






## Heralded nonlocal quantum gates for distributed quantum computation in a decoherence-free subspace

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We propose a heralded protocol for implementing nontrivial quantum gates on two stationary qubits coupled to spatially separated cavities. By dynamically controlling the evolution of the composite system, nonlocal two-qubit quantum (e.g., CPHASE and CNOT) gates can be achieved without real excitations of either cavity modes or atoms. The success of our protocol is conditioned on projecting an auxiliary atom onto a postselected state, which simultaneously removes various detrimental effects of dissipation on the gate fidelity. In principle, the success probability of the gate can approach unity as the single-atom cooperativity becomes sufficiently large. Furthermore, we show its application for implementing single- and two-qubit gates within a decoherence-free subspace that is immune to a collective dephasing noise. This faithful, heralded, and nonlocal protocol could, therefore, be useful for distributed quantum computation and scalable quantum networks.

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### I. INTRODUCTION

Quantum computation exploiting quantum systems for information processing has attracted a great deal of attention [1–9] due to its promising advantages over classical computation [10–12], and has been experimentally demonstrated with its superiority in handling well-defined tasks. These include implementing algorithms based on quantum gates [13,14] and quantum annealing [15] using superconducting quantum processors, and performing boson sampling using linear-optical interferometers [16–18]. Nontrivial two-qubit quantum gates in combination with general single-qubit rotations in principle enable implementing various quantum algorithms for practical applications. The two-qubit quantum gates always involve direct or indirect interactions between the systems which they are applied on. So far, two-qubit quantum gates have been proposed for different physical systems, such as photons [19–24], trapped ions [25,26], color centers [27–33], quantum dots [34–37], and superconducting circuits [38–40]. However, the scalability of quantum computation is challenging due to the inevitable presence of noise and decoherence. Fortunately, their influence on the evolution of quantum systems can be suppressed by the use of, e.g., dynamical decoupling [41,42], holonomic manipulation [43–45], and decoherence-free subspaces (DFSs) [46–49]. Moreover, a certain amount

of noise and decoherence can be tolerated by harnessing quantum error-correction codes [50,51], in which the overheads and the complexity considerably increase with the error rate.

For some specific dominant noise or decoherence [46], DFSs can provide an efficient method for protecting the logical qubits against noise by encoding quantum information in a DFS [52–56]. A fundamental and dominant noise in stationary systems is dephasing due to the random fluctuations of external fields [55], which destroy the coherence between two computational basis states. A simple DFS for tackling this issue can be constructed by properly encoding a logical qubit with two physical qubits, which simultaneously suffers from the same phase noise (i.e., collective dephasing noise) [46]. Exploiting DFS for quantum computation has been widely studied using various platforms [57–71]. For these protocols, a DFS can work in a deterministic way by dynamically controlling the evolution of systems, or in a heralded way with the detection of single photons scattered by cavity-coupled platforms. Furthermore, some significant experimental efforts have been made for the realization of quantum gates acting on decoherence-free systems [72–76].

Recently, a heralded method for achieving effective quantum computation [77–79] has been presented by dynamically controlling the evolution rather than by scattering and measuring single photons. Borregaard *et al.* [77] proposed a heralded, near-deterministic protocol for performing quantum gates on natural atoms trapped in a single optical cavity. Qin *et al.*

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[78] presented heralded, controlled-phase (CPHASE) gates on superconducting qubits coupled to the same cavity, and introduced a spatially separated cavity coupled to an auxiliary qubit for a heralding operation. These protocols provide a *quadratic fidelity improvement* compared to previous deterministic cavity-based gates, and can find their applications in long-distance entanglement distribution and quantum computation [79–82].

However, it is noteworthy that nontrivial two-qubit gates applied on spatially separated stationary qubits coupled to different optical cavities are useful for connecting several distinct quantum information processors, which constitute the backbone for distributed quantum computation [83–86] and scalable quantum repeater networks [87–93]. Hence, it is important to generalize the heralded schemes of Refs. [77,78] to the *nonlocal* case, where *nontrivial two-qubit quantum gates applied on two spatially separated qubits can be generated in a heralded architecture by dynamically controlling and measuring the auxiliary atom*. For simplicity of notation, we refer to quantum gates applied on spatially separated qubits as *nonlocal gates* when there is no ambiguity.

In this paper, we propose a heralded method for implementing nontrivial quantum gates acting on spatially separated stationary qubits coupled to different cavities by dynamically controlling the evolution of cavity-coupled systems. The cavities can be connected by short fibers or superconducting coaxial cables [94]. A four-level auxiliary atom is coupled to an additional cavity as both a virtual-photon source and a detector for heralding the success of the quantum gate [77,78]. According to the results of a proper measurement on the auxiliary atom, *the gate errors introduced by atomic spontaneous emission and cavity photon loss can be inherently removed, leading to faithful two-qubit nonlocal gates*. As a result, the detected errors simply lower the success probability of the gate rather than its fidelity, which is extremely important for practical applications [83–93].

We show that *the fidelity of our nonlocal two-qubit gate can be further improved by applying proper single-qubit operations to the qubits before completing the two-qubit gate*. Furthermore, we propose an approach for performing a heralded nontrivial two-qubit gate in a DFS immune to collective dephasing noise. Each logical qubit consisting of two physical qubits couples to an individual cavity and suffers from different dephasing noises. Combining the advantages of heralded inherent error detection and error-avoiding DFS, our protocol for implementing nonlocal quantum gates can directly find its applications in distributed quantum computation and quantum networks.

The remainder of the paper is organized as follows: In Sec. II, we describe the physical model and mechanism for implementing a heralded nonlocal two-qubit gate on two spatially separated qubits. In Sec. III, we introduce the effective Hamiltonian and Lindblad operators after conditionally excluding dissipative quantum jumps. In Sec. IV, we describe an implementation of a heralded nonlocal CPHASE gate and analyze its performance both analytically, using the effective Hamiltonian and Lindblad operators in Sec. III, and numerically through a master equation simulation. In Sec. V, we present heralded nonlocal two-qubit gates operating on logical

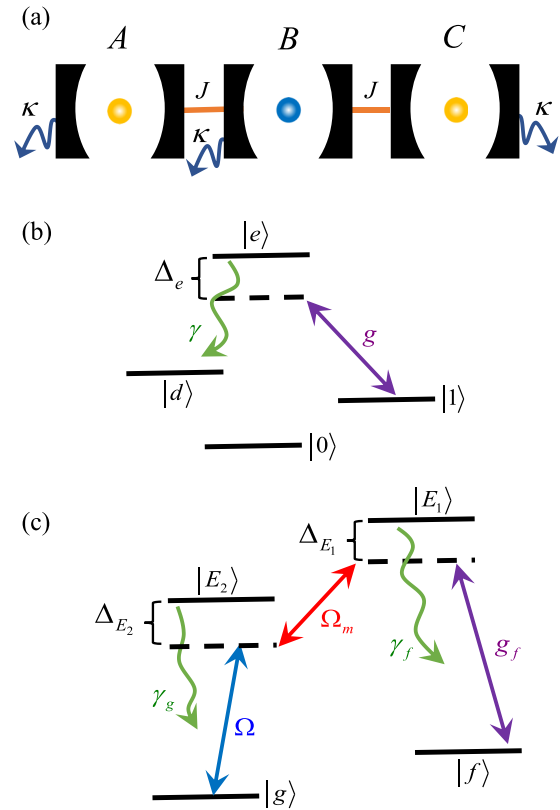


FIG. 1. Schematics of a heralded nonlocal two-qubit quantum gate. (a) Implementation of the gate with a cavity-coupled system. Two stationary qubits are distributed in two separated cavities that are connected to an auxiliary cavity via short fibers or superconducting coaxial cables. (b) Level structure of two qubit-encoding atoms coupled to cavities A and C. (c) Level structure of the auxiliary atom that couples to cavity B and works as a heralding system.

qubits in a DFS immune to collective dephasing noise. Finally, we conclude with a brief discussion and summary in Sec. VI.

## II. PHYSICAL MECHANISM AND CONFIGURATION FOR IMPLEMENTING HERALDED NONLOCAL TWO-QUBIT GATES

An essential building block for implementing heralded nonlocal two-qubit gates is the use of cavity-coupled systems [94]. They can be implemented by various natural or artificial atoms [4] coupled to optical cavities (including transmission-line resonators), which can be connected by short optical fibers (or superconducting coaxial cables).

The schematics of our heralded nonlocal protocol is shown in Fig. 1. Two qubit-encoding atoms couple to two separated cavities A and C, respectively, which are connected via short optical fibers, and an auxiliary atom couples to cavity B in the middle. The effective coupling between two neighboring cavities can be described by a coupling rate  $J$  when the fiber length  $L$  is small and two cavities are resonant [95,96].

A *collective* normal mode can be formed as a linear combination of these cavity modes. It interacts *simultaneously* with all the atoms, when all the cavity modes are resonant and strongly interact with the neighboring cavity

modes through photon exchange. A distributed quantum gate, operating on spatially separated qubit-encoding atoms, as shown in Fig. 1(a), can be simplified to a quantum gate acting on the atoms coupled to the same cavity mode [77].

Each qubit-encoding atom has two ground levels ( $|0\rangle$  and  $|1\rangle$ ), which can encode a qubit, and one excited level  $|e\rangle$ , shown in Fig. 1(b). We assume that the transition  $|1\rangle \leftrightarrow |e\rangle$  of both qubit-encoding atoms is coupled to the cavity mode with a coupling rate  $g$  and a detuning  $\Delta_e$ , and that the excited level  $|e\rangle$  decays to a level  $|d\rangle$ , which may or may not be  $|0\rangle$  or  $|1\rangle$ .

The auxiliary atom has two ground states ( $|g\rangle$  and  $|f\rangle$ ) and two excited states ( $|E_1\rangle$  and  $|E_2\rangle$ ), shown in Fig. 1(c). The excited states  $|E_1\rangle$  and  $|E_2\rangle$  spontaneously decay to the ground states  $|f\rangle$  and  $|g\rangle$  with rates  $\gamma_f$  and  $\gamma_g$ , respectively. In addition, the  $|f\rangle \leftrightarrow |E_1\rangle$  transition couples to the cavity mode  $a_B$  with a coupling rate  $g_f$  and a detuning  $\Delta_{E_1}$ . The transition between states  $|E_2\rangle$  and  $|E_1\rangle$  ( $|g\rangle$  and  $|E_2\rangle$ ) is driven by a classical field with frequency  $\omega_m$  ( $\omega_L$ ) and the Rabi frequency  $\Omega_m$  ( $\Omega$ ). Therefore, a three-photon resonant transition, resulting in a flip of the two ground states of the auxiliary atom, can be achieved by tuning the driving frequencies  $\omega_m$  and  $\omega_L$ .

In general, the auxiliary atom involves three independent transitions from the ground state  $|g\rangle$ : the single-photon transition  $|g\rangle \leftrightarrow |E_2\rangle$ , the two-photon transition  $|g\rangle \leftrightarrow |E_1\rangle$ , and the three-photon transition  $|g\rangle \leftrightarrow |f\rangle$ . When all qubit-encoding atoms decouple from the collective mode and the three-photon resonance transition is achieved, the auxiliary atom can evolve into a dark zero-energy state after removing the Stark shift of the ground state  $|g\rangle$  that is introduced by the nonresonant single-photon transition. Note that the auxiliary atom remains almost unchanged, and the excitation of the collective mode is negligible for weak driving fields.

Conversely, when the qubit-encoding atoms couple to the collective mode, the frequency of the collective mode is shifted, and the three-photon resonance condition is no longer satisfied. As a result, the combined state of the system mainly experiences the single-photon and two-photon transitions for weak driving fields with large detunings. The two-photon transition introduces an additional energy shift of the ground state  $|g\rangle$ , which is nearly independent of the number of the coupled qubit-encoding atoms due to the weak excitation of the collective mode. By appropriately adjusting the driving pulse length, a relative phase shift of  $\pi$  can be introduced for the decoupled state of the qubit-encoding atoms compared to the case of all the coupled states.

The decay of either the atoms or the cavity modes leads to the collapse of the auxiliary atom into state  $|f\rangle$ . This collapse can be heralded by measuring the auxiliary atom, other than relying on the null detection of the photons leaving the cavity. By postselecting state  $|g\rangle$  of the measurement on the auxiliary atom as a heralding signal, errors introduced by finite decay rates are then converted into a nonunity probability of success. Moreover, the excitations of the cavity modes and the excited states of the atoms are negligible and can be adiabatically eliminated, when the system is initially prepared in the ground-state subspace for weak driving fields and large detunings. Consequently, we can concentrate on the evolution of the ground state and describe the corresponding dynamics

using an effective Hamiltonian that excludes the dissipation of atomic and cavity excitations.

The total Hamiltonian of the composite system, consisting of the three atoms and three cavities, can be written as

$$H_T = H_0 + H_1, \quad (1)$$

where  $H_0$  and  $H_1$  represent the free and interaction Hamiltonians, respectively. The free Hamiltonian  $H_0$  is

$$H_0 = \sum_{k=1,2} (\omega_e |e\rangle_k \langle e| + \omega_1 |1\rangle_k \langle 1| + \omega_0 |0\rangle_k \langle 0| + \omega_{E_1} |E_1\rangle \langle E_1| + \omega_{E_2} |E_2\rangle \langle E_2| + \omega_f |f\rangle \langle f| + \omega_g |g\rangle \langle g| + \omega_c (a_A^\dagger a_A + a_B^\dagger a_B + a_C^\dagger a_C), \quad (2)$$

where  $\omega_x$  is the frequency of the atomic level  $|x\rangle$ , except  $\omega_c$ , which is the common resonance frequency of the three cavities. The interaction Hamiltonian  $H_1$  (including the cavity-cavity coupling, the atom-cavity coupling, and the classical driving) becomes

$$H_1 = [g(a_A |e\rangle_1 \langle 1| + a_C |e\rangle_2 \langle 1|) + g_f a_B |E_1\rangle \langle f| + \frac{1}{2} (\Omega e^{-i\omega_L t} |E_2\rangle \langle g| + \Omega_m e^{-i\omega_m t} |E_1\rangle \langle E_2|) + J(a_A a_B^\dagger + a_C a_B^\dagger)] + \text{H.c.}, \quad (3)$$

where H.c. represents the Hermitian conjugate, and we have assumed a symmetric coupling between the two qubit-encoding atoms and their corresponding cavities.

In order to explicitly describe the dynamics of the composite system, we perform a transformation for the three cavity modes and introduce three delocalized normal modes as

$$\begin{aligned} c_1 &= \frac{1}{2}(a_A - \sqrt{2}a_B + a_C), \\ c_2 &= \frac{1}{2}(a_A + \sqrt{2}a_B + a_C), \\ c_3 &= \frac{1}{\sqrt{2}}(a_A - a_C). \end{aligned} \quad (4)$$

The total Hamiltonian in the new basis can be described, in a proper rotating frame, as

$$H_T = H_e + V + V^\dagger, \quad (5)$$

where  $H_e$  and  $V$  describe the evolution of the single-excitation subspace and its coupling to the ground space, respectively. Specifically, they can be expressed as

$$H_e = \Delta_{E_1} |E_1\rangle \langle E_1| + \Delta_{E_2} |E_2\rangle \langle E_2| + \left[ \frac{\Omega_m}{2} |E_1\rangle \langle E_2| + \text{H.c.} \right] + H_{e1}, \quad (6)$$

where

$$H_{e1} = \sum_{k=1,2} \left\{ \left[ \frac{g}{2} (c_1 + c_2 + \sqrt{2} S_k c_3) |e\rangle_k \langle 1| + \text{H.c.} \right] + \Delta_e |e\rangle_k \langle e| \right\} + \left[ \frac{g_f}{\sqrt{2}} (c_2 - c_1) |E_1\rangle \langle f| + \text{H.c.} \right] + \sum_i^3 \Lambda_i c_i^\dagger c_i, \quad (7)$$

with  $S_k = (-1)^{k+1}$ ,  $\Lambda_1 = \omega_c - \sqrt{2}J$ ,  $\Lambda_2 = \omega_c + \sqrt{2}J$ ,  $\Lambda_3 = \omega_c$ , and  $V = \frac{\Omega}{2} |E_2\rangle \langle g|$ . Here, for simplicity, we have defined

some detunings as follows:

$$\begin{aligned}\Delta_{E_1} &= \omega_{E_1} - \omega_L - \omega_m - \omega_g, \\ \Delta_{E_2} &= \omega_{E_2} - \omega_L - \omega_g, \\ \Delta_e &= \omega_e - \omega_L - \omega_m + \omega_f - \omega_g - \omega_1.\end{aligned}\quad (8)$$

For large detunings (i.e.,  $\Delta_{E_1} \gg \Omega$  and  $\Delta_{E_2} \gg \Omega_m$ ) and a large coupling strength (i.e.,  $J \gg g_f$ ) between two neighboring cavities, we can effectively eliminate the excited states  $|E_1\rangle$  and  $|E_2\rangle$  and then obtain a three-photon resonant Raman transition  $|g\rangle \rightarrow |f\rangle$ , which is mediated by mode  $c_1$  rather than modes  $c_{2,3}$  if the driving field frequency is tuned to

$$\omega_L = \omega_c - \omega_m + \omega_f - \omega_g - \sqrt{2}J, \quad (9)$$

i.e.,  $\Lambda_1 = 0$ . The evolution of the composite system consisting of two qubit-encoding atoms, a single auxiliary atom, and three cavities connected by optical fibers can in principle be identical to that of two qubit-encoding atoms and one auxiliary atom, all directly coupled to the same cavity mode [77].

By adiabatically eliminating state  $|E_2\rangle$  of the auxiliary atom and moving into a proper rotating frame, the effective Hamiltonian of the composite system can be described by  $H'_T = H'_e + V' + V'^{\dagger}$ , with an effective three-level auxiliary atom,

$$H'_e = \left( \Delta_{E_1} - \frac{\Omega_m^2}{4\Delta_{E_2}} \right) |E_1\rangle\langle E_1| + H_{e1}, \quad (10)$$

and

$$V' = -\tilde{\Omega}|E_1\rangle\langle g|, \quad \tilde{\Omega} = \frac{\Omega_m\Omega}{2\Delta_{E_2}}, \quad (11)$$

where the energy of the ground state  $|g\rangle$  has been shifted by  $\Omega^2/(4\Delta_{E_2})$ , which can be achieved by using a laser that couples to  $|g\rangle$  nonresonantly with an additional level.

When all qubit-encoding atoms are in state  $|0\rangle$  that is decoupled from mode  $c_1$ , an adiabatic excitation of the auxiliary atom results in the dark zero-energy state,

$$|\psi\rangle_d = \frac{1}{\sqrt{g_f^2 + 2\tilde{\Omega}^2}} (g_f|0, 0, 0, g\rangle - \sqrt{2}\tilde{\Omega}|1, 0, 0, f\rangle), \quad (12)$$

where  $|0, 0, 0, g\rangle$  represents the three normal modes in the vacuum state and the auxiliary state is  $|g\rangle$ , while  $|1, 0, 0, f\rangle$  represents that mode  $c_1$  has a single photon, modes  $c_2$  and  $c_3$  are in the vacuum state, and the auxiliary atom is in state  $|f\rangle$ . For weak driving fields with large detunings, the dark state  $|\psi\rangle_d$  approaches  $|0, 0, 0, g\rangle$ , and the excitation of the normal modes can be considered negligible with a probability approximately zero,  $[\Omega_m\Omega/(\Delta_{E_2}g)]^2 \sim 0$ .

In contrast, when either or both qubit atoms are in state  $|1\rangle$ , they couple to mode  $c_1$ , thereby distorting the three-photon resonant condition. This introduces the ac Stark shifts arising from the nonresonant one- and two-photon transitions and leads to dynamical phases upon applying the driving fields. Therefore, all the qubit states except the uncoupled one acquire a phase that is determined by the duration of the driving field, which is essential for constructing various heralded non-local quantum gates (as shown below).

### III. EFFECTIVE HAMILTONIAN AND LINDBLAD OPERATORS FOLLOWING THE CONDITIONAL EXCLUSION OF DISSIPATIVE QUANTUM JUMPS

So far, we have provided a qualitative description of the physical model and mechanism for the implementation of the heralded nonlocal two-qubit gates, focusing particularly on the ideal scenario, where the composite system remains decoupled from its environment. In this section, we proceed to a quantitative analysis of the physical mechanism, where we derive an effective Hamiltonian with qubit-state-dependent energy shifts. Additionally, we introduce effective Lindblad operators to model the conditional states of the qubit atoms and the corresponding probabilities by postselecting state  $|g\rangle$  of the auxiliary atom.

We assume that the dissipation of the system is described by the Lindblad operators:  $L_{c_l} = \sqrt{\kappa}c_l$ , with  $l = 1, 2, 3$  representing the photon loss of the cavity modes with the same dissipation rate  $\kappa$ ;  $L_f = \sqrt{\gamma_f}|f\rangle\langle E_1|$  and  $L_g = \sqrt{\gamma_g}|g\rangle\langle E_2|$  describe the decay of the auxiliary atom with rates  $\gamma_f$  and  $\gamma_g$ , respectively; and  $L_k = \sqrt{\gamma}|d\rangle\langle e|$  ( $k = 1, 2$ ) describes the decay of the qubit-encoding atoms with rate  $\gamma$ . We assume that the excited level  $|e\rangle$  decays to some level  $|d\rangle$ , which, in fact, may or may not be  $|1\rangle$  or  $|0\rangle$ , since the decay of either a cavity or an excited atom leads to a heralded error.

The standard master equation in the Lindblad form for the composite system described by the Hamiltonian in Eq. (5) can be given by [97,98]

$$\begin{aligned}\dot{\rho}_T(t) &= i[\rho_T(t), H_T] + \frac{1}{2} \sum_j [2L_j\rho_T(t)L_j^\dagger \\ &\quad - \rho_T(t)L_j^\dagger L_j - L_j^\dagger L_j\rho_T(t)],\end{aligned}\quad (13)$$

where  $\rho_T(t)$  represents the density matrix of the total system. Alternatively, the standard Lindblad master equation can be recast in the form with the non-Hermitian Hamiltonian  $H_{\text{NH}}^T = H_T - \frac{i}{2} \sum_j L_j^\dagger L_j$  and the quantum-jump terms  $\sum_j L_j\rho_T(t)L_j^\dagger$ , as it is done in quantum-trajectory approaches [99–101], as follows:

$$\begin{aligned}\dot{\rho}_T(t) &= \mathcal{L}\rho_T \\ &= -i[H_{\text{NH}}^T\rho_T(t) - \rho_T(t)H_{\text{NH}}^{T\dagger}] + \sum_j L_j\rho_T(t)L_j^\dagger,\end{aligned}\quad (14)$$

which can be used to study the effect of quantum jumps in relation to quantum exceptional points [102] and to analyze the postselection on the number of quantum jumps within the hybrid-Liouvillian formalism [103].

For a weak classical driving field, i.e.,  $\{\Omega/\Delta_{E_2}, \Omega/g\} \ll 1$ , the excitations of the cavity modes and the excited states of the atoms can be adiabatically eliminated, when the system is initially prepared in the ground-state subspace. Therefore, the ground-state evolution of the composite system can be described by an effective master equation as follows [104,105]:

$$\begin{aligned}\dot{\rho} &= i[\rho, H_{\text{eff}}] + \frac{1}{2} \sum_j \{2L_{\text{eff}}^j\rho(L_{\text{eff}}^j)^\dagger \\ &\quad - [(L_{\text{eff}}^j)^\dagger L_{\text{eff}}^j\rho + \rho(L_{\text{eff}}^j)^\dagger L_{\text{eff}}^j]\}.\end{aligned}\quad (15)$$

Here,  $\rho$  denotes the ground-space density matrix of the composite system,  $H_{\text{eff}}$  represents an effective Hamiltonian given by

$$H_{\text{eff}} = -\frac{1}{2}V^\dagger[H_{\text{NH}}^{-1} + (H_{\text{NH}}^{-1})^\dagger]V, \quad (16)$$

and  $L_{\text{eff}}^j$  are the effective Lindblad operators with

$$L_{\text{eff}}^j = L_j H_{\text{NH}}^{-1} V, \quad (17)$$

while the non-Hermitian Hamiltonian  $H_{\text{NH}}$  governing the dynamics of the decaying excited states [105,106] can be given, in the quantum jump formalism, as

$$\begin{aligned} H_{\text{NH}} &= H_e - \frac{i}{2} \sum_j L_j^\dagger L_j \\ &= \sum_{k=1,2} \left[ \frac{\bar{\Delta}_e}{2} |e\rangle_k \langle e| + \frac{g}{2} (c_1 + c_2 + \sqrt{2} S_k c_3) |e\rangle_k \langle 1| \right. \\ &\quad \left. + \text{H.c.} \right] + \bar{\Delta}_{E_1} |E_1\rangle \langle E_1| + \bar{\Delta}_{E_2} |E_2\rangle \langle E_2| \\ &\quad - \frac{i\kappa}{2} c_1^\dagger c_1 + \frac{gf}{\sqrt{2}} [(c_2 - c_1) |E_1\rangle \langle f| + \text{H.c.}] \\ &\quad + \sum_{l=2,3} \bar{J}_l c_l^\dagger c_l + \frac{\Omega_m}{2} (|E_1\rangle \langle E_2| + \text{H.c.}). \end{aligned} \quad (18)$$

Here, the auxiliary parameters are as follows:

$$\begin{aligned} \bar{\Delta}_{E_1} &= \Delta_{E_1} - i\gamma_f/2, \\ \bar{\Delta}_{E_2} &= \Delta_{E_2} - i\gamma_g/2, \\ \bar{\Delta}_e &= \Delta_e - i\gamma/2, \\ \bar{J}_2 &= 2\sqrt{2}J - i\kappa/2, \\ \bar{J}_3 &= \sqrt{2}J - i\kappa/2. \end{aligned} \quad (19)$$

To achieve the nonlocal heralded gate, the composite system is confined within the zero- and single-excitation subspaces. The effective Hamiltonian  $H_{\text{eff}}$  and the effective Lindblad operators  $L_{\text{eff}}^j$  can be directly derived from Eqs. (16)–(18). Specifically,  $H_{\text{eff}}$  is given as follows:

$$H_{\text{eff}} = |g\rangle \langle g| \otimes \sum_{N=0}^2 \Delta_N \mathcal{P}_N, \quad (20)$$

where  $\mathcal{P}_N$  is a projection operator that projects the two qubit-encoding atoms onto a state with  $N$  qubits in  $|1\rangle$ , while  $\Delta_N$  represents the  $N$ -dependent ac Stark shift, which can be expressed as

$$\begin{aligned} \Delta_N &= -\frac{\Omega^2}{\gamma} \text{Re} \left\{ \frac{1}{\mathcal{X}_N} [C \bar{\Delta}_e (m+n) (S_1 + \bar{J}_2 S_2) \right. \\ &\quad \left. - 2\bar{\Delta}_e^2 \bar{J}_2 S_1 - 2mnC^2 S_2] \right\}, \end{aligned} \quad (21)$$

where  $\text{Re}$  denotes the real part of an argument, and  $m(n) \in \{0, 1\}$  denotes the number of the qubit-encoding atoms in state  $|1\rangle$  and coupled to cavity A (C). Moreover, the auxiliary

parameters are as follows:

$$\begin{aligned} C &= g^2/(\gamma\kappa), \\ C_f &= g_f^2/(\gamma\kappa), \\ \tilde{\Omega}_m &= \Omega_m/\gamma, \\ \tilde{J}_1 &= 2\sqrt{2}J/\kappa - i/2, \\ \tilde{J}_2 &= \sqrt{2}J/\kappa - i/2, \\ \tilde{\Delta}_e &= \Delta_e/\gamma - i/2, \\ \tilde{\Delta}_{E_1} &= \Delta_{E_1}/\gamma - i\gamma_f/(2\gamma), \\ \tilde{\Delta}_{E_2} &= \Delta_{E_2}/\gamma - i\gamma_g/(2\gamma), \\ S_1 &= C_f(2i\tilde{J}_1 + 1) - 2\tilde{\Delta}_{E_1}\tilde{J}_1, \\ S_2 &= 4iC_f - \tilde{\Delta}_{E_1}(2i\tilde{J}_1 + 1), \\ Z &= 4\tilde{\Delta}_{E_1}\tilde{\Delta}_{E_2} - \tilde{\Omega}_m^2, \\ \mathcal{X}_N &= C_f\tilde{\Delta}_{E_2}R_2 - R_1Z, \\ R_1 &= \tilde{\Delta}_e C(m+n)(\tilde{J}_2 + 2\tilde{J}_1 + 2i\tilde{J}_1\tilde{J}_2) \\ &\quad - 2C^2mn(2i\tilde{J}_1 + 1) - 4\tilde{\Delta}_e^2\tilde{J}_1\tilde{J}_2, \\ R_2 &= 4\tilde{\Delta}_e C(m+n)[2i(\tilde{J}_1 + 2\tilde{J}_2) + 1] \\ &\quad - 32iC^2mn - 8\tilde{\Delta}_e^2\tilde{J}_2(2i\tilde{J}_1 + 1). \end{aligned} \quad (22)$$

The effective Lindblad operators are expressed as follows:

$$\begin{aligned} L_{\text{eff}}^g &= |g\rangle \langle g| \otimes \sum_{N=0}^2 r_{g,N} \mathcal{P}_N, \\ L_{\text{eff}}^f &= |f\rangle \langle g| \otimes \sum_{N=0}^2 r_{f,N} \mathcal{P}_N, \\ L_{\text{eff}}^{c_l} &= |f\rangle \langle g| \otimes \sum_{N=0}^2 r_{c_l,N} \mathcal{P}_N, \\ L_{\text{eff}}^k &= |f\rangle \langle g| \otimes \sum_{N=1}^2 r_{k,N} |d\rangle_k \langle 1| \mathcal{P}_N, \end{aligned} \quad (23)$$

where  $k = 1$  ( $k = 2$ ) labels the qubit-encoding atom coupled to cavity A (C) in state  $|1\rangle$ . The corresponding effective decay rates  $r_{g,N}$ ,  $r_{f,N}$ ,  $r_{c_l,N}$ , and  $r_{k,N}$  are given by

$$\begin{aligned} r_{g,N} &= \frac{2\Omega\sqrt{\gamma_g}}{\gamma\mathcal{X}_N} [C\tilde{\Delta}_e(m+n)(S_1 + \bar{J}_2 S_2) - 2\tilde{\Delta}_e^2 \bar{J}_2 S_1 - 2mnC^2 S_2], \\ r_{f,N} &= \Omega\tilde{\Omega}_m R_1 \sqrt{\gamma_f}/\gamma\mathcal{X}_N, \\ r_{c_1,N} &= 2\sqrt{2}i\delta [\tilde{\Delta}_e C(\tilde{J}_1 + \tilde{J}_2)(m+n) - 2\tilde{\Delta}_e^2 \tilde{J}_1 \tilde{J}_2 - 2C^2 mn], \\ r_{c_2,N} &= \sqrt{2}\delta [2\tilde{\Delta}_e^2 \tilde{J}_2 + 4iC^2 mn - C\tilde{\Delta}_e(1 + 2i\tilde{J}_2)(m+n)], \\ r_{c_3,N} &= C\delta [\tilde{\Delta}_e(1 - 2i\tilde{J}_1)(m-n)], \\ r_{1,N} &= \sqrt{2C}\delta [(1 - 2i\tilde{J}_1)(nC - \tilde{\Delta}_e \tilde{J}_2)], \\ r_{2,N} &= \sqrt{2C}\delta [(1 - 2i\tilde{J}_1)(mC - \tilde{\Delta}_e \tilde{J}_2)], \\ \delta &= \sqrt{C_f} \Omega \tilde{\Omega}_m / (\sqrt{\gamma} \mathcal{X}_N). \end{aligned} \quad (24)$$

For a weak field, driving the transition  $|E_2\rangle \rightarrow |E_1\rangle$  with  $\Omega_m/\Delta_{E_2} \ll 1$ , the ac Stark shift  $\Delta_N$  and the effective decay

rates  $r_{i,N}$ , shown in Eqs. (21) and (24), can be simplified:

$$\begin{aligned}\Delta_N &= -\frac{\Omega^2}{4\Delta_{E_2}} - \frac{\tilde{\Omega}^2}{4\gamma} \operatorname{Re}\left(\frac{Q}{C_f R + \tilde{\Delta}_{E_1} Q}\right), \\ r_{f,N} &= -\frac{\tilde{\Omega} Q \sqrt{\gamma_f}}{2\gamma(C_f R + \tilde{\Delta}_{E_1} Q)}, \\ r_{g,N} &= \frac{\Omega \sqrt{\gamma_g}}{2\Delta_{E_2}} + \frac{\tilde{\Omega} Q \sqrt{\gamma_g}}{2\gamma(C_f R + \tilde{\Delta}_{E_1} Q)}, \\ r_{c_1,N} &= 2\sqrt{2}\delta'[2\tilde{\Delta}_e^2 \tilde{J}_1 \tilde{J}_2 + 2C^2 mn - C\tilde{\Delta}_e(\tilde{J}_1 + \tilde{J}_2)(m+n)], \\ r_{c_2,N} &= \sqrt{2}\delta'[2i\tilde{\Delta}_e^2 \tilde{J}_2 - 4C^2 mn + C\tilde{\Delta}_e(2\tilde{J}_2 - i)(m+n)], \\ r_{c_3,N} &= \delta'[C\tilde{\Delta}_e(i + 2\tilde{J}_1)(m-n)], \\ r_{1,N} &= \alpha'\sqrt{2C}\delta[(1 - 2i\tilde{J}_1)(nC - \tilde{\Delta}_e \tilde{J}_2)], \\ r_{2,N} &= \alpha'\sqrt{2C}\delta[(1 - 2i\tilde{J}_1)(mC - \tilde{\Delta}_e \tilde{J}_2)], \\ \alpha' &= i\tilde{\Omega} \mathcal{X}_N / [2\Omega \Omega_m (C_f R + \tilde{\Delta}_{E_1} Q)], \\ \delta' &= \tilde{\Omega} \sqrt{C_f} / [2\sqrt{\gamma}(C_f R + \tilde{\Delta}_{E_1} Q)], \\ R &= 2\tilde{\Delta}_e^2(-i + 2\tilde{J}_1)\tilde{J}_2 + 8C^2 mn \\ &\quad - C\tilde{\Delta}_e(-i + 2\tilde{J}_1 + 4\tilde{J}_2)(m+n), \\ Q &= 4i\tilde{\Delta}_e \tilde{J}_1 \tilde{J}_2 + 2C^2(i - 2\tilde{J}_1)mn \\ &\quad + C\tilde{\Delta}_e[2\tilde{J}_1 \tilde{J}_2 - i(\tilde{J}_2 + 2\tilde{J}_1)](m+n).\end{aligned}\quad (25)$$

We note that  $\tilde{\Omega} = \Omega \Omega_m / (2\Delta_{E_2})$  is the effective Rabi frequency of the transition  $|g\rangle \rightarrow |E_1\rangle$  and  $\tilde{\gamma}_g = \gamma_g \Omega_m^2 / (2\Delta_{E_2})^2$  is an effective decay rate of the excited state  $|E_1\rangle$  to  $|g\rangle$ .

In practice, the auxiliary and the qubit-encoding atoms can be different. Their atom-cavity cooperativities and decay rates can be parametrized by  $C_f = \alpha C$  and  $\gamma_f = \beta \gamma$ . For simplicity, we set  $\alpha = \beta = 1$  in all our numerical simulations to show the influence of the cooperativity  $C$  on the system evolution. In this case,  $\Delta_N$  and  $r_{g,N}$  can be further simplified as

$$\begin{aligned}\Delta_N &= -\frac{\tilde{\Omega}^2}{4\gamma} \operatorname{Re}\left(\frac{Q}{C_f R + \tilde{\Delta}_{E_1} Q}\right), \\ r_{g,N} &= \frac{\Omega \sqrt{\gamma_g}}{2\Delta_{E_2}},\end{aligned}\quad (26)$$

where the first term,  $-\Omega^2/(4\Delta_{E_2})$ , of  $\Delta_N$  in Eq. (25) has been removed, because it is independent of the state of the qubits and thus has no influence on the phase gates. Furthermore, the second term of  $r_{g,N}$  has also been removed for  $\tilde{\gamma}_g \ll 1$ , because the decay of the auxiliary-atom excited state to  $|g\rangle$  is suppressed by the large detuning  $\Delta_{E_2}$ .

Each Lindblad operator shown in Eq. (23), except  $L_{\text{eff}}^g$  (i.e., the dephasing of  $|g\rangle$ ), represents various effective *dissipative processes*, leading to the transition  $|g\rangle \rightarrow |f\rangle$ . These are the dominant error factors that drive the system out of its effective subspace. Fortunately, the errors introduced by these dissipative processes can be inherently detected, because the success of each nonlocal two-qubit gate is heralded by the measurement result  $|g\rangle$  of the auxiliary atom. For heralded

gates, these detectable decays have no effect on the fidelity, but decrease their success probability.

The *success probability*  $P$  of detecting the auxiliary atom in state  $|g\rangle$  can be obtained by solving the effective Lindblad master equation, given in Eq. (15), with the following definition:

$$P = \sum_{N=0}^2 \operatorname{Tr}[(|g\rangle\langle g| \otimes \mathcal{P}_N)\rho(t)], \quad (27)$$

where  $\operatorname{Tr}$  is the trace operation over the subspace spanned by the ground states of the auxiliary and qubit-encoding atoms.

After the measurement on the auxiliary atom, the *conditional* density operator of the two qubit-encoding atoms is reduced to

$$\begin{aligned}\rho_{\text{qubit}}(t) &= \frac{1}{P} \sum_{N,N'=0}^2 e^{-i(\Delta_N - \Delta_{N'})t} e^{-(\Gamma_N + \Gamma_{N'})t/2} \\ &\quad \times \mathcal{P}_N[|g\rangle\langle g| \rho(0)] \mathcal{P}_{N'}.\end{aligned}\quad (28)$$

Here, the *total decay rate*  $\Gamma_N$  for  $N$  qubit-encoding atoms in state  $|1\rangle$  is found to be

$$\Gamma_N = |r_{f,N}|^2 + \sum_{l=1}^3 |r_{c_l,N}|^2 + m|r_{1,N}|^2 + n|r_{2,N}|^2, \quad (29)$$

where  $r_{g,N}$ ,  $r_{f,N}$ ,  $r_{c_l,N}$ , and  $r_{k,N}$  are the effective decay rates given in Eq. (24). By properly controlling the evolution time and measuring the auxiliary atom, we can in principle achieve a two-qubit nonlocal CPHASE gate in a heralded way, as described below. The success probability of the gate is equal to that of projecting the auxiliary atom onto state  $|g\rangle$ . All basic symbols used in this paper are shown in Table I.

#### IV. HERALDED NONLOCAL CPHASE GATE AND ITS PERFORMANCE

The effective Hamiltonian in Eq. (20) shows that the energy shift depends on the number of qubit-encoding atoms in state  $|1\rangle$  when the auxiliary atom is in state  $|g\rangle$ . Therefore, the time evolution under this effective Hamiltonian gives rise to different dynamical phases for the two qubits in states  $|00\rangle$ ,  $|10\rangle$ ,  $|01\rangle$ , and  $|11\rangle$ . By choosing a suitable evolution time and then performing single-qubit transformations, we can achieve a phase flip of the qubit state  $|11\rangle$ , while leaving the other three states unchanged, which achieves the *heralded nonlocal* CPHASE gate on the two spatially separated atom qubits.

The detrimental effect of *dissipative processes* on the CPHASE gate, represented by the state flip of the auxiliary atom, can be inherently removed by projecting the auxiliary atom onto state  $|g\rangle$ , while the state-dependent decay rate  $\Gamma_N$  of the qubit-encoding atoms and the finite spontaneous decay rate  $\tilde{\gamma}_g > 0$  can introduce extra errors. Therefore, we can improve the gate fidelity by modifying the system to achieve a state-independent decay rate, i.e.,  $\Gamma_0 = \Gamma_1 = \Gamma_2$ . The state-independent total decay rate  $\Gamma_N$ , in the limit  $\{G, C\} \gg 1$ , where  $G = J/\kappa$ , can be given by

$$\Gamma_N = \Gamma = \frac{\tilde{\Omega}^2}{2\gamma} \frac{1}{\alpha C}, \quad (30)$$

TABLE I. Basic notations used in this paper.

Notation	Meaning
$\omega_x$	Frequency of the atomic state $ x\rangle$
$\omega_c$	Common resonance frequency of the cavities A, B, and C
$\omega_L, \omega_m$	Frequencies of the classical driving fields
$\Omega_L, \Omega$	Rabi frequencies of the classical driving fields
$g (g_f)$	Coupling strength between the qubit (auxiliary) atom and the cavity
$J$	Intercavity coupling strength
$\gamma, \gamma_g, \gamma_f$	Decay rates of atomic excited states
$\kappa$	Cavity decay rate
$C = g^2/(\kappa\gamma)$	Atom-cavity cooperativity
$\Delta_{E_1}, \Delta_{E_2}$	Detunings for the one- and two-photon transitions in the auxiliary atom
$\Delta_e$	Detuning of the qubit-encoding atom from the normal mode $c_1$
$\mathcal{P}_N$	Operators projecting the qubit-encoding atoms onto a state with $N$ qubits in $ 1\rangle$
$\Delta_N$	$N$ -dependent ac Stark shifts
$L_{\text{eff}}^\zeta$	Effective Lindblad operators for $\zeta = f, g, c_l$ , and $k$
$r_{\zeta, N}$	Effective decay rates of $L_{\text{eff}}^\zeta$

where the detunings are changed to

$$\begin{aligned} \frac{\Delta_{E_1}}{\gamma} &= \alpha CD/\sqrt{2}, \\ \frac{\Delta_e}{\gamma} &= \frac{-2 + C(\bar{G}^2 - 4D\bar{G})}{2\sqrt{2}(\bar{G} - 2D)}, \end{aligned} \quad (31)$$

where  $\bar{G} = 1/G$  and  $D = \sqrt{\beta/\alpha C}$  are two auxiliary parameters. The corresponding energy shift can be rewritten as

$$\begin{aligned} \Delta_0 &= -\Gamma \frac{(4D - \bar{G})}{8\sqrt{2}}, \\ \Delta_1 &= -\frac{\Gamma}{\sqrt{2}} \frac{2D - \bar{G}}{2/C + \bar{G}^2 - D\bar{G} + 2D^2}, \\ \Delta_2 &= -\frac{\Gamma}{\sqrt{2}} \frac{2D - \bar{G}}{1/C + \bar{G}^2/2 - D\bar{G} + 2D^2}, \end{aligned} \quad (32)$$

where  $\Delta_0$  approaches zero for  $\{G, C\} \gg 1$ , while  $\Delta_1$  and  $\Delta_2$  are nonzero and approximately equal to each other. This property can be used to achieve a heralded nonlocal CPHASE gate by a driving pulse with duration

$$T_\pi = \frac{\pi}{|\Delta_2|}. \quad (33)$$

In practice, we can further decrease the *gate error* to arbitrarily small by performing unitary single-qubit rotations on each qubit-encoding atom, which depends on the dynamical evolution of the composite system. The duration of the driving pulse length is chosen to be

$$t_{\text{CZ}} = \frac{\pi}{|\Delta_2 - 2\Delta_1 + \Delta_0|}, \quad (34)$$

and the single-qubit rotation on each qubit after applying the pulse reads

$$\begin{aligned} \mathcal{U}|0\rangle &= \exp(i\Delta_0 t_{\text{CZ}}/2)|0\rangle, \\ \mathcal{U}|1\rangle &= \exp[i(2\Delta_1 - \Delta_0)t_{\text{CZ}}/2]|1\rangle. \end{aligned} \quad (35)$$

These processes result in a *phase flip* of state  $|11\rangle$ , while leaving the other three states (i.e.,  $|00\rangle$ ,  $|10\rangle$ , and  $|01\rangle$ ) unchanged.

The *success probability of the heralded nonlocal CPHASE gate* equals that of finding the auxiliary atom in state  $|g\rangle$  at the end of the gate operation, and can be given by

$$P_{\text{CZ}} = \exp(-\Gamma t_{\text{CZ}}). \quad (36)$$

It can be further approximated as

$$P_{\text{CZ}} = 1 - Z_p \frac{\pi}{\sqrt{C}} \quad (37)$$

for  $\{C, G\} \gg 1$ , where the scaling factor  $Z_p$ , with  $\lambda = G/\sqrt{C}$  and  $d = \sqrt{\beta/\alpha}$ , can be given as

$$Z_p = \sqrt{2}d + \frac{(1 + 2\lambda^2)^2}{\sqrt{2}d\lambda^2(1 - 2d\lambda)^2} + \frac{3 + 6\lambda^2}{\sqrt{2}\lambda(2d\lambda - 1)}. \quad (38)$$

As long as  $\lambda \gg 1$ , the success probability  $P_{\text{CZ}}$  remains almost constant for a given  $C$ . In fact, we need to select appropriate parameters to ensure that the success probability of the heralded nonlocal CPHASE gate remains relatively high, while its error is arbitrarily small.

To demonstrate the feasibility of our protocol, we perform numerical simulations of the evolution of the composite system with the full master equation in Eq. (13), instead of the effective master equation in Eq. (15). The initial state of our composite system is assumed to be

$$|\Psi\rangle_{\text{ini}} = |\Phi\rangle_{\text{ini}} \otimes |\text{vac}\rangle, \quad (39)$$

where  $|\Phi\rangle_{\text{ini}}$  represents the initial state of the auxiliary and qubit-encoding atoms, given by

$$|\Phi\rangle_{\text{ini}} = |g\rangle \left[ \prod_{k=1}^2 |+\rangle_k \right], \quad (40)$$

where  $|+\rangle_k = (|0\rangle_k + |1\rangle_k)/\sqrt{2}$ ,  $|\text{vac}\rangle$  is the vacuum state of the three coupled cavities. We solve the master equation with the QUTIP package [107,108], and calculate the *success probability* ( $P_{\text{CZ}}$ ) and *fidelity* ( $F_{\text{CZ}}$ ) of the gate with the following

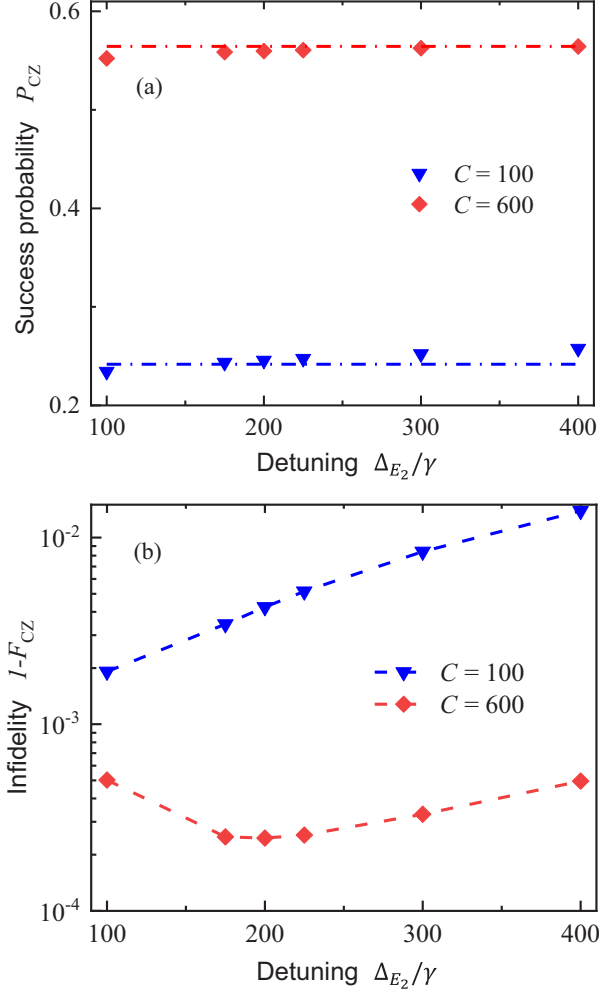


FIG. 2. Numerical simulations for the success probability and infidelity of the heralded nonlocal CPHASE gate with two cooperativities  $C = 100$  (blue down-triangles) and  $C = 600$  (red diamonds). (a) The success probability  $P_{CZ}$  of the gate as a function of the detuning  $\Delta_{E_2}$ . Simultaneously, we also plot the analytical success probability (curves), which is in good agreement with the numerical values. (b) Infidelity ( $1 - F_{CZ}$ ) of the CPHASE gate vs the detuning  $\Delta_{E_2}$ . In both panels, we have set  $\lambda = 10$ ,  $\gamma_g = \gamma_f = \gamma = 0.1\kappa$ ,  $g = g_f$ ,  $C = g^2/(\kappa\gamma)$ ,  $\lambda = J/(\kappa\sqrt{C})$ ,  $\alpha = \beta = 1$ ,  $\Omega = \Delta_{E_2}/(6C^{1/4})$ , and  $\Omega_m = 4\gamma C^{1/4}$ .

expressions:

$$P_{CZ} = \sum_{N=0}^2 \text{Tr}[(|g\rangle\langle g| \otimes \mathcal{P}_N \otimes \mathcal{I})\rho_T(t_{CZ})], \quad (41)$$

$$F_{CZ} = \langle \psi | (\mathcal{U} \otimes \mathcal{U}) \rho_{\text{qubit}}(t_{CZ}) (\mathcal{U} \otimes \mathcal{U})^\dagger | \psi \rangle, \quad (42)$$

$$\rho_{\text{qubit}}(t_{CZ}) = \frac{1}{P_{CZ}} \text{Tr}_{\text{cav}}[|g\rangle\langle g| \rho_T(t_{CZ}) |g\rangle], \quad (43)$$

where  $\text{Tr}$  and  $\text{Tr}_{\text{cav}}$  are trace operations over the composite system and the cavities, respectively, and  $\mathcal{I}$  is the identity operator for the three cavities.

The success probability  $P_{CZ}$  and the gate error (infidelity),  $1 - F_{CZ}$ , are shown in Fig. 2 as a function of the detuning  $\Delta_{E_2}/\gamma$  for two different cooperativities  $C = 100$  and  $C =$

600. In our numerical simulations, we set  $\lambda = 10$  to reduce the influence of the off-resonant modes  $c_2$  and  $c_3$  on the gate error. Meanwhile, we assume that  $\gamma_g = \gamma_f$ ,  $\kappa = 10\gamma$ ,  $\alpha = \beta = 1$ ,  $\Omega = \Delta_{E_2}/(6C^{1/4})$ , and  $\Omega_m = 4\gamma C^{1/4}$ .

The detunings  $\Delta_{E_1}$  and  $\Delta_e$ , given in Eq. (31), are tuned to achieve a *total qubit-independent decay rate*. The numerical results (marked by symbols) of the success probability  $P_{CZ}$  are in agreement with the analytical ones determined by Eq. (37), as shown in Fig. 2(a). The success probability  $P_{CZ}$  is almost constant for a given cooperativity  $C$  and gradually increases with increasing  $C$ . For the aforementioned parameters,  $P_{CZ} = 0.56$  can be achieved for  $C = 600$ .

The fidelity of the heralded nonlocal two-qubit gate, which is conditional on the detection of the auxiliary atom in state  $|g\rangle$ , can approach unity in principle. The finite length of the driving field in combination with the finite effective decay from  $|E_2\rangle$  to  $|g\rangle$  can introduce undetectable errors. Theoretically, the former error leads to a nonadiabatic error of the gate, but which can be suppressed by properly tuning the Rabi frequency  $\Omega$  of the driving field. At the same time, the latter error can be decreased by increasing the detuning  $\Delta_{E_2}$ . For a cooperativity  $C = 100$ , the gate error increases with the detuning  $\Delta_{E_2}$ , due to the increase in  $\Omega$  and thus in the nonadiabatic error, and can be less than  $2 \times 10^{-3}$  for  $\Delta_{E_2}/\gamma = 100$ . For a larger cooperativity  $C = 600$ , the gate error first decreases and then increases with increasing detuning  $\Delta_{E_2}/\gamma$ . A gate error below  $3 \times 10^{-4}$  can be achieved for  $C = 600$  and  $\Delta_{E_2}/\gamma = 180$ , as shown in Fig. 2(b).

## V. HERALDED NONLOCAL QUANTUM GATES ENCODED IN A DECOHERENCE-FREE SUBSPACE

In this section, we focus on the implementation of heralded single- and two-qubit gates on logical qubits that are robust against collective random dephasing errors, stemming from the fluctuations of the external fields and, thus, resulting in uncontrolled energy shifts [46]. In the case of collective dephasing, the symmetry properties of the errors allow to identify a DFS in the Hilbert space of a two-physical-qubit system [52–56], where the two logical basis states can be  $|0_L\rangle = |01\rangle$  and  $|1_L\rangle = |10\rangle$ , and a memory-time enhancement of two orders of magnitude has been experimentally demonstrated for ion-trap systems [55].

Suppose that the qubit-encoding atoms 1 and 2 (3 and 4) are coupled to cavity A (C) and encode a logical qubit. Cavities A and C interact with cavity B through two short fibers or superconducting coaxial cables, as shown in Fig. 3. We assume that there is an auxiliary atom coupled to cavity B. The coupling rate between cavities A (C) and B is  $J_1$  ( $J_2$ ), and all three cavities decay with the same rate  $\kappa$ .

In principle, a CPHASE gate  $U_L^{CZ}$  on these two logical qubits, given by  $U_L^{CZ} = \exp(i\pi |1_L 1_L\rangle\langle 1_L 1_L|)$ , can be achieved with a heralded nonlocal CPHASE gate  $U_{1,3}^{CZ}$  on the atom pair (1, 3) from two logical qubits. The gate  $U_{1,3}^{CZ}$  can be implemented with the same method described in the previous sections, while the other two atoms need to be decoupled from the cavities (i.e., by modifying their detunings) during the CPHASE gate operation. Furthermore, the controlled-NOT (CNOT) gate on two nonlocal logical qubits can be constructed by sandwiching the CPHASE gate with two Hadamard operations on



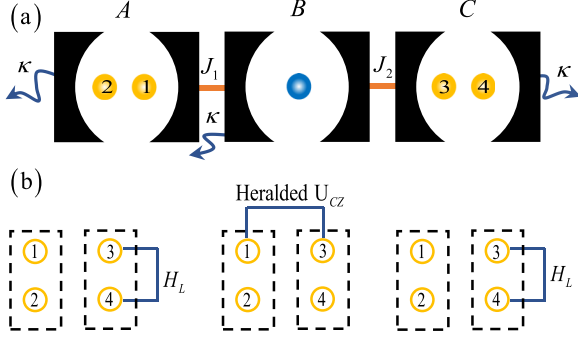


FIG. 3. (a) Schematic diagram of heralded nonlocal two-qubit quantum gates within a decoherence-free subspace. (b) Implementation scheme of the CNOT gate.  $H_L$  represents the Hadamard operation on a logical qubit consisting of atoms 3 and 4, and  $U_{CZ}$  represents the nonlocal CPHASE gate on atoms 1 and 3 that couple to cavities A and C, respectively.

the same logical qubit as follows:

$$\text{CNOT}_L = (I \otimes H_L) \times (U_{13}^{CZ}) \times (I \otimes H_L), \quad (44)$$

where  $I$  is the identity on the first logical qubit,  $U_{13}^{CZ}$  is a nonlocal CPHASE gate performed on the atom pair (1, 3), and  $H_L$  performs the Hadamard transformation on the second logical qubit, as shown in Fig. 3.

The operation of the Hadamard gate on a logical qubit is nontrivial and changes the entanglement between two physical atoms encoding a logical qubit. The logical Hadamard gate can be implemented by a two-qubit CNOT gate in combination with single-qubit rotations on two qubit-encoding atoms as follows [64]:

$$H_L = [(HSHZ) \otimes (HSH)] \text{CNOT}_{34} [(HSX) \otimes X], \quad (45)$$

where the gate  $S = \text{diag}(1, i)$ , in the computational basis  $\{|0\rangle, |1\rangle\}$ , denotes a rotation around the  $z$  axis by an angle  $\pi/2$ ;  $H$  is the standard Hadamard transformation on a single physical qubit; while  $X$  and  $Z$  are Pauli operators. The  $\text{CNOT}_{34}$  gate, with the control atom 3 and the target atom 4, can be implemented by

$$\text{CNOT}_{34} = H_4 U_{34}^{CZ} H_4, \quad (46)$$

where  $H_4$  represents the Hadamard transform on qubit 4, and  $U_{34}^{CZ}$  is the heralded CPHASE gate acting on qubits 3 and 4 that are coupled to the same cavity.

The heralded CPHASE gate  $U_{34}^{CZ}$  acting on qubits 3 and 4 can be achieved in a setup similar to that shown in Fig. 1, except that cavity A is decoupled from cavity B, i.e.,  $J_1 = 0$  and  $J_2 = J$ , and the heralded nonlocal CPHASE gate is modified to become a *compact* one, as described in Ref. [78].

In order to explicitly describe the dynamics of the *composite system* consisting of two cavities and three atoms, we perform a transformation for the two cavity modes and introduce the symmetric and antisymmetric optical modes,  $a_{\pm} = (a_B \pm a_C)/\sqrt{2}$ . The total Hamiltonian is  $H_T = H_e + V + V^\dagger$ ,

where  $V$  is the same as in Eq. (7), while  $H_e$  is changed to

$$\begin{aligned} H_e = \sum_{k=1}^2 \left\{ \Delta_e |e\rangle_k \langle e| + \frac{g}{\sqrt{2}} [(a_+ - a_-) |e\rangle_k \langle 1| + \text{H.c.}] \right\} \\ + \Delta_{E_1} |E_1\rangle \langle E_1| + \Delta_{E_2} |E_2\rangle \langle E_2| + 2J a_+^\dagger a_+ \\ + \frac{g_f}{\sqrt{2}} [(a_+ + a_-) |E_1\rangle \langle f| + \text{H.c.}] \\ + \frac{\Omega_m}{2} (|E_1\rangle \langle E_2| + \text{H.c.}). \end{aligned} \quad (47)$$

For *large detunings* ( $\Delta_{E_1} \gg \Omega$  and  $\Delta_{E_2} \gg \Omega_m$ ) and a *large coupling strength* ( $J \gg g_f$ ) between cavities B and C, we can adiabatically eliminate the excited states  $|E_1\rangle$  and  $|E_2\rangle$  and then obtain a three-photon resonant Raman transition from  $|g\rangle$  to  $|f\rangle$ , by choosing a driving field with frequency

$$\omega_L = \omega_c - \omega_m + \omega_f - \omega_g - J. \quad (48)$$

Such a *three-photon resonant Raman transition* is resonantly mediated by the antisymmetric mode  $a_-$ , while detuned by  $2J$  from the symmetric mode  $a_+$ .

Following the procedure in Sec. II, we can implement the heralded near-deterministic CPHASE gate on the qubit-encoding atoms 3 and 4 in the same cavity, which has been discussed in dissipative circuit QED systems [78]. We can completely remove the gate errors introduced by the qubit-dependent decay rate by modifying the detunings  $\Delta_e$  and  $\Delta_{E_1}$  to be

$$\frac{\Delta_e}{\gamma} = \frac{1}{2(2D_1 + \bar{G})}, \quad (49)$$

$$\frac{\Delta_{E_1}}{\gamma} = \alpha C(D_1 + \bar{G}), \quad (50)$$

where  $D_1 = \sqrt{[\bar{G}^2 + \beta/(\alpha C)]/2}$ . In the limit  $\{G, C\} \gg 1$ , the effective Hamiltonian driving the evolution of the *composite system* can be described as

$$H_{\text{eff}} = |g\rangle \langle g| \otimes \sum_{n=0}^2 \Delta'_n \mathcal{P}_n, \quad (51)$$

where  $\mathcal{P}_n$  is a projector onto the states with  $n$  qubit-encoding atoms in state  $|1\rangle$ . The corresponding energy shift  $\Delta'_n$  is given by

$$\Delta'_0 = -\frac{\Gamma D_1}{2}, \quad (52)$$

$$\Delta'_{n>0} = -\frac{\tilde{\Omega}^2}{2\gamma} \frac{n(2D_1 + \bar{G})}{\alpha C(4nD_1^2 + 2nD_1\bar{G} + 1/C)}, \quad (53)$$

where  $\Delta'_0$  approaches *zero*, while  $\Delta'_1 \simeq \Delta'_2$  with  $|\Delta'_1| \simeq |\Delta'_2| \gg |\Delta'_0|$  for  $\{G, C\} \gg 1$ . Therefore, we can implement a CPHASE gate on atoms 3 and 4 by properly tuning the duration of the driving pulse in combination with the single-qubit rotations, according to Eqs. (34) and (35), after replacing  $\Delta_n$  with  $\Delta'_n$ .

The success probability  $P'_{CZ}$  and the error  $(1 - F'_{CZ})$  of the CPHASE gate on two qubit-encoding atoms coupled to the same cavity are of the same formalism as those described in Eqs. (42) and (43), while the density matrix describes the composite system consisting of three atoms and two cavities.

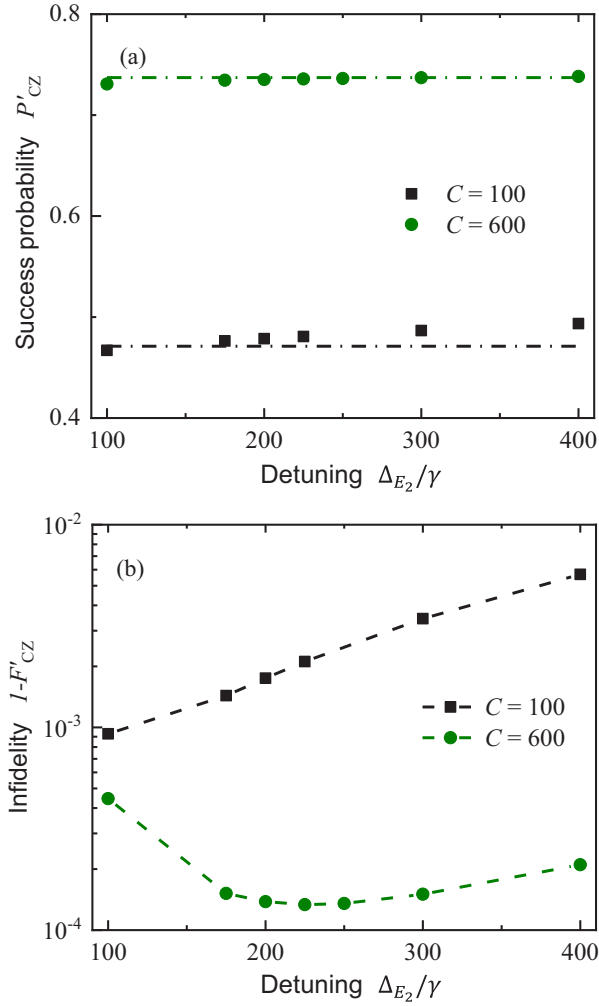


FIG. 4. Numerical simulations for the heralded CPHASE gate on two qubit-encoding atoms, the logical qubit, with two cooperativities:  $C = 100$  (black squares) and  $C = 600$  (olive solid circles). (a) The success probability  $P'_{CZ}$  as a function of the detuning  $\Delta_{E_2}/\gamma$ . Simultaneously, we also plot the analytical results (shown by curves), which match well with the numerical ones. (b) Infidelity  $1 - F'_{CZ}$  vs the detuning  $\Delta_{E_2}/\gamma$ . All the system parameters and the initial state are the same as those assumed in Fig. 2, except  $\lambda = 1.84$ .

We numerically calculate  $P'_{CZ}$  and  $(1 - F'_{CZ})$  and demonstrate their dependence on the detuning  $\Delta_{E_2}/\gamma$  for different cooperativities ( $C = 100$  and  $C = 600$ ), shown in Fig. 4. All the system parameters and the initial state are the same as those assumed in Fig. 2, except for  $\lambda = 1.84$ . The success probability  $P'_{CZ}$  increases with increasing  $C$  and can be larger than that of the heralded nonlocal CPHASE gate with  $P'_{CZ} = 0.74$  for  $C = 600$ . Meanwhile, the gate error decreases with  $C$  and shows a dependence on detuning  $\Delta_{E_2}/\gamma$ , similar to that of the nonlocal CPHASE gate. For  $C = 600$ , the gate error  $1 - F'_{CZ}$  can be suppressed to  $1.2 \times 10^{-4}$  for  $\Delta_{E_2}/\gamma = 220$ . Therefore, the Hadamard gate in combination with the nonlocal CPHASE gate can be faithfully implemented with the cavity-coupled system in a heralded way.

## VI. DISCUSSION AND SUMMARY

Our protocol generalizes the previous proposal of heralded CPHASE gates [77,78] on qubits coupled to the same cavity to a nonlocal case by dynamically controlling the evolution rather than by scattering and measuring single photons. The integrated error detection eliminates the limitation of single-photon sources and measurements [94], and enables a high fidelity of the heralded CPHASE gates at the cost of a smaller success probability. Furthermore, we apply our heralded nonlocal CPHASE gate to heralded single- and two-qubit quantum gates within a DFS that is immune to collective dephasing noise. The heralded nonlocal CPHASE gates on qubits belonging to different cavities are suitable for interconnecting individual quantum processors for distributed quantum computing [85] and quantum repeater networks [92,93].

Our protocol can be experimentally implemented with neutral or artificial atoms coupled to various cavities [4]. As an example, we consider ultracold  $^{87}\text{Rb}$  atoms coupled to optical cavities [77]. The relevant energy levels can be encoded as the two ground states  $|g\rangle$  ( $|0\rangle$ ) and  $|f\rangle$  ( $|1\rangle$ ), corresponding to the atomic levels  $|F = 1, m_f = 1\rangle$  and  $|F = 2, m_f = 2\rangle$  of  $5^2S_{1/2}$ , respectively; and the two excited states  $|E_2\rangle$  and  $|E_1\rangle$  ( $|e\rangle$ ), corresponding to  $|F = 2, m_f = 2\rangle$  and  $|F = 3, m_f = 3\rangle$  of  $5^2P_{3/2}$ , respectively.

Optical cavities with high- $Q$  factors have recently been widely used for quantum information technology [109–111]. The coupling strength  $g$  between a cavity and an atom depends inversely on the cavity mode volume, i.e.,  $g \propto 1/\sqrt{V}$ , and can thus be significantly enhanced for small mode volume cavities, such as fiber Fabry-Perot cavities [112], photonic crystal cavities [22], and whispering gallery mode cavities [113]. A single-atom cooperativity  $C > 500$  for a strong single atom-photon coupling can be achieved for microring resonators [114].

In practice, our protocol is designed for short-distance distributed quantum computation. The length of the fiber channel  $L_{fc}$  connecting two neighboring cavities is within the short-fiber limit [95,96], ensuring that the interaction time between spatially separated cavities is sufficiently short compared to the cavity mode lifetime [115]. The effective interaction between two spatially separated qubits is mediated by the vacuum field, without exciting the atoms or the cavity modes due to the nonresonant couplings in our protocol, except that a single excitation of the normal mode  $c_1$  occurs when both qubits decouple from their respective cavity modes. Thus, the presence of fiber attenuation increases the effective decay rates.

Fortunately, the intrinsic loss induced by fiber attenuation can be calculated as  $\kappa_{fc} = -c \ln(1 - \alpha_l)/(2L_{fc})$  [116], where  $c$  represents the speed of light in the fiber and  $\alpha_l$  denotes the single-pass loss of the fiber channel. The impact of the intrinsic loss  $\kappa_{fc}$  on the performance of our protocol can be considered to be negligible, given that  $\kappa_{fc}$  is approximately  $10^{-3}$  of the decay rate of the atomic excited state for a short fiber length of  $L_{fc} < 1$  m.

In summary, we have proposed a scheme for implementing a heralded nonlocal CPHASE gate on spatially separated stationary qubits coupled to different cavities. We can

faithfully implement a nonlocal CPHASE gate in a heralded way by dynamically controlling the evolution of a composite system and projecting the auxiliary atom onto a postselected state. We have further showed its application for implementing quantum gates on logical qubits within a DFS. All these distinct characteristics make these quantum gates useful for distributed quantum computation and quantum networks.

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