on the exponent γ , the transverse magnetoconductivity exhibits different scaling with magnetic field in the ultraquantum limit (max{ μ, T } $\ll \Omega$)

$$\sigma_{xx} = \begin{cases} \frac{e^2 v^2}{r_0^4} g_1(c/eH) \sim H^{-1}, & -1 < \gamma < 0, \\ e^2 v^2 \alpha^2 \ln(1/\alpha)(c/eH) \sim H^{-1}, & \gamma = 0, \\ e^2 v^2 n_{\rm imp} u_0^2 \left(\frac{eHr_0^2}{c}\right)^{\gamma} (c/eH) \sim H^{\gamma-1}, & 0 < \gamma < 1, \end{cases}$$
(2)

where μ , T are chemical potential and temperature, respectively, e is the electron charge, v is the Fermi velocity, c is the velocity of light, H is the applied magnetic field,

$$\Omega = v\sqrt{2eH/c} \tag{3}$$

is the energy scale associated with the magnetic field (the distance between the zeroth and first Landau levels), g_1 is a numerical constant dependent on the type of the potential, n_{imp} is the concentration of impurities, and u_0 is the characteristic amplitude of the impurity potential in the coordinate space.

Therefore, the scaling of the transverse magnetoresistance with magnetic field in the ultraquantum regime reveals the information on the nature of disorder. In the opposite, so-called *semiclassical* limit, for which $\Omega \ll \max\{\mu, T\}$, we obtain general formula for the transverse magnetoconductivities and Hall conductivities valid for an arbitrary ratio of μ and T, as well as for an arbitrary $\tau_{tr} \Omega^2 / T$ ratio, where τ_{tr} is the transport scattering time.

The paper is organized as follows. In Sec. II, we formulate the model and introduce all the necessary parameters. In Sec. III, we analyze the components of the electrical conductivity tensor and their magnetic field dependence in the ultraquantum limit, for which the dominant contribution comes from the zeroth Landau level. In Sec. IV, we consider the magnetotransport at the semiclassical limit, for which the temperature is high enough and a large number of Landau levels come into play. Both in Secs. III and IV, we put the main emphasis on the magnetotransport in the context of longrange impurity potentials (the exact conditions are specified in these sections). In Sec. V, we discuss the obtained results. The details of our calculations are presented in Appendices A, B, and C.

II. MODEL AND CHARACTERISTIC PARAMETERS

Our study is aimed at the analysis of the conductivity tensor of the Weyl semimetal (WSM) with impurities under the effect



FIG. 1. Disorder vertex for the perturbation theory. Here, the dashed line represents the disorder correlation function, while solid tails are fermion lines.

of an applied transverse magnetic field (i.e., the magnetic field direction is perpendicular to that of the electric current). We start from the low-energy Hamiltonian for the WSM in its conventional form

$$H = H_0 + H_{imp},$$

$$H_0 = v \int \psi^{\dagger}(\mathbf{r}) \sigma \left(\mathbf{p} - \frac{e}{c} \mathbf{A}\right) \psi(\mathbf{r}) d\mathbf{r},$$

$$H_{imp} = \int \psi^{\dagger}(\mathbf{r}) u(\mathbf{r}) \psi(\mathbf{r}) d\mathbf{r},$$
(4)

where H_0 is the Hamiltonian of noninteracting Weyl fermions and H_{imp} describes the interactions with the impurity potential; $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices acting in the pseudospin space of Weyl fermions, $\mathbf{p} = -i\nabla_{\mathbf{r}}$ is the momentum operator, v is the Fermi velocity, and $u(\mathbf{r})$ is the impurity potential.

The specific form of the disorder potential is irrelevant to us. Its correlation function is assumed to be a smooth function of coordinates, with power-law ultraviolet asymptotics in the coordinate space. The exact restrictions are discussed below (see Sec. III).

Of particular importance to the experiment is the screened Coulomb impurity potential. As was argued in Ref. [23], there exists a regime, in which the Coulomb impurity scattering dominates over the electron-electron interaction (see the corresponding discussion in Sec. V).

Throughout the paper, we set $\hbar = k_B = 1$. We also neglect the influence of different Weyl cones on each other, concentrating on the low-energy physics. The vector potential of the magnetic field **H** is chosen in the asymmetric gauge

$$\mathbf{A} = (0, Hx, 0). \tag{5}$$

In this paper, we will use the Kubo-type diagrammatic approach. The impurity potential thus enters the formalism in terms of its correlation function averaged over the impurity positions. The relevant Feynman diagram is shown in Fig. 1.

The disorder correlation function is written in terms of the dimensionless function g (see its exact definition in Appendix A), which is introduced in momentum space from the very beginning. The disorder characteristic momentum is p_0 . The p_0^6 factor is introduced from dimensional considerations. Momentum p_0 can be identified with inverse correlation length r_0^{-1} . Potential amplitude u_0 is introduced in Appendix A.

The ultraquantum case corresponds to the limit

$$\max\{T,\mu\} \ll \Omega. \tag{6}$$

In this case, the long-range disorder condition means that its correlation length satisfies the following inequalities:

$$l_H \ll r_0 \ll \lambda, \tag{7}$$

where

$$\lambda = v/\max\{T, \mu\}, \ l_H = \sqrt{c/(eH)}$$
(8)

are the characteristic particle wavelength and the magnetic length, respectively.

In the opposite *semiclassical* limit $\Omega \ll \max\{T, \mu\}$, the respective condition for the disorder correlation length is changed as

$$\lambda \ll r_0 \ll l_H. \tag{9}$$

Limit (7) intuitively appeals to the physical picture where the center of the magnetic orbit moves along the impurity potential line, while limit (9) corresponds to the proper particle motion along the impurity potential line.

III. MAGNETOTRANSPORT AT max{ T, μ } $\ll \Omega$ (ULTRAQUANTUM LIMIT)

A. Computation of σ_{xx}

This limit means that the first of the Landau levels

$$\varepsilon_n = \sqrt{n\Omega^2 + p_z^2} \tag{10}$$

is high enough and only the ground state contributes to the magnetotransport (actually, the first excited state also contributes to the conductivity due to the long-range nature of disorder, as we will see below). The xx component of the conductivity tensor is determined by the respective Kubo formula [16] (see Appendix A for the derivation):

$$\sigma_{xx} = e^2 v^2 \int \frac{d\varepsilon \, d\mathbf{p} \, dx'}{(2\pi)^3} \frac{df(\varepsilon)}{d\varepsilon} \\ \times \langle \text{Im} G_{11}^R(x, x'; \varepsilon, \mathbf{p}) \text{Im} G_{22}^R(x', x; \varepsilon, \mathbf{p}) \rangle, \quad (11)$$

where angular brackets denote the averaging over disorder, f is the Fermi distribution function, and the retarded Green's functions are defined in the matrix form as follows:

$$G^{R}(x, x'; \varepsilon, \mathbf{p}) = \sum_{n=0}^{\infty} S_{n}(x_{p_{y}})G(\varepsilon, \mathbf{p})S_{n}^{\dagger}(x'_{p_{y}}),$$

$$S_{n}(s) = \begin{pmatrix} \chi_{n}(s) & 0\\ 0 & \chi_{n-1}(s) \end{pmatrix},$$

$$G(\varepsilon, \mathbf{p}) = \frac{\varepsilon + \upsilon \boldsymbol{\sigma} \cdot \mathbf{p}_{n}}{(\varepsilon + i0)^{2} - \varepsilon_{n}^{2}},$$

$$x_{p_{y}} = x - p_{y}l_{H}^{2}.$$
(12)

Here, $\chi_n(s)$ is the normalized oscillator wave function of the *n*th state, and

$$\mathbf{p}_n = (0, \sqrt{2n/l_H}, p_z)$$
 (13)

is the effective 2D momentum.

We are using perturbation theory and the dimensionless expansion parameter characterizing the disorder strength is



FIG. 2. Three contributions to the conductivity in first-order perturbation theory. The disorder vertices are represented by dashed lines defined in Fig. 1. The indices i, j become x or y depending on the type of conductivity which is calculated.

assumed to be small:

$$\frac{1}{\varepsilon\tau} \sim \frac{n_{\rm imp}u_0^2}{v^2 p_0^5} \ll 1,\tag{14}$$

where $\varepsilon = \max{\{\mu, T\}}$ is the characteristic energy scale for charge carriers and τ is its impurity scattering time [see below Eq. (40)].

The analysis (see, e.g., Ref. [15]) shows, that unlike the ordinary Drude conductivity proportional to the disorder scattering time (inverse disorder strength), the magnetoconductivity in the ultraquantum limit is, in fact, perturbative in the disorder strength.

Here is a short explanation. In the absence of magnetic field (spatially uniform case) and the absence of disorder, the momentum is conserved and the conductivity is infinite. The introduction of disorder scattering, however small, makes the conductivity finite. As a result, the conductivity is nonperturbative in the disorder strength.

The magnetic field radically changes the system. In the absence of disorder, the application of an external transverse electric field can be gauged out by changing the reference system moving along the direction perpendicular to the electric and magnetic fields. This leads to a vanishing σ_{xx} in the absence of disorder. Consequently, the transverse magneto-conductivity is perturbative in the disorder strength.

As a result, it is enough to compute σ_{xx} in the first order of perturbation in disorder. There are three possible diagrams (see Fig. 2). In the limit of very long-range disorder, $r_0 \rightarrow \infty$, all three diagrams vanish due to the fact that in the ultraquantum limit, they are proportional to the spectral density of the zeroth Landau level (LL) $\delta(\varepsilon - vp_z)$, multiplied by the combination of Green's functions of the first LL. The disorder scattering with finite characteristic length $r_0 \gg l_H$ effectively smears out the particle spectral density in momentum space leading to the enhanced contribution from all three diagrams. This was first stated explicitly for the case of short-range disorder in Ref. [16].

The expression for the conductivity σ_{xx} is obtained along the same lines for the general type of disorder as is done in Ref. [15] for the case of Coulomb disorder. In the ultraquantum limit and in the long-range disorder case $l_H \ll r_0$ [Eq. (7)], only the zeroth and first Landau level (LL) in Fig. 2 contribute to the conductivity. For the Green's function G_{11}^R in (11), it is enough to take only the contribution of the zeroth LL,

$$\operatorname{Im} G_{11}(\varepsilon, \mathbf{p}) = -\pi \delta(\varepsilon - p_z v). \tag{15}$$

Next, we expand all three diagrams in the small parameter of our problem

$$(q_y l_H \ll 1, q_x l_H) \sim l_H / r_0 \ll 1,$$
 (16)

where q_x , q_y is the momentum carrier by the disorder line (see Fig. 2).

The result is given by the following integral:

$$\sigma_{xx} = \frac{e^2 v^2}{4\Omega^2} \int \frac{dq_x dq_y}{(2\pi)^2} (q_x^2 + q_y^2) g_{2,q_{xy}},$$
 (17)

where $g_{2,q_{xy}}$ is the effective two-dimensional disorder correlation function defined as

$$g_{2,p_{xy}} = \int g(\mathbf{p}) \frac{dp_z}{2\pi} \equiv g(p_{xy}) \bigg|_{z=0}.$$
 (18)

Here, $p_{xy} = (p_x, p_y)$ is a two-dimensional (2D) momentum and $g(\mathbf{p})$ is defined in Fig. 1. In Ref. [15], Eq. (17) was analyzed only in the case of the Coulomb impurity potential. We, however, come to the conclusion that for different types of disorder, this formula gives a qualitatively different *H* dependence.

Before we proceed, let us make the following observation. The integral in Eq. (17) of the 2D disorder correlation function determining the conductivity can become divergent at high momenta (short-range case). However, in our calculations, we used the long-range disorder approximation, implying that $ql_H \ll 1$, where q is the characteristic disorder momentum. Therefore, $q \sim l_H^{-1}$ is the natural short-range cutoff scale. As we will see below, the system exhibits a qualitatively different magnetic field dependence of the conductivity for different short-range behavior of the disorder $p_0 \ll q \ll l_H^{-1}$.

B. σ_{xx} for different short-range behaviors of the impurity potential

In this paper, we restrict our attention to disorder potentials with the power-law short-range asymptotics

$$u(r) = \frac{u_0}{(p_0 r)^{1+\gamma}}, \ l_H \ll r \ll p_0^{-1}, \ -1 < \gamma < 1.$$
(19)

Here, the natural constraint $\gamma < 1$ means that we are not considering pathological cases of potentials leading to the "falling to the center" phenomenon. On the other hand, the $\gamma < -1$ constraint should exclude the unphysical case of decaying at r = 0. Then, the disorder correlation function in momentum space reads as (see Appendix A for the details)

$$g(p) = \frac{n_{\rm imp} u_0^2}{p_0^6 (p_0/p)^{4-2\gamma}}, \quad p_0 \ll p.$$
(20)

The question we now address is as follows: What is the behavior of the conductivity as a function of H for different values of γ ? To answer this question, we analyze expression (17) for various cases discussed below.

(a) $-1 < \gamma < 0$. We call this the "regular disorder" case. The integral in (17) is convergent, and the convergence region is $p \sim p_0$. In this case, we have

$$\sigma_{xx} = \frac{ec}{16\pi H p_0^2} n_{\rm imp} u_0^2 g_1, \quad \gamma < 0, \tag{21}$$

where $g_1 = \int_0^\infty g(x^2) x^3 dx$, with $x = p/p_0$, is a numerical constant, which depends on the details of the shape of the disorder distribution function.

As we are going to see below, the behavior corresponding to Eq. (21) is identical to that characteristic of the Coulomb disorder $\gamma = 0$.

(b) $\gamma = 0$. For the Coulomb disorder, the integral determining the conductivity in (17) is log-divergent. In the Coulomb case, the inverse Debye radius reads as

$$p_0 = \sqrt{\alpha} l_H^{-1} \tag{22}$$

for the case $\{T, \mu\} \ll \Omega$, where

$$\alpha = \frac{e^2}{\hbar v \kappa} \tag{23}$$

is the WSM fine structure constant and κ is the permittivity. One recovers the result [15]

$$\sigma_{xx} = \frac{e^3 c \pi}{H} n_{\rm imp} \ln \frac{1}{\alpha}, \quad \gamma = 0.$$
 (24)

(c) $0 < \gamma < 1$. We call this the "singular disorder" case due to its short-range behavior. The integral in (17) is then divergent at high momenta q and an appropriate cutoff $q \sim l_H^{-1}$ should be introduced. In this case, we have a nontrivial result for σ_{xx} :

$$\sigma_{xx} = \frac{ec}{16\pi H p_0^2} n_{\rm imp} u_0^2 \left(\frac{eH}{c p_0^2}\right)^{\gamma}, \quad 0 < \gamma < 1.$$
(25)

The above results can be summarized by formula (2), presented in the Introduction. We see that the *H* dependence of the conductivity is affected by the nature of the disorder. In particular, if the correlation function has stronger than Coulomb power-law growth at short distances, the corresponding exponent γ enters the conductivity.

The parameter most relevant to many experiments is the magnetoresistance. To calculate it, we need to know the Hall conductivity σ_{xy} .

C. Hall conductivity σ_{xy}

The Hall conductivity is given by the sum of two terms:

$$\sigma_{xy} = \sigma_{xy}^{\mathrm{I}} + \sigma_{xy}^{\mathrm{II}}.$$
 (26)

The first term in (26), σ_{xy}^{I} , is the so-called normal contribution, which is given by the following relation [24]:

$$\sigma_{xy}^{I} = \frac{e^{2}\Omega^{2}}{4\pi^{2}} \int \frac{d\varepsilon}{2\pi} \frac{df(\varepsilon)}{d\varepsilon} \sum_{n} \left[G_{22}^{R} \operatorname{Im} G_{11}^{R} - G_{11}^{R} \operatorname{Im} G_{22}^{R} - \operatorname{Im} G_{22}^{R} G_{11}^{A} + \operatorname{Im} G_{11}^{R} G_{22}^{A} \right].$$
(27)

As is seen from Eq. (27), it comes from the vicinity of the Fermi surface, as it is proportional to $df/d\varepsilon$. In the absence of disorder, it is easily verified that $\sigma_{xy}^{I} = 0$. Therefore, it is perturbative in the disorder strength. The second term in Eq. (26)

is the so-called anomalous contribution. It is proportional to the derivative of the charge-carrier density with respect to the applied magnetic field H and, as such, comes from the entire volume inside the Fermi surface. As is understood from the definition of the anomalous part

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$$\sigma_{xy}^{II} = ec \frac{dN(H, \mu, I)}{dH},$$
$$N(H, \mu) = \int_{-\infty}^{\infty} v(\varepsilon) f_{\varepsilon - \mu} d\varepsilon,$$
(28)

it is nonzero even in the absence of disorder. Here, the density of states reads as

$$\nu(\varepsilon) = \operatorname{tr} \int \frac{dp_{yz}}{(2\pi)^2} \operatorname{Im} G(\varepsilon, p_y, p_z, x, x)$$
$$= \frac{1}{2\pi l_H^2} \sum_n \int \frac{dp_z}{2\pi} \operatorname{Im} G_n(\varepsilon, p_z).$$
(29)

Thus, from perturbative arguments, we understand that $\sigma_{xy} = \sigma_{xy}^{II}$, i.e., it is determined by the anomalous part.

In our case (long-range disorder), it is even possible to compute σ_{xy}^{II} at all orders of perturbation theory in the strength of the disorder, in the limit $p_0^{-1} \rightarrow \infty$. The result is independent of the disorder strength and is given by the disorder-free expression

$$\sigma_{xy} = \sigma_{xy}^{\rm II} = \frac{e^2 \mu}{4\pi^2 v} = e^2 n_0 l_H^2, \tag{30}$$

where n_0 is the charge-carrier density. In experiments, n_0 is usually a fixed parameter stemming from the charge-neutrality condition due to the imbalance of donor and acceptor impurities in WSMs. Therefore, to compare with experiments, one needs to express the chemical potential in terms of n_0 .

The formula for the resistivity is as follows:

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}.$$
(31)

Taking into account the expressions for σ_{xx} [Eq. (2)] and σ_{xy} [Eq. (30)], we obtain the following results for the field dependence of the resistivity in the ultraquantum limit:

$$\rho_{xx} \sim \begin{cases}
H, & -1 < \gamma < 0, \\
H, & \gamma = 0 \\
H^{1+\gamma}, & 0 < \gamma < 1,
\end{cases}$$
(32)

at fixed n_0 . The expression (32) is an important result of our paper. It shows that *measuring* $\rho_{xx}(H)$ of the WSM in the ultraquantum regime, one can extract information about disorder correlations and the form of the impurity potential.

IV. MAGNETOTRANSPORT AT max{ T, μ } $\gg \Omega$ (SEMICLASSICAL LIMIT)

The opposite limit, which allows for an analytical treatment, is when the temperature or chemical potential of the WSM is much larger than Ω . Here, we focus on the most experimentally viable case when the magnetic length is being much larger than the disorder correlation length

$$l_H^{-1} \ll p_0 \ll \frac{\max\{T, \mu\}}{v}.$$
 (33)

In the case of the Coulomb potential, p_0 is the inverse Debye screening length, and the right-hand side condition in Eq. (33) is equivalent to $\alpha \ll 1$ [which is true for a typical WSM, like Ca₂As₃ (see Refs. [25,26]), where the α value is estimated [27] as ~0.05]. The left-hand side condition in (33) in this case should be substituted by

$$\Omega \ll \sqrt{\alpha} \max\{T, \mu\}. \tag{34}$$

Therefore, the temperatures should not be too low.

A. Semiclassical perturbation theory

In this regime, the transport physics of the system is governed by highly excited LLs, as follows from the Fermifunction shape entering the expression for the conductivity (11). This time the computation of the conductivity requires the summation of an infinite Drude-type diagrammatic series.

The long-range nature of the disorder $p_0 l_H \gg 1$ allows us to simplify the perturbation series for the conductivity. It turns out that the diagrammatic series in this regime can be built in a manner similar to the spatially uniform case, albeit with an effective 2D disorder correlation function.

Here, we present the qualitative arguments for the simplification mentioned, directing the reader to Appendix B for all the technical details. The most important terms in the Green's function are those close to its poles, where the energy $\varepsilon \approx \varepsilon_n$. This means that for not very large $p_z \ll \varepsilon/v$ [we will see that the weight of terms with $p_z \approx \varepsilon/v$ in (11) is small],

$$n \sim \frac{\varepsilon^2}{\Omega^2} \gg 1.$$
 (35)

The fact that we are interested in large-n terms in the Green's function (12) allows for a natural separation of scales in the problem. The crucial observation is that all the integrals entering the Green's functions and Dyson equations in this case are essentially orthogonality equations sometimes *spoiled* by the potential enveloping function.

On the *slow* scale of disorder correlation length r_0 , the asymptotics of the LL wave functions $\psi_n(x)$ are highly oscillatory and can be presented as a modulated combination of plane waves: $\psi_n(x) \sim \exp(\pm i\sqrt{nx}/l_H)$. The corresponding length $l_H/\sqrt{n} \sim \lambda$ sets the *fast* scale of the problem. Here, we remind that λ is the particle wavelength with energy $\max\{T, \mu\}$.

Physically, the fast oscillating functions of the LLs are a signature of the semiclassical regime, where a large amount of wavelengths can be accommodated by an envelope over the wave function (corresponding to a semiclassical magnetic orbit of radius $r_c = l_H \sqrt{n}$). The respective matrix elements of the disorder correlation function get averaged over the position of the center of the orbit (formally determined by $l_H^2 p_y$ in our asymmetric gauge). Therefore, only the 2D potential correlation function enters all the scattering rates in the problem. The hopping between levels n and $n + \Delta n$, $\Delta n/n \ll 1$ implies the change of the linear scale:

$$\Delta r_c \sim \frac{\Delta n l_H}{\sqrt{n}} \sim \lambda \Delta n, \qquad (36)$$

and due to the smoothness of disorder on this scale, its matrix elements are equivalent to the ones of Fourier components with momentum transfer $[\sqrt{n + \Delta n} - \sqrt{n}]/l_H$.

Once we take into account the disorder averaging of the Green's function, we obtain the following result:

$$G(x, x') \approx \sum_{n} \chi_n(x_{p_y}) [G^{-1}(\mathbf{p}_n) - \Sigma(\mathbf{p}_n)]^{-1} \chi_n(x'_{p_y}), \quad (37)$$

where \mathbf{p}_n is introduced in (12). The self-energy $\Sigma(\mathbf{p}_n)$ is computed with the help of the 2D disorder correlation function

$$g_{2,p_{xz}} = \int g(\mathbf{p}) \frac{dp_y}{2\pi p_0} \equiv g(p_{xz}) \Big|_{y=0}$$
 (38)

and is equal to

$$\Sigma(\mathbf{p}_n) = -\delta\mu + \delta v(\mathbf{p}_n(\sigma)) - \frac{i}{2\tau_0} - \frac{i\mathbf{n}\sigma}{2\tau_1}, \qquad (39)$$

where $\delta \mu$ and δv are the corrections to the chemical potential and Fermi velocity, respectively, and $\mathbf{n} \equiv \mathbf{p}_n/|\mathbf{p}_n|$ is a unit vector in momentum direction.

We also define the effective two-dimensional scattering times τ_l (l = 0, 1, 2) according to

$$\frac{1}{2\tau_l} = n_{\rm imp} u_0^2 \frac{\varepsilon}{2\upsilon^2 p_0^5} \int g_{2,p_\varepsilon(\mathbf{n}-\mathbf{n}')} \cos^l(\theta-\theta') \frac{d\theta'}{2\pi}.$$
 (40)

Here, $p_{\varepsilon} = \varepsilon/v$ and $\mathbf{n}^{(\prime)} = (\cos \theta^{(\prime)}, \sin \theta^{(\prime)})$.

The very fact that the whole physics of the problem can be reformulated in terms of the 2D potential has a beautiful physical interpretation. Let us recall that in the Landau gauge (5), the center of the orbit is given by $p_y l_H^2$. That is, the effective scattering rates (40) entering the perturbation theory are essentially ordinary scattering rates but averaged over the positions of the center of the Landau orbit. With these perturbative building blocks, we are ready to compute the conductivity tensor.

B. General expressions for conductivities

The conductivities σ_{xx} and σ_{xy} , in leading order of the expansion parameter (14), are given by the following Kubo expressions [17]:

$$\sigma_{x,x[y]} = \frac{e^2 \Omega^2}{4\pi^2 v} \sum_n \int \operatorname{Re}[\operatorname{Im}] \langle G_{n,11}^R G_{n+1,22}^A \rangle \frac{df}{d\varepsilon} \, d\varepsilon \, dp_z.$$
(41)

Here, we discard the $G^R G^R$ and $G^A G^A$ terms as subleading in the $1/(\tau \varepsilon)$ disorder expansion. Also, by σ_{xy} , we mean the normal part σ_{xy}^{I} of the Hall conductivity (see Sec. IV C below for the full computation of the Hall conductivity).

We switch from the summation over LLs to integration over *n*. This is possible under the condition that the broadening of the LL, $(1/\tau)(\varepsilon^2/\Omega^2)$, [16] is larger than the distance between LLs: Ω^2/ε . That is, $\varepsilon \equiv \max\{T, \mu\} \gtrsim \Omega(\Omega\tau)^{1/3}$. Since $\Omega\tau$ is an arbitrary finite parameter of our problem, this additional condition is assumed to be satisfied in the semiclassical limit. We substitute $1 = dn = v^2 p_y dp_y/\Omega^2$, and turn to polar coordinates: $p_y dp_y dp_z = p^2 \sin\theta \, d\theta \, dp \, (p_y = p \sin\theta)$. Now, we need to find the nonperturbative vertex renormalization responsible for the difference between $\langle G^R G^A \rangle$ and $\langle G^R \rangle \langle G^A \rangle$. As shown in Appendix B, we are able to perform the integration over the modulus of the momentum p in (41) and end up with only an angular integral. As a result, the conductivity tensor (41) can be rewritten in the following form:

$$\sigma_{x,x[y]} = \int \frac{d\varepsilon}{2\pi} \frac{df(\varepsilon)}{d\varepsilon} \int \frac{d\theta \,\sin\theta}{2\pi} \,\mathrm{tr} \big\{ \Gamma_x^{RA}(\theta) \tilde{\sigma}_{x[y]} \big\}, \quad (42)$$

where $\Gamma_x(\theta)$ is the so-called angular vertex function

$$\Gamma_{x}^{RA}(\theta) = \sum_{n} \int \frac{dp_{z}dp_{y}}{(2\pi)^{2}} \delta\left(\cos\theta - \frac{p_{z}}{p}\right) \\ \times \left\langle G_{x,x'}^{R}(\varepsilon, \mathbf{p}) \,\tilde{\sigma} \, G_{x',x}^{A}(\varepsilon, \mathbf{p}) \right\rangle dx.$$
(43)

It is essentially a vertex function, integrated over the modulus of the momentum at fixed vp_x/ε ratio. The notation $\tilde{\sigma}_{x,y}$ in the right-hand side of Eq. (42) stands for Pauli σ matrices. Then, we plug in the vertex expressions from (B20) and take the angular integral to obtain

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{xy}^{\mathrm{I}} \end{pmatrix} = \frac{e^2}{2\pi v} \int \frac{d\varepsilon \,\varepsilon^2}{2\pi} \frac{df}{d\varepsilon} \frac{\varepsilon^2 \tau_{\mathrm{tr}}(\varepsilon)}{\tau_{\mathrm{tr}}(\varepsilon)^2 \Omega^4 + \varepsilon^2} \begin{pmatrix} 1 \\ \Omega^2 \tau_{\mathrm{tr}}/\varepsilon \end{pmatrix}.$$
(44)

Here,

$$\tau_{\rm tr}^{-1} \equiv \tau_0^{-1} - \tau_2^{-1} \tag{45}$$

is the transport scattering rate. Equation (44) is quite an important result. It shows that *in the long-range disorder limit, the conductivity is effectively recast in terms of the 2D Drude-type expression.*

Similar formulas for the δ -correlated disorder were obtained in Ref. [17]. However, the magnetoconductance in Ref. [17] is expressed in terms of the three-dimensional (3D) scattering rates. This is somewhat predictable since the δ -correlated disorder has zero correlation length and the scattering rate is not affected by any other scale, including the magnetic length, which is responsible for the change in the geometry of the problem.

For the disorder of the general type, with the correlation radius independent of the characteristic energy of the host system, we have for the transport scattering time

$$\tau_{\rm tr}^{-1}(\varepsilon) = \frac{u_0^2 T_{\rm imp}^3}{g_1 \varepsilon^2 (p_0 v)^2}, \quad T_{\rm imp} = n_{\rm imp}^{1/3} v.$$
(46)

Here, $g_1 = \int_0^\infty g(x^2)x^2 dx$ is the numerical constant determined by the type of disorder. This way, we obtain the *general* expression for the longitudinal conductivity for any relation between the chemical potential and temperature:

$$\sigma_{xx} = \frac{e^2}{g_1 v} \frac{u_0^2 T^2 T_{\rm imp}^3}{(p_0 v)^2 \Omega^4} f\left(\frac{T}{\tau_{\rm tr}(T) \Omega^2}\right), \text{ where}$$

$$f(a) = \frac{4\pi^2}{3} - 4\left(a^2 - \frac{\mu^2}{T^2}\right) + \frac{2a^3}{\pi} \operatorname{Re}\left[\psi'\left(\frac{1}{2} + \frac{a + i\mu/T}{2\pi}\right)\right],$$

$$a = \frac{u_0^2 T_{\rm imp}^3}{g_1(p_0 v)^2 \Omega^2 T}.$$
(47)



FIG. 3. Conductivity σ_{xx} given by the exact Eq. (47) (solid curve) and by the approximate Eq. (48) (dashed curve) expressions as a function of dimensionless parameter $\Omega^2 \tau_{tr}/T \sim H$.

Here, $\psi(x)$ is the Euler's digamma function. The dimensionless parameter *a* plays the role of the relative strength of the disorder.

The exact formula (47) can be somewhat simplified by the interpolation expression (which becomes exact in the limits $\Omega \rightarrow 0$ and $\Omega \rightarrow \infty$) making it more useful for experimental purposes:

$$\sigma_{xx} = \frac{e^2}{v} \tau_{\text{tr}} \max\left\{T^2, \mu^2\right\} \left[1 + \frac{7\pi^2}{5} \frac{\Omega^4 \tau_{\text{tr}}^2}{\max\left\{T^2, \mu^2\right\}}\right]^{-1}.$$
 (48)

Here, the transport scattering time τ_{tr} should be taken at the energy $\varepsilon = \max{\{\mu, T\}}$. To give the reader an idea of how well the interpolation formula represents the exact result (47), we plot it in Fig. 3. Equations (47) and (48) reproduce the T^4 dependence at $\Omega \rightarrow 0$, obtained in Refs. [28] and [23] in the zero-field limit. The interpolation formula (48) for the conductivity σ_{xx} effectively recasts it in the form of the familiar Drude-type metallic expression

$$\sigma \propto \frac{\tau_{\rm tr}}{1 + \omega_c^2 \tau_{\rm tr}^2},\tag{49}$$

where $\omega_c = \Omega^2/2\varepsilon$ is the semiclassical cyclotron frequency at energy ε . The same kind of Drude representation, but with 3D scattering times, was obtained in Ref. [17] for δ -correlated disorder. The conductivity $\sigma_{xx}(H)$ for different values of μ/T is shown in Fig. 4.

The behavior of the conductivity σ_{xx} can be conveniently shown in the phase diagram (see Fig. 5). The upper left red corner of this phase diagram corresponds to the ultraquantum regime $T \ll \Omega$, where depending on the characteristic exponent γ of the impurity potential, we expect a γ -dependent scaling of σ_{xx} . The lower right corner is divided into the regimes of weak and strong disorder. The brown area corresponds to a strong disorder, and is described by Eq. (48) in the

$$\tau_{\rm tr} \ll \max\{T, \mu\}/\Omega^2 \tag{50}$$

limit. One could also refer it to as a *weak* magnetic field regime, where σ_{xx} exhibits predominantly the T^4 dependence characteristic of a zero-field system with a correction proportional to H^2 . The green area depicts the opposite weak-disorder limit (or that of high magnetic field), where the



FIG. 4. Conductivity σ_{xx} given by Eq. (47) as a function of μ/T and the dimensionless parameter $\Omega^2 \tau_{tr}/T \sim H$. Here, we assume that $\mu \ll T$.

transport of charge carriers is strongly affected by the magnetic field.

Next, we calculate the Hall conductivity.

C. Hall conductivity σ_{xy}

As usual, the Hall conductivity is split into two parts: anomalous and normal ones. Let us first calculate the anomalous part. As before, we make use of Eq. (28). To regularize the expression for the charge-carrier density, we subtract the respective density at zero chemical potential, thus eliminating the contribution of the Fermi sea

$$n_0(H,\mu,T) = \frac{\Omega^2}{4\pi^2 v^2} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \sum_n (f(\varepsilon_n - \mu) - f(\varepsilon_n + \mu)).$$
(51)



FIG. 5. Phase diagram for the conductivity σ_{xx} for the non-Coulomb disorder [see Eqs. (2) and (47)]. Here, g = 0 for $\gamma \leq 0$, and $g = \gamma/2$ for $0 < \gamma < 1$. See detailed explanations in the text.

In the $\Omega \ll (T, \mu)$ limit, we use the Euler-MacLaurin summation formula

$$\sum_{n=0}^{\infty} F(a+n) \approx \frac{F(a)}{2} + \int_{a}^{\infty} F(x) \, dx.$$
 (52)

Then, we obtain

$$n_0(\mu, T) = \frac{1}{4\pi^2 v^3} \Big(\Omega^2 \mu + \frac{2}{3} [\mu^3 + \pi^2 \mu^2 T] \Big).$$
(53)

Only the first term in Eq. (53) is field dependent. Despite its smallness, it is this term that contributes to σ_{xy}^{II} . This way, we arrive at the following expression:

$$\sigma_{xy}^{\mathrm{II}} = \frac{e^2 \mu}{2v}.$$
(54)

In experiments, the charge-carrier density is constant for each sample of WSM. Hence, the chemical potential is almost field independent in the high-temperature regime (see Discussion section for the relevant estimates).

Note here that the anomalous contribution to the Hall conductivity (which is independent of disorder and retains its value for clean WSM) is totally due to the Berry curvature. In our paper, we focus on the effects of disorder, and the the Berry phase itself does not lead to additional effects. However, it is not so in the case of strong spin-orbit coupling of charge carriers with impurities, which we do not consider here (see, e.g., Ref. [29]).

Next, we calculate the normal part of σ_{xy}^{I} given by (44). The conductivity σ_{xy}^{I} can be computed exactly at any value of μ (see the corresponding integral derived in Appendix C):

$$\sigma_{xy}^{I} = \frac{e^{2}T^{2}\mu}{\pi^{2}v\Omega^{2}}f_{1}(a),$$

$$f_{1}(a) = \frac{\mu^{2}}{T^{2}} + \pi^{2} - a^{2} - \frac{a^{4}T}{2\pi\mu}\text{Im}\Big[\psi^{(1)}\Big(\frac{a}{2\pi} + \frac{1}{2} + \frac{i\mu/T}{2\pi}\Big)\Big],$$

(55)

where a is defined in Eq. (47).

As before, we concoct a Drude-type interpolation formula from $\Omega \to 0$ to $\Omega^2 \gg \tau_{tr}^{-1} \max\{T, \mu\}$ using formula (55):

$$\sigma_{xy}^{\rm I} \approx \frac{c_1 e^2 \mu}{v} \frac{\Omega^2 \tau_{\rm tr}^2}{1 + \frac{c_2 \tau_{\rm tr}^2 \Omega^4}{\max\{T^2, \mu^2\}}},$$
(56)

where $c_{1,2} = 1$ if $\mu \gg T$ and $c_{1,2} = 7\pi^{2(4)}/3$ if $\mu \ll T$. Here, $\tau_{tr}(\varepsilon)$ is taken at $\varepsilon = \max\{\mu, T\}$. Depending on the factor $\omega \tau_{tr}$, the normal part may be a leading or subleading contribution to the Hall conductivity. Qualitatively, σ_{xy} can be described by the following identity:

$$\sigma_{xy} = \frac{e^2 \mu}{v} \begin{cases} d_1, & \Omega \tau_{\text{tr}} \ll 1, \\ d_2 \Omega^2 \tau_{\text{tr}}^2, & 1 \ll \Omega \tau_{\text{tr}} \ll \frac{\max\{T, \mu\}}{\Omega}, \\ d_3 \frac{\max\{T^2, \mu^2\}}{\Omega^2}, & \frac{\max\{T, \mu\}}{\Omega} \ll \Omega \tau_{\text{tr}}. \end{cases}$$
(57)

Here, d_i are numerical factors following from relations (54) and (56). The plot illustrating the accuracy of the interpolation formula (56) is presented in Fig. 6.

Interestingly, the case of Coulomb impurities does not lead to different results, despite the fact that the Debye radius



FIG. 6. Hall conductivity σ_{xy} given by Eq. (55) (solid curve) and by Eq. (56) (dashed curve) as a function of the dimensionless parameter $\Omega^2 \tau_{tr}/T \sim H$.

depends on temperature. The relatively slow decay of the Coulomb correlation function leads to a trivial logarithmic enhancement of the respective 2D scattering rate:

$$\tau_{\rm tr}^{-1} = 2\pi^2 \frac{\alpha^2 T_{\rm imp}^3}{\varepsilon^2} \ln \frac{1}{\alpha}.$$
 (58)

Otherwise, the whole temperature and field dependence remains the same.

The plot of the Hall conductivity σ_{xy} is presented in Fig. 7 as a function of the magnetic field *H* and chemical potential μ .

V. DISCUSSION

In this paper, we have performed a detailed analysis of the effect of the long-range disorder introduced by impurities on the magnetotransport in WSMs. Our study is mainly focused



FIG. 7. Conductivity σ_{xy}^{I} given by Eq. (47) as a function of μ/T and the dimensionless parameter $\Omega^{2} \tau_{tr}/T \sim H$.



FIG. 8. Electron self-energy.

on the magnetic field and temperature dependence of the transverse magnetoconductivity.

Two important limiting cases are considered: (i) the ultraquantum limit, corresponding to low temperatures (or high magnetic field), for which the main contribution comes from the zeroth Landau level, and (ii) the opposite semiclassical limit, when a large number of Landau levels are involved in the transport phenomena.

We have completely discarded the effects of the internode charge transfer. In principle, this effect can be important at sufficiently high fields, as argued in Ref. [30]. The necessary condition for the applicability of our approach is

$$\tau_{\rm inter}^{-1} \ll \tau_{\rm intra}^{-1},\tag{59}$$

where for finding the scattering rates, we can use, e.g., Eqs. (40) or (46). As derived in Ref. [30], this condition is equivalent to

$$H \ll \alpha^{-3/2} e Q^2 / v, \tag{60}$$

where Q is the distance between the Weyl nodes in momentum space.

However, for a typical WSM like TaAs [31–33], we extract the separation between Weyl nodes as $Q = 0.01 \text{ Å}^{-1}$, while the Fermi velocity $v \approx 3 \times 10^5 \text{ m/s}$, which gives the respective field estimate of $H \sim 50$ T even for the fine structure constant in TaAs (unknown to us at the moment) $\alpha \sim 1$. Therefore, we safely discard this effect.

The long-range impurity potential is chosen in a rather general form. We show that in the ultraquantum limit the nonlinear magnetoresistivity dependence on the magnetic field is the manifestation of the singular (non-Coulomb) short-range behavior of the impurity potential and its correlation function. In the semiclassical limit, we have demonstrated that unlike the short-range disorder case [17], the long-range disorder makes the scattering in the system essentially two dimensional. We derived general formulas for $\sigma_{xx}(H)$ and $\sigma_{xy}(H)$ valid within a wide range of values of temperature and chemical potential.

In typical experiments [13], the doping levels in WSMs are rather high (~10 meV), which corresponds to $\mu \sim 100$ K. The typical magnetic fields $H \sim 1$ T correspond to the gap between the zeroth and first Landau levels ~10 K. Reference [34], however, reports the observation of WSM in an almost undoped regime $\mu \ll T$. Therefore, both the $\mu \ll T$ and $\mu \gg T$ regimes seem experimentally viable and the relation between μ , H, and T can be quite general. Hence, our results obtained in both limits can be relevant.

We can also mention the numerical work in Ref. [19]. In this paper, Coulomb impurities are correctly identified as the long-range disorder, and the high-temperature limit is explored. The nontrivial result of Ref. [19] is the scaling of the magnetoconductance $\sigma_{xx} \propto H^{-5/3}$ in the low-field regime $\Omega \leq T$. Our analytical study addresses the case $\Omega \ll T$, and the aforementioned regime cannot be accessed in our semiclassical computation, where $\Omega/T \ll 1$ is the essential expansion parameter.

In the semiclassical regime $T \gg \Omega$, our results match the results reported in Ref. [16] in the low-impurity concentration regime $\tau_{\rm tr} \gg T \Omega^{-2}$, but differ in the opposite limit. We attribute this to the effect of the long-range disorder correlations.

In conclusion, the diagrammatic approach within the Kubo formalism used in our paper can be applied not only to the specific problems studied here, but also have a wider range of applicability. This was clearly demonstrated in our earlier work [27] on the conductivity of anisotropic Weyl semimetals, and we believe that it could also allow going beyond the self-consistent Born approximation in the analysis of Weyl semimetals with different kinds of impurities [35], and even in more general problems of electron transport and optics in linear and nonlinear regimes [36,37].

Note also that the problem under study has a deep analogy with the detailed analysis (in Refs. [38–40]) of the effect of the interplay of quantum interference and disorder on the magnetoresistance in systems with hopping conductivity.

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APPENDIX A: KUBO FORMULA AND DISORDER AVERAGING $\sigma_{\alpha\alpha}$

1. Disorder correlation function

The disorder correlation function entering the diagrammatic series is obtained from the standard averaging over impurity position:

$$g(\mathbf{r}_{1} - \mathbf{r}_{2}) = \frac{1}{V} \sum_{\mathbf{r}_{a}} \int d\mathbf{r}_{a} u(\mathbf{r}_{1} - \mathbf{r}_{a}) u(\mathbf{r}_{2} - \mathbf{r}_{a}) d\mathbf{r}_{a} = \frac{1}{V} \sum_{\mathbf{r}_{a}} \int d\mathbf{r}_{a} \frac{d\mathbf{p} \, d\mathbf{q}}{(2\pi)^{6}} u_{\mathbf{p}} e^{i\mathbf{p}(\mathbf{r}_{1} - \mathbf{r}_{a})} u_{\mathbf{q}} e^{i\mathbf{q}(\mathbf{r}_{2} - \mathbf{r}_{a})} d\mathbf{r}_{a}$$
$$= n_{\rm imp} \int |u_{\mathbf{p}}|^{2} e^{i\mathbf{p}(\mathbf{r}_{1} - \mathbf{r}_{2})} \frac{d\mathbf{p}}{(2\pi)^{3}}.$$
(A1)

We parametrize the disorder correlation function introducing the dimensionless function g and characteristic momentum p_0 as $|u_p|^2 = |u_0|^2 / p_0^6 g(p^2/p_0^2)$.

2. General expression for the σ_{xx} conductivity

The general expression for the conductivity reads as

$$\sigma_{\alpha\beta}(\omega) = e^2 \frac{\Pi^R_{\alpha\beta}(\omega)}{i\omega}, \qquad \Pi^R_{\alpha\beta}(\omega) = i \int dt \, d\mathbf{r} \langle j_\alpha(t, \mathbf{r}) j_\beta(t', \mathbf{r}') \rangle e^{i\omega(t-t')} \theta(t). \tag{A2}$$

As follows from the Ward identity, $\Pi_{\alpha\beta}^{R}(0) = 0$ (a vector potential uniform in space and time does not create the field). Therefore, we will use the slightly more suitable formula for the conductivity

$$\sigma_{\alpha\beta}(\omega) = e^2 \frac{\Pi^R_{\alpha\beta}(\omega) - \Pi^R_{\alpha\beta}(0)}{i\omega},\tag{A3}$$

and the expression for the polarization operator

$$\Pi_{\alpha\beta}(\omega) = -v^{2} \operatorname{tr} \int \frac{d\varepsilon}{4\pi i} \frac{d\mathbf{p}}{(2\pi)^{3}} \left\{ \langle G^{A}(\varepsilon, \mathbf{p}) \sigma_{\alpha} G^{R}(\varepsilon + \omega) \sigma_{\beta} \rangle \left[\tanh \frac{\varepsilon + \omega}{2T} - \tanh \frac{\varepsilon}{2T} \right] \right. \\ \left. + \langle G^{R}(\varepsilon, \mathbf{p}) \sigma_{\alpha} G^{R}(\varepsilon + \omega, \mathbf{p}) \sigma_{\beta} \rangle \tanh \frac{\varepsilon}{2T} - \langle G^{A}(\varepsilon, \mathbf{p}) \sigma_{\alpha} G^{A}(\varepsilon + \omega, \mathbf{p}) \sigma_{\beta} \rangle \tanh \frac{\varepsilon + \omega}{2T} \right\}.$$
(A4)

Since we are interested in the $\omega \to 0$ limit, we expand the expression in ω :

$$\begin{aligned} \sigma_{\alpha\beta}(0) &\equiv \sigma_{\alpha\beta} = e^2 v^2 \mathrm{tr} \int \frac{d\varepsilon}{4\pi} \frac{d\mathbf{p}}{(2\pi)^3} \bigg\{ \langle G^A(\varepsilon, \mathbf{p}) \sigma_\alpha G^R(\varepsilon, \mathbf{p}) \sigma_\beta \rangle \partial_\varepsilon \tanh \frac{\varepsilon}{2T} \\ &+ \langle G^R(\varepsilon, \mathbf{p}) \sigma_\alpha \partial_\varepsilon G^R(\varepsilon, \mathbf{p}) \sigma_\beta \rangle \tanh \frac{\varepsilon}{2T} - \langle G^A(\varepsilon, \mathbf{p}) \sigma_\alpha \partial_\varepsilon G^A(\varepsilon, \mathbf{p}) \sigma_\beta \rangle \tanh \frac{\varepsilon}{2T} - \langle G^A(\varepsilon, \mathbf{p}) \sigma_\alpha \partial_\varepsilon G^A(\varepsilon, \mathbf{p}) \sigma_\beta \rangle \partial_\varepsilon \tanh \frac{\varepsilon}{2T} \bigg\} \\ &= i e^2 v^2 \mathrm{tr} \int \frac{d\varepsilon}{2\pi} \frac{d\mathbf{p}}{(2\pi)^3} \bigg\{ \langle G^A(\varepsilon, \mathbf{p}) \sigma_\alpha \mathrm{Im} G^R(\varepsilon, \mathbf{p}) \sigma_\beta \rangle \partial_\varepsilon \tanh \frac{\varepsilon}{2T} + \langle \mathrm{Im} [G^R(\varepsilon, \mathbf{p}) \sigma_\alpha \partial_\varepsilon G^R(\varepsilon, \mathbf{p}) \sigma_\beta] \rangle \tanh \frac{\varepsilon}{2T} \bigg\}. \end{aligned}$$
(A5)

If we are interested in symmetric combinations of the conductivity tensor σ_{ii} , then the expression for the conductivity can be further simplified to the extent that all the contributions to the integral over energy only originate from the vicinity of the Fermi surface. Let us rewrite the expression for the conductivity

$$\sigma_{\alpha\alpha} = ie^2 v^2 \operatorname{tr} \int \frac{d\varepsilon}{2\pi} \frac{d\mathbf{p}}{(2\pi)^3} \bigg\{ \langle G^A(\varepsilon, \mathbf{p}) \sigma_\alpha \operatorname{Im} G^R(\varepsilon, \mathbf{p}) \sigma_\alpha \rangle \partial_\varepsilon \tanh \frac{\varepsilon}{2T} + \langle \operatorname{Im} [G^R(\varepsilon, \mathbf{p}) \sigma_\alpha \partial_\varepsilon G^R(\varepsilon, \mathbf{p}) \sigma_\alpha] \rangle \tanh \frac{\varepsilon}{2T} \bigg\}.$$
(A6)

The term I is fine since it is proportional to the derivative of the Fermi function and cuts a slice of the order of T from the Fermi surface.

Term II should be rearranged:

$$\operatorname{tr}\left[G^{R}(\varepsilon,\mathbf{p})\sigma_{\alpha}\partial_{\varepsilon}G^{R}(\varepsilon,\mathbf{p})\sigma_{\alpha}\right] = \frac{1}{2}\operatorname{tr}\partial_{\varepsilon}\left[G^{R}(\varepsilon,\mathbf{p})\sigma_{\alpha}G^{R}(\varepsilon,\mathbf{p})\sigma_{\alpha}\right] \to -\frac{1}{2}\operatorname{tr}\left[G^{R}(\varepsilon,\mathbf{p})\sigma_{\alpha}G^{R}(\varepsilon,\mathbf{p})\sigma_{\alpha}\right]\partial_{\varepsilon}.$$
(A7)

As a result, we obtain for the conductivity

$$\sigma_{\alpha\alpha} = e^2 v^2 \text{tr} \int d\Gamma_p \frac{d\varepsilon}{2\pi} \bigg\{ \langle \text{Im} G^R(\varepsilon, \mathbf{p}) \sigma_\alpha \text{Im} G^R(\varepsilon, \mathbf{p}) \sigma_\alpha \rangle \partial_\varepsilon \tanh \frac{\varepsilon}{2T} \bigg\},$$
(A8)

where $d\Gamma_p = \frac{eH}{2\pi c} \frac{dp_z}{2\pi}$. Expanding the trace we recover Eq. (11).

APPENDIX B: PERTURBATION THEORY FOR $\max\{T, \mu\} \gg \Omega$

1. Self-energy and Dyson series for the Green's function

The diagram representing the first-order correction to the Green's function is presented in Fig. 8 and is given by the following analytical expression:

$$\delta G(x,x') = \sum_{n,m,k} \int \frac{dq_x dq_y dq_z}{(2\pi)^3} e^{iq_x(x^1 - x^2)} g(q_x, q_{yz}) S_n(x_{p_y}) G_n(p_z) S_n^{\dagger}(x_{p_y}^1) S_m(x_{p_y+q_y}^1) G_m(p_z + q_z) S_m^{\dagger}(x_{p_y+q_y}^2) S_k(x_{p_y}^2) G_k(p_z) S_k^{\dagger}(x_{p_y}').$$
(B1)

The correction to the Green's function is a matrix, where each term contains a product of $\chi_l(x_{p_v})$ functions.

It is essential that for large *n*, we discard the difference between *n* and *n* + 1, when computing the correlation function. This allows us to treat the matrices $S_n(x_{p_y})$, $S_n^{\dagger}(x_{p_y})$ as proportional to unit ones: $S_n(x_{p_y})$, $S_n^{\dagger}(x_{p_y}) = \chi_n(x_{p_y}) \cdot 1$. The leading contribution to the conductivity comes from $n \gg 1$ [in fact, we will later see that the contribution comes from $n \sim (T/\Omega)^2$] and use the asymptotic relation for the Hermite polynomials [41]:

$$e^{-x^2/2}H_n(x) \approx \left(\frac{2n}{e}\right)^{\frac{n}{2}}\sqrt{2}\cos\left(x\sqrt{2n} - \frac{n\pi}{2}\right)\left(1 - \frac{x^2}{2n+1}\right)^{-\frac{1}{4}}.$$
 (B2)

The question is how to simplify the product

$$\chi_m(x_{p_y+q_y}^1)\chi_m(x_{p_y+q_y}^2)\,dq_y$$
(B3)

entering (B1). We split the product of two cosine functions in each Hermite polynomial into

$$\cos\left[\sqrt{2m}(x_{p_y}^1 - x_{p_y}^2)/l_H\right] + \cos\left(\sqrt{2m}\left[(x_{p_y}^1 + x_{p_y}^2)/l_H - 2q_y l_H\right]\right). \tag{B4}$$

Then, we perform integration over q_y . The first term gives just the integral

$$\int g(q_x, q_{yz}) dq_y / 2\pi = g_{2,xz}.$$
(B5)

It is simply an effective 2D potential (18). The second term in (B4) is proportional to $U(\sqrt{2m}l_H, q_{xz})$. However, the correlation radius of the potential obeys the inequality $r_0 \ll l_H \ll \sqrt{2m}l_H$. As a result, the term proportional to $\cos(\sqrt{2m}[(x_{p_y}^1 + x_{p_y}^2)/l_H - 2q_y l_H])$ is suppressed.

Now, we are able to perform the next estimate:

$$\delta G(x,x') \approx \sum_{n,m,k} \chi_n(x_{p_y}) \chi_k(x'_{p_y}) \int \frac{dq_z}{2\pi} G_n(p_z) G_m(p_z+q_z) G_k(p_z) \int \frac{dq_x}{2\pi} e^{iq_x(x^1-x^2)} g_2(q_{xz}) \\ \times \frac{1}{\pi l_H \sqrt{2m}} \cos\left[\sqrt{2m} (x_{p_y}^1 - x_{p_y}^2)/l_H\right] \frac{1}{\pi l_H} \left(\frac{4}{nk}\right)^{1/4} \frac{\cos[\sqrt{2n} x_{p_y}^1/l_H]}{\sqrt[4]{1-\frac{x_{p_y}^{1/2}}{2n}}} \frac{\cos[\sqrt{2k} x_{p_y}^2/l_H]}{\sqrt[4]{1-\frac{x_{p_y}^{2/2}}{2k}}} dx^1 dx^2.$$
(B6)

In Eq. (B6), it is important to discern the difference between the fast-oscillating cosine-type terms in the numerator and the slow algebraic factors in the denominator. To perform the integration over x^1 , x^2 , we change x^1 , $x^2 \rightarrow r = x^1 - x^2$, x^2 . We obtain many fast-oscillating terms (the relevant *n*, *m*, and *k* are large). For example, performing integration over x^2 , we obtain

$$\frac{1}{2} \int dx^2 \frac{\cos[\sqrt{2n}r + (\sqrt{2n} + \sqrt{2k})x^2/l_H] + \cos[\sqrt{2n}r + (\sqrt{2n} - \sqrt{2k})x^2/l_H]}{\sqrt[4]{1 - \frac{(r+x_{p_y}^2)^2}{2nl_H^2}}\sqrt[4]{1 - \frac{x_{p_y}^{2,2}}{2kl_H^2}}}.$$
(B7)

As we see from the structure of the integral of (B7), the nominator is a fast-oscillating function of $x_{p_y}^2$. As a result, the integral is suppressed unless n = k. Thus, the integral is $\propto \delta_{nk}$ in the main order for $1/\sqrt{n-k}$. Let us compute it for n = k. The nominator does not oscillate anymore, and we should analyze the denominator. The dominant range is $r \sim p_0^{-1}$, which comes from $g_2(q_{xz})$. On the other hand, the main contribution comes from $x_{p_y}^2 \sim \sqrt{n}l_H$. We see that $r \ll x_{p_y}^2$ for the denominator. Therefore, we integrate over $x_{p_y}^2$ trivially. We are then left with the following expression:

$$\delta G(x, x') \approx \sum_{n,m} \chi_n(x_{p_y}) \chi_n(x'_{p_y}) \int \frac{dq_z}{2\pi} G_n(p_z) G_m(p_z + q_z) G_n(p_z) \int \frac{dq_x dr}{2\pi} e^{iq_x r} g_2(q_{xz}) \frac{1}{\pi l_H \sqrt{2m}} \cos(\sqrt{2mr}/l_H) \cos(\sqrt{2nr}/l_H).$$
(B8)

While integrating over r, we obtain the combination of four δ functions. The only relevant ones are $\delta(q_x + \sqrt{2m}l_H^{-1} - \sqrt{2n}l_H^{-1})$ and $\delta(q_x - \sqrt{2m}l_H^{-1} + \sqrt{2n}l_H^{-1})$. Consequently, we have the following final formula for the correction to the Green's function:

$$\delta G(x,x') \approx \sum_{n,m} \chi_n(x_{p_y}) \chi_n(x'_{p_y}) \int \frac{dq_z}{2\pi} G_n(p_z) G_m(p_z+q_z) G_n(p_z) \frac{1}{2\pi l_H \sqrt{2m}} g_2 \Big[(\sqrt{2m} - \sqrt{2n}) l_H^{-1}, q_z \Big]. \tag{B9}$$

Using the fact that $\sqrt{n}l_H^{-1} \sim \max\{T, \mu\} \gg p_0$, we understand that *m* in the last sum is actually very close to *n*. Indeed, we see that

$$(\sqrt{m} - \sqrt{n}) \sim p_0 l_H \gg 1$$
, while $\frac{\sqrt{m} - \sqrt{n}}{\sqrt{m} + \sqrt{n}} \sim \frac{p_0 v}{\max\{T, \mu\}} \ll 1.$ (B10)

From the last inequalities, we see that the terms of the sum over *m* are smooth functions of *m*, and the sum can be turned into an integral. Introducing the effective momentum $p'_{y} = \sqrt{2m}l_{H}^{-1}$, $dm \equiv 1 = q_{y}dq_{y}l_{H}^{2}$, and $p'_{z} = p_{z} + q_{z}$, we obtain

$$\delta G(x, x') \approx \sum_{n} \chi_n(x_{p_y}) \left[\int \frac{d\mathbf{p}'}{(2\pi)^2} G(\mathbf{p}_n) G(\mathbf{p}') G(\mathbf{p}_n) g_2(\mathbf{p}' - \mathbf{p}_n) \right] \chi_n(x'_{p_y}).$$
(B11)

Here, \mathbf{p}_n is introduced in (12). The expression in the square brackets in (B11) allows us to build the ordinary 2D Dyson series for the Green's function as well as vertex functions determining the conductivity tensor. Indeed, it coincides with the standard expression of perturbation theory without magnetic field with the effective 2D potential $g_2(\mathbf{p}' - \mathbf{p}_n)$. Therefore, we can write the momentum-dependent self-energy as

$$\Sigma(\mathbf{p}_n) = \int \frac{d\mathbf{p}'}{(2\pi)^2} G(\mathbf{p}') g_2(\mathbf{p}' - \mathbf{p}_n).$$
(B12)

The resummed Green's function then reads as

$$G(x, x') \approx \sum_{n} \chi_n(x_{p_y}) [G^{-1}(\mathbf{p}_n) - \Sigma(\mathbf{p}_n)]^{-1} \chi_n(x'_{p_y}).$$
(B13)

The resulting expression yields an irrelevant part, which can be absorbed into the renormalized chemical potential and Fermi velocity, and the dissipative part. The dissipative part reads as

$$\Sigma^{R}(\mathbf{p}_{n}) = v.p. \int \frac{d\mathbf{p}'}{(2\pi)^{2}} \frac{\varepsilon + \mathbf{p}'\sigma}{(\varepsilon + i0)^{2} - \mathbf{p}_{n}^{2}} g_{2}(\mathbf{p}' - \mathbf{p}_{n}) - i\pi \int \frac{d\mathbf{p}'}{(2\pi)^{2}} [\delta(\varepsilon - \varepsilon_{n}) + \delta(\varepsilon + \varepsilon_{n})] \left[\frac{1}{2} + \frac{\mathbf{p}'\sigma}{2\varepsilon}\right] g_{2}(\mathbf{p}' - \mathbf{p}_{n})$$

$$= -\delta\mu + \delta v \mathbf{p}_{n}\sigma - \frac{i}{2\tau} - \frac{i\mathbf{n}\sigma}{2\tau_{1}},$$

$$\frac{1}{\tau} = \frac{p_{n}}{4\pi} \int g_{2}(\mathbf{nn}')d\mathbf{n}', \qquad \frac{1}{\tau_{1}} = \frac{\varepsilon}{4\pi} \int (\mathbf{nn}')g_{2}(\mathbf{nn}')d\mathbf{n}'.$$
(B14)

2. Vertex renormalization and the conductivity tensor

The Dyson equation for the vertex is built in a more subtle way. In this case, the built-in magnetic anisotropy of the problem takes its toll. What we are going to do now is to introduce a slightly unusual definition of the vertex. We define the mass-shell vertex according to the following equation:

$$\Gamma_x^{RA}(\theta) = \sum_n \int \frac{dp_z dp_y}{(2\pi)^2} \delta\Big(\cos\theta - \frac{p_z}{p}\Big) \big\langle G_{x,x'}^R(\varepsilon, \mathbf{p}) \,\sigma_x \, G_{x',x}^A(\varepsilon, \mathbf{p}) \big\rangle dx. \tag{B15}$$

Then, the conductivity tensor assumes the form in Eq. (42). In the zeroth-order ladder approximation, we change $\langle G^R G^A \rangle = \langle G^R \rangle \langle G^A \rangle$, and the vertex becomes

$$\Gamma_x^{RA,0} = \frac{1}{2\pi l_H^2} \sum_n \frac{dp_z}{2\pi} \delta \left(\cos \theta - \frac{p_z}{p} \right) \left(\begin{array}{cc} 0 & G_{11,n}^R G_{22,n+1}^A \\ G_{22,n}^R G_{11,n-1}^A & 0 \end{array} \right). \tag{B16}$$

Changing the sum and the integral using the semiclassical approximation

$$\sum_{n} \int \frac{dp_z}{2\pi} \delta\left(\cos\theta - \frac{p_z}{p}\right) = l_H^2 \int \frac{p^2 dp}{2\pi},\tag{B17}$$

we obtain

$$\Gamma_x^{RA,0} = \frac{n_y^2}{2\pi v^3} \begin{pmatrix} 0 & \left[\frac{1}{\tau} + \frac{1}{\tau_1} + \frac{i\Omega^2}{\varepsilon}\right]^{-1} \\ \left[\frac{1}{\tau} + \frac{1}{\tau_1} - \frac{i\Omega^2}{\varepsilon}\right]^{-1} & 0 \end{pmatrix}.$$
 (B18)

In the first order of perturbation theory, the picture changes slightly, and we obtain the first stair of the ladder series

$$\Gamma_x^{RA,1} = \frac{n_y^2}{2\pi v^3} \begin{pmatrix} 0 & \frac{\frac{1}{r_1} + \frac{1}{r_2}}{\left[\frac{1}{r} + \frac{1}{r_1} + \frac{i\Omega^2}{s}\right]^2} \\ \frac{\frac{1}{r_1} + \frac{1}{r_2}}{\left[\frac{1}{r} + \frac{1}{r_1} - \frac{i\Omega^2}{s}\right]^2} & 0 \end{pmatrix}.$$
 (B19)

In higher orders of the perturbation theory, the pattern repeats itself. As a result, we are able to perform the full disorder ladder summation

$$\Gamma_x^{RA} = \frac{n_y^2}{2\pi v^3} \begin{pmatrix} 0 & \left[\frac{1}{\tau_{\rm tr}} + \frac{i\Omega^2}{\varepsilon}\right]^{-1} \\ \left[\frac{1}{\tau_{\rm tr}} - \frac{i\Omega^2}{\varepsilon}\right]^{-1} & 0 \end{pmatrix}.$$
 (B20)

Using vertex (B20), we are ready to obtain the expression for the conductivity:

$$\sigma_{x,x[y]} = \int \frac{d\varepsilon}{\pi} \frac{df(\varepsilon)}{d\varepsilon} \int \frac{d\theta \sin^3 \theta}{2\pi} \operatorname{Re}[\operatorname{Im}] \left[\frac{1}{\tau_{\text{tr}}} + \frac{i\Omega^2}{\varepsilon} \right]^{-1}.$$
(B21)

In expression (B21), we, as usual, take into account only the normal part of σ_{xy} .

APPENDIX C: CALCULATION OF THE INTEGRAL FOR THE CONDUCTIVITY AT max $\{T, \mu\} \gg \Omega$

The integral that enters the upper matrix element in the expressions for the conductivities in (44) has the following form:

$$I(a) = \int_{-\infty}^{\infty} \frac{1}{x^2 + a^2} \frac{x^4 dx}{\cosh^2 \frac{x - \mu}{2}} = \int_{-\infty}^{\infty} \frac{x^2 + \mu^2 - a^2}{\cosh^2 \frac{x}{2}} dx + a^4 \int_{-\infty}^{\infty} \frac{1}{\cosh^2 \frac{x}{2}} \frac{dx}{(x + \mu)^2 + a^2}$$

The last integral is equal to

$$\int_{-\infty}^{\infty} \frac{1}{\cosh^2 \frac{x}{2}} \frac{dx}{(x+\mu)^2 + a^2} = \frac{2}{\pi a} \operatorname{Re} \left[\psi' \left(\frac{1}{2} + \frac{a-i\mu}{2\pi} \right) \right].$$
(C1)

As a result, we recover Eq. (47). In the same manner, we perform the integration for σ_{xy}^1 in the lower part of (44) to obtain the exact expression (55).

- P. Hosur and X. Qi, Recent developments in transport phenomena in Weyl semimetals, C. R. Phys. 14, 857 (2013).
- [2] H.-Z. Lu and S.-Q. Shen, Quantum transport in topological semimetals under magnetic fields, Front. Phys. 12, 127201 (2017).
- [3] D. T. Son and B. Z. Spivak, Chiral anomaly and classical negative magnetoresistance of Weyl metals, Phys. Rev. B 88, 104412 (2013).
- [4] A. A. Burkov, Chiral Anomaly and Diffusive Magnetotransport in Weyl Metals, Phys. Rev. Lett. 113, 247203 (2014).
- [5] X. Huang, L. Zhao, Y. Long, P. Wang, D. Chen, Z. Yang, H. Liang, M. Xue, H. Weng, Z. Fang, X. Dai, and G. Chen, Observation of the Chiral-Anomaly-Induced Negative Magnetoresistance in 3D Weyl Semimetal TaAs, Phys. Rev. X 5, 031023 (2015)
- [6] C. Shekhar, A. K. Nayak, Y. Sun, M. Schmidt, M. Nicklas, I. Leermakers, U. Zeitler, W. Schnelle, J. Grin, C. Felser, and B. Yan, Extremely large magnetoresistance and ultrahigh mobility in the topological Weyl semimetal candidate NbP, Nat. Phys. 11, 645 (2015).
- [7] C. Z. Li, L. X. Wang, H. W. Liu, J. Wang, Z. M. Liao, and D. P. Yu, Giant negative magnetoresistance induced by the

chiral anomaly in individual Cd₃As₂ nanowires, Nat. Commun. **6**, 10137 (2015).

- [8] A. V. Andreev and B. Z. Spivak, Longitudinal Negative Magnetoresistance and Magnetotransport Phenomena in Conventional and Topological Conductors, Phys. Rev. Lett. **120**, 026601 (2018).
- [9] H. Z. Lu and S. Q. Shen, Weak antilocalization and localization in disordered and interacting Weyl semimetals, Phys. Rev. B 92, 035203 (2015).
- [10] H.-Z. Lu and S.-Q. Shen, Weak antilocalization and interactioninduced localization of Dirac and Weyl fermions in topological insulators and semimetals, Chin. Phys. B 25, 117202 (2016).
- [11] T. Liang, Q. Gibson, M. N. Ali, M. Liu, R. J. Cava, and N. P. Ong, Ultrahigh mobility and giant magnetoresistance in the Dirac semimetal Cd₃As₂, Nat. Mater. 14, 280 (2015).
- [12] J. Feng, Y. Pang, D. Wu, Zhijun W., Hongming Weng, J. Li, X. Dai, Z. Fang, Y. Shi, and L. Lu, Large linear magnetoresistance in Dirac semimetal Cd₃As₂ with Fermi surfaces close to the Dirac points, Phys. Rev. B 92, 081306(R) (2015).

- magnetoresistance in the Dirac semimetal TIBiSSe, Phys. Rev. B **91**, 041203(R) (2015).
- [14] Y. Zhao, H. Liu, C. Zhang, H. Wang, J. Wang, Z. Lin, Y. Xing, H. Lu, J. Liu, Y. Wang, S. M. Brombosz, Z. Xiao, S. Jia, X. C. Xie, and J. Wang, Anisotropic Fermi Surface and Quantum Limit Transport in High Mobility Three-Dimensional Dirac Semimetal Cd₃As₂, Phys. Rev. X 5, 031037 (2015).
- [15] A. A. Abrikosov, Quantum magnetoresistance, Phys. Rev. B 58, 2788 (1998).
- [16] J. Klier, I. V. Gornyi, and A. D. Mirlin, Transversal magnetoresistance in Weyl semimetals, Phys. Rev. B 92, 205113 (2015).
- [17] J. Klier, I. V. Gornyi, and A. D. Mirlin, Transversal magnetoresistance and Shubnikov-de Haas oscillations in Weyl semimetals, Phys. Rev. B 96, 214209 (2017).
- [18] D. A. Pesin, E. G. Mishchenko, and A. Levchenko, Density of states and magnetotransport in Weyl semimetals with longrange disorder, Phys. Rev. B 92, 174202 (2015).
- [19] J. C. W. Song, G. Refael, and P. A. Lee, Linear magnetoresistance in metals: Guiding center diffusion in a smooth random potential, Phys. Rev. B 92, 180204(R) (2015).
- [20] X. Xiao, K. T. Law, and P. A. Lee, Magnetoconductivity in Weyl semimetals: Effect of chemical potential and temperature, Phys. Rev. B 96, 165101 (2017).
- [21] V. Könye and M. Ogata, Magnetoresistance of a threedimensional Dirac gas, Phys. Rev. B 98, 195420 (2018).
- [22] V. Könye and M. Ogata, Thermoelectric transport coefficients of a Dirac electron gas in high magnetic fields, Phys. Rev. B 100, 155430 (2019).
- [23] Ya. I. Rodionov and S. V. Syzranov, Conductivity of a Weyl semimetal with donor and acceptor impurities, Phys. Rev. B 91, 195107 (2015).
- [24] P. Středa, Theory of quantised Hall conductivity in two dimensions, J. Phys. C: Solid State Phys. 15, L717 (1982).
- [25] Z. K. Liu, J. Jiang, B. Zhou, Z. J. Wang, Y. Zhang, H. M. Weng, D. Prabhakaran, S.-K. Mo, H. Peng, P. Dudin, *et al.*, A stable three-dimensional topological Dirac semimetal Cd₃As₂, Nat. Mater. **13**, 677 (2014).
- [26] J.-P. Jay-Gerin, M. J. Aubin, and L. G. Caron, The electron mobility and the static dielectric constant of Cd₃As₂ at 4.2 K, Solid State Commun. 21, 771 (1977).
- [27] Ya. I. Rodionov, K. I. Kugel, and F. Nori, Effects of anisotropy and disorder on the conductivity of Weyl semimetals, Phys. Rev. B 92, 195117 (2015).
- [28] S. D. Sarma, E. H. Hwang, and H. Min, Carrier screening, transport, and relaxation in 3D Dirac semimetals, Phys. Rev. B 91, 035201 (2015).

- [29] N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald, and N. P. Ong, Anomalous Hall effect, Rev. Mod. Phys. 82, 1539 (2010).
- [30] G. Bednik, K. S. Tikhonov, S. V. Syzranov, Magnetotransport and internodal tunneling in Weyl semimetals, Phys. Rev. Res. 2, 023124 (2020).
- [31] S.-Y. Xu, I. Belopolski, N. Alidoust, M. Neupane, G. Bian, C. Zhang, R. Sankar, G. Chang, Z. Yuan, C.-C. Lee, S.-M. Huang, H. Zheng, J. Ma, D. S. Sanchez, B.-K. Wang, A. Bansil, F. Chou, P. P. Shibayev, H. Lin, S. Jia, and M. Z. Hasan, Discovery of a Weyl fermion semimetal and topological Fermi arcs, Science 349, 613 (2015).
- [32] B. Q. Lv, N. Xu, H. M. Weng, J. Z. Ma, P. Richard, X. C. Huang, L. X. Zhao, G. F. Chen, C. E. Matt, F. Bisti, V. N. Strocov, J. Mesot, Z. Fang, X. Dai, T. Qian, M. Shi, and H. Ding, Observation of Weyl nodes in TaAs, Nat. Phys. 11, 724 (2015).
- [33] B. J. Ramshaw, K. A. Modic, A. Shekhter, Y. Zhang, Eun-Ah Kim, Philip J. W. Moll, Maja D. Bachmann, M. K. Chan, J. B. Betts, F. Balakirev, A. Migliori, N. J. Ghimire, E. D. Bauer, F. Ronning, and R. D. McDonald, Quantum limit transport and destruction of the Weyl nodes in TaAs, Nat. Commun. 9, 2217 (2018).
- [34] P. G. LaBarre, L. Dong, J. Trinh, T. Siegrist, and A. P. Ramirez, Evidence for undoped Weyl semimetal charge transport in Y₂Ir₂O₇, J. Phys.: Condens. Matter **32**, 02LT01 (2019).
- [35] J. Klier, I. V. Gornyi, and A. D. Mirlin, From weak to strong disorder in Weyl semimetals: Self-consistent Born approximation, Phys. Rev. B 100, 125160 (2019).
- [36] C. J. Tabert, J. P. Carbotte, and E. J. Nicol, Optical and transport properties in three-dimensional Dirac and Weyl semimetals, Phys. Rev. B 93, 085426 (2016).
- [37] Z. Li and F. Nori, Nonlinear response in a noncentrosymmetric topological insulator, Phys. Rev. B 99, 155146 (2019).
- [38] Y. L. Lin and F. Nori, Strongly Localized Electrons in a Magnetic Field: Exact Results on Quantum Interference and Magnetoconductance, Phys. Rev. Lett. 76, 4580 (1996).
- [39] Y. L. Lin and F. Nori, Analytical results on quantum interference and magnetoconductance for strongly localized electrons in a magnetic field: Exact summation of forward-scattering paths, Phys. Rev. B 53, 15543 (1996).
- [40] Y. L. Lin and F. Nori, Quantum interference from sums over closed paths for electrons on a three-dimensional lattice in a magnetic field: Total energy, magnetic moment, and orbital susceptibility, Phys. Rev. B 53, 13374 (1996).
- [41] Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1972).