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Supplementary information

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SUPPLEMENTARY INFORMATION

Transverse spinning of unpolarized light

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1. Theoretical calculations

1.1. Spin and polarization in 2D and 3D fields

The spin angular momentum density in a monochromatic electromagnetic field in free space is given by [17,33]

$$\mathbf{S} = \frac{1}{16\pi\omega} \operatorname{Im}\left(\mathbf{E}^* \times \mathbf{E} + \mathbf{H}^* \times \mathbf{H}\right) \equiv \mathbf{S}^{(e)} + \mathbf{S}^{(m)}, \tag{S1}$$

where we use Gaussian units and take into account both electric and magnetic field contributions, $\mathbf{S}^{(e)}$ and $\mathbf{S}^{(m)}$. Notice that the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ are complex because we are using the analytic signal representation; the real fields are the real part of the complex fields. The intensity of the field can be determined by the ratio of its energy density to the frequency: $I = W/\omega$, which yields

$$I = \frac{1}{16\pi\omega}\left(|\mathbf{E}|^2 + |\mathbf{H}|^2\right) \equiv I^{(e)} + I^{(m)}. \tag{S2}$$

Let us consider the polarization properties of the electric wave field $\mathbf{E}$, the properties of the magnetic field $\mathbf{H}$ following a similar description. Polarization of a 2D paraxial field propagating in the positive-z direction is determined by the 2×2 polarization (density) matrix or, equivalently, by 4 real Stokes parameters $\vec{s} = (s_0, s_1, s_2, s_3)$ [1,2]:

$$\hat{\mathbf{\Phi}}^{\mathrm{2D(e)}} = \begin{pmatrix} \langle E_x^* E_x \rangle & \langle E_x^* E_y \rangle \\ \langle E_y^* E_x \rangle & \langle E_y^* E_y \rangle \end{pmatrix} = \frac{1}{2} \begin{pmatrix} s_0^{(\mathrm{e})} + s_1^{(\mathrm{e})} & s_2^{(\mathrm{e})} + i s_3^{(\mathrm{e})} \\ s_2^{(\mathrm{e})} - i s_3^{(\mathrm{e})} & s_0^{(\mathrm{e})} - s_1^{(\mathrm{e})} \end{pmatrix}, \tag{S3}$$

where $\langle ... \rangle$ denotes time averaging. The corresponding degree of 2D polarization is given by

$$P^{\mathrm{2D(e)}} = \frac{\sqrt{\sum_{i=1}^{3} s_i^{(\mathrm{e})2}}}{s_0^{(\mathrm{e})}} \in [0,1]. \tag{S4}$$

From Eqs. (S1)–(S4) and their magnetic-field counterparts, it is easy to find the following relations:

$$\frac{\langle \mathbf{S}^{(\mathrm{e,m})} \rangle}{\langle I^{(\mathrm{e,m})} \rangle} = \frac{s_3^{(\mathrm{e,m})}}{s_0^{(\mathrm{e,m})}} \bar{\mathbf{z}}, \tag{S5}$$

where the overbar denotes the unit vector of the corresponding axis. For totally polarized light, $P^{\mathrm{2D(e,m)}} = 1$, the averaging brackets can be omitted, and Eq. (S5) yields the usual relation between the plane-wave spin and degree of circular polarization (helicity) [2,17]. For totally unpolarized 2D field, $P^{\mathrm{2D(e,m)}} = 0$, the Stokes vector is $\vec{s}^{(\mathrm{e,m})} \propto (1,0,0,0)$, and the time-averaged spin vanishes: $\langle \mathbf{S}^{(\mathrm{e,m})} \rangle = \mathbf{0}$.

The polarization of a generic 3D nonparaxial field at a given point is determined by the 3×3 polarization (density) matrix or, equivalently, by 9 real polarization parameters $\vec{\Lambda} = (\Lambda_0, \Lambda_1, ..., \Lambda_8)$ [4–12]:

$$\hat{\mathbf{\Phi}}^{\mathrm{3D(e)}} = \begin{pmatrix} \langle E_x^* E_x \rangle & \langle E_x^* E_y \rangle & \langle E_x^* E_z \rangle \\ \langle E_y^* E_x \rangle & \langle E_y^* E_y \rangle & \langle E_y^* E_z \rangle \\ \langle E_z^* E_x \rangle & \langle E_z^* E_y \rangle & \langle E_z^* E_z \rangle \end{pmatrix} = \frac{1}{3} \begin{pmatrix} \Lambda_0^{(\mathrm{e})} + \Lambda_3^{(\mathrm{e})} + \frac{\Lambda_8^{(\mathrm{e})}}{\sqrt{3}} & \Lambda_1^{(\mathrm{e})} - i\Lambda_2^{(\mathrm{e})} & \Lambda_4^{(\mathrm{e})} - i\Lambda_5^{(\mathrm{e})} \\ \Lambda_1^{(\mathrm{e})} + i\Lambda_2^{(\mathrm{e})} & \Lambda_0^{(\mathrm{e})} - \Lambda_3^{(\mathrm{e})} + \frac{\Lambda_8^{(\mathrm{e})}}{\sqrt{3}} & \Lambda_6^{(\mathrm{e})} - i\Lambda_7^{(\mathrm{e})} \\ \Lambda_4^{(\mathrm{e})} + i\Lambda_5^{(\mathrm{e})} & \Lambda_6^{(\mathrm{e})} + i\Lambda_7^{(\mathrm{e})} & \Lambda_0^{(\mathrm{e})} - \frac{2\Lambda_8^{(\mathrm{e})}}{\sqrt{3}} \end{pmatrix}. \tag{S6}$$

The corresponding degree of 3D polarization can be defined as [5,10–12,40,41]

$$P^{\mathrm{3D(e)}} = \frac{\sqrt{\sum_{i=1}^{8} \Lambda_i^{(\mathrm{e})2}}}{\sqrt{3}\Lambda_0^{(\mathrm{e})}} \in [0,1]. \tag{S7}$$

From Eqs. (S1), (S2), (S7), and its magnetic-field counterpart, we find the following relations between the spin density and 3D polarization parameters:

$$\frac{\langle \mathbf{S}^{(\mathrm{e,m})} \rangle}{\langle I^{(\mathrm{e,m})} \rangle} = -\frac{2\Lambda_7^{(\mathrm{e,m})}}{3\Lambda_0^{(\mathrm{e,m})}} \bar{\mathbf{x}} + \frac{2\Lambda_5^{(\mathrm{e,m})}}{3\Lambda_0^{(\mathrm{e,m})}} \bar{\mathbf{y}} - \frac{2\Lambda_2^{(\mathrm{e,m})}}{3\Lambda_0^{(\mathrm{e,m})}} \mathbf{z}. \tag{S8}$$

For totally polarized light, $P^{\mathrm{2D(e,m)}} = 1$, the spin density is still associated with the ellipticity of the polarization ellipse in 3D space and directed normally to its plane [14,17]. For completely unpolarized 3D light (e.g., in the interior of a black-body cavity), $P^{\rm 3D(e,m)}=0$, the polarization parameters are $\vec{\Lambda}^{\rm (e,m)}\propto\left(1,0,0,...,0\right)$, and the time-averaged spin vanishes: $\left\langle \mathbf{S}^{\rm (e,m)}\right\rangle=\mathbf{0}$.

Here we should make an important remark about the electric and magnetic field characteristics. In paraxial light, the electric and magnetic parameters are equal: $\vec{s}^{\rm (e)}=\vec{s}^{\rm (m)}\equiv\vec{s}$, $\mathbf{S}^{\rm (e)}=\mathbf{S}^{\rm (m)}=\dfrac{\mathbf{S}}{2}$, $I^{\rm (e)}=I^{\rm (m)}=\dfrac{I}{2}$. This is a consequence of the relation $\mathbf{H}=\bar{\mathbf{z}}\times\mathbf{E}$ between the fields. For nonparaxial light, on the other hand, the electric and magnetic fields are not locally locked with each other, and therefore in general $\vec{\Lambda}^{\rm (e)}\neq\vec{\Lambda}^{\rm (m)}$, $\mathbf{S}^{\rm (e)}\neq\mathbf{S}^{\rm (m)}$, $I^{\rm (e)}\neq I^{\rm (m)}$ [17,18,20,26,32,33].

1.2. Unpolarized plane wave

A paraxial 2D field at a given point can be approximated by a plane wave. A totally unpolarized plane wave can be considered as a field with equal amplitudes of the orthogonal and mutually incoherent components E_x and E_y. With vanishing z-component, $E_z=0$, this yields the following 2D and 3D polarization matrices (S3) and (S6):

$$\hat{\mathbf{\Phi}}^{\rm 2D(e)}\propto\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad \hat{\mathbf{\Phi}}^{\rm 3D(e)}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}, \tag{S9}$$

The corresponding 2D and 3D polarization parameters are:

$$\vec{s}^{\rm (e)}\propto\left(1,0,0,0\right),\quad \vec{\Lambda}^{\rm (e)}\propto\left(1,0,0,0,0,0,0,0,\sqrt{3}/2\right), \tag{S10}$$

which yield the degrees of polarization (S4) and (S7):

$$P^{\rm 2D(e,m)}=0,\quad P^{\rm 3D(e,m)}=\frac{1}{2}. \tag{S11}$$

Thus, a totally unpolarized plane wave is *half*-polarized in the 3D sense [5,11,12]. This is related to the fact that the third component of a paraxial field vanishes, and the 3D polarization matrix is not proportional to the unit 3×3 matrix. The spin density vanishes in an unpolarized plane wave, $\left\langle \mathbf{S}^{\rm (e,m)}\right\rangle=\mathbf{0}$, but its partial polarization in the 3D sense means that the spin can appear after optical transformations generating a nonzero z-component in the field.

1.3. Focused field from an unpolarized source

A nonparaxial focused field can be described by a simple model of a Gaussian beam with the inclusion of its longitudinal field components $\left\{E_z,H_z\right\}\simeq ik^{-1}\left\{\mathbf{\nabla}_\perp\cdot\mathbf{E}_\perp,\mathbf{\nabla}_\perp\cdot\mathbf{H}_\perp\right\}$, where the subscript "$\perp$" indicates transverse $\left(x,y\right)$ components. In this manner, a polarized Gaussian beam in the focal plane is described by the fields [17]

$$\left\{\mathbf{E}_\perp,\mathbf{H}_\perp\right\}\propto\frac{1}{\sqrt{1+\left|m\right|^2}}\left\{\bar{\mathbf{x}}+m\bar{\mathbf{y}},-m\bar{\mathbf{x}}+\bar{\mathbf{y}}\right\}e^{ikz-kr^2/2z_R},\quad \left\{E_z,H_z\right\}\simeq-i\tilde{r}\left\{E_r,H_r\right\}. \tag{S12}$$

Here, m is a complex parameter describing the polarization state of the wave, $k = \omega / c$ is the wave number, (r, φ, z) are the cylindrical coordinates, $\tilde{r} = r / z_R$, and z_R is the Rayleigh range. The intensity distribution (S2) of the beam (S12) is given by $I \propto \left(1 + \tilde{r}^2 / 2\right) e^{-kr^2/z_R}$.

The focusing process can be considered as a transition from a plane-wave limit $z_R = \infty$, $E_z = H_z = 0$, to a given finite z_R. The incident paraxial field is characterized by the Stokes parameters $\vec{s} \propto \left(1, \dfrac{1-|m|^2}{1+|m|^2}, \dfrac{2\,\mathrm{Re}(m)}{1+|m|^2}, \dfrac{2\,\mathrm{Im}(m)}{1+|m|^2}\right)$ and normalized spin density $\mathbf{S}_0 / I_0 = (s_3 / s_0)\bar{\mathbf{z}}$. For the focused field with a finite z_R, the normalized spin density is described by Eq. (2) in the main text:

$$\frac{\mathbf{S}}{I} \simeq \frac{1}{1+\tilde{r}^2/2}\left[\frac{\mathbf{S}_0}{I_0} + \tilde{r}\bar{\boldsymbol{\varphi}}\right] \equiv \frac{\mathbf{S}_{\parallel}}{I} + \frac{\mathbf{S}_{\perp}}{I}. \tag{S13}$$

We emphasize that this expression is valid for the total (electric plus magnetic) spin (S1) and (S2), while the electric and magnetic parts, $\mathbf{S}^{(\mathrm{e})}$ and $\mathbf{S}^{(\mathrm{m})}$, have more complicated forms and are generally not equal to each other [17,26,32].

Let us now consider the focused beam produced from a totally unpolarized paraxial (plane) wave. This field can be considered as an *incoherent* superposition of two beams (S12) with same intensities and orthogonal polarizations (e.g., with $m = 0$ and $m = \infty$). Such incoherent superposition means additive properties of quadratic field quantities, including the spin (S1), intensity (S2), and polarization matrix (S6). Direct calculations with fields (S12) result in the following polarization matrix:

$$\hat{\boldsymbol{\Phi}}^{\mathrm{3D(e)}} = \hat{\boldsymbol{\Phi}}^{\mathrm{3D(m)}} \propto \begin{pmatrix} 1 & 0 & -i\tilde{x} \\ 0 & 1 & -i\tilde{y} \\ i\tilde{x} & i\tilde{y} & \tilde{r}^2 \end{pmatrix}, \tag{S14}$$

where $\tilde{x} = x / z_R$ and $\tilde{y} = y / z_R$. Notably, for the nonparaxial 3D field produced from an unpolarized source, the electric- and magnetic-field-based polarization parameters are equivalent, similarly to the paraxial case. Equation (S14) with Eq. (S6) corresponds to the polarization parameters Eq. (3) in the main text: $\Lambda_1 = \Lambda_2 = \Lambda_3 = \Lambda_4 = \Lambda_6 = 0$,

$$\frac{\Lambda_8}{\Lambda_0} = \frac{\sqrt{3}}{2}\frac{1-\tilde{r}^2}{1+\tilde{r}^2/2}, \quad \frac{\Lambda_5}{\Lambda_0} = \frac{3}{2}\frac{\tilde{x}}{1+\tilde{r}^2/2}, \quad \frac{\Lambda_7}{\Lambda_0} = \frac{3}{2}\frac{\tilde{y}}{1+\tilde{r}^2/2}. \tag{S15}$$

From here, the 3D degree of polarization (S7) is found to be

$$P^{\mathrm{3D}} = \frac{\sqrt{1+2\tilde{r}^2}}{2+\tilde{r}^2}. \tag{S16}$$

For $\tilde{r} \ll 1$, this reduces to $P^{\mathrm{3D}} \simeq \dfrac{1}{2}\left(1 + \dfrac{\tilde{r}^2}{2}\right)$.

Substituting Eq. (S15) into Eq. (S8), we obtain the time-averaged spin density in the focused field from an unpolarized source:

$$\frac{\langle \mathbf{S}\rangle}{\langle I\rangle}=\frac{\langle \mathbf{S}^{(\mathrm{e})}\rangle}{\langle I^{(\mathrm{e})}\rangle}=\frac{\langle \mathbf{S}^{(\mathrm{m})}\rangle}{\langle I^{(\mathrm{m})}\rangle}=\frac{\tilde{x}\bar{\mathbf{y}}-\tilde{y}\bar{\mathbf{x}}}{1+\tilde{r}^2/2}=\frac{\tilde{r}\bar{\boldsymbol{\varphi}}}{1+\tilde{r}^2/2}=\frac{\mathbf{S}_\perp}{I}. \tag{S17}$$

Thus, depolarization of the incident paraxial wave eliminates the longitudinal spin determined by the Stokes parameters, $\mathbf{S}_0/I_0=(s_3/s_0)\bar{\mathbf{z}}$, (because we consider an *incoherent* superposition of states with opposite s_3), while the transverse spin $\mathbf{S}_\perp$ remains unaffected. The only new feature of the transverse spin from an unpolarized source is that it now has equal electric and magnetic field contributions; for polarized light, these contributions depend on the Stokes parameters of the incident field [17,26,32].

1.4. Evanescent wave from an unpolarized source

We now consider an evanescent wave, which can be generated via total internal reflection of a paraxial incident field (plane wave). As before, we start with the polarized evanescent wave, which has the electric and magnetic fields [17,20]:

$$\{\mathbf{E},\mathbf{H}\}\propto\frac{1}{\sqrt{1+|m|^2}}\left\{\bar{\mathbf{x}}+m\frac{k}{k_z}\bar{\mathbf{y}}-i\frac{\kappa}{k_z}\bar{\mathbf{z}},-m\bar{\mathbf{x}}+\frac{k}{k_z}\bar{\mathbf{y}}+im\frac{\kappa}{k_z}\bar{\mathbf{z}}\right\}e^{ik_z z-\kappa x}, \tag{S18}$$

where $k_z>k$ and $\kappa=\sqrt{k_z^2-k^2}$ are the propagation and decay constants of the wave. The intensity distribution is given by $I\propto e^{-2\kappa x}$.

Assuming, for simplicity, that the transmission coefficients of the total internal reflection are polarization-independent, the generation of the evanescent field (S18) can be regarded as a transition from the plane-wave limit $\kappa=0$, $k_z=k$, to the given finite κ (assuming the local *z*-axis to be aligned with the propagation direction in both incident and evanescent fields). The incident plane wave has the same properties as in the focused-field case, including the normalized spin density $\mathbf{S}_0/I_0=(s_3/s_0)\bar{\mathbf{z}}$. In the evanescent field with finite κ, the normalized spin density is described by Eq. (4) of the main text:

$$\frac{\mathbf{S}}{I}=\frac{k}{k_z}\frac{\mathbf{S}_0}{I_0}+\frac{\kappa}{k_z}\bar{\mathbf{y}}\equiv\frac{\mathbf{S}_\parallel}{I}+\frac{\mathbf{S}_\perp}{I}. \tag{S19}$$

As before, the evanescent wave produced from a totally unpolarized plane wave can be described as an incoherent superposition of two waves (S18) with orthogonal polarizations (e.g., with $m=0$ and $m=\infty$). Direct calculations with fields (S18) result in the following polarization matrix (S6):

$$\hat{\boldsymbol{\Phi}}^{3\mathrm{D(e)}}=\hat{\boldsymbol{\Phi}}^{3\mathrm{D(m)}}\propto\begin{pmatrix}1 & 0 & -i\kappa/k_z\\ 0 & k^2/k_z^2 & 0\\ i\kappa/k_z & 0 & \kappa^2/k_z^2\end{pmatrix}. \tag{S20}$$

As in the case of the focused field, the electric- and magnetic-field-based polarization parameters are equivalent. Equation (S20) with Eq. (S6) corresponds to the polarization parameters Eq. (5) in the main text: $\Lambda_1=\Lambda_2=\Lambda_4=\Lambda_6=\Lambda_7=0$,

$$\frac{\Lambda_8}{\Lambda_0}=\frac{\sqrt{3}}{2}\frac{k^2-\kappa^2/2}{k_z^2},\quad\frac{\Lambda_3}{\Lambda_0}=\frac{3}{4}\frac{\kappa^2}{k_z^2},\quad\frac{\Lambda_5}{\Lambda_0}=\frac{3}{2}\frac{\kappa}{k_z}. \tag{S21}$$

From here, the 3D degree of polarization (S7) equals

$$P^{\mathrm{3D}} = \frac{\sqrt{1+3\kappa^4/k_z^4}}{2}. \tag{S22}$$

For $\kappa/k_z \ll 1$, it behaves as $P^{\mathrm{3D}} \simeq \frac{1}{2}\left(1+\frac{3}{2}\frac{\kappa^4}{k_z^4}\right)$.

Substituting Eq. (S21) into Eq. (S8), we obtain the time-averaged spin density in the evanescent wave generated from an unpolarized source:

$$\frac{\langle \mathbf{S} \rangle}{\langle I \rangle} = \frac{\langle \mathbf{S}^{(\mathrm{e})} \rangle}{\langle I^{(\mathrm{e})} \rangle} = \frac{\langle \mathbf{S}^{(\mathrm{m})} \rangle}{\langle I^{(\mathrm{m})} \rangle} = \frac{\kappa}{k_z}\bar{\mathbf{y}} = \frac{\mathbf{S}_\perp}{I}. \tag{S23}$$

As before, depolarization of the incident paraxial wave eliminates the longitudinal spin determined by the Stokes parameters, while the transverse spin $\mathbf{S}_\perp$ remains unaffected, and with equal electric- and magnetic-field contributions. For polarized evanescent waves, these contributions depend on the Stokes parameters of the incident field [17,20].

2. Details of the evanescent-wave experiment

2.1. Stokes parameters retrieval

In the evanescent wave experiment, only quantities related to the electric field were actually measured. We therefore omit the corresponding superscript "(e)" in what follows. In order to reconstruct the Stokes parameters describing the polarization state of the scattering signal from a single nanoparticle, a set of 12 measurements were recorded. The parameters s_0, s_1, and s_2 were measured by sending the scattering signal through a linear polarizer and recording the scattering intensity for 4 angles of the polarizer: 0° (along the *z*-axis), 45°, 90° and 135°. The remaining Stokes parameter s_3 was measured by inserting a quarter-wave plate (QWP), with its fast axis set along the *z*-axis, just before the linear polarizer and recording the scattering intensity for the linear polarizer set at 45° and 135°. The same set of 6 measurements was repeated on an area of the sample without a nanoparticle (background measurements), in order to remove any contributions from parasite signals to the scattering signal of the nanoparticle.

We use the scattering intensity of the nanoparticle $I^{\mathrm{sc}}(\phi,\theta) = I_p^{\mathrm{sc}}(\phi,\theta) - I_0^{\mathrm{sc}}(\phi,\theta)$, where I_p^{sc} and I_0^{sc} are the scattering intensities with and without the nanoparticle, respectively, ϕ is the angle of the fast axis of the QWP (defined as "$\varnothing$" in the case where the QWP is not present), and θ is the angle of the linear polarizer. Then, the Stokes parameters s_0, s_1, and s_2 are calculated as

$$\begin{aligned} s_0 &= I^{\mathrm{sc}}(\varnothing,0^\circ) + I^{\mathrm{sc}}(\varnothing,90^\circ), \\ s_1 &= I^{\mathrm{sc}}(\varnothing,0^\circ) - I^{\mathrm{sc}}(\varnothing,90^\circ), \\ s_2 &= I^{\mathrm{sc}}(\varnothing,45^\circ) - I^{\mathrm{sc}}(\varnothing,135^\circ). \end{aligned} \tag{S24}$$

Due to the imperfect retardance of the QWP in the whole visible range used in the experiment, a correction based on Ref. [S1] was applied to the calculation of the Stokes parameter s_3, given by

$$s_3 = \frac{1}{\cos\delta}\left[I^{sc}\left(0°,45°\right) - I^{sc}\left(0°,135°\right) + s_2 \sin\delta \right], \tag{S25}$$

where δ is the phase difference error of the QWP, obtained from the manufacturer's specifications.

2.2. Theoretical modelling

Here we describe theoretical modelling of the evanescent-wave experiment (inserts in Fig. 3b). In order to model unpolarized light, we performed two separate calculations using two orthogonal states of incident light. We then computed the Stokes parameters of the scattered light for each of the two orthogonal polarizations and added them up. Since the Stokes parameters are quadratic forms of fields, this is an incoherent superposition describing the unpolarized light similar to the Supplemental sections 1.3 and 1.4. We confirmed that our results are invariant with respect to which two pairs of mutually orthogonal incident polarizations we chose for the analysis. For the two chosen orthogonal polarizations, the following steps were done in order to calculate the theoretical insets on Fig. 3b in the main text.

1. Incident light undergoes total internal reflection in the glass-air interface $z = 0$, with an experimentally measured incident angle of 47° inside the glass, generating an evanescent wave whose 3D field $\mathbf{E}^{ev}$ can be theoretically calculated including the Fresnel transmission coefficients of the interface.

2. The evanescent wave interacts with a point-polarizable particle located at $\mathbf{r}_0 = z_0\overline{\mathbf{z}}$ and induces an electric dipole: $\mathbf{p} = \alpha^{eff}\mathbf{E}^{ev}$. The value of the effective polarizability α^{eff} takes into account multiple reflections from the surface, following an approach similar to Ref. [S2]:

$$\mathbf{p} = \alpha\,\mathbf{E}^{inc} \equiv \alpha\left(\mathbf{E}^{ev} + \mathbf{E}^{ref}\right) = \alpha\left(\mathbf{E}^{ev} + \hat{\mathbf{G}}^{ref}\mathbf{p}\right), \tag{S26}$$

so that

$$\mathbf{p} = \left[\hat{\mathbf{I}} - \alpha\,\hat{\mathbf{G}}^{ref}\right]^{-1} \alpha\,\mathbf{E}^{ev} \equiv \alpha^{eff}\,\mathbf{E}^{ev}. \tag{S27}$$

Here $\mathbf{E}^{inc}$ is the total incident field, including the incident evanescent wave $\mathbf{E}^{ev}$ and the reflected fields of the dipole $\mathbf{E}^{ref}$ acting back on itself, which is expressed via Green's tensor of the surface reflected field, $\mathbf{E}^{ref}\left(\mathbf{r}\right) = \hat{\mathbf{G}}^{ref}\left(\mathbf{r},\mathbf{r}_0\right)\mathbf{p}$, and α is the polarizability of the particle in a homogeneous medium. Using the quasistatic approximation for the metallic particle of permittivity ε_p surrounded by the medium with permittivity ε, $\alpha = 4\pi R^3 \varepsilon_0 \dfrac{\varepsilon_p - \varepsilon}{\varepsilon_p + 2\varepsilon}$ (in the SI units) [S3], where R is the particle's radius. The calculation of $\hat{\mathbf{G}}^{ref}$ is detailed later.

3. Once the induced dipole $\mathbf{p}$ is known, the far-field radiation in the upper half space is calculated by coherently adding the fields radiated by the dipole towards the upper half space and the fields radiated by the dipole towards the bottom half-space and subsequently reflected from the glass surface:

$$\mathbf{E}^{sc}\left(\mathbf{r}\right) = \mathbf{E}^{0}\left(\mathbf{r}\right) + \mathbf{E}^{ref}\left(\mathbf{r}\right) = \left[\hat{\mathbf{G}}^{0}\left(\mathbf{r},\mathbf{r}_0\right) + \hat{\mathbf{G}}^{ref}\left(\mathbf{r},\mathbf{r}_0\right)\right]\mathbf{p}, \tag{S28}$$

The Green function for the reflected field, $\hat{\mathbf{G}}^{ref}$, required in the steps 2 and 3, can be obtained from the angular spectrum decomposition of the dipolar fields as derived in Refs. [S4,S5]:

$$\mathbf{E}^{\text{ref}}(\mathbf{r}) = \hat{\mathbf{G}}^{\text{ref}}(\mathbf{r},\mathbf{r}_0)\,\mathbf{p} = \iint \frac{ik^2}{8\pi^2\varepsilon k_z}\left[R_s\left(\bar{\mathbf{e}}_s\cdot\mathbf{p}\right)\bar{\mathbf{e}}_s + R_p\left(\bar{\mathbf{e}}_p^-\cdot\mathbf{p}\right)\bar{\mathbf{e}}_p^+\right]e^{\mathbf{k}\cdot(\mathbf{r}+\mathbf{r}_0)}dk_x dk_y\,. \tag{S29}$$

Here we used the unit vectors for the s and p polarizations defined over the **k**-space as $\bar{\mathbf{e}}_s = \left(-\frac{k_y}{k_\perp}, \frac{k_x}{k_\perp}, 0\right)$ and $\bar{\mathbf{e}}_p^\pm = \left(\pm\frac{k_x k_z}{k_\perp k}, \pm\frac{k_y k_z}{k_\perp k}, -\frac{k_z}{k}\right)$, $k_\perp = \sqrt{k_x^2+k_y^2} = \sqrt{k^2-k_z^2}$, $k = n\omega/c = \sqrt{\varepsilon/\varepsilon_0}\,\omega/c$, as well as the corresponding Fresnel reflection coefficients of the air-glass interface, $R_{s,p}\left(k_x,k_y\right)$. Using the identity $\left(\bar{\mathbf{e}}_s\cdot\mathbf{p}\right)\bar{\mathbf{e}}_s = \left(\bar{\mathbf{e}}_s\otimes\bar{\mathbf{e}}_s\right)\mathbf{p}$, where $\otimes$ denotes the outer product, we arrive at the Green function required for step 2:

$$\hat{\mathbf{G}}^{\text{ref}}(\mathbf{r},\mathbf{r}_0) = \iint \frac{ik^2}{8\pi^2\varepsilon k_z}\left[R_s\left(\bar{\mathbf{e}}_s\otimes\bar{\mathbf{e}}_s\right) + R_p\left(\bar{\mathbf{e}}_p^-\otimes\bar{\mathbf{e}}_p^+\right)\right]e^{\mathbf{k}\cdot(\mathbf{r}+\mathbf{r}_0)}dk_x dk_y\,, \tag{S30}$$

The Green function for the radiated dipolar fields in the upper half space (without reflection) is

$$\hat{\mathbf{G}}^{0}(\mathbf{r},\mathbf{r}_0) = \iint \frac{ik^2}{8\pi^2\varepsilon k_z}\left[\bar{\mathbf{e}}_s\otimes\bar{\mathbf{e}}_s + \bar{\mathbf{e}}_p^+\otimes\bar{\mathbf{e}}_p^+\right]e^{\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_0)}dk_x dk_y\,, \tag{S31}$$

We emphasize that this model accounts for both (i) multiple reflections acting on the particle and modifying its effective polarizability, and (ii) reflection of the scattered fields adding up to the direct scattering from the particle. Both of these effects are required to achieve the close match to experiment. In particular, surface reflections are responsible for breaking the mirror symmetry $k_z \to -k_z$ in the scattering patterns in Fig. 3b.

In order to compute the far-field radiation patterns and polarization from the angular spectrum, we use the known relation

$$\mathbf{E}^{\text{far}}(\mathbf{r}) = \mathbf{E}^{\text{pattern}}(\vartheta,\varphi)\frac{e^{ikr}}{r} = -2\pi i k_z \mathbf{E}\left(k_x,k_y\right)\frac{e^{ikr}}{r}\,, \tag{S32}$$

where $\mathbf{E}\left(k_x,k_y\right) = E_p\left(k_x,k_y\right)\bar{\mathbf{e}}_p^+\left(k_x,k_y\right) + E_s\left(k_x,k_y\right)\bar{\mathbf{e}}_s\left(k_x,k_y\right)$ corresponds to the angular spectrum $\mathbf{E}(\mathbf{r}) = \iint \mathbf{E}\left(k_x,k_y\right)e^{i\mathbf{k}\cdot\mathbf{r}}dk_x dk_y$, and $\mathbf{k} = \left(k\sin\vartheta\cos\varphi, k\sin\vartheta\sin\varphi, k\cos\vartheta\right)$.

Supplementary References

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