



## Squeezing-enhanced quantum sensing with quadratic optomechanics

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Cavity optomechanical (COM) sensors, enhanced by quantum squeezing or entanglement, have become powerful tools for measuring ultra-weak forces with high precision and sensitivity. However, these sensors usually rely on linear COM couplings, a fundamental limitation when measurements of the mechanical energy are desired. Very recently, a giant enhancement of the signal-to-noise ratio was predicted in a quadratic COM system. Here we show that the performance of such a system can be further improved surpassing the standard quantum limit by using quantum squeezed light. Our approach is compatible with available engineering techniques of advanced COM sensors and provides new opportunities for using COM sensors in tests of fundamental laws of physics and quantum metrology applications.

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### 1. INTRODUCTION

The development of quantum-enhanced sensors aimed at the sensitive measurement of time, temperature, pressure, or electromagnetic fields has witnessed considerable progress in recent years [1,2], with a broad spectrum of approaches including the use of elementary particles [3–6], superconducting circuits [7–9], optical systems [10–12], and solid-state mechanical devices [13,14]. In particular, cavity optomechanical (COM) [15–17] and electromechanical sensors [18–20] are remarkably well suited for the measurement of weak forces or very small displacements [21]. Importantly, their standard quantum limit (SQL), which results from the combined effects of backaction noise and photon shot noise, can be broken by the use of optical fields with appropriate quantum correlations (see e.g. [22,23]). For example, in an impressive recent experiment, the sub-SQL displacement measurement in a COM system with a macroscopic 40 kg mirror was achieved by injecting squeezed light in the otherwise empty port of the system [24].

COM displacement sensors typically rely on the linear coupling between the displacement of the mechanical element and the electromagnetic field. However, such a coupling is not appropriate for energy or phonon number measurements, which require instead an optomechanical coupling that is quadratic in the mechanical displacement [25]. This coupling also allows

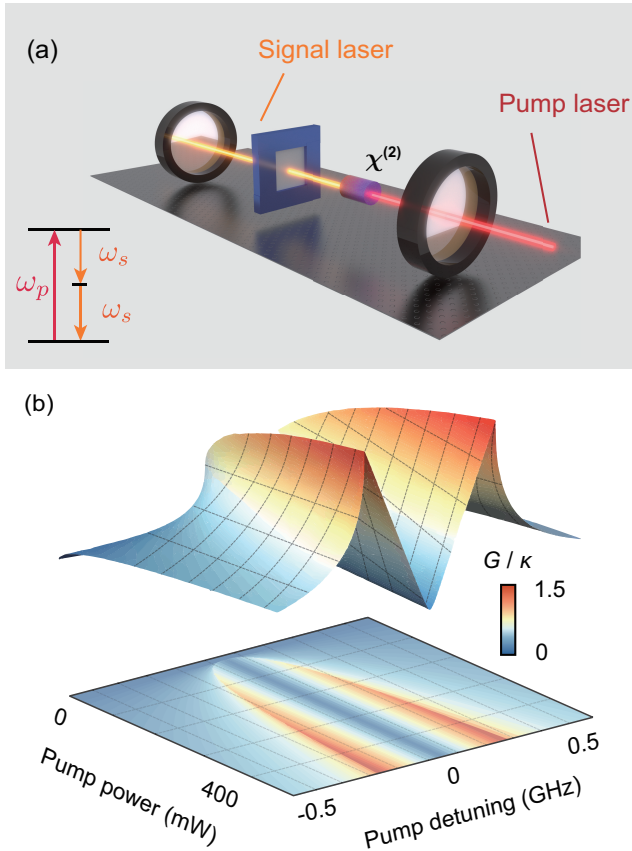
for applications such as two-phonon cooling [26], and a variety of quantum non-demolition (QND) measurements [27–32]. Quadratic COM systems (where the cavity detuning is proportional to the square of the mechanical displacement, i.e.,  $\omega_{\text{cav}}(x) \propto x^2$  [33]) have been demonstrated using, e.g., levitated nanospheres [34], membrane-in-the-middle cavities [33,35–37], photonic crystals [38,39], and atomic gases [40]. Also, selective linear or quadratic COM coupling was achieved via homodyne measurements and utilized to create non-Gaussian mechanical states [41,42]. However one known issue of quadratic coupling is the linear dissipative coupling typically associated with it and there has been significant interest in exploiting quantum noise interference to cancel the residual linear backaction in the bad-cavity limit, allowing one to make QND measurements of mechanical energy using a quadratic COM system [43]. A recent publication proposed a novel geometry that significantly solves this problem and results in a dramatic reduction of backaction noise [44].

In this paper we expand on the study of quadratic optomechanical sensors [28,43,44] and demonstrate theoretically that the inclusion of intracavity optical squeezing [48–50] can result in a remarkable improvement in their sensitivity. Our proposed scheme, which is compatible with other available techniques of fabricating and engineering advanced COM sensors, provides

a way to further enhance the power of quadratic COM sensors for applications ranging from quantum metrology to tests of fundamental laws of physics.

## 2. SQUEEZED QUADRATIC OPTOMECHANICS

We consider an ideal membrane-in-the-middle (MIM) Fabry-Pérot cavity with a thin dielectric membrane located either at a node or antinode of the standing wave mode and coupled quadratically to the field [33,51], allowing for quantum non-demolition readout of the membrane's phonon numbers [35]. An additional nonlinear  $\chi^{(2)}$  medium, coupled quadratically to the cavity field, induces intracavity squeezing, integrated with an intracavity. It is driven by a pump field of frequency  $\omega_p$  at twice the signal frequency  $\omega_s$ , [52], see Fig. 1(a). We limit our considerations to the case where the membrane has a low enough reflection that it will not split the cavity into two sub-cavities [53,54].



**Fig. 1.** Squeezing-enhanced quadratic COM sensing. (a) Schematic representation of the generation of intracavity squeezing within a Fabry-Pérot cavity. A thin dielectric membrane, at a node or antinode of the standing waves of the cavity, is coupled quadratically with the cavity field, allowing for quantum non-demolition readout of the membrane's phonon numbers [35]. The nonlinear  $\chi^{(2)}$  medium induces intracavity squeezing. (b) The parametric gain  $G/\kappa$  versus the pump power and the detuning of the pump mode. Here, we focus on a red detuning of the pump to generate the required parametric gain [45]. Thus,  $G/\kappa$  is decreased at zero-pump detuning, which does not follow the Lorentzian response of the cavity.

The intracavity second-order nonlinear optical process is described by the Hamiltonian [55]

$$\hat{H}_{\chi^{(2)}} = \hbar\Delta_c\hat{a}^\dagger\hat{a} + \hbar\Delta_p\hat{a}_p^\dagger\hat{a}_p + i\hbar\chi^{(2)}(\hat{a}^{\dagger 2}\hat{a}_p e^{i\theta} - \hat{a}^2\hat{a}_p^\dagger e^{-i\theta}), \quad (1)$$

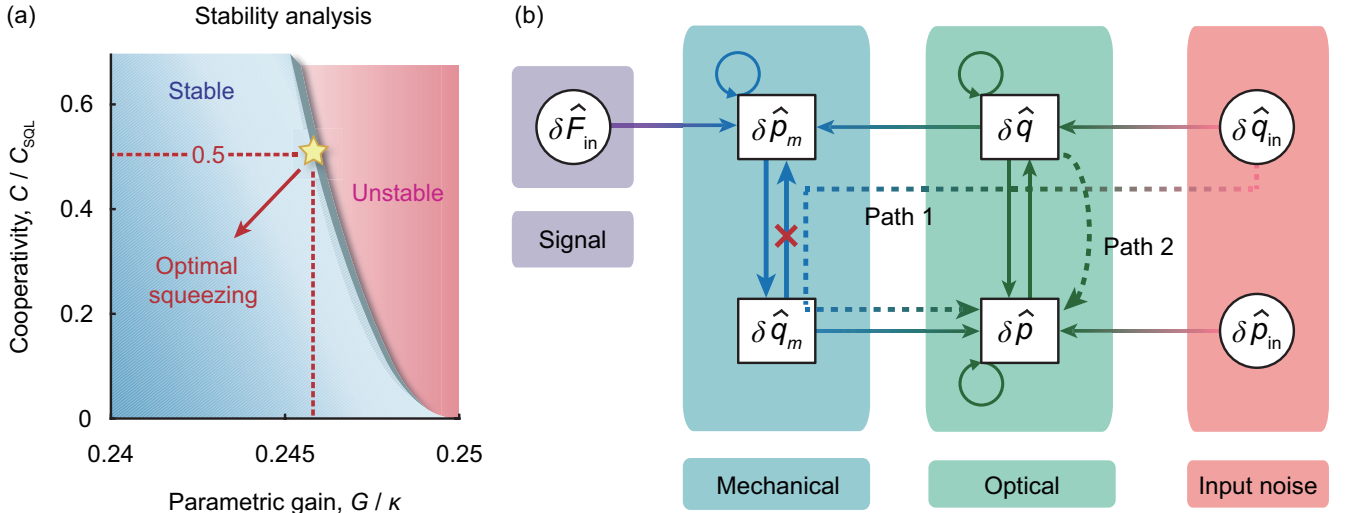
where  $\hat{a}_p$  and  $\hat{a}$  are the boson operators of pump and signal modes, of frequencies  $\omega_p = 2\omega_s$ ;  $\Delta_p$  is the detuning between the pump drive and the nearest cavity mode frequencies;  $\Delta_c$  is the detuning between the signal and the nearest cavity mode frequencies, and  $\theta$  is the associated phase of  $\chi^{(2)}$ .

We assume that the pump field is strong enough that it can be treated classically, and characterized by a large mean 'photon number'  $n_p$ . Eliminating the associated optomechanical interaction adiabatically and including the driving  $\mathcal{E}_c$  of the signal mode, we obtain the effective model Hamiltonian at its simplest level [33,45]:

$$\begin{aligned} \hat{H} = & \hbar\Delta_c\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}\Omega_m(\hat{q}_m^2 + \hat{p}_m^2) - \hbar g_0\hat{a}^\dagger\hat{a}\hat{q}_m^2 \\ & + i\hbar G(\hat{a}^{\dagger 2}e^{i\theta} - \hat{a}^2e^{-i\theta}) + i\hbar(\mathcal{E}_c\hat{a}^\dagger - \mathcal{E}_c^*\hat{a}), \end{aligned} \quad (2)$$

where  $\hat{q}_m$  and  $\hat{p}_m$  are the position and momentum operators of mechanical mode at frequency  $\Omega_m$ ;  $g_0$  represents single-photon COM coupling strength, which quantifies the interaction between a single phonon and a single photon;  $G = \chi^{(2)}\sqrt{n_p}$  is the nonlinear gain coefficient, and  $\mathcal{E}_c$  is the driving amplitude. We stress that both the quadratic COM coupling and the squeezing-enhanced COM systems were already well-established in experiments. For examples, a high-finesse MIM system was utilized for direct measurements of the membrane's displacement [33] and, by tuning the suitable position of the membrane, the quadratic coupling strength can be greatly enhanced for 3 orders of magnitude, indeed reaching a purely quadratic COM system [35,56]. Such a quadratic COM system was also experimentally demonstrated by levitating a nanosphere in a suitable potential [34]. We also note that in a recent experiment, by using an intra-cavity parametric amplifier, phase-sensitive manipulations of an input squeezed vacuum were demonstrated [57]. Similarly, loss suppressions and thus giant enhancement of sensitivities were also demonstrated in experiments by inserting such optical amplifiers into interferometers [58,59]. Indeed, the merits of quantum squeezing in enhancing linear COM sensors have already been confirmed in experiments, and the main purpose of our present work is to confirm that such a merit also exists for a quadratic COM system. Hence it is reasonable to expect that even for a hybrid COM system with both linear and quadratic couplings, the positive effects of quantum squeezing will still exist, which we plan to further study in our future work (we note that in a very recent work, the linear coupling was confirmed to be not detrimental for quantum entanglement emerging in such a hybrid COM system [60]).

Here we use the experimentally feasible parameter values, i.e., the cavity quality factor  $Q = 1 \times 10^7$  [61], the total optical decay rate  $\kappa/2\pi = 3$  MHz [61], including both the decay rate  $\kappa_{\text{ex}}$  at the input mirror and the intra-cavity decay rate  $\kappa_0$ , with 'efficiency'  $\eta_c = \kappa_{\text{ex}}/(\kappa_0 + \kappa_{\text{ex}})$ , and the mechanical quality factor  $Q_m = 5 \times 10^8$ , with the mechanical frequency  $\Omega_m/2\pi = 1$  MHz [61], and the effective mass  $m_{\text{eff}} = 1$  ng [61], and the associated decay rate  $\Gamma_m$  [61]. We note that a second-order nonlinearity of  $\chi^{(2)}/2\pi = 80$  kHz was realized [52], confirming the feasibility of  $G = 0.246\kappa$ . Very recently, a new optomechanical experiment using an optical crystal with third-order nonlinearity has



**Fig. 2.** (a) System stability versus multi-photon cooperativity and parametric gain (see Supplement 1 [46] for the details of calculations). The parameters chosen in this paper are confirmed to be within the stable region. The stable and unstable regions can be tuned by altering the parametric gain or the cooperativity, while the optimally sensitive case is close to the border between the stable and unstable regimes. (b) Flow chart representation of Eq. (3), as done in Ref. [47].  $\delta\hat{q}$  and  $\delta\hat{p}$  are the ‘position’ and ‘momentum’-like operators of the optical field, respectively, and  $\delta\hat{q}_{in}$  and  $\delta\hat{p}_{in}$  are the associated input noise, respectively. The coefficient from  $\delta\hat{q}_m$  to  $\delta\hat{p}_m$  is zero (indicated by a Red ‘X’). The original backaction noise in path 1 is eliminated by introducing intracavity squeezing in the antinoise path 2. The path from  $\delta\hat{q}_m$  to  $\delta\hat{p}_m$  is canceled in quadratic COM systems, which results in the enhancement of the destructive interference in the backaction noise. Compared with no squeezing case, the correlation between  $\delta\hat{q}$  and  $\delta\hat{p}$  results in the destructive interference, achieving the suppression of the backaction noise. The experimental parameters are chosen as  $\Omega_m/2\pi = 1$  MHz,  $m_{eff} = 1$  ng,  $Q_m = 5 \times 10^8$ ,  $\kappa/2\pi = 3$  MHz.

demonstrated that with this nonlinearity-assisted system, optical spring effect can be enhanced [62]. Figure 1(b) shows that the nonlinear gain coefficient  $G$  increases with the pump laser power and the second-order nonlinearity, indicating the required parametric gain occurs at large pump detunings [45].

Neglecting the higher-order nonlinear terms [63] in the quantum fluctuations results in coupled linear equations

$$\begin{aligned} \delta\dot{\hat{a}} &= -\left(i\Delta + \frac{\kappa}{2}\right)\delta\hat{a} + 2g\delta\hat{q}_m + 2Ge^{i\theta}\hat{a}^\dagger \\ &\quad + \sqrt{\eta_c}\kappa\delta\hat{f}_{a,in} + \sqrt{(1-\eta_c)\kappa}\delta\hat{f}_{a,0}, \\ \delta\dot{\hat{q}}_m &= \Omega_m\delta\hat{p}_m, \\ \delta\dot{\hat{p}}_m &= -\Gamma_m\delta\hat{p}_m + 2g\delta\hat{q} + \sqrt{2\Gamma_m}\hat{F}_{in}, \end{aligned} \quad (3)$$

where  $g = g_0\bar{q}_m|\alpha|$  is the effective optomechanical coupling constant (see Supplement 1 for the detailed classical mean value equations of motion);  $\hat{f}_{a,in}$  and  $\hat{f}_{a,0}$  are the noise operators associated with the input cavity mirror and the internal losses, and  $\Delta = \Delta_c - g_0\bar{q}_m^2$  is the effective optical detuning. The flowchart of Fig. 2(b) illustrates the various couplings involved in Eq. (3). A variable on the right-hand side of an equation of motion is connected to a variable on the left-hand side by arrows, showing that  $\delta\hat{p}_m$  is indeed independent of  $\delta\hat{q}_m$ , a consequence of the cancellation of the associated coefficient, i.e.,  $-\Omega_m + 2g_0n_c = 0$ , where  $n_c = \Omega_m/(2g_0)$ .

Direct measurements of intracavity fields are typically challenging, and one often measures the field that escapes the resonator instead. The relationship between the input field and the output field is given by the input-output relation  $\hat{a}_{out} = \sqrt{\eta_c}\kappa\hat{a} - \hat{a}_{in}$  [63]. As illustrated in Fig. 2(a-b), the parameters used in our work are indeed in the optimally sensitive regime at the border between the stable and unstable regions. Figure 2(b) shows that in the quadratic COM system under consideration

the flow of signal and noise between  $\delta\hat{q}_m$  and  $\delta\hat{p}_m$  is unidirectional, in contrast to the situation for linear COM systems. This causes the mechanical susceptibility of the quadratic COM sensor to differ from the expression  $\Omega_m/(\Omega_m^2 - \Omega^2 - i\Omega\Gamma_m)$  of those systems [63].

One way to measure the frequency-dependent force noise is homodyne detection [64], whereby the output signal is mixed at a 50:50 beam splitter with a local oscillator, with a phase  $\phi$  between the signal and the reference field. The photocurrent  $\hat{I}_\phi$  at the output of the balanced detector is then proportional to a rotated field quadrature

$$\delta\hat{q}_{out}^\phi[\Omega] = \delta\hat{q}_{out}\cos\phi + \delta\hat{p}_{out}\sin\phi. \quad (4)$$

Introducing the correlation functions [65]

$$\begin{aligned} \langle\delta\hat{q}_u[\omega]\delta\hat{q}_u[\Omega]\rangle &= \langle\delta\hat{p}_u[\omega]\delta\hat{p}_u[\Omega]\rangle = \frac{1}{2}\delta(\omega + \Omega), \\ \langle\delta\hat{q}_u[\omega]\delta\hat{p}_u[\Omega]\rangle &= -\langle\delta\hat{p}_u[\omega]\delta\hat{q}_u[\Omega]\rangle = \frac{i}{2}\delta(\omega + \Omega), \\ \langle\delta\hat{F}_{th}[\omega]\delta\hat{F}_{th}[\Omega]\rangle &= \bar{n}_m\delta(\omega + \Omega), \end{aligned} \quad (5)$$

where  $u = in, 0$  and  $\bar{n}_m = [\exp(\hbar\Omega_m/k_B T) - 1]^{-1}$  denotes the thermal phonon occupancy. The output amplitude and phase quadrature spectrum can be expressed as [64]

$$\begin{aligned} \bar{S}_{qq}^{out}[\Omega] &= \frac{1}{2}\langle\{\delta\hat{q}_{out}[\Omega]\delta\hat{q}_{out}[-\Omega]\}\rangle \\ &= \frac{1}{2}\mathcal{K}_-[\Omega] + \bar{n}_m|\mathcal{N}_-[\Omega]|^2, \\ \bar{S}_{pp}^{out}[\Omega] &= \frac{1}{2}\langle\{\delta\hat{p}_{out}[\Omega]\delta\hat{p}_{out}[-\Omega]\}\rangle \\ &= \frac{1}{2}\mathcal{K}_+[\Omega] + \bar{n}_m|\mathcal{N}_+[\Omega]|^2, \end{aligned} \quad (6)$$

in the above two equations, we introduced the following definitions

$$\begin{aligned}\mathcal{K}_- &= |\mathcal{A}_-|^2 + |\mathcal{B}_-|^2 + |\mathcal{C}_-|^2 + |\mathcal{D}_-|^2, \\ \mathcal{K}_+ &= |\mathcal{A}_+|^2 + |\mathcal{B}_+|^2 + |\mathcal{C}_+|^2 + |\mathcal{D}_+|^2.\end{aligned}\quad (7)$$

The parameters  $\mathcal{A}_\pm$ ,  $\mathcal{B}_\pm$ ,  $\mathcal{C}_\pm$ ,  $\mathcal{D}_\pm$ , and  $\mathcal{N}_\pm$  can be derived through straightforward algebraic calculations (see Supplement 1 [46] for their lengthy expressions).  $\mathcal{K}_\pm$  denotes the contributions of shot noise and backaction noise to the output amplitude or phase quadrature spectrum  $\bar{S}_{\text{qq,pp}}^{\text{out}}$ , while  $\mathcal{N}_\pm$  is from the noise imprinted by mechanical motion. The symmetrized cross-correlation spectrum is then written as

$$\begin{aligned}\bar{S}_{\text{pq}}^{\text{out}}[\Omega] &= \frac{1}{2} \langle \{ \delta \hat{q}_{\text{out}}[\Omega] \delta \hat{p}_{\text{out}}[-\Omega] \} \rangle \\ &= \Re \left\{ \frac{1}{2} \mathcal{K}_{\text{co}}[\Omega] + \bar{n}_m \mathcal{N}[\Omega] \right\},\end{aligned}\quad (8)$$

with  $\mathcal{K}_{\text{cr}} = \mathcal{B}_- \mathcal{A}_+^* - \mathcal{A}_- \mathcal{B}_+^* + \mathcal{D}_- \mathcal{C}_+^* - \mathcal{C}_- \mathcal{D}_+^*$ , and  $\mathcal{K}_{\text{si}} = \mathcal{A}_- \mathcal{A}_+^* + \mathcal{B}_- \mathcal{B}_+^* + \mathcal{C}_- \mathcal{C}_+^* + \mathcal{D}_- \mathcal{D}_+^*$ . Here  $\mathcal{K}_{\text{co}} = \mathcal{K}_{\text{cr}} + i\mathcal{K}_{\text{si}}$ , which contains the squeezed-dependent correlations between shot noise and backaction noise, and  $\mathcal{N} = \mathcal{N}_+^* \mathcal{N}_-$  [66]. The output spectrum thus contains amplitude or phase vacuum noises, thermal occupations, and quantum correlations [64], viz.,

$$\begin{aligned}\bar{S}_{\text{II}}[\Omega] &= \frac{1}{2} \langle \{ \delta \hat{q}_{\text{out}}^\phi[\Omega] \delta \hat{q}_{\text{out}}^\phi[-\Omega] \} \rangle \\ &= \bar{S}_{\text{qq}}^{\text{out}} \cos^2 \phi + \bar{S}_{\text{pp}}^{\text{out}} \sin^2 \phi + \bar{S}_{\text{pq}}^{\text{out}} \sin(2\phi) \\ &= \mathcal{R}_m[\Omega] (\bar{n}_m + \bar{n}_{\text{add}}[\Omega]).\end{aligned}\quad (9)$$

By tuning the squeezed parameters  $G$  and  $\theta$ , the cross term  $\mathcal{K}_{\text{co}}$  in  $\bar{S}_{\text{pq}}^{\text{out}}$  can become negative, allowing for cancellation of backaction noise and shot noise. The mechanical response of our quadratic COM sensor to the detected force signal is derived as

$$\mathcal{R}_m = |\mathcal{N}_-|^2 \cos^2 \phi + |\mathcal{N}_+|^2 \sin^2 \phi + \Re \mathcal{N} \sin(2\phi). \quad (10)$$

The value of  $\mathcal{R}_m$  can be tuned with the squeezing parameters  $G$  and  $\theta$ , leading to effective amplification of the force signal when  $\mathcal{R}_m > 1$  [67]. The added noise is

$$\bar{n}_{\text{add}} = \frac{\mathcal{K}_- \cos^2 \phi + \mathcal{K}_+ \sin^2 \phi + \Re \mathcal{K}_{\text{co}} \sin(2\phi)}{2\mathcal{R}_m}. \quad (11)$$

The added noise includes both the shot noise and the backaction noise, contributing to the total force noise spectrum for quantifying the sensitivity of the force measurement

$$\bar{S}_{\text{FF}}[\Omega] = 2\hbar m_{\text{eff}} \Gamma_m \Omega_m (\bar{n}_m + \bar{n}_{\text{add}}). \quad (12)$$

As detailed in Ref. [23], quantum correlations, arranging destructive interference of the imprecision noise and the quantum backaction noise, can be observed in the measured spectrum by detecting rotated quadratures, including amplitude and phase fluctuations, as opposed to standard phase measurements. Also, the thermal noise, subtracted to reveal quantum noise, can be suppressed by considering a feasible bath temperature of 0.2 K with a cavity placed inside a dilution refrigerator [68].

### 3. SQUEEZING-ENHANCED SENSING

Quantum squeezing is known to be capable of increasing the COM sensitivity [11]. In the absence of a  $\chi^{(2)}$  medium (or with  $\theta = 0$ ), the force sensitivity is limited by the SQL where in

the limit of  $\kappa \gg \Omega$  the symmetrized noise spectrum takes the simplified form

$$\bar{n}_{\text{add}}[\Omega] = C + \frac{1}{16\eta_c C \Gamma_m^2 |\chi_m|^2}, \quad (13)$$

where the multi-photon cooperativity is defined as

$$C = \frac{4g^2}{\kappa \Gamma_m}. \quad (14)$$

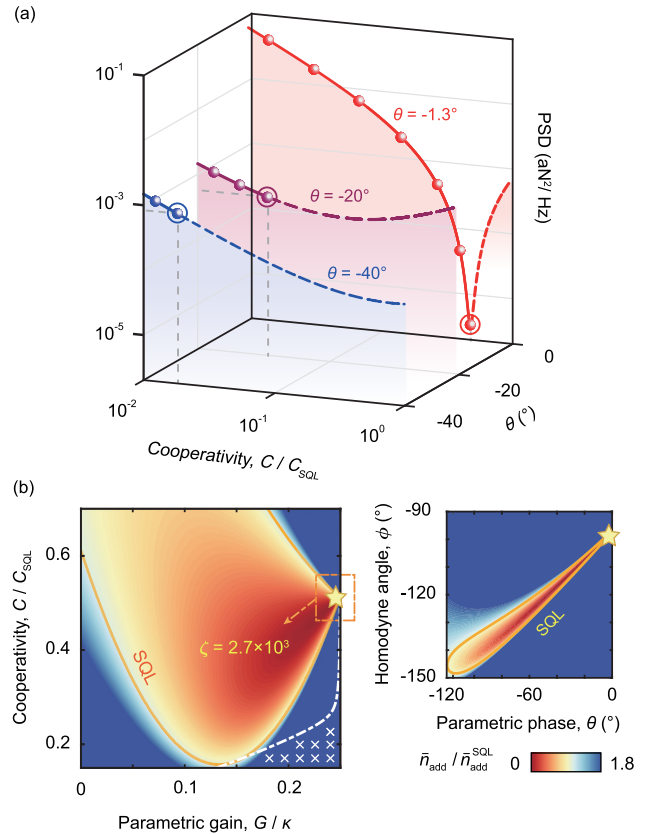
It is minimized to

$$\bar{n}_{\text{add}}^{\text{SQL}}[\Omega] = \frac{1}{2\sqrt{\eta_c} \Gamma_m |\chi_m|}, \quad (15)$$

for

$$C \equiv C_{\text{SQL}} = \frac{1}{4\sqrt{\eta_c} \Gamma_m |\chi_m|}, \quad (16)$$

where  $\chi_m = -\Omega_m / (\Omega^2 + i\Omega \Gamma_m)$  is the mechanical susceptibility of the system, which quantifies the response of the oscillator to



**Fig. 3.** Performance of the squeezing-enhanced quadratic COM sensor. (a) Power spectral density (PSD) as a function of cooperativity and  $\theta$ . The solid and dashed curves denote the PSD in the stable or unstable region, respectively, and the circles give the minimum PSD in the stable region. The mechanical parameters are  $\Omega_m/2\pi = 1$  MHz,  $m_{\text{eff}} = 1$  ng,  $Q_m = 5 \times 10^8$ . (b) Quantum noise below the SQL, with suitable squeezed parameters. The white dashed curve denotes the mechanical response of  $\mathcal{R}_m[\Omega] = 1$ . The signal is amplified except for the marked region ('x'). Added noise  $\bar{n}_{\text{add}}/\bar{n}_{\text{add}}^{\text{SQL}}$  as a function of the phase of the local oscillator  $\phi$  and the parametric phase  $\theta$ , and the remaining parameters are  $G/\kappa = 0.246$ ,  $C/C_{\text{SQL}} = 0.5$ .

external forces. So that in the absence of squeezing the minimum output force noise is given by

$$\bar{S}_{\text{FF}}^{\text{SQL}}[\Omega] = 2\hbar m_{\text{eff}}\Gamma_m\Omega_m(\bar{n}_m + \bar{n}_{\text{add}}^{\text{SQL}}). \quad (17)$$

It is clear from Fig. 2(b) that a direct way to counter the effect of the backaction noise is to introduce another path from  $\delta\hat{q}$  to  $\delta\hat{p}$  using intracavity squeezing [47]. Then, without standard phase detection, the imprecision and backaction noises can be correlated by tuning the parametric phase of  $\chi^{(2)}$  medium. Thus, a decreased parametric phase corresponds to a lower detection sensitivity in the stable region because of the narrower range for the multi-photon cooperativity [Fig. 3(a)].

To simultaneously achieve quantum noise suppression and force signal amplification, the values of the scaled cooperativity ( $C/C_{\text{SQL}}$ ) and the squeezed parameters should be chosen within the stable region [Fig. 3(b)]. For the parameters of our numerical examples it yields the quantum noise that is 3.5 decibels below the SQL [see Fig. 3(b)]. We note that in a very recent experiment, using a linear COM system assisted by quantum correlations [24], a joint quantum uncertainty that is 3 decibels below the SQL was shown after subtracting thermal noises. Here, we define the degree of the squeezing as

$$\sigma = \lg(S_{\text{FF}}/S_{\text{SQL}}). \quad (18)$$

Quantum-enhanced force measurement can be simply characterized by the enhancement factor due to the squeezing

$$\zeta = \frac{\min \bar{S}_{\text{FF}}(G=0, \theta=0)}{\min \bar{S}_{\text{FF}}(G \neq 0, \theta \neq 0)}. \quad (19)$$

When the thermal noise of the system has been significantly reduced, for instance by utilizing dilution refrigeration or pre-cooling, further enhancement could be further improved by injecting squeezed vacuum into the optical cavity [48,69].

Notably, the mechanical susceptibility can transduce force into the displacement of the membrane and quantify the response of the mechanical resonator to the detected force [63]. In the quadratic COM system, the mechanical response [derived from Eq. (10)] to the detected force is significantly enhanced [Fig. 4(a)] due to the larger mechanical susceptibility, enabling

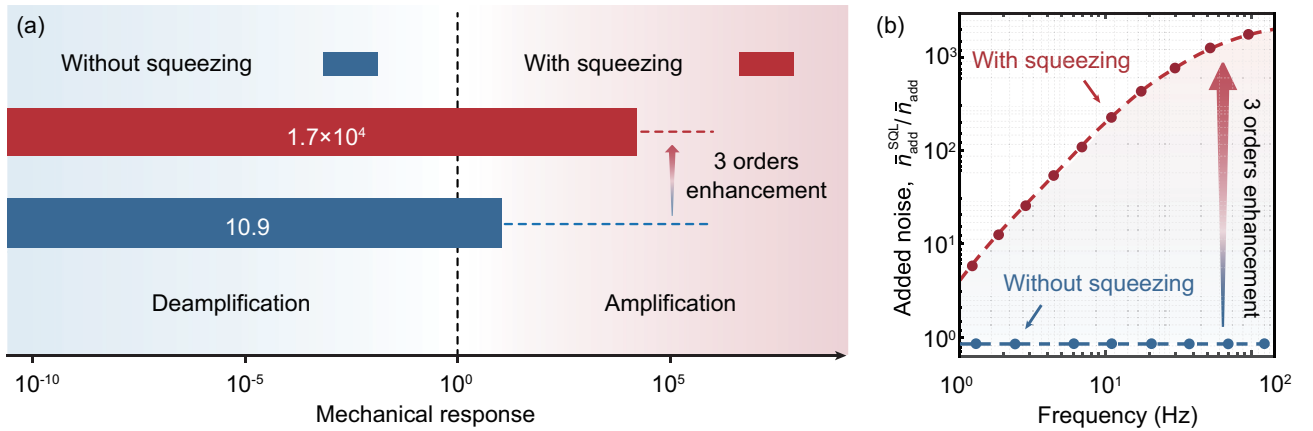
a remarkable amplification of the force signal and corresponding to a low quantum noise given by Eq. (11). Hence, the enhanced mechanical response is important to achieve better measurement sensitivity. As shown in Fig. 4(a), the optimal mechanical response derived from Eq. (10) for the quadratic COM sensor can be further enhanced by introducing intracavity squeezing. Therefore, from the analyses made above, according to Eq. (19), combined with the additional merit of quantum squeezing, the quadratic COM systems can be more beneficial by incorporating the additional merit of quantum squeezing [Fig. 4(b)].

The advantage of the quadratic COM system is mainly manifested in quantum-noise-dominated situations, which becomes marginal with increasing thermal noises. The high sensitivity is predicted close to the boundary between the stable and unstable regimes [70], as shown in Fig. 2(a). The sensitivity of force measurements is mainly limited by the thermal Langevin force, with the PSD given by [71]

$$\bar{S}_{\text{FF}}^{\text{th}} = 2m_{\text{eff}}k_{\text{B}}T\frac{\Omega_m}{Q_m}, \quad (20)$$

where  $k_{\text{B}}$  is Boltzmann's constant, and  $T$  is the bath temperature. In practice, thermal noise can lower the measurement sensitivity. Nevertheless, we estimate that under realistic conditions, the force sensitivity can still reach  $(10.2 \text{ aN})^2/\text{Hz}$  even at room temperature (which can be optimized as  $(0.26 \text{ aN})^2/\text{Hz}$  at cryogenic temperatures), approaching the level of the state-of-the-art sensors with force noises in the range of  $10 - 100 \text{ aN Hz}^{-1/2}$  at room temperature (or less than  $1 \text{ aN Hz}^{-1/2}$  at cryogenic temperatures) [72]. We estimate that by using the state-of-the-art membrane [73], the force noise even can be reduced to  $(9.9 \text{ zN})^2/\text{Hz}$  at the temperature of 0.2 K.

For a highly reflective membrane, another practical concern is the backaction arising from the underlying linearity of hybridized modes [28,43,44]. However, this technical challenge has not prevented the advances in quadratic COM systems [28,43,44]. In fact, linear backaction can be effectively suppressed in practice through structural design or active feedback [42,44], or by using highly tunable COM systems such as levitated particles, photonic crystals, electromechanical devices, and cold atoms [34,39,40,74,75]. Indeed, the merits of quantum



**Fig. 4.** (a) A comparison of the optimal mechanical responses for standard and squeezed quadratic COM sensors. The mechanical response describes the amplification ( $\mathcal{R}_m > 1$ ) or the deamplification ( $\mathcal{R}_m < 1$ ) of the force signal imprinted on the output quadratures [67]. The optimal mechanical response for the quadratic COM sensor can be further enhanced by introducing intracavity squeezing. (b) For the quadratic COM sensors, the force sensitivity can be significantly enhanced by intracavity squeezing (with the same experimentally accessible parameters) [61]. The remaining parameters are  $\Omega_m/2\pi = 1 \text{ MHz}$ ,  $m_{\text{eff}} = 1 \text{ ng}$ ,  $Q_m = 5 \times 10^8$ ,  $\kappa/2\pi = 3 \text{ MHz}$ ,  $G/\kappa = 0.246$ ,  $C/C_{\text{SQL}} = 0.5$ ,  $\theta = 1.3^\circ$ , and  $\phi = -98^\circ$ . We produce Fig. 4(a-b) according to Eq. (10) and Eq. (19), respectively.

squeezing in enhancing linear COM sensors have already been confirmed in experiments, and the main purpose of our present work is to confirm that such a merit also exists for a quadratic COM system. Hence it is reasonable to expect that even for a hybrid COM system with both linear and quadratic couplings, the positive effects of quantum squeezing will still exist—a specific topic we plan to further calculate and verify in our next work.

#### 4. CONCLUSION

In summary, we have shown that the performance of quadratic COM sensors can be significantly enhanced by intracavity squeezing. We find that the mechanical response to weak force signals can be significantly amplified with considerably reduced quantum noise in these systems, promising sub-SQL force measurements with experimentally accessible parameters. We expect that by combining it with other existing techniques of fabricating and operating COM-based sensors, such as those involving feedback control [15,76] or advanced materials with much higher mechanical  $Q$  factors [73,77], it is possible to further improve its performance in practice. Such an improved COM sensor can be useful for a wide range of applications requiring ultrahigh sensitivity [78–83]. It is our hope that these results will stimulate further efforts toward building and utilizing quantum-squeezing-enhanced sensors, such as those based on levitated spheres [34], cold atoms [40], dissipative or near-field COM systems [42].

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**Data availability.** Data underlying the results presented in this paper may be obtained from the authors upon reasonable request.

**Supplemental document.** See Supplement 1 for supporting content.

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