STRUCTURED LIGHT

# Longitudinal fields and transverse rotations

Electromagnetic fields in light waves are mainly transverse to propagation direction but actually also have longitudinal components, which may give rise to unexpected optical phenomena involving the angular momentum of light, such as transverse spin and optical torques.

Filippo Cardano and Lorenzo Marrucci

Light waves, as all electromagnetic waves, are said to be 'transverse', meaning that the electric and magnetic fields oscillate in a plane that is orthogonal to the wave propagation direction. However, this is strictly true only for an infinitely extended plane wave. Any realistic, finite wave deviates from this simple rule and also exhibits, usually small, 'longitudinal' oscillating field components, parallel to the wave propagation direction. The interplay of these often-neglected components with the ordinary transverse ones, in turn, underlies a number of interesting optical phenomena, typically involving the angular momentum of light and its transfer to matter. Two striking examples of such phenomena[1,2] are now reported in *Nature Photonics*.

The first is a new development on the intriguing phenomenon known as transverse spin, occurring in all laterally confined electromagnetic waves[3]. This is a continuous rotation of the optical fields in time, analogous to that occurring for circularly or elliptically polarized light that is associated with the ordinary 'longitudinal' spin. In the case of transverse spin, however, the field rotation is taking place around an axis that is orthogonal to the wave propagation, as opposed to parallel. Hence, it needs the simultaneous presence of transverse and longitudinal field components, oscillating in quadrature to each other. Interestingly, the rotation direction of this transverse spin is locked to the wave propagation direction[4,5]. Reporting in *Nature Photonics*, Jörg Eismann et al. now demonstrate that even fully unpolarized waves, when laterally confined, exhibit a transverse spin[1]. The elegant general theory of three-dimensional polarization developed in their work shows that this transverse form of polarization is indeed essentially unaffected by the random phases between the two polarization components of the input unpolarized light. The appearance of the transverse spin has been confirmed in two separate experiments, distinguished by the kind of transverse confinement; one experiment involved a strongly focused beam, the other an evanescent field of a totally reflected wave[1]. In both cases, the local transverse spin is then revealed by scanning the space within the beam with a nanoparticle and detecting the degree of circular polarization of the resulting laterally scattered waves.

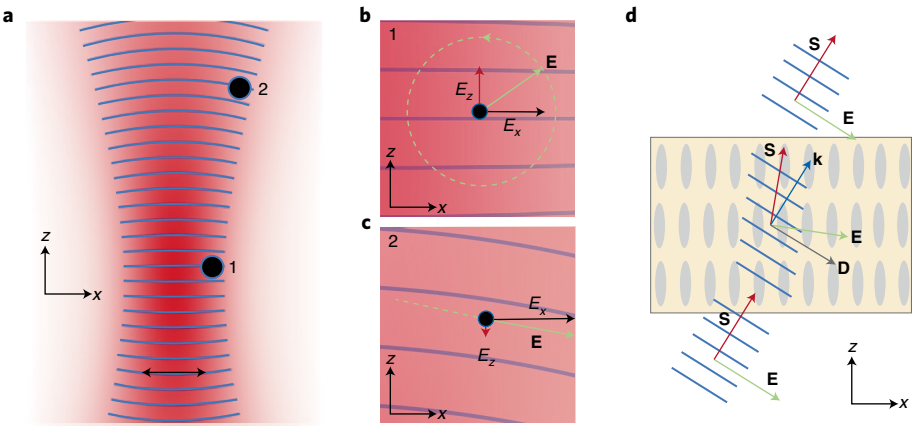


**Fig. 1 | Longitudinal and transverse components of optical fields, in free space and in anisotropic media. a**, A Gaussian beam, polarized along the *x* direction, propagates along the *z* direction. Wavefronts are sketched as violet lines. **b**, In the focal plane (position 1 in panel **a**), $E_z$ (red arrow) oscillates in quadrature with $E_x$ (black arrow), giving rise to a total electric field (green arrow) that rotates counterclockwise. This corresponds to a spin angular momentum density in the *y* direction, that is a transverse spin. **c**, At position 2 (see panel **a**), wavefronts are tilted with respect to the main propagation direction while the amplitude gradient is negligible. Hence $E_z$ oscillates approximately in phase with $E_x$, with the total electric field varying linearly along a unique direction. **d**, An optical wave propagates through an anisotropic medium (coloured rectangle) with the wavevector oblique with respect to the optic axis (*z* direction). Within the medium, the angle between fields **E** and **D** leads to an optical torque on the molecules. The same angle between **S** and **k** originates a lateral walk-off of the local energy flow, yielding in turn a variation of the light orbital angular momentum density, opposite to the torque.

Writing in *Nature Photonics*, Mohamed El Ketara and colleagues report the discovery of a new feature in an 'old' system, which has been widely investigated since the early 1980s — a weakly focused linearly polarized light beam propagating, at normal incidence, through a film of nematic liquid crystal (NLC)[2]. The NLC molecules have a common average orientation, as in a uniaxial crystal, but unlike crystals they can be easily reoriented by external fields, which generate internal torques on the molecules. The NLC is prepared with 'homeotropic' alignment, that is, its molecules are initially oriented (on average) perpendicular to the containing glasses and parallel to the beam propagation direction. This specific geometry is known to exhibit a threshold phenomenon named optical Fréedericksz transition[6]. The optical field is initially orthogonal to the molecules and hence its optical torque vanishes. If the light intensity is raised above a certain threshold, an instability sets in that allows random fluctuations in the molecular orientation to grow, ultimately leading to a large reorientation of the whole medium. When intensity is well below the threshold, as in the experiments conducted by El Ketara et al., nothing is expected to occur. Surprisingly, El Ketara et al. found that, when the NLC sample is located somewhat off the focal region, a distinct pattern of molecular reorientation is induced by

light, as revealed via its birefringent action on a probe beam. Even more interesting patterns are observed when the impinging beam is a vector-vortex beam, with a space-variant polarization around the beam axis. These reorientation patterns must correspond to analogous patterns of an optical torque in the medium. But, given that intensity is below threshold, why is the torque non-vanishing at all? As explained by El Ketara et al., the reason is again linked to the presence of longitudinal field components. Here, however, as we will see, it is essential that these components oscillate in phase with the transverse ones.

To better understand the two phenomena, let us first discuss the general mechanism by which longitudinal field components appear in the light wave and why their oscillation is either in-phase or in-quadrature with respect to that of transverse components. Consider a paraxial light beam of wavelength $\lambda$ propagating along the $z$ axis. The wave electric field distribution can be generally represented (in complex-number notation) as $\mathbf{E} = \mathbf{A}(x,y,z)e^{i(kz-\omega t)}$, where $\omega$ is the (angular) frequency, $k{=}2\pi/\lambda$ the wave number, and $\mathbf{A}$ is the complex vector amplitude, taken to be slowly varying on the scale $\lambda$. Inserting this expression in the local Gauss law, we obtain (for a homogeneous isotropic medium)

$$\nabla\cdot\mathbf{E} \approx \left(ikA_z + \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}\right)e^{i(kz-\omega t)} = 0, \tag{1}$$

where in the second expression we have neglected the small $z$ derivative of the longitudinal amplitude $A_z$. This equation links the longitudinal ($z$ component) field with the transverse ones ($x$,$y$ components). In particular, if we orient the $x$ axis along the local transverse gradient of the field, so that the $y$ derivative vanishes, we obtain the following expression for the longitudinal field complex amplitude:

$$A_z = \frac{i}{k}\frac{\partial A_x}{\partial x} = \frac{i}{k}\frac{\partial |A_x|}{\partial x}e^{i\varphi_x} - \frac{1}{k}\frac{\partial \varphi_x}{\partial x}|A_x|e^{i\varphi_x}, \tag{2}$$

where in the second expression we set $A_x = |A_x|e^{i\varphi_x}$ so as to distinguish the effects of amplitude versus phase gradients. We conclude that the $z$ component of the field is linked only to the $x$ component of the transverse field, that is, the component along the local gradient, while it is totally independent of the $y$ component, orthogonal to the local gradient. This explains why a random phase relation between the $x$ and $y$ components of the wave, as in unpolarized light, does not affect the phase relation between the $x$ and $z$ components, hence allowing even a totally unpolarized wave to develop a transverse spin. Moreover, we can see from Eq. (2) that a local amplitude gradient leads to an in-quadrature phase relation between the longitudinal and the transverse fields, while a phase gradient (as in curved wavefront) leads to an in-phase relation. These field relations are pictorially illustrated in Fig. 1a–c. We stress that this simple analysis applies to a paraxial field, while non-paraxial regimes typical of nano-optics require a more complete theory.

Once the fields are known, the local (time-averaged, electric-field-related) density of spin angular momentum for a monochromatic wave can be computed by the expression $\mathbf{s} = \frac{\varepsilon_0}{4\omega}\mathrm{Im}\left[\mathbf{E}^*\times\mathbf{E}\right]$ (ref. [3]). It is immediate to verify from this expression that only an in-quadrature phase relation between orthogonal field components can give rise to a non-vanishing spin density. In particular, a transverse spin may arise from the field $z$ component in a direction that is locally orthogonal to the field transverse gradient ($x$ axis). For example, using Eq. (2), we obtain a $y$-oriented spin density given by the following expression:

$$s_y = \frac{\varepsilon_0}{2\omega}\mathrm{Im}\left[E_z^*E_x\right] = -\frac{\varepsilon_0}{\omega k}\left(\frac{\partial|A_x|^2}{\partial x}\right). \tag{3}$$

It should be noticed in particular that the transverse spin is specifically linked to the existence of transverse gradients of field amplitude (or intensity), as they give rise to the in-quadrature phase relation between longitudinal and transverse fields.

On the other hand, the (time-averaged) optical torque per unit volume generated by light in an anisotropic dielectric medium is given by the general expression $\mathbf{\Gamma} = \frac{1}{2}\mathrm{Re}\left[\mathbf{D}^*\times\mathbf{E}\right]$, where $\mathbf{D} = \boldsymbol{\varepsilon}\cdot\mathbf{E}$ is the electric displacement field, $\boldsymbol{\varepsilon}$ being the permittivity tensor. For a uniaxial medium with optic axis specified by the unit vector $\mathbf{n}$ we have in particular $\mathbf{D} = \varepsilon_\perp\mathbf{E} + \Delta\varepsilon(\mathbf{n}\cdot\mathbf{E})\mathbf{n}$, where $\varepsilon_\perp$ ($\varepsilon_\parallel$) is the permittivity for field perpendicular (parallel) to $\mathbf{n}$ and $\Delta\varepsilon = \varepsilon_\parallel - \varepsilon_\perp$. Using again Eq. (2) (adjusted to take into account the medium dielectric anisotropy) and the fact that $\mathbf{n}$ is oriented along the $z$ axis, we then obtain a $y$-oriented torque given by the following expression:

$$\Gamma_y = \frac{\Delta\varepsilon}{2}\mathrm{Re}\left[E_z^*E_x\right] = -\frac{\varepsilon_\perp\Delta\varepsilon}{2k\varepsilon_\parallel}\left(\frac{\partial\varphi_x}{\partial x}\right)|A_x|^2. \tag{4}$$

As anticipated, while the transverse spin probes the in-quadrature longitudinal component, the optical torque is found to be proportional to the in-phase longitudinal field component. The torque hence requires transverse phase gradients to be present in the field, or equivalently wavefronts that are locally tilted with respect to the medium optic axis. This explains why the torque arises only outside the beam focal region, where the light wavefront is curved.

It is interesting to notice here that this torque does not originate from the local spin density, whose transfer to matter would necessarily require light absorption, while the dielectric medium is transparent. Yet the optical torque must still correspond to an exchange of angular momentum between light and matter: if it is not spin, then what is it? The answer involves the orbital angular momentum of the impinging light (more specifically its 'extrinsic' part; see, for example, ref. [7]). Indeed, the combination of medium anisotropy and wavefront inclination with respect to the optic axis implies that the electric field $\mathbf{E}$ is tilted relative to the displacement vector $\mathbf{D}$, leading in turn to a non-vanishing torque. At the same time, the Poynting vector $\mathbf{S}{=}\mathbf{E}{\times}\mathbf{H}$ describing the flow of wave energy is also locally tilted relative to the wavevector $\mathbf{k}$, where the latter fixes the wave (linear) momentum (more precisely, $\mathbf{k}/\omega$ is the momentum per unit energy). Upon propagation, this angle between $\mathbf{S}$ and $\mathbf{k}$ leads to a (local) transverse shift of the wave energy distribution (Fig. 1d), which in turn implies a variation of the light orbital angular momentum density $\frac{d\mathbf{L}}{dt} = \mathbf{S}\times\frac{\mathbf{k}}{\omega}$ that exactly balances the torque on the molecules.

Where do these two works leave us? In terms of pure knowledge, we now more fully understand certain subtle aspects of the vector structure of light, of its related angular momentum and its interplay with matter. At the same time, these findings may help us push forward the emerging technology of structured light, particularly in the area of spin–orbit interactions[8–10]. Transverse spin, in particular, may be useful for coupling light unidirectionally in and from optical fibres or waveguides, with possible implications for future quantum networks[5,11]. The optical torque and other similar photoinduced media-reconfiguration phenomena in soft matter[12] may also be interesting for photo-lithographic applications, and could be used in future to achieve a relatively simple imaging technique of the full vector structure of light, including its elusive longitudinal field components. ❐

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Competing interests
The authors declare no competing interests.



QUANTUM OPTICS

# Entanglement-enhanced sensor networks

A network of quantum sensors for estimating phase shifts is shown to operate with superior sensitivity when delocalized highly entangled states are employed.

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In 1935, the Austrian physicist Erwin Schrödinger coined the term 'quantum entanglement' to indicate a form of non-local correlation predicted by quantum mechanics that has no classical analogue[1]. The counterintuitive behaviour of entanglement leads to paradoxical physical situations that have been puzzling generations of physicists since the founding fathers of quantum mechanics[2,3]. However, in the past few decades, it has been recognized that entanglement is not only a strange "spooky action at a distance" phenomenon, as identified by Albert Einstein in a famous 1947 letter to Max Born[4], but also a powerful physical resource to underpin a new generation of quantum technologies.

Sensing is an important example of technology — with applications ranging from gravitational wave detection, to imaging and biological probing — that can highly benefit from entanglement. Yet, not all entangled states are useful to enhance the performance of quantum sensors and not all of them are equally useful[5]. The coherent equal superposition of two maximally different quantum states of several compound subsystems is particularly valuable. This superposition state is generally known as the Greenberger–Horne–Zeilinger (GHZ) state[6] and has extreme non-classical properties. When applying a phase shift to each subsystem, the GHZ state acquires a phase shift that is 'amplified' proportionally to the number of entangled subsystems[5,7,8]. This amplification effect associated with the entanglement has no classical counterpart and can be used to increase substantially the sensitivity in the estimation of the phase shift.

Now, writing in *Nature Photonics*, Pan and co-workers report the creation and manipulation GHZ states in a sensor network[9]. The states are realized by an ensemble of photons in a coherent quantum superposition where they are all simultaneously vertically and horizontally polarized. The sensor network is composed by a set of nodes (or modes) located at different spatial positions (with a typical separation distance of tens of centimetres on an optical table in this experiment). Each mode is occupied by one or more photons that play the role of individual sensors (schematically depicted as antennas in Fig. 1). At each mode, the polarization of the photon(s) is rotated by a different angle (corresponding to the phase shift that one wants to estimate), independently from all the other modes. Interestingly, the paper investigates a rich variety of possibilities to generate entanglement between the photons in the network (Fig. 1), which, in turn, correspond to different sensing limits[10].

The technological state-of-the-art configuration of the network is that of independent sensors. In this case each photon has no relationship neither with its fellow(s) occupying the same mode, nor with other photons in different modes: a configuration indicated here and in Fig. 1 as mode-separable particle-separable (MsPs). This situation is easily conceivable by our common 'classical' experience where each photon detects a single phase shift independently from all the other photons. The uncertainty in the estimation of any linear combination of the phases follows the shot-noise limit, $1/\sqrt{N}$, corresponding to a straightforward statistical scaling proportional to the total number of sensors, $N$. While this uncertainty can be, in principle, arbitrarily decreased by increasing $N$, it is not difficult to encounter situations where the number of sensors is limited, for instance by spatial constraints. In this case, preparing the sensors in a proper entangled state is a viable method to increase further the sensitivity of the network.

Liu et al. have first prepared a GHZ states of $N/M$ photons in each of the $M$ modes (in the experiment[9], as well as in Fig. 1, $N = 6$ and $M = 3$). These GHZ states are independent from each other and are used to sense locally a single phase. Within this strategy (indicated as mode-separable particle-entangled, MsPe) linear combinations of the $M$ phases can be estimated with an uncertainty $\sqrt{M}/N$. This overcomes the shot-noise limit by a factor $\sqrt{N/M}$ (note that $M<N$ here) that is directly proportional to the number of entangled photons in each GHZ state[5,7,8].

The possibility to enhance a local sensitivity with entangled states has already been demonstrated in photonic as well as in atomic systems (for example,