

## Reconciling quantum and classical spectral theories of ultrastrong coupling: role of cavity bath coupling and gauge corrections: erratum

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This erratum contains typographical corrections to our recent paper [Opt. Quantum 2, 133 (2024)]. These corrections do not alter any of our previous results and conclusions.

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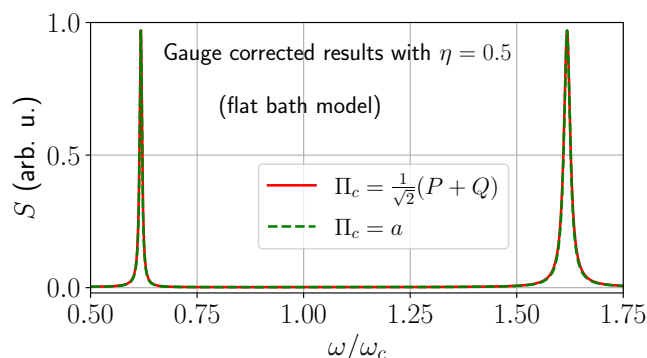
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In our recent paper [1], we noticed small typographical errors in the renormalized dipole frequency and atom-cavity coupling rate for finite cavity-emitter detunings, which should read  $\tilde{\omega}_0 = \omega_0(1 + 4\eta^2\omega_c/\omega_0)$  and  $\tilde{g}^2 = g^2/(1 + 4\eta^2\omega_c/\omega_0)^{\frac{1}{2}}$ . In addition, the  $g^2$  appearing in Eq. (5) should be replaced by  $g^2\omega_0/\omega_c$ . No results in our paper are affected by this change, and the resonance frequencies between quantum and classical models agree as stated.

Additionally, we described the quadrature coupling  $\Pi_c = (Q \pm P)/\sqrt{2}$  in the system-reservoir Hamiltonian as “phase-insensitive.” Although this coupling is symmetric with regards to the  $Q$  and  $P$  quadratures, it is not phase-insensitive, which can be verified by considering a unitary transformation which changes the phase of the cavity operators. However, an alternative form of the coupling Hamiltonian which is phase-insensitive is  $\Pi_c = a$ . We did not consider this form in the paper, but in fact we find that it gives almost identical results to those of the quadrature choices  $(Q \pm P)/\sqrt{2}$ , which we show in Fig. 1 (with normalized coupling rate  $\eta = g/\omega_c = 0.5$ ), and thus also exhibits correspondence with the classical model in the ultrastrong coupling regime.

As such, our argument about phase-insensitive phenomenological couplings giving correspondence with the classical phase-insensitive phenomenological dissipation model remains accurate, though it should be more properly interpreted as applying to the coupling operator choice  $\Pi_c = a$ , which also arises naturally in the recently proposed ab initio theory of loss in quantized cavities [2].

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**Fig. 1.** Example spectra computed for the quantized harmonic oscillator emitter coupled to a cavity mode, with  $g = 0.1$  and  $\kappa = 0.05g$ , but now showing  $\Pi_c = a$  as well as  $\Pi_c = (Q + P)/\sqrt{2}$  [which yields the same answer as  $\Pi_c = (Q - P)/\sqrt{2}$ ]. We can see that these models are in agreement, and thus both models also agree with the classical spectral results as shown in our original article [1].

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**Disclosures.** The authors declare no conflicts of interest.

**Data availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

## REFERENCES

1. S. Hughes, C. Gustin, and F. Nori, "Reconciling quantum and classical spectral theories of ultrastrong coupling: role of cavity bath coupling and gauge corrections," *Opt. Quantum* **2**, 133–139 (2024).
2. C. Gustin, J. Ren, S. Franke, *et al.*, "Dissipation in the broadband and ultrastrong coupling regimes of cavity quantum electrodynamics: an ab initio quantized quasinormal mode approach," [arXiv](#) (2025).