Rapid Communications

Nonreciprocal ground-state cooling of multiple mechanical resonators

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The simultaneous ground-state cooling of multiple degenerate or near-degenerate mechanical modes coupled to a common cavity-field mode has become an outstanding challenge in cavity optomechanics. This is because the dark modes formed by these mechanical modes decouple from the cavity mode and prevent extracting energy from the dark modes through the cooling channel of the cavity mode. Here we propose a universal and reliable dark-mode-breaking method to realize the simultaneous ground-state cooling of two degenerate or nondegenerate mechanical modes by introducing a phase-dependent phonon-exchange interaction, which is used to form a loop-coupled configuration. We find an asymmetrical cooling performance for the two mechanical modes and expound this phenomenon based on the nonreciprocal energy transfer mechanism, which leads to the directional flow of phonons between the two mechanical modes. We also generalize this method to cool multiple mechanical modes. The physical mechanism in this cooling scheme has general validity and this method can be extended to break other dark-mode and dark-state effects in physics.

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Introduction. Mechanical resonators in cavity optomechanical systems [1–3] have the advantages of easy resonance, wide compatibility, and tunable coupling to diverse physical devices. These resonators not only provide a promising platform for investigating macroscopic mechanical coherence [4–15], quantum many-body effects [16–21], and topological energy transfer [22], but also can be used as high-performance sensors [23–25], transducers [26], and mechanical computers [27,28]. To suppress thermal noise in those applications, the simultaneous ground-state cooling of these mechanical resonators becomes an obligatory and important task. Though great advances have been made in ground-state cooling of a single mechanical resonator [29–41], the simultaneous ground-state cooling of multiple mechanical resonators remains an outstanding challenge in cavity optomechanics [42–44]. The physical origin behind this obstacle is the existence of the dark-mode effect [8,42–45] induced by the multiple mechanical resonators (modes) coupled to a common cavity field, as demonstrated theoretically [42,43] and experimentally [44,45].

In this Rapid Communication, we propose a reliable method to realize the *simultaneous ground-state cooling* of multiple mechanical modes by *breaking the dark-mode effect* in an optomechanical system consisting of a cavity mode coupled to two mechanical modes. This is realized by introducing a phase-dependent phonon-exchange interaction between the two mechanical modes [46]. Owing to the phase-dependent phonon-exchange interaction in this loop-coupled system, there is no dark mode anymore, and asymmetrical ground-state cooling of the two mechanical resonators is realized via an interference effect. We find that the asymmetrical cooling performance is caused by nonreciprocal excitation transfer between the two mechanical modes [51–65]. We also extend this method to the simultaneous cooling of N mechanical resonators and this advance will be helpful for the miniaturization of quantum devices [66,67]. This dark-mode-breaking mechanism is universal and can be generalized to break the dark-state or dark-mode effects in other physical systems [46].

System. We consider a three-mode optomechanical structure [Fig. 1(a)] consisting of a cavity field optomechanically coupled to two mechanical modes, which are coupled with each other via a phase-dependent phonon-exchange interaction [46]. A monochromatic driving field with frequency ω_L and amplitude Ω is applied to the optical cavity. In a rotating frame defined by $\exp(-i\omega_L t a^\dagger a)$, the system Hamiltonian reads ($\hbar = 1$) [46]

$$H_I = \Delta_c a^\dagger a + \sum_{l=1,2}[\omega_l b_l^\dagger b_l + g_l a^\dagger a(b_l + b_l^\dagger)] + (\Omega a + \Omega^* a^\dagger) + \eta(e^{i\theta} b_1^\dagger b_2 + e^{-i\theta} b_2^\dagger b_1), \qquad (1)$$

where a ($a^\dagger$) and $b_{l=1,2}$ ($b_l^\dagger$) are, respectively, the annihilation (creation) operators of the cavity mode (ω_c) and the lth

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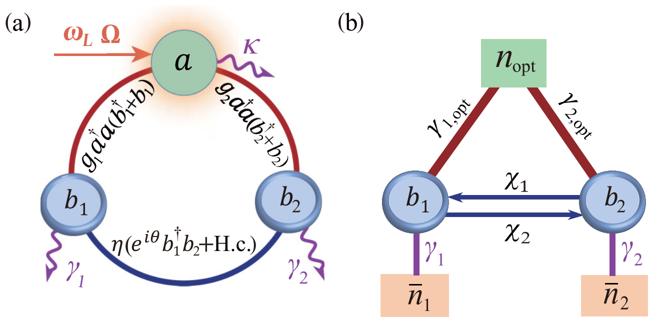

FIG. 1. (a) A loop-coupled optomechanical system consists of one cavity-field mode a optomechanically coupled to two mechanical modes b_1 and b_2, which are coupled with each other via a phase-dependent phonon-exchange coupling (with the coupling strength η and phase θ). (b) The reduced two-mechanical-mode system with the effective phonon-exchange channel ($\chi_{l=1,2}$), the common optomechanical-cooling channel ($\gamma_{l,\rm opt}$, $n_{\rm opt}$), and the mechanical dissipations ($\gamma_{l=1,2}$, $\bar{n}_l$).

mechanical mode (ω_l). The $g_{l=1,2}$ terms describe the optomechanical couplings. The Ω term denotes the cavity-field driving with detuning $\Delta_c = \omega_c - \omega_L$, and the η term describes a phase-dependent phonon-exchange interaction between the two mechanical resonators, with the real coupling strength η and phase θ. Note that this model can be implemented with either circuit electromechanical systems [8,23] or photonic crystal optomechanical cavity systems [51]. The phase-dependent phonon-hopping coupling in the electromechanical system can be indirectly induced by coupling to a charge qubit [46]. In the photonic crystal optomechanical setup, the phase-dependent phonon-hopping coupling has been suggested by using two assistant cavity fields [51]. In addition, we mention that the two mechanical modes could be either bare mechanical modes in individual mechanical resonators or supermodes of coupled mechanical resonators [68,69]. For the latter case, the phase-dependent phonon-exchange coupling should be implemented between these supermodes accordingly.

By expressing the operators $o \in \{a, b_{l=1,2}, a^\dagger, b_{l=1,2}^\dagger\}$ with their steady-state average values and fluctuations $o = \langle o\rangle_{\rm ss} + \delta o$, the system can be linearized in the strong-driving regime, and the linearized Hamiltonian in the rotating-wave approximation (RWA) reads

$$H_{\rm RWA} = \Delta\delta a^\dagger\delta a + \sum_{l=1,2}[\omega_l\delta b_l^\dagger\delta b_l + G_l(\delta a\delta b_l^\dagger + \delta b_l\delta a^\dagger)] + \eta(e^{i\theta}\delta b_1^\dagger\delta b_2 + e^{-i\theta}\delta b_2^\dagger\delta b_1), \quad (2)$$

where Δ is the normalized driving detuning and $G_{l=1,2} = g_l\alpha$ are the linearized optomechanical-coupling strengths. The displacement $\alpha \equiv \langle a\rangle_{\rm ss} = -i\Omega^*/(\kappa + i\Delta)$ is assumed to be real by choosing a proper driving amplitude Ω, where κ is the decay rate of the cavity field. When $\omega_1 = \omega_2$ and $\eta = 0$, there exists a bright mode B_+ and a dark mode B_- defined by [46]

$$B_\pm = (G_{1(2)}\delta b_1 \pm G_{2(1)}\delta b_2)/\sqrt{G_1^2 + G_2^2}. \quad (3)$$

Then $H_{\rm RWA} = \Delta\delta a^\dagger\delta a + \omega_+ B_+^\dagger B_+ + \omega_- B_-^\dagger B_- + G_+(\delta a B_+^\dagger + B_+\delta a^\dagger)$ with $G_+ = \sqrt{G_1^2 + G_2^2}$. Here the dark mode B_- decouples from the cavity mode and the ground-state cooling of the two resonators is unaccessible.

Ground-state cooling by breaking the dark mode. To analyze the action of the phonon-exchange interaction, we introduce two bosonic modes $\tilde{B}_+ = f\delta b_1 - e^{i\theta}h\delta b_2$ and $\tilde{B}_- = e^{-i\theta}h\delta b_1 + f\delta b_2$, where the coefficients are given by $f = |\tilde{\omega}_- - \omega_1|/\sqrt{(\tilde{\omega}_- - \omega_1)^2 + \eta^2}$ and $h = \eta f/(\tilde{\omega}_- - \omega_1)$, with the resonance frequencies $\tilde{\omega}_\pm = \frac{1}{2}(\omega_1 + \omega_2 \pm \sqrt{(\omega_1 - \omega_2)^2 + 4\eta^2})$ and the coupling strengths $\tilde{G}_+ = fG_1 - e^{-i\theta}hG_2$ and $\tilde{G}_- = e^{i\theta}hG_1 + fG_2$. The linearized optomechanical Hamiltonian becomes

$$H_{\rm RWA} = \Delta\delta a^\dagger\delta a + \tilde{\omega}_+\tilde{B}_+^\dagger\tilde{B}_+ + \tilde{\omega}_-\tilde{B}_-^\dagger\tilde{B}_- + (\tilde{G}_+^*\delta a\tilde{B}_+^\dagger + \tilde{G}_+\tilde{B}_+\delta a^\dagger) + (\tilde{G}_-^*\delta a\tilde{B}_-^\dagger + \tilde{G}_-\tilde{B}_-\delta a^\dagger). \quad (4)$$

In the degenerate-resonator ($\omega_1 = \omega_2 = \omega_m$) and symmetric-coupling ($G_1 = G_2 = G$) cases, the coupling strengths become $\tilde{G}_+ = \sqrt{2}G(1 + e^{-i\theta})/2$ and $\tilde{G}_- = \sqrt{2}G(1 - e^{i\theta})/2$. When $\theta = n\pi$ for an integer n, the cavity field is decoupled from one of the two hybrid mechanical modes $\tilde{B}_-$ (for even n) and $\tilde{B}_+$ (for odd n). However, in the general case $\theta \neq n\pi$, the dark-mode effect is broken [46], and then the simultaneous ground-state cooling becomes accessible under proper parameter conditions. We emphasize that the dark-mode-breaking mechanism is universal and it can be proved by analyzing the eigenstates of a 3×3 matrix, which is used to describe either a three-mode system or a three-level system [46].

To study the cooling performance of the two mechanical resonators, we calculate the final average phonon numbers n_1^f and n_2^f by solving the steady-state covariance matrix governed by the Lyapunov equation [46]. Figure 2(a) shows the phonon numbers n_1^f and n_2^f as functions of the driving detuning Δ when the system works in both the dark-mode-unbreaking ($\eta = 0$) and -breaking ($\eta/\omega_m = 0.05$ and $\theta = \pi/2$) regimes. The results indicate that ground-state cooling of the two mechanical resonators is unfeasible when the system possesses the dark mode [the upper solid curves in Fig. 2(a)]. When the dark mode is broken by adding the phonon-exchange coupling [the dashed curves in Fig. 2(a)], the emergence of the valley corresponds to ground-state cooling ($n_{1,2}^f \ll 1$). The phonon-exchange coupling provides the physical origin for breaking the dark mode and builds the channel to transfer the excitation energy between the two mechanical resonators. The optimal driving detuning is located at $\Delta = \omega_m$, which is consistent with a typical resolved-sideband cooling [29–31,35,36], because the phonons exactly compensate the energy mismatch between the scattered photons and the driving light.

When the phonon-exchange coupling is absent, though the dark mode exists theoretically only in the degenerate-resonator case (i.e., $\omega_1 = \omega_2$), the dark-mode effect actually works for a wider detuning range in the near-degenerate-resonator case [as marked by the shadow area in Fig. 2(b)] [46]. The width of the shadow area can be characterized by the effective mechanical linewidth Γ_l ($\Delta\omega = |\omega_2 - \omega_1| \leqslant \Gamma_l$). The cooling of the individual mechanical resonators is suppressed in this region, i.e., the individual

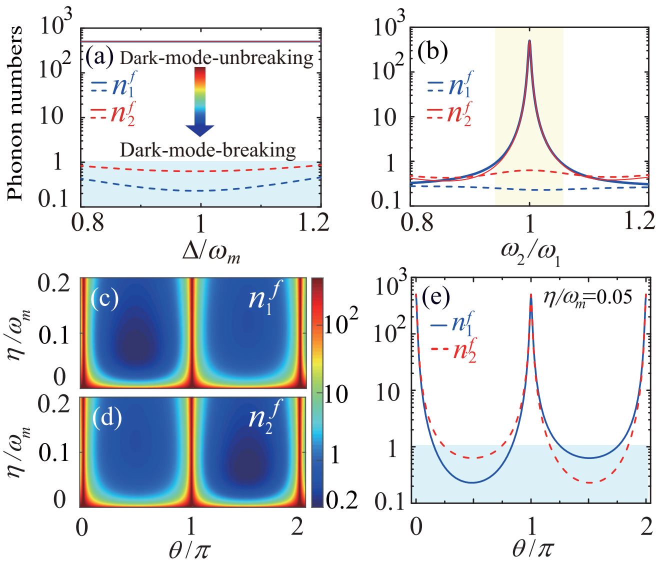

FIG. 2. (a) The final average phonon numbers n_1^f (blue curves) and n_2^f (red curves) in the two mechanical resonators versus the effective driving detuning Δ in the dark-mode-unbreaking ($\eta = 0$, solid curves) and -breaking ($\eta = 0.05\omega_m$ and $\theta = \pi/2$, dashed curves) cases when $\omega_1 = \omega_2 = \omega_m$. (b) n_1^f and n_2^f as functions of ω_2/ω_1 in both the dark-mode-unbreaking (solid curves) and -breaking (dashed curves) cases when $\Delta = \omega_1$. (c) n_1^f and (d) n_2^f vs η and θ under the optimal driving $\Delta = \omega_m$ and $\omega_1 = \omega_2 = \omega_m$. (e) n_1^f and n_2^f vs θ at $\eta = 0.05\omega_m$. Other used parameters are given by $G_1/\omega_m = G_2/\omega_m = 0.1$, $\gamma_1/\omega_m = \gamma_2/\omega_m = 10^{-5}$, $\kappa/\omega_m = 0.2$, and $\bar{n}_1 = \bar{n}_2 = 10^3$.

mechanical resonators have significant spectral overlap and become effectively degenerate. When the phonon-exchange coupling is applied, the dark-mode effect is broken and the ground-state cooling for the degenerate and near-degenerate resonators becomes feasible [the dashed curves in Fig. 2(b)].

The dependence of the final average phonon numbers n_1^f and n_2^f on the phonon-exchange parameters η and θ is displayed in Figs. 2(c) and 2(d). The ground-state cooling of the two mechanical resonators is achievable in the region $0 < \theta < \pi$ ($\pi < \theta < 2\pi$) for a wide range of η, and the cooling performance of the first (second) resonator is better than the other one $n_1^f < n_2^f$ ($n_1^f > n_2^f$). In particular, at $\theta = n\pi$, the two mechanical resonators cannot be cooled to their ground states, which corresponds to the dark-mode-unbreaking case, as shown in Figs. 2(c)–2(e).

Nonreciprocal phonon transfer. To explain the asymmetrical cooling phenomenon in Fig. 2(e), we introduce a relative resonant-phonon-scattering rate $\Lambda_{vw} = (T_{vw} - T_{wv})/(T_{vw})_{\max}$ corresponding to the transfer of a phonon with frequency ω_m from modes w to v, where T_{vw} denotes the transmittance from modes w to v [$v, w \in \{b_1, b_2\}$]. The relative resonant-phonon-scattering rates can be expressed as [46]

$$\Lambda_{b_2b_1} = \frac{4\sqrt{\Pi}\sin\theta}{(1+\sqrt{\Pi})^2}\left[1 + \frac{4\Pi\cos^2\theta}{\left(\frac{\mathcal{C}_1+\mathcal{C}_2+1}{\mathcal{C}_1\mathcal{C}_2} + \Pi\right)^2}\right]^{-1}, \qquad (5)$$

and $\Lambda_{b_1b_2} = -\Lambda_{b_2b_1}$, where $\Pi = \mathcal{C}_3/(\mathcal{C}_1\mathcal{C}_2)$ with $\mathcal{C}_{l=1,2} = G_l^2/\gamma_l\kappa$ and $\mathcal{C}_3 = \eta^2/\gamma_1\gamma_2$ being the cooperativities associated with the optomechanical couplings and the phonon-exchange

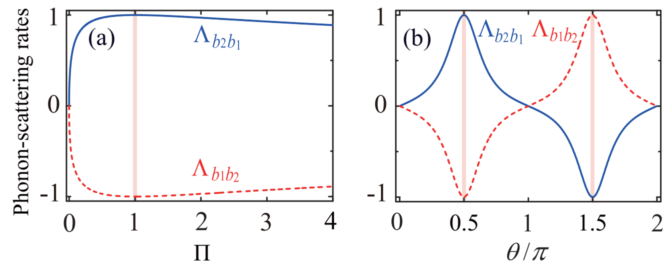

FIG. 3. The relative resonant-phonon-scattering rate $\Lambda_{b_2b_1}$ (blue solid curves) and $\Lambda_{b_1b_2}$ (red dashed curves) versus (a) the ratio Π of the optomechanical cooperativities when $\theta = \pi/2$ and (b) the phase θ when $\Pi = 1$. Here $\Delta = \omega_m$ and $\omega_1 = \omega_2 = \omega_m$. Other parameters used are the same as those in Fig. 2.

coupling ($\gamma_{l=1,2}$ denoting the decay rate of the lth resonator), respectively. The dependence of the relative resonant-phonon-scattering rates Λ_{vw} on the ratio Π of the optomechanical cooperativities and the phase θ is shown in Fig. 3. In panel (a), we find that in the region $0 < \Pi < 1$ ($\Pi > 1$), $\Lambda_{b_2b_1}$ increases (decreases) with increasing Π, and the optimal nonreciprocity ($\Lambda_{b_2b_1} = 1$) emerges at $\Pi = 1$, which indicates directional flow of phonons between the two mechanical resonators. As shown in Fig. 3(b), when $0 < \theta < \pi$, $\Lambda_{b_2b_1} > 0$, i.e., $T_{b_2b_1} > T_{b_1b_2}$, the phonon transmission from mechanical mode b_1 to b_2 is enhanced, while the transmission in the backward direction is suppressed (see blue solid curves). In the range $\pi < \theta < 2\pi$, it exhibits $\Lambda_{b_1b_2} > 0$, i.e., $T_{b_1b_2} > T_{b_2b_1}$ (see red dashed curves). Meanwhile, the phonon transmission satisfies the Lorentz reciprocal theorem [$\Lambda_{b_2b_1} = \Lambda_{b_1b_2} = 0$, i.e., $T_{b_1b_2} = T_{b_2b_1}$] at $\theta = n\pi$. Moreover, the transmittance is optimal for the process from b_1 (b_2) to b_2 (b_1) and is zero for the opposite process when $\theta = \pi/2$ ($\theta = 3\pi/2$). We see from Eq. (5) that, when $\Pi = 1$ and $\theta = \pi/2$, an excellent nonreciprocal phonon transfer ($\Lambda_{b_2b_1} = 1$) is realized.

Cooling limits. The cooling limits can be analytically obtained in the large cavity-field-decay regime, in which the cavity field is eliminated adiabatically such that the three-mode optomechanical system is reduced to a two-mode system described by the Hamiltonian $\tilde{H}_{\text{eff}} = \sum_{l=1}^{2}(\Omega_l - i\Gamma_l)b_l^\dagger b_l + i\xi_1 b_1^\dagger b_2 + i\xi_2 b_2^\dagger b_1$ [Fig. 1(b)], where $\Gamma_l = \gamma_l + \gamma_{l,\text{opt}}$ and $\Omega_l = \omega_l - \omega_{l,\text{opt}}$ are, respectively, the effective decay rate and resonance frequency for the lth mechanical resonator, with the optical induced decay rates $\gamma_{l,\text{opt}} = G_l^2/\kappa$ and mechanical frequency shifts $\omega_{l,\text{opt}} = G_l^2/2\omega_l$. In addition, $i\xi_l$ is the effective phonon-exchange coupling strength between the two mechanical modes with $\xi_{1(2)} = -[G_1G_2/\kappa + i(\eta e^{\pm i\theta} - G_1G_2/2\omega_{2(1)})]$. The mechanical mode b_l is contacted to an effective optomechanical cooling bath ($\gamma_{l,\text{opt}}$ and n_{opt}) and a heat bath (γ_l and $\bar{n}_l$). Considering the parameter relations $\omega_{1,2} \gg \kappa \gg G_{1,2} \gg \{\gamma_{1,\text{opt}} \approx \gamma_{2,\text{opt}}\} \gg \gamma_{1,2}$, the final average phonon occupations can be obtained as [46]

$$n_{l=1,2}^f \approx \frac{\gamma_l\bar{n}_l + \gamma_{l,\text{opt}}n_{\text{opt}}}{\Gamma_l + \chi_+} + \frac{(-1)^{l-1}\sqrt{\chi_l}}{\Gamma_l + \chi_-} \times(\sqrt{\chi_1}n_{\chi_1} - \sqrt{\chi_2}n_{\chi_2}), \qquad (6)$$

where $n_{\text{opt}} = 4\kappa^2/(\omega_1 + \omega_2 + 2\Delta)^2$, $n_{\chi_{1(2)}} = 2(\gamma_{2(1)}\bar{n}_{2(1)} + \gamma_{2(1),\text{opt}}n_{\text{opt}})/(\Gamma_1 + \Gamma_2 + 2\chi_+)$, and $\chi_\pm = \mp\sqrt{\chi_1\chi_2} -$

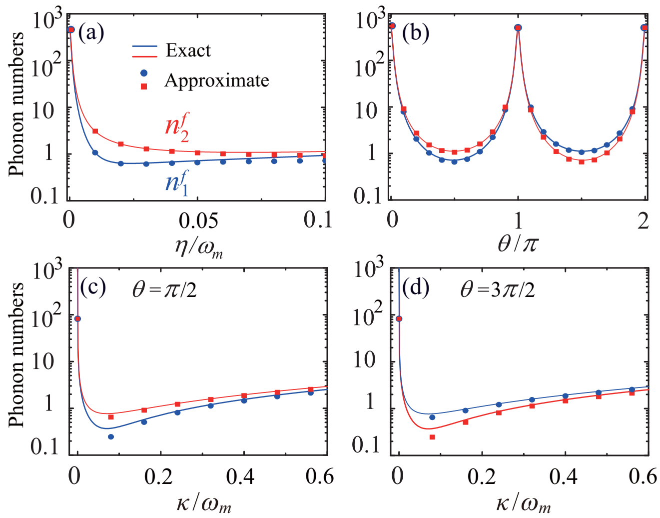

FIG. 4. The exact and approximate final average phonon numbers n_1^f (blue) and n_2^f (red) versus (a) the phonon-exchange coupling strength η when $\theta = \pi/2$ and $\kappa/\omega_m = 0.2$, (b) the phase θ when $\eta/\omega_m = 0.05$ and $\kappa/\omega_m = 0.2$, and (c),(d) the cavity-field decay rate κ when $\eta/\omega_m = 0.05$ for (c) $\theta = \pi/2$ and (d) $\theta = 3\pi/2$. The solid curves and the symbols correspond to the exact (numerical) and approximate (analytical) results, respectively. Here $\Delta/\omega_m = 1$ and $G_1/\omega_m = G_2/\omega_m = 0.05$. Other parameters used are the same as those in Fig. 2.

$\mathrm{Re}[\xi_1\xi_2/(\Gamma_1+\Gamma_2)]$, with $\chi_{l=1,2} = |\xi_l|^2/(\Gamma_1+\Gamma_2)$ being the effective phonon-transfer rate from b_2 (b_1) to b_1 (b_2). The cooling limits (n_l^{lim}) are obtained at $\Delta = \omega_l$. In Fig. 4, we plot the exact final average phonon numbers (solid lines) and the cooling limits (symbols) given by Eq. (6) as functions of the phonon-exchange parameters η and θ. Figure 4 shows asymmetrical ground-state cooling and excellent agreement between numerical and analytical results.

The first term in Eq. (6) is caused by the thermal bath and the effective optical bath connected by the lth mechanical mode, while the phonon extraction by the phonon-exchange channel is described by the last term. Physically, the nonreciprocity of the phonon transfer is determined by the phonon-exchange rate χ_l which depends on the phase θ. For the case $\bar{n}_1 \approx \bar{n}_2$ and $\gamma_1 \approx \gamma_2$, we have $n_{\chi_1} \approx n_{\chi_2} = n_\chi$ and thus $(\sqrt{\chi_1}n_{\chi_1} - \sqrt{\chi_2}n_{\chi_2}) \approx (\sqrt{\chi_1} - \sqrt{\chi_2})n_\chi$ [see Eq. (6)]. In the range $0 < \theta < \pi$ ($\pi < \theta < 2\pi$), we obtain $\sqrt{\chi_1} < \sqrt{\chi_2}$ ($\sqrt{\chi_1} > \sqrt{\chi_2}$). This means that the phonon-transfer efficiency from b_1 (b_2) to b_2 (b_1) is larger than that for the opposite case, i.e., $n_1^f < n_2^f$ ($n_1^f > n_2^f$) [see Fig. 4(b)]. When $\theta = \pi/2$ ($3\pi/2$) and $\sqrt{\mathcal{C}_1\mathcal{C}_2} = \sqrt{\mathcal{C}_3}$, the unidirectional flow of the phonons between the two mechanical resonators is obtained [$\chi_1 \approx 0$ ($\chi_2 \approx 0$)]. For $\theta = n\pi$, the phonon transfer between the two mechanical resonators is reciprocal ($\sqrt{\chi_1} = \sqrt{\chi_2}$), due to the emergence of the dark mode. In the absence of the phonon-transfer interaction ($\eta = 0$), the ground-state cooling is unfeasible due to the invalid effective cooling channel ($\Gamma_l + \chi_+ \rightarrow \gamma_l$) [see Fig. 4(a)]. In the absence of the optomechanical cooling channels ($G_{1,2} = 0$), Eq. (6) becomes $n_{l=1,2}^f \approx \bar{n}_l + (-1)^{l-1}(n_{\chi_1} - n_{\chi_2})/2$, which indicates quantum thermalization in the coupled mechanical system.

Cooling N mechanical resonators. Our proposal can be extended to the cooling of a net-coupled system: a cavity mode coupled to $N \geqslant 3$ mechanical modes via the optomechanical couplings $H_{\mathrm{opc}} = \sum_{j=1}^{N} g_j a^\dagger a(b_j + b_j^\dagger)$, and the nearest-neighboring mechanical modes are coupled through the phase-dependent phonon-exchange couplings $H_{\mathrm{pec}} = \sum_{j=1}^{N-1} \eta_j(e^{i\theta_j}b_j^\dagger b_{j+1} + \mathrm{H.c.})$. We find that the function of these phases in the optomechanical interactions is determined by the term $\sum_{v=1}^{j-1}\theta_v$ [46] and hence, for convenience, we assume $\theta_1 = \pi/2$ and $\theta_j = 0$ for $j = 2$ - $(N-1)$ in our simulations. In the dark-mode-unbreaking case ($\eta_j = 0$), the ground-state cooling of the mechanical resonators is unfeasible, with the final average phonon numbers $\bar{n}(N-1)/N$ in the case of $\bar{n}_j = \bar{n}$ [46]. When the dark modes are broken, simultaneous ground-state cooling can be realized in this system ($n_j^f < 1$).

Conclusions. We proposed a dark-mode-breaking method to realize simultaneous ground-state cooling of multiple mechanical modes coupled to a common cavity mode by constructing a loop-coupled optomechanical system with a phase drop. We found an asymmetric cooling phenomenon and expounded it using the nonreciprocal phonon-exchange mechanism. The present physical mechanism is universal and hence it will motivate the manipulation of various dark-state-related physical effects.

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[1] T. J. Kippenberg and K. J. Vahala, Cavity optomechanics: Back-action at the mesoscale, Science **321**, 1172 (2008).

[2] P. Meystre, A short walk through quantum optomechanics, Ann. Phys. (Berlin) **525**, 215 (2013).

[3] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. **86**, 1391 (2014).

[4] J.-Q. Liao and L. Tian, Macroscopic Quantum Superposition in Cavity Optomechanics, Phys. Rev. Lett. **116**, 163602 (2016).

[5] S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, Entangling Macroscopic Oscillators Exploiting Radiation Pressure, Phys. Rev. Lett. **88**, 120401 (2002).

[6] L. Tian and P. Zoller, Coupled Ion-Nanomechanical Systems, Phys. Rev. Lett. **93**, 266403 (2004).

[7] M. J. Hartmann and M. B. Plenio, Steady State Entanglement in the Mechanical Vibrations of Two Dielectric Membranes, Phys. Rev. Lett. **101**, 200503 (2008).

[8] F. Massel, S. U. Cho, J.-M. Pirkkalainen, P. J. Hakonen, T. T. Heikkilä, and M. A. Sillanpää, Multimode circuit optomechanics near the quantum limit, Nat. Commun. **3**, 987 (2012).

[9] Z. L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, Rev. Mod. Phys. **85**, 623 (2013).

[10] A. Mari, A. Farace, N. Didier, V. Giovannetti, and R. Fazio, Measures of Quantum Synchronization in Continuous Variable Systems, Phys. Rev. Lett. **111**, 103605 (2013).

[11] M. H. Matheny, M. Grau, L. G. Villanueva, R. B. Karabalin, M. C. Cross, and M. L. Roukes, Phase Synchronization of Two Anharmonic Nanomechanical Oscillators, Phys. Rev. Lett. **112**, 014101 (2014).

[12] M. Zhang, S. Shah, J. Cardenas, and M. Lipson, Synchronization and Phase Noise Reduction in Micromechanical Oscillator Arrays Coupled through Light, Phys. Rev. Lett. **115**, 163902 (2015).

[13] R. Riedinger, A. Wallucks, I. Marinković, C. Löschnauer, M. Aspelmeyer, S. Hong, and S. Gröblacher, Remote quantum entanglement between two micromechanical oscillators, Nature (London) **556**, 473 (2018).

[14] C. F. Ockeloen-Korppi, E. Damskägg, J.-M. Pirkkalainen, M. Asjad, A. A. Clerk, F. Massel, M. J. Woolley, and M. A. Sillanpää, Stabilized entanglement of massive mechanical oscillators, Nature (London) **556**, 478 (2018).

[15] O. D. Stefano, A. Settineri, V. Macrì, A. Ridolfo, R. Stassi, A. F. Kockum, S. Savasta, and F. Nori, Interaction of Mechanical Oscillators Mediated by the Exchange of Virtual Photon Pairs, Phys. Rev. Lett. **122**, 030402 (2019).

[16] G. Heinrich, M. Ludwig, J. Qian, B. Kubala, and F. Marquardt, Collective Dynamics in Optomechanical Arrays, Phys. Rev. Lett. **107**, 043603 (2011).

[17] A. Xuereb, C. Genes, and A. Dantan, Strong Coupling and Long-Range Collective Interactions in Optomechanical Arrays, Phys. Rev. Lett. **109**, 223601 (2012).

[18] M. Ludwig and F. Marquardt, Quantum Many-Body Dynamics in Optomechanical Arrays, Phys. Rev. Lett. **111**, 073603 (2013).

[19] A. Xuereb, C. Genes, G. Pupillo, M. Paternostro, and A. Dantan, Reconfigurable Long-Range Phonon Dynamics in Optomechanical Arrays, Phys. Rev. Lett. **112**, 133604 (2014).

[20] A. Xuereb, A. Imparato, and A. Dantan, Heat transport in harmonic oscillator systems with thermal baths: Application to optomechanical arrays, New J. Phys. **17**, 055013 (2015).

[21] O. Černotík, S. Mahmoodian, and K. Hammerer, Spatially Adiabatic Frequency Conversion in Optoelectromechanical Arrays, Phys. Rev. Lett. **121**, 110506 (2018).

[22] H. Xu, D. Mason, L. Jiang, and J. G. E. Harris, Topological energy transfer in an optomechanical system with exceptional points, Nature (London) **537**, 80 (2016).

[23] F. Massel, T. T. Heikkilä, J.-M. Pirkkalainen, S. U. Cho, H. Saloniemi, P. J. Hakonen, and M. A. Sillanpää, Microwave amplification with nanomechanical resonators, Nature (London) **480**, 351 (2011).

[24] P. Huang, P. Wang, J. Zhou, Z. Wang, C. Ju, Z. Wang, Y. Shen, C. Duan, and J. Du, Demonstration of Motion Transduction Based on Parametrically Coupled Mechanical Resonators, Phys. Rev. Lett. **110**, 227202 (2013).

[25] P. Huang, L. Zhang, J. Zhou, T. Tian, P. Yin, C. Duan, and J. Du, Nonreciprocal Radio Frequency Transduction in a Parametric Mechanical Artificial Lattice, Phys. Rev. Lett. **117**, 017701 (2016).

[26] P. Rabl, S. J. Kolkowitz, F. H. L. Koppens, J. G. E. Harris, P. Zoller, and M. D. Lukin, A quantum spin transducer based on nanoelectromechanical resonator arrays, Nat. Phys. **6**, 602 (2010).

[27] S. C. Masmanidis, R. B. Karabalin, I. D. Vlaminck, G. Borghs, M. R. Freeman, and M. L. Roukes, Multifunctional Nanomechanical Systems via Tunably Coupled Piezoelectric Actuation, Science **317**, 780 (2007).

[28] I. Mahboob and H. Yamaguchi, Bit storage and bit flip operations in an electromechanical oscillator, Nat. Nanotechnol. **3**, 275 (2008).

[29] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Theory of Ground State Cooling of a Mechanical Oscillator Using Dynamical Backaction, Phys. Rev. Lett. **99**, 093901 (2007).

[30] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Quantum Theory of Cavity-Assisted Sideband Cooling of Mechanical Motion, Phys. Rev. Lett. **99**, 093902 (2007).

[31] C. Genes, D. Vitali, P. Tombesi, S. Gigan, and M. Aspelmeyer, Ground-state cooling of a micromechanical oscillator: Comparing cold damping and cavity-assisted cooling schemes, Phys. Rev. A **77**, 033804 (2008);

[32] K. Xia and J. Evers, Ground State Cooling of a Nanomechanical Resonator in the Nonresolved Regime via Quantum Interference, Phys. Rev. Lett. **103**, 227203 (2009).

[33] L. Tian, Ground state cooling of a nanomechanical resonator via parametric linear coupling, Phys. Rev. B **79**, 193407 (2009).

[34] A. Xuereb, T. Freegarde, P. Horak, and P. Domokos, Optomechanical Cooling with Generalized Interferometers, Phys. Rev. Lett. **105**, 013602 (2010).

[35] J. Chan, T. P. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Groeblacher, M. Aspelmeyer, and O. Painter, Laser cooling of a nanomechanical oscillator into its quantum ground state, Nature (London) **478**, 89 (2011).

[36] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Sideband cooling of micromechanical motion to the quantum ground state, Nature (London) **475**, 359 (2011).

[37] Y.-C. Liu, Y.-F. Xiao, X. Luan, and C. W. Wong, Dynamic Dissipative Cooling of a Mechanical Resonator in Strong Coupling Optomechanics, Phys. Rev. Lett. **110**, 153606 (2013).

[38] J. B. Clark, F. Lecocq, R. W. Simmonds, J. Aumentado, and J. D. Teufel, Sideband cooling beyond the quantum backaction limit with squeezed light, Nature (London) **541**, 191 (2017).

[39] X. Xu, T. Purdy, and J. M. Taylor, Cooling a Harmonic Oscillator by Optomechanical Modification of Its Bath, Phys. Rev. Lett. **118**, 223602 (2017).

[40] M. Rossi, N. Kralj, S. Zippilli, R. Natali, A. Borrielli, G. Pandraud, E. Serra, G. D. Giuseppe, and D. Vitali, Enhancing Sideband Cooling by Feedback-Controlled Light, Phys. Rev. Lett. **119**, 123603 (2017).

[41] O. Černotík, A. Dantan, and C. Genes, Cavity Quantum Electrodynamics with Frequency-Dependent Reflectors, Phys. Rev. Lett. **122**, 243601 (2019).

[42] C. Genes, D. Vitali, and P. Tombesi, Simultaneous cooling and entanglement of mechanical modes of a micromirror in an optical cavity, New J. Phys. **10**, 095009 (2008).

[43] C. Sommer and C. Genes, Partial Optomechanical Refrigeration via Multimode Cold-Damping Feedback, Phys. Rev. Lett. **123**, 203605 (2019).

[44] C. F. Ockeloen-Korppi, M. F. Gely, E. Damskägg, M. Jenkins, G. A. Steele, and M. A. Sillanpää, Sideband cooling of nearly degenerate micromechanical oscillators in a multimode optomechanical system, Phys. Rev. A **99**, 023826 (2019).

[45] A. B. Shkarin, N. E. Flowers-Jacobs, S. W. Hoch, A. D. Kashkanova, C. Deutsch, J. Reichel, and J. G. E. Harris, Optically Mediated Hybridization between Two Mechanical Modes, Phys. Rev. Lett. **112**, 013602 (2014).

[46] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.102.011502, which includes Refs. [47–50], for analyzing and breaking the dark-mode effect, ground-state cooling of the two mechanical resonators, nonreciprocal phonon transfer, derivation of the cooling limits, ground-state cooling of N mechanical resonators, discussing the parameter condition of the rotating-wave approximation; simultaneous cooling of the mechanical supermodes; breaking the dark state in a Lambda-type three-level system, and a possible physical implementation of the present model and detailed derivation of a phase-dependent phonon-hopping interaction.

[47] Y.-x. Liu, J. Q. You, L. F. Wei, C. P. Sun, and F. Nori, Optical Selection Rules and Phase-Dependent Adiabatic State Control in a Superconducting Quantum Circuit, Phys. Rev. Lett. **95**, 087001 (2005).

[48] M. Nakahara and T. Ohmi, *Quantum Computing: From Linear Algebra to Physical Realizations* (CRC Press, Boca Raton, 2008).

[49] H. Fröhlich, Theory of the superconducting state. i. the ground state at the absolute zero of temperature, Phys. Rev. **79**, 845 (1950).

[50] S. Nakajima, Perturbation theory in statistical mechanics, Adv. Phys. **4**, 363 (1953).

[51] K. Fang, J. Luo, A. Metelmann, M. H. Matheny, F. Marquardt, A. A. Clerk, and O. Painter, Generalized non-reciprocity in an optomechanical circuit via synthetic magnetism and reservoir engineering, Nat. Phys. **13**, 465 (2017).

[52] N. R. Bernier, L. D. Tóth, A. Koottandavida, M. A. Ioannou, D. Malz, A. Nunnenkamp, A. K. Feofanov, and T. J. Kippenberg, Nonreciprocal reconfigurable microwave optomechanical circuit, Nat. Commun. **8**, 604 (2017).

[53] D. Malz, L. D. Tóth, N. R. Bernier, A. K. Feofanov, T. J. Kippenberg, and A. Nunnenkamp, Quantum-Limited Directional Amplifiers with Optomechanics, Phys. Rev. Lett. **120**, 023601 (2018).

[54] J. P. Mathew, J. d. Pino, and E. Verhagen, Synthetic gauge fields for phonon transport in a nano-optomechanical system, Nat. Nanotechnol. **15**, 198 (2020).

[55] K. Fang, Z. Yu, and S. Fan, Realizing effective magnetic field for photons by controlling the phase of dynamic modulation, Nat. Photonics **6**, 782 (2012).

[56] K. Fang, Z. Yu, and S. Fan, Photonic Aharonov-Bohm Effect Based on Dynamic Modulation, Phys. Rev. Lett. **108**, 153901 (2012).

[57] M. Hafezi and P. Rabl, Optomechanically induced non-reciprocity in microring resonators, Opt. Express **20**, 7672 (2012).

[58] A. Metelmann and A. A. Clerk, Nonreciprocal Photon Transmission and Amplification via Reservoir Engineering, Phys. Rev. X **5**, 021025 (2015).

[59] Z. Shen, Y.-L. Zhang, Y. Chen, C.-L. Zou, Y.-F. Xiao, X.-B. Zou, F.-W. Sun, G.-C. Guo, and C.-H. Dong, Experimental realization of optomechanically induced non-reciprocity, Nat. Photonics **10**, 657 (2016).

[60] G. A. Peterson, F. Lecocq, K. Cicak, R. W. Simmonds, J. Aumentado, and J. D. Teufel, Demonstration of Efficient Nonreciprocity in a Microwave Optomechanical Circuit, Phys. Rev. X **7**, 031001 (2017).

[61] S. Kim, X. Xu, J. M. Taylor, and G. Bahl, Dynamically induced robust phonon transport and chiral cooling in an optomechanical system, Nat. Commun. **8**, 205 (2017).

[62] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, Nature (London) **541**, 473 (2017).

[63] Z. Shen, Y.-L. Zhang, Y. Chen, F.-W. Sun, X.-B. Zou, G.-C. Guo, C.-L. Zou, and C.-H. Dong, Reconfigurable optomechanical circulator and directional amplifier, Nat. Commun. **9**, 1797 (2018).

[64] A. Seif, W. DeGottardi, K. Esfarjani, and M. Hafezi, Thermal management and non-reciprocal control of phonon flow via optomechanics, Nat. Commun. **9**, 1207 (2018).

[65] H. Xu, L. Jiang, A. A. Clerk, and J. G. E. Harris, Nonreciprocal control and cooling of phonon modes in an optomechanical system, Nature (London) **568**, 65 (2019).

[66] M. Partanen, K. Y. Tan, J. Govenius, R. E. Lake, M. K. Mâkelä, T. Tanttu, and M. Möttönen, Quantum-limited heat conduction over macroscopic distances, Nat. Phys. **12**, 460 (2016).

[67] S. Barzanjeh, M. Aquilina, and A. Xuereb, Manipulating the Flow of Thermal Noise in Quantum Devices, Phys. Rev. Lett. **120**, 060601 (2018).

[68] D. Ramos, I. W. Frank, P. B. Deotare, I. Bulu, and M. Lončar, Non-linear mixing in coupled photonic crystal nanobeam cavities due to cross-coupling opto-mechanical mechanisms, Appl. Phys. Lett. **105**, 181121 (2014).

[69] S. Barzanjeh and D. Vitali, Phonon Josephson junction with nanomechanical resonators, Phys. Rev. A **93**, 033846 (2016).