nature portfolio

# Peer Review File

# Nonreciprocal quantum synchronization

Corresponding Author: Dr Deng-Gao Lai

**This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.**

**Attachments originally included by the reviewers as part of their assessment can be found at the end of this file.**

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
What are the noteworthy results?

The authors study a model of coupled photons/phonons/magnons motivated by a setup consisting of a spinning silica microsphere and a YIG sphere. Counterclockwise/clockwise optical modes are excited in the silica sphere by lasers from the left/right directions. The sign of the magnon-Kerr coefficient of the YIG sphere is controlled by an applied magnetic field. The dynamics of the system is calculated by writing down phenomenological Langevin equations. Synchronization effects are investigated by calculating the continuous-variable measure introduced by Mari et al. [67] for the mechanical modes of the two spheres. The degree of synchronization can be controlled by the magnetic field and the directions of the lasers and is robust to imperfections and noise. The coupling may be nonreciprocal because of the interplay of Kerr effect and Sagnac effect.

Will the work be of significance to the field and related fields? How does it compare to the established literature? If the work is not original, please provide relevant references.

New setups for quantum synchronization are significant. However, Caption Fig. 1 says that the phonon-phonon interaction between the two spheres is established via a direct physical contact. How is it possible if the silica sphere is spinning (in the simulation in Fig. 1 with Omega up to 8 kHz). How is this compatible with direct physical contact?

Does the work support the conclusions and claims, or is additional evidence needed?
Are there any flaws in the data analysis, interpretation and conclusions? Do these prohibit publication or require revision?

omega_1 introduced after (6) appears to be an arbitrary reference unit, i.e., the axis labels in the figures are in arbitrary units? It is hard to judge whether the parameters used are realistic.

Is the methodology sound? Does the work meet the expected standards in
your field?

The bibliography appears sloppy. E.g., p. 1, "one-way quantum processors [30-33]".
But I cannot find one-way quantum processors in [33].
Many references in the SM appear 2x (e.g., S3 = S41, S9 = S37, S16 = S53, S17 = S54, S18 = S55, etc.)

Is there enough detail provided in the methods for the work to be reproduced?

The main part of the paper is hard to read because many details are missing. The SM provides many details.

Reviewer #2

(Remarks to the Author)
Please see the attached report.

[Editorial Note: This is displayed at the end of the file]

Reviewer #3

(Remarks to the Author)
The manuscript by Lai and colleagues is devoted to a study of nonreciprocal quantum synchronization. They consider a specific model of a hybrid system that consists of a pair of microspheres which allows one to coherently couple phonons, magnons and photons as presented in [53]. By combining the Sagnac effect and Kerr nonlinearity, they show the occurrence of nonreciprocal synchronization between phonons by using the continuous variable quantum synchronization parameter of [67]. They further show that synchronization is more resilient to noise in the nonreciprocal phase and relate this remarkable property to squeezing.

I find the obtained results interesting and timely, but would like to a few points to be clarified before I can make a recommendation.

PROS:

1) the authors consider a novel quantum synchronization system (besides nonlinear oscillators and qubits)

2) the considered system has been realized experimentally in [53].

3) the occurrence of nonreciprocal quantum synchronization is in my opinion important and novel. However, [78] has already appeared in PRX in January and I feel that this should be acknowledged and not just in mentioned in an added note: the present manuscript does not seem to have been uploaded on the arXiv (I wonder why) and there is hence no way to verify that the authors did this study at the same time as those of [78]. But there is almost no overlap between the two papers, so I don't see this as an issue.

4) the enhanced stability in the nonreciprocal phase is notable and could be important for concrete applications.

CONS:

1) it is difficult to tell how generic the obtained results really are, and hence judge their potential impact to a wider field.

2) the paper is not clearly written: it is too dense, there are too many abbreviations, it is not written for people not familiar with (all) the topics of the manuscript. I mean, the paper combines many different fields (quantum synchronization, nonreciprocal interactions, hybrid quantum systems), which makes it really interesting - potentially. Yet, the authors do not try at all to make it accessible to an interdisciplinary audience. Concretely, the choice (and importance) of the chosen system in not really emphasized or explained, Sagnac and Kerr are supposed to be well known (and are not explained), quantum synchronization is neither introduced nor explained.

3) the discussion of the quantum synchronization parameter on page 3 almost follows verbatim that of [67].

4) the captions are suboptimal (especially those of Figs. 2 and 3): there is absolutely no way to learn something about the results of the paper by simply reading the captions.

5) the paragraph about the robustness against random fabrication is not understandable. What have rho and K to do with that.

6) Potential applications are not addressed at all. What are these results good for (possibly)?

To conclude, I feel that the manuscript is potentially publishable in Nature Communications in view of the importance and novelty of the obtained results. However, in my judgement, the authors have done a bad job at presenting them in a clear and accessible way. I therefore invite them to address the above comments.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
In the revised manuscript, the concerns that I raised in my previous report have been addressed.

Shouldn't the new phrase in the caption of Fig. 1 read "The phonon-phonon coupling chi originates from direct physical contact between a spinning silica microsphere and a COUNTER-rotating YIG sphere, both maintained at constant angular velocities Omega and -Omega"?
I am still somewhat skeptical about the experimental realization of a direct contact between two rotating silica and YIG

spheres at angular velocities of 6 kHz but leave this for the community to judge.

A last remark about the new section "Discussions and conclusions": the authors refer repeatedly to RMP 91, 025001 (2019) [Ref. 80] in the context of quantum synchronization and coupled quantum oscillators. However, Ref. 80 is a review about quantum resource theories and does not address quantum synchronization. The same remark applies to Section F and Ref. S31 of the Supplementary Material.

The revised manuscript is substantially improved and ready for publication.

Reviewer #2

(Remarks to the Author)
The authors have fully addressed all my comments. The revised manuscript has also been significantly improved. I can recommend a publication now.

Reviewer #3

(Remarks to the Author)
The authors have clarified all the issues that I have raised to my satisfaction. I therefore recommend publication in Nature Communications.

## *** REPORT OF REFEREE A – NCOMMS-25-04564 ***

### COMMENT A1

*What are the noteworthy results?*

*The authors study a model of coupled photons/phonons/magnons motivated by a setup consisting of a spinning silica microsphere and a YIG sphere. Counterclockwise/clockwise optical modes are excited in the silica sphere by lasers from the left/right directions. The sign of the magnon-Kerr coefficient of the YIG sphere is controlled by an applied magnetic field. The dynamics of the system is calculated by writing down phenomenological Langevin equations. Synchronization effects are investigated by calculating the continuous-variable measure introduced by Mari et al. [67] for the mechanical modes of the two spheres. The degree of synchronization can be controlled by the magnetic field and the directions of the lasers and is robust to imperfections and noise. The coupling may be nonreciprocal because of the interplay of Kerr effect and Sagnac effect.*

### OUR REPLY TO COMMENT A1

We thank Referee A for the concise and excellent summary of our work, and for the constructive comments and suggestions that can significantly improve our manuscript. We also appreciate Referee A's acknowledgment of the noteworthy nature of our findings. Below, we provide point-by-point responses to all of Referee A's insightful comments and suggestions.

We here highlight four noteworthy results of our manuscript.

**(i) Nonreciprocal quantum synchronization of phonon modes remains unexplored.** —To our knowledge, we are the first to study nonreciprocal quantum synchronization via the synergy of the Sagnac and magnon-Kerr effects. While the use of the Sagnac effect to achieve the nonreciprocity of the optical transmission [R1] and photon blockade [R2] has been studied, its application to quantum synchronization has not been explored to date. Inspired by the Sagnac-effect-induced nonreciprocity mechanism [R1], we introduce a fundamentally different nonreciprocity mechanism based on the magnon-Kerr effect and demonstrate the first realization of nonreciprocal quantum synchronization, revealing its counterintuitive immunity against both random fabrication imperfections and thermal noise of practical devices.

**(ii) Our idea is not a simple synergy of the Sagnac and magnon-Kerr effects, but rather the generation of novel nonreciprocal quantum phenomena and addressing an outstanding challenge, i.e., quantum synchronization is extremely sensitive to random fabrication imperfections and thermal noise of practical devices.** —Specifically, quantum synchronization of phonon modes is generally deteriorated or even completely destroyed by thermal noise and random fabrication imperfections. Surprisingly, our proposal overcomes this obstacle and generates a unique one-way quantum

# Broad applicability and universality

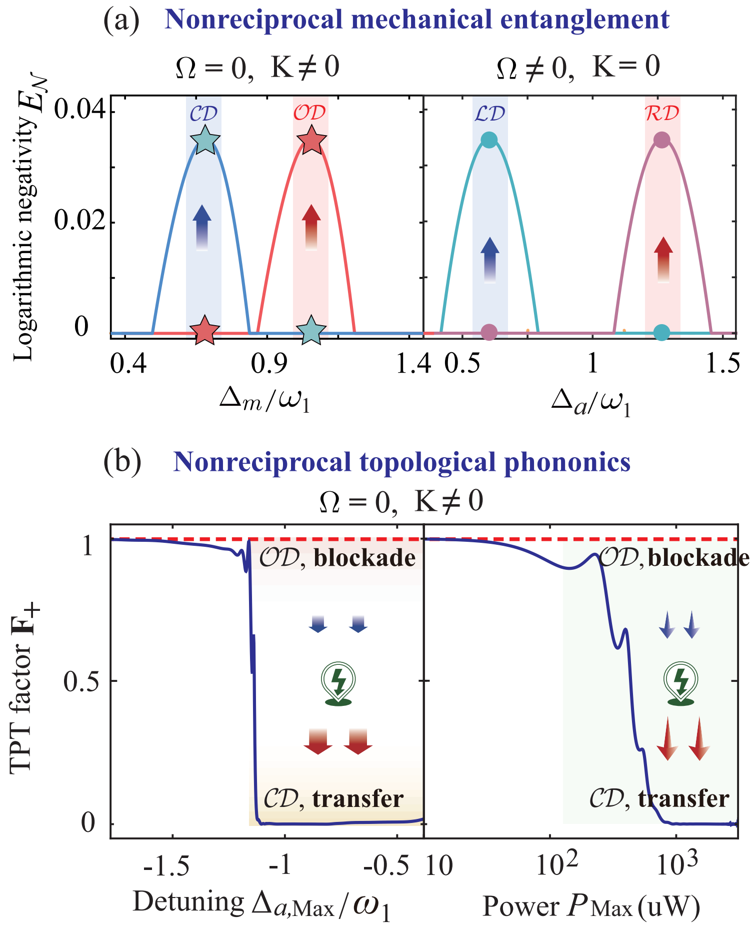


FIG. R1. Broad applicability and universality of our results. (a) Nonreciprocal quantum entanglement: Nonreciprocal phonon-phonon entanglement quantified by the logarithmic negativity $E_{\mathcal{N}}$ versus the scaled magnon detuning $\Delta_m$ when $\Omega = 0$ and $\mathrm{K} \neq 0$, and versus the scaled optical detuning $\Delta_a$ when $\Omega \neq 0$ and $\mathrm{K} = 0$. (b) Nonreciprocal topological phonon transfer (TPT) and its blockade: TPT quality factor $\mathbf{F}_+$ versus the maximal driving detuning $\Delta_{\mathrm{Max}}$ and power $P_{\mathrm{Max}}$ when the magnetic field is injected from the $\mathcal{CD}$ or $\mathcal{OD}$.

synchronization immune to these detrimental factors, without the need of using any high-cost, low-loss materials or noise filters at the expense of the system complexity.

**(iii) In a broader view, our study presents an innovative approach to reversing the intrinsically detrimental effects of practical devices.**—It paves a general route to pioneering nonreciprocal quantum resources, with immunity against both random fabrication imperfections and thermal noise of practical devices.

**(iv) Our approach has a broad applicability and universality.** —Our framework is not limited to a specific unidirectional quantum effect such as nonreciprocal quantum synchronization, but extends to a broader class of one-way quantum phenomena, including nonreciprocal quantum entanglement and unidirectional topological phonon transfer. For example: (1) our framework naturally extends to nonreciprocal quantum entanglement between phonon modes [see Fig. R1(a) and Fig. R2], and (2) it can be generalized to explore nonreciprocal topological phononics and photonics [see Fig. R1(b) and Fig. R2].

We next illustrate the broad applicability and universality of our approach through two ongoing projects:

(1) Our proposed physical framework naturally extends to the study of nonreciprocal quantum entanglement between two phonon modes, as explored in our ongoing work Ref. [Deng-Gao Lai, Adam Miranowicz, and Franco Nori, Nonreciprocal Mechanical Quantum Entanglement, in preparation (2025)].

Quantum entanglement of mechanical resonators serves as a key resource for quantum information processing and memory. Mechanical entanglement generation, however, is generally suppressed or even fully destroyed by thermal noise and/or random fabrication imperfections of practical devices. The proposed method can be extended to generate nonreciprocal mechanical entanglement by harnessing the Sagnac effect in combination with magnon-Kerr nonlinearity.

We find that two mechanical modes are *entangled* in one chosen direction of the laser (magnetic field) but *separable* in the other, as shown in Fig. R1(a) and and Fig. R2. Remarkably, the threshold thermal phonon number required to preserve mechanical entanglement in our approach is much larger than that of the standard method. These findings are broadly applicable and pave the way for advances in one-way quantum resources that are resilient to thermal noise and fabrication imperfections.

(2) Our approach can be generalized to investigate nonreciprocal topological phononics and photonics, as explored in our ongoing work Ref. [Deng-Gao Lai, Adam Miranowicz, and Franco Nori, Nonreciprocal Topological Phononics, in preparation (2025)].

Overcoming the strict dependence on the encircling direction of an exceptional-point (EP) and reversing the inherent detrimental effects caused by random fabrication imperfections (e.g., large masses and losses) in practical devices remain significant challenges in topological physics. The proposed method can be used to overcome these obstacles, enabling a versatile yet unique nonreciprocal topological phonon transfer (TPT). This is possible by employing the magnon-Kerr nonlinearity effect stemming from magnetocrystalline anisotropy, leading to turning detrimental imperfections into benefits, having no correspondence in previous studies.

Specifically, TPT occurs when the system is driven from one chosen direction of the magnetic field but not the other, giving rise to a profoundly different unidirectional TPT independent of the EP-winding direction, as shown in Fig. R1(b) and Fig. R2. Unlike conventional schemes, where TPT is generally suppressed or even completely destroyed in large-mass and/or large-damping regimes, our approach not only directly turns suppression (detriments) into revival (benefits), but also immunizes TPT in these domains, with a *seven-orders-of-magnitude* enhancement in the TPT performance, as shown in Fig. R3. The study presents an innovative approach to reversing the intrinsically detrimental effects of device imperfections, and paves a general route to pioneering nonreciprocal topology independent of the EP-encircling direction.

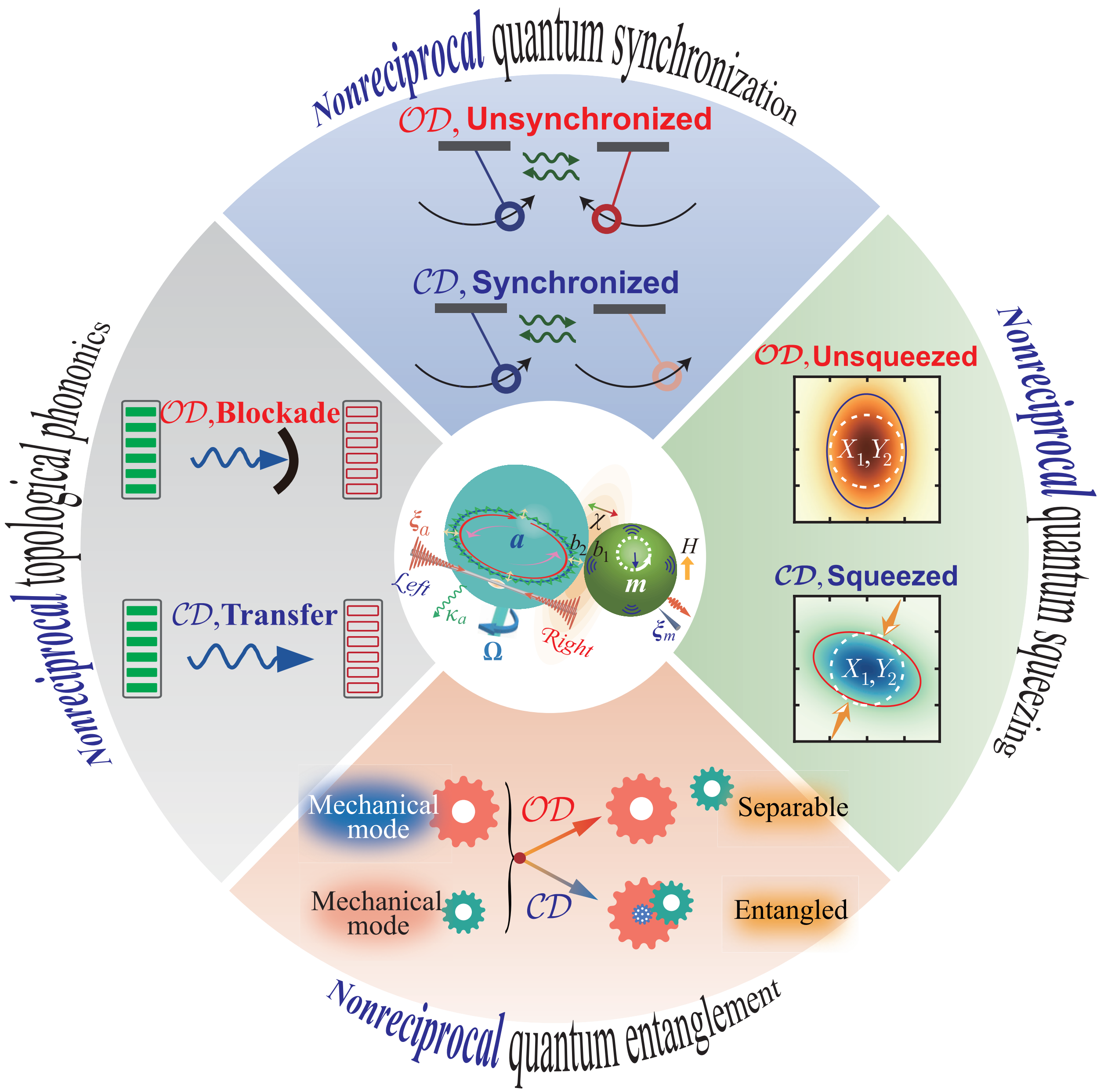


FIG. R2. Broad applicability and universality of our model. The proposed framework naturally extends to a range of nonreciprocal quantum phenomena, including nonreciprocal quantum synchronization, nonreciprocal quantum squeezing, nonreciprocal quantum entanglement, and nonreciprocal topological phononics.

To address these points more explicitly, we have included a dedicated section of "Noteworthiness, Significance, and Advantages" in the revised main text and Supplemental Materials. Please see the last paragraph of page 6 and the first paragraph of page 7 in the revised main text, as well as Sec. I of the revised Supplemental Materials.

## COMMENT A2

*Will the work be of significance to the field and related fields? How does it compare to the established literature? If the work is not original, please provide relevant references.*

## OUR REPLY TO COMMENT A2

We thank Referee A for this important and thoughtful three questions. Below, we provide a detailed response to each point in turn.

**(1) Originality and Novelty:** Nonreciprocal physics is garnering enormous attention in both classical and quantum research fields. Surprisingly, previous demonstrations have not explored nonreciprocal quantum synchronization of phonons, one of the most obvious examples of nonreciprocal quantum resources. Here we fill this gap to demonstrate the possibility of nonreciprocal quantum synchronization, revealing its counterintuitive immunity against random fabrication imperfections and thermal noise of practical devices. The study lays the foundation for generating fragile-to-robust nonreciprocal quantum resources.

**(2) Significance to the field:** (i) We believe our work offers both conceptual and technical advances that are broadly relevant to the field of cavity opto-magnon-mechanics. The demonstrated ability to achieve nonreciprocal quantum synchronization via both the Sagnac effect and magnon-Kerr nonlinearity opens a new pathway for an active control of one-way nonequilibrium quantum dynamics in hybrid quantum platforms. These results are expected to be of interest also to researchers in quantum phononics, nonlinear quantum dynamics, and quantum information, where robust and tunable quantum synchronization and its quantum nonreciprocity are highly desirable. (ii) Our work presents an innovative approach to reversing the intrinsically detrimental effects of practical devices, and paves a general route to pioneering nonreciprocal quantum resources, with immunity against both random fabrication imperfections and thermal noise of practical devices. (iii) Our study exhibits broad applicability and universality. Beyond nonreciprocal quantum synchronization, our approach applies broadly to one-way quantum phenomena, including nonreciprocal entanglement [as shown in Fig. R1(a) and Fig. R2] and nonreciprocal topological phonon transfer [as shown in Fig. R1(b) and Fig. R2].

**(3) Comparison with Existing Literatures:** Prior studies have investigated quantum synchronization in optomechanical systems [R3] [PRL 111, 103605 (2013)] nonreciprocal transport of information or photons using the Sagnac effect [R1, R2] [Nature 558, 569 (2018); PRL 121, 153601 (2018)], and magnon-Kerr nonlinearity in YIG-based systems [R4–R6] [PRL 120, 057202 (2018); PRL 127, 183202 (2021); PRL 132, 156901 (2024)]. However, our work brings together these ingredients in a previously unexplored regime:

**(i)** The sign and strength of the magnon-Kerr nonlinearity are tuned in situ via the external magnetic field, allowing dynamical control over the coupling landscape.

**(ii)** Nonreciprocal quantum synchronization is modulated by optical driving directions or external magnetic field directions, enabled by the Sagnac effect in a spinning microsphere or the magnon-Kerr effect in the YIG sphere, respectively.

**(iii)** Quantum synchronization becomes effectively nonreciprocal, without requiring additional gain or engineered reservoirs.

To our knowledge, no previous study has demonstrated this level of nonreciprocal control over quantum synchronization and unidirectional phononic coupling via combined photonic, phononic, and magnonic pathways. We are neither aware of any prior established demonstration that reports this combination of these mechanisms, nor one that realizes tunable nonreciprocal quantum synchronization in this manner.

To clarify these points, we have added a dedicated section entitled "Noteworthiness, Significance, and Advantages" in both the revised main text and the Supplementary Materials. Please refer to the last paragraph of page 6 and the first three paragraphs of page 7 in the revised main text, as well as Sec. I of the revised Supplemental Materials.

## COMMENT A3

*New setups for quantum synchronization are significant. However, Caption Fig. 1 says that the phonon-phonon interaction between the two spheres is established via a direct physical contact. How is it possible if the silica sphere is spinning (in the simulation in Fig. 1 with Omega up to 8 kHz). How is this compatible with direct physical contact?*

## OUR REPLY TO COMMENT A3

We thank Referee A for recognizing the significance of our proposed physical setup for quantum synchronization. In addition, we are sorry for causing any confusion due to an unclear explanation of the physical-contact-induced coupling. In our system, the phonon-phonon interaction is enabled by a direct contact between the co-rotating silica and YIG spheres, both maintained at a same spinning angular velocity.

Motivated by Referee A's insightful comment, we have clarified this point by adding the following sentence to the revised main text:

"The phonon-phonon coupling $\chi$ originates from direct physical contact between a spinning silica microsphere and a co-rotating YIG sphere, both maintained at a constant angular velocity $\Omega$." Please see the third sentence of the caption of Fig. 1 in the revised main text.

## COMMENT A4

*Does the work support the conclusions and claims, or is additional evidence needed? Are there any flaws in the data analysis, interpretation and conclusions? Do these prohibit publication or require revision?*

## OUR REPLY TO COMMENT A4

We thank Referee A for raising these essential questions concerning the validity and completeness of our analysis and conclusions. All claims made in the manuscript are directly supported by our analytical derivations and numerical simulations based on a set of coupled quantum Langevin equations, which capture the essential physics of the hybrid photon-phonon-magnon system. Each of the central results arises naturally from the hybrid quantum opto-magnon-mechanical model and has been systematically tested across a broad and experimentally realistic (state-of-the-art) parameter spaces, as shown in Tab. I and Fig. R4. To further illustrate this point, we plot the quantum synchronization measure $\mathcal{S}_{\mathcal{Q}}$ as a function of each relevant parameters (decay rates $\gamma_j$ and $\kappa_{a,m}$ and thermal noise $\bar{n}_j$) when magnon-Kerr strengths $\mathrm{K} = 0$, $\mathrm{K} < 0$, and $\mathrm{K} > 0$, as shown in Fig. R4. Our results reveal that the effectiveness of the scheme extends well beyond the previous parameter choices [R3], demonstrating robust performance over a wide range of parameter space. Please see our reply to Comments A5 and C9 for more details.

Following the constructive comments and suggestions from Referee A, we have substantially revised the manuscript, leading to significant improvements in both clarity and scientific rigor. Moreover, we have carefully re-examined our data analysis and interpretation under state-of-the-art experimental conditions, as shown in Tab. I, and found no methodological flaws that undermine our conclusions. The system parameters, boundary conditions, and dynamical regimes are clearly stated, and the outcomes are consistent with the underlying physical mechanisms.

To further aid clarity, we have revised several sections of the manuscript to more explicitly highlight the logical flow from assumptions to conclusions. We therefore believe that the existing evidence fully supports the claims of the work and that no additional simulations or analysis are required. We hope these clarifications address Referee A's concerns and reinforce the robustness and originality of the study.

## COMMENT A5

*$\omega_1$ introduced after (6) appears to be an arbitrary reference unit, i.e., the axis labels in the figures are in arbitrary units? It is hard to judge whether the parameters used are realistic.*

## OUR REPLY TO COMMENT A5

We apologize for any confusion caused by the inadvertent omission of certain parameter values.

"In the revised version, all system parameters used in the numerical simulations have been explicitly provided, closely consistent with those values reported in previous studies [R1–R4, R7–R9]. Note that **we set $\omega_1 = 2\pi \times 10$ MHz as a reference unit of frequency**, and all physical parameters used in the simulations are listed in Tab. I.

| **Symbols** | **Physical quantities** | **Simulation parameters [R2, R3, R7]** | **Experimental parameters [R1, R4, R8, R9]** |
|---|---|---|---|
| $c$ | Vacuum light speed | $3 \times 10^8$ m/s | $3 \times 10^8$ m/s |
| $\lambda$ | Laser wavelength | 1550 nm | 1550 nm |
| $\gamma$ | Gyromagnetic ratio | $2\pi \times 2.8$ MHz/Oe | $2\pi \times 2.8$ MHz/Oe |
| $H$ | Bias magnetic field of the YIG sphere | Above its saturation magnetization ($H > 1750$ Oe) | Above its saturation magnetization ($H > 1750$ Oe) |
| $\omega_1/2\pi$ | Resonance frequency of the first resonator | 10 MHz | 15.25 MHz |
| $\omega_2/2\pi$ | Resonance frequency the second resonator | 10.05 MHz | 15.367 MHz |
| $\Delta_a/\omega_1$ | Optical detuning | $-1.005$ | $\pm 1.008$ |
| $\Delta_m/\omega_1$ | Magnonic detuning | $-1$ | $\pm 1$ |
| $\mathrm{K}/\omega_1$ | Magnon-Kerr coefficient | $-5 \times 10^{-5}$ to $5 \times 10^{-5}$ | 0 |
| $\kappa_a/\omega_1$ | Optical decay rate | 0.15 | 0.66 |
| $\kappa_m/\omega_1$ | Magnon decay rate | 0.2 | 0.066 |
| $\gamma_j/\omega_1$ | Mechanical damping rates | 0.005 | 0.0003 (0.0004) |
| $m_j$, $m_0$ | Effective masses of resonators | 100 ng | 100 (50) ng |
| $\bar{n}_j$ | Bath mean phonon numbers | 0 to $10^4$ | Not shown |
| $N_a$ | Mean photon number | $N_a = \langle a^\dagger a \rangle$ | $N_a = \langle a^\dagger a \rangle$ |
| $N_m$ | Mean magnon number | $N_m = \langle m^\dagger m \rangle$ | $N_m = \langle m^\dagger m \rangle$ |
| $G_a/\omega_1 = g_a\sqrt{N_a}/\omega_1$ | Effective photon-phonon coupling strength | $0 \sim 0.2$ | $0 \sim 0.1$ |
| $G_m/\omega_1 = g_m\sqrt{N_m}/\omega_1$ | Effective magnon-phonon coupling strength | $0 \sim 0.2$ | $0 \sim 0.1$ |
| $\chi/\omega_1$ | Phonon-phonon coupling rate | 0.02 | 0.0003 |
| $\xi_{a(m)}/\omega_1$ | Driving intensity | 35 | Not shown |
| $\Omega$ | Spinning angular velocity | 0 to 10 kHz | 0 |
| $\varepsilon_{0(j=1,2)}$ | Dielectric constants of air (taper, silica sphere) | 1 (3.9) | 1 (3.9) |
| $\zeta$ | Refractive indexes of silica sphere | 1.486 | 1.486 |
| $r$ | Silica microsphere radii | 1.1 mm | 0.2 mm |
| $r_0$ | YIG microsphere radii | 0.5 mm | 0.2 mm |
| E | Young modulus of silica | 75 GPa | 75 GPa |
| $\Upsilon$ | Elastic limit of silica | 9 GPa | 9 GPa |

TABLE I. Parameters of the hybrid quantum devices set in our simulations [R2, R3, R7] and in reported experiments [R1, R4, R8, R9]. Columns 1 and 2 present the parameter symbols and their physical meanings, respectively. The parameters in columns 3 and 4 are used in our numerical simulations [R2, R3, R7] and the state-of-the-art experiments [R1, R4, R8, R9], respectively. The close agreement between experimentally reported parameters and those used in our simulations demonstrates the experimental feasibility of the proposed phenomena, highlighting their relevance to current state-of-the-art platforms.

Although what we have proposed a purely theoretical scheme, our approach is completely experimentally feasible, using the state-of-the-art experimental conditions (see Tab. I). Table I demonstrates the consistency between the parameters used in our numerical simulations [R2, R3, R7] and those reported in realistic experiments [R1, R4, R8, R9], indicating that the proposed phenomena are experimentally accessible with current state-of-the-art platforms.”

To further substantiate this point, we consulted with several experimental groups in cavity optomechanics (including Prof. Şahin K. Özdemir in the Pennsylvania State University, USA; Prof. Yasunobu Nakamura in RIKEN & University of Tokyo, Japan), who unanimously affirmed that our theoretical scheme is fully compatible with state-of-the-art experimental conditions.

In response to Referee A's insightful comment, we have added some paragraphs to the revised main text and included Sec. II entitled "System parameters" to the revised Supplemental Materials. Please see the last sentence of the caption of Fig. 1 of the revised main text, the last two paragraphs of page 3 and the first two paragraphs of page 4 of the revised main text, and see Sec. II entitled "System parameters" in the revised Supplemental Materials.

## COMMENT A6

*Is the methodology sound? Does the work meet the expected standards in your field?*

## OUR REPLY TO COMMENT A6

We appreciate Referee A's inquiry regarding the methodological soundness and overall standards of the work.

Our theoretical framework employed in the manuscript is based on a set of quantum Langevin equations that capture the coupled dynamics of optical, mechanical, and magnonic modes in the presence of both intrinsic dissipation and external driving. This approach is well established in the studies of cavity optomechanics and cavity optomagnonics, and has been carefully adapted here to combine the key features of our hybrid platform, including the magnon-Kerr nonlinearity, the optical Sagnac effect, and the direct phonon-phonon contact coupling.

We have verified the validity of our methodology by performing extensive numerical simulations across a wide range of state-of-the-art realistic experimental parameters (see Tab. I and Fig. R4), ensuring that our main effects are not artifacts of fine-tuning. Furthermore, the quantum synchronization dynamics are quantified using a continuous-variable measure consistent with the prior literature [R3] [Phys. Rev. Lett. 111, 103605 (2013)], and the emergence of purely quantum nonreciprocity is traced analytically to the combination of the Sagnac effect and magnon-Kerr nonlinearity.

We believe that the methodology not only meets but also extends current standards in the field of quantum synchronization, by providing a unified framework to study hybrid, nonlinear, and unidirectional quantum phenomena in a tunable opto-magno-mechanical system. We hope our response adequately addresses Referee A's concerns and highlights the soundness and novelty of our approach.

Please see the second paragraph of page 4 of the revised main text, and the last paragraph on page 8 and the first two paragraphs on page 9 in the revised Supplemental Materials.

## COMMENT A7

*The bibliography appears sloppy. E.g., p. 1, "one-way quantum processors [30-33]". But I cannot find one-way quantum processors in [33]. Many references in the SM appear 2x (e.g., S3 = S41, S9 = S37, S16 = S53, S17 = S54, S18 = S55, etc.)*

## OUR REPLY TO COMMENT A7

We thank Referee A for carefully reviewing the bibliography and for pointing out these important inconsistencies. We have now carefully reviewed and corrected the reference list in both the main text and the Supplementary Materials. Specifically:

(i) We have relocated Ref. [33] to a more appropriate position in the revised manuscript, as it is not directly relevant to one-way quantum processors. The corrected citation now reads "[30–32]", which accurately reflects the intended sources.

(ii) We identified and eliminated all duplicate entries in the Supplementary Materials bibliography. As noted by Referee A, references such as S3/S41, S9/S37, S16/S53, S17/S54, and S18/S55 were repeated due to an earlier formatting error. These have been consolidated and relabeled accordingly to ensure each cited work appears only once.

(iii) We also conducted a full consistency check between all in-text citations and the final bibliography to ensure that each reference is properly cited, correctly numbered, and contextually relevant.

We sincerely apologize for these oversights and appreciate Referee A's diligence in bringing them to our attention. The revised manuscript and Supplementary Materials now reflect a corrected and professionally formatted bibliography.

## COMMENT A8

*Is there enough detail provided in the methods for the work to be reproduced?*

## OUR REPLY TO COMMENT A8

We thank Referee A for highlighting the importance of methodological transparency and reproducibility. In preparing the main manuscript and Supplementary Materials, we have made a concerted effort to ensure that all key steps in our modeling and simulation procedures are presented with sufficient transparency to allow for independent reproduction. The set of quantum Langevin equations governing the coupled photon-phonon-magnon dynamics is fully specified, including all relevant damping rates, coupling constants, nonlinear coefficients, and external driving terms. All used parameters are clearly defined relative to a natural frequency scale, and the dynamical regimes explored are explicitly outlined.

We confirm that the main manuscript and Supplementary Material provide all the necessary details to enable independent recalculation and reproduction of our results. Specifically:

(i) The full set of quantum Langevin equations governing the system dynamics is explicitly provided, along with a clear definition of all parameters and their physical meaning.

(ii) The initial conditions, numerical integration schemes, parameter values, and the criteria used to quantify synchronization [based on the continuous-variable measure introduced in PRL 111, 103605 (2013)] are stated in detail in the main text and Supplementary Material.

(iii) We have further expanded the Supplementary Materials to provide a step-by-step account of both the analytical and numerical procedures, ensuring full reproducibility using standard quantum optics frameworks and computational tools.

(iv) To enable experimental benchmarking, all parameters in our numerical simulations are grounded in physically realistic values drawn from state-of-the-art experiments, as detailed in Table I.

We hope these clarifications and additions address Referee A's concern, and demonstrate our commitment to transparency and reproducibility.

## COMMENT A9

*The main part of the paper is hard to read because many details are missing. The SM provides many details.*

## OUR REPLY TO COMMENT A9

We thank Referee A for this valuable comment. We agree that the clarity and self-containment of the main text are essential for accessibility, especially for readers who may not immediately consult the Supplementary Material. Inspired by Referee A's insightful comment, we have revised the Main Text to better balance conceptual exposition and technical completeness and to restore and highlight several key details that were previously deferred to the Supplementary Material. Specifically:

(i) We have reintegrated into the main text several essential definitions, modeling assumptions, and representative parameter choices that were previously deferred to the Supplementary Material.

(ii) We provide a more self-contained overview of the system architecture, key coupling mechanisms, and physical intuition behind the observed quantum synchronization and related quantum nonreciprocity effects.

(iii) To avoid overwhelming the main narrative, we continue to retain analytical derivations and extended numerical details in the Supplementary Material, but now explicitly guide the reader to them at appropriate points in the main text.

We believe these adjustments significantly improve the readability and coherence of the manuscript, while preserving its focus and conciseness. We are grateful to Referee A for pointing this out and hope the revised version meets the expected standard of clarity.

In summary, we thank Referee A for carefully reading our manuscript and providing a highly valuable report, which has been extremely helpful in improving our manuscript. Thanks to these comments and suggestions, the quality of our manuscript has been significantly improved. We therefore hope that our paper is now suitable for publication in Nature Communications.

### *** REPORT OF REFEREE B – NCOMMS-25-04564 ***

### COMMENT B1

*Quantum nonreciprocity is of interest in fundamental physics and many important applications. It has been widely studied in optical systems. Recently, it is explored in phononic systems, in particular, the optomechanical resonators. By spinning resonators, this manuscript theoretically investigates nonreciprocal quantum synchronization of two mechanical modes and their one-way quantum squeezing.* ***The model is clear and reasonable. The results may reveal new physics in phonons. The manuscript is well organized and written in English. Before recommendation of publication, I have some comments for the authors:***

### OUR REPLY TO COMMENT B1

We thank Referee B for the thorough and careful reading of our manuscript and for providing an excellent summary of our work. We also appreciate Referee B for acknowledging the novelty and originality of our work, and for recommending our manuscript for publication in Nature Communications.

Combining Referee B's insightful comments and suggestions, we have made every effort to improve the manuscript. Each of Referee B's comments and suggestions is addressed in detail below.

### COMMENT B2

*1. As claimed by the authors, the phononic coupling is created via a direct physical contact between the spinning silica microsphere and the YIG sphere. How can a physical contact be made between a moving (spinning) part and a static part, if the YIG sphere is not spinning?*

### OUR REPLY TO COMMENT B2

We apologize for any confusion resulting from the absence of a clear description of direct physical contact between the silica microsphere and the YIG sphere. In our work, the phononic coupling is realized using physical contact between a spinning silica microsphere and a spinning YIG sphere, both maintaining constant angular velocities throughout the process.

Motivated by Referee B's insightful and valuable comment, we have clarified this point by adding the following sentences to the revised main text:

"The phonon-phonon coupling $\chi$ originates from direct physical contact between a spinning silica microsphere and a co-rotating YIG sphere, both maintained at a constant angular velocity $\Omega$."

Please see the third sentence of the caption of Fig. 1 in the revised main text.

## COMMENT B3

*2. How to drive the mechanical mode and the magnon mode? Is it easy in experiment?*

## OUR REPLY TO COMMENT B3

Our hybrid quantum platform comprises a YIG sphere (serving as a magnomechanical cavity) and a silica microsphere (serving as an optomechanical cavity), both of which are coherently coupled to each other via direct physical contact. In the silica microsphere, the phonon mode is driven by the radiation-pressure interaction from circulating optical fields; whereas in the YIG sphere, it is excited via magnetostrictive forces mediated by microwave-driven magnons.

Specifically, a uniform magnon mode, supported by the YIG sphere under an external magnetic field, couples to a phonon mode via magnetostrictive interaction [R9, R10], enabling microwave excitation of phonons in the YIG sphere. In the silica microsphere supporting a radiation-pressure-induced mechanical radial breathing mode, the optical mode and the mechanical radial mode are intrinsically coupled through radiation pressure and the photoelastic effect [R11, R12], forming a canonical optomechanical interaction. Meanwhile, the direct physical contact between the silica and YIG microspheres establishes an effective mechanical coupling of their localized phonon modes.

In the YIG sphere, microwave driving of the magnon mode excites the phonon mode via the magnomechanical effect [R13]. Bringing the silica microsphere into direct contact with the YIG sphere establishes a mechanical coupling between their spatially separated phonon modes. Simultaneously, radiation-pressure-induced optomechanical coupling in the silica cavity plays a key role in the hybrid quantum dynamics. The process involves a synergistic interplay of optomechanics, magnomechanics, phonon interference, and quantum mechanical effects [R14, R15], wherein microwave and optical signals are coherently mapped onto two nearly degenerate mechanical modes, enabling their interference [R16, R17].

According to Referee B's insightful comment, we have added a section on "Experimental realization" to both the revised main text and Supplemental Materials. Please see the section entitled "Experimental realization" on page 7 and the first paragraph of Sec. III-B of the revised Supplemental Materials.

### COMMENT B4

*3. I can't find the definition of $m_0$.*

### OUR REPLY TO COMMENT B4

We apologize for the confusion caused by the definition of $m_0$ in our original manuscript. In our work, the parameter $m_0$ is the mass of the microsphere resonators. Inspired by Referee B's insightful comment, we have clarified the definition of $m_0$ in the revised manuscript. Remarkably, we have included Tab. I in the updated Supplementary Material to provide detailed definitions of all parameters used.

Please see the last sentence of the caption of Fig. 1 of the revised main text, the last two paragraphs of page 3 and the first two paragraphs of page 4 of the revised main text, and see Sec. II entitled "System parameters" and Tab. I in the revised Supplemental Materials.

### COMMENT B5

*4. As shown in Fig. 4(a), the two-mode squeezing is very weak. What limits the squeezing degree? Can the squeezing be improved?*

### OUR REPLY TO COMMENT B5

We thank Referee B for this insightful question. As shown in Fig. 4, the two-mode mechanical squeezing is indeed very weak, which is primarily limited by a small Kerr nonlinearity. "Under experimentally realistic conditions (i.e., state-of-the-art experimental conditions), the Kerr coefficient must remain much smaller than the optomechanical coupling strength to ensure both system stability and experimental feasibility, thereby constraining the achievable squeezing degree.

In principle, quantum squeezing could be enhanced by increasing the Kerr nonlinearity, but this would require parameter regimes that remain experimentally inaccessible with current state-of-the-art techniques. Alternative strategies to enhance two-mode mechanical squeezing include engineering stronger effective nonlinearities via auxiliary modes or tailored driving schemes, as well as harnessing the power of an optical parametric amplifier (OPA), which is an intriguing direction for future exploration."

In light of Referee B's suggestion, and given that the squeezing effect is negligibly small, we have removed its discussion from the present manuscript. A detailed study will be pursued in our future work entitled "Nonreciprocal quantum squeezing", where we plan to address why the squeezing effect is so weak, what factors limit its extent, and what strategies might enable a significant enhancement of the quantum squeezing effect.

## COMMENT B6

*5. What is the difference between nonreciprocal quantum synchronization and quantum steering?*

## OUR REPLY TO COMMENT B6

| Characteristics | Nonreciprocal Quantum Synchronization | Nonreciprocal Quantum Steering |
|---|---|---|
| **Definition** | Unidirectional phase, amplitude, and frequency locking between two quantum systems | One-way quantum correlations allowing state inference via local measurements |
| **Nonreciprocity origin** | Asymmetric interaction or control (via Kerr nonlinearity, Sagnac effect) | Asymmetric violation of local hidden state (LHS) models |
| **Observables** | Dynamical quantities (e.g., phase, spectrum) | Conditional measurement outcomes and steering inequalities |
| **Directionality** | One system influences another's dynamics without feedback | One party (Alice) can steer the other (Bob), but not vice versa |
| **Applications** | One-way quantum control, nonreciprocal quantum synchronization, chiral networks | One-sided quantum cryptography, entanglement certification, quantum information tasks |

TABLE II. Comparison between nonreciprocal quantum synchronization and nonreciprocal quantum steering [R18, R19].

"Nonreciprocal quantum synchronization [R18] and nonreciprocal quantum steering [R19] both exhibit unidirectional quantum behavior in quantum systems, but arise from fundamentally different physical mechanisms.

Nonreciprocal quantum synchronization refers to asymmetric quantum dynamical locking, such as phase or frequency entrainment, between coupled quantum oscillators [R18]. This unidirectionality stems from engineered asymmetries in the considered quantum system via the Kerr nonlinearity or rotation-induced Sagnac effect, leading to one-way quantum coherence in time-domain observables, as shown in Tab. II. The resulting unidirectional quantum coherence emerges in the time evolution of system observables and reveals asymmetric quantum synchronization, as shown in Tab. II.

In stark contrast, nonreciprocal quantum steering is a form of asymmetric quantum correlation and a measurement-based manifestation of quantum nonlocality, wherein one party (Alice) can nonlocally affect quantum state of another's part (Bob) through measurement [R19], but not vice versa, as shown in Tab. II. This irreversibility reflects a directional violation of local hidden state models and underpins one-sided device-independent quantum protocols. While both phenomena break reciprocity, quantum synchronization concerns quantum dynamical behavior, whereas quantum steering reflects the structure of quantum measurement correlations. That means that unlike quantum synchronization, quantum steering does not arise from quantum dynamical evolution but from the structure of quantum measurements and conditional states, as shown in Tab. II."

Inspired by Referee B's helpful comment, we have added a new section entitled "Discussions and conclusions" to the revised main text and Sec. V-F entitled "Difference of nonreciprocal quantum synchronization and quantum steering" to the revised Supplemental Materials. Please see last paragraph of page 7 of the revised main text, and Tab. III and Sec. V-F of the revised Supplemental Materials.

We appreciate Referee B for the thorough review of our manuscript and for providing insightful comments and constructive suggestions. Incorporating this valuable feedback has greatly enhanced the clarity and rigor of our work. We now believe that our manuscript meets the high standards necessary for publication in Nature Communications.

***** REPORT OF REFEREE C – NCOMMS-25-04564 *****

**COMMENT C1**

*The manuscript by Lai and colleagues is devoted to a study of nonreciprocal quantum synchronization. They consider a specific model of a hybrid system that consists of a pair of microspheres which allows one to coherently couple phonons, magnons and photons as presented in [53]. By combining the Sagnac effect and Kerr nonlinearity, they show the occurrence of nonreciprocal synchronization between phonons by using the continuous variable quantum synchronization parameter of [67]. They further show that synchronization is more resilient to noise in the nonreciprocal phase and relate this remarkable property to squeezing.*

**OUR REPLY TO COMMENT C1**

We thank Referee C for the thoughtful summary and for the insightful comments and constructive suggestions, which have helped improve the clarity and quality of our manuscript.

**COMMENT C2**

***I find the obtained results interesting and timely, but would like to a few points to be clarified before I can make a recommendation.***

**OUR REPLY TO COMMENT C2**

We sincerely thank Referee C for the positive assessment of our work as "interesting and timely", and especially for recommending our manuscript for publication in Nature Communications. Referee C's insightful comments and suggestions have been very helpful in improving the manuscript. We have carefully addressed all the points raised and revised the manuscript accordingly. Detailed point-by-point responses are provided below.

**COMMENT C3**

*PROS:*

*1) the authors consider a novel quantum synchronization system (besides nonlinear oscillators and qubits)*

## OUR REPLY TO COMMENT C3

We thank Referee C for acknowledging the novelty of our scheme on nonreciprocal quantum synchronization.

## COMMENT C4

*2) the considered system has been realized experimentally in [53].*

## OUR REPLY TO COMMENT C4

We sincerely appreciate Referee C's recognition of the experimental accessibility and practical relevance of our model, which is designed to align closely with current state-of-the-art experimental capabilities.

## COMMENT C5

*3) the occurrence of nonreciprocal quantum synchronization is in my opinion important and novel. However, [78] has already appeared in PRX in January and I feel that this should be acknowledged and not just in mentioned in an added note: the present manuscript does not seem to have been uploaded on the arXiv (I wonder why) and there is hence no way to verify that the authors did this study at the same time as those of [78]. But there is almost no overlap between the two papers, so I don't see this as an issue.*

## OUR REPLY TO COMMENT C5

We thank Referee C for recognizing the importance and novelty of our results. Following Referee C's helpful suggestion, we now acknowledge in the revised manuscript that Ref. [78] (published in PRX in January 2025) reported a study on nonreciprocal synchronization of active quantum spins. We note, however, that the scope, methodology, and conclusions of that work are fundamentally different from ours, and there is minimal overlap between the two studies.

Inspired by Referee C's insightful suggestion, we have added the following sentence to the "Introduction" of the revised main text:

"Very recently, nonreciprocal synchronization of active quantum spins has been demonstrated through engineered nonreciprocal coupling [R20]." Please see the third sentence of the last paragraph of page 1 in the revised main text.

Although we originally considered posting this manuscript on arXiv, we have chosen not to do so at this stage, in light of our intention to develop a series of follow-up studies grounded in the same framework, as shown in Figs. R1, R2, and R3. Please see our Reply to Comment C7 for more details.

## COMMENT C6

*4) the enhanced stability in the nonreciprocal phase is notable and could be important for concrete applications.*

## OUR REPLY TO COMMENT C6

We thank Referee C for highlighting the enhanced stability in the nonreciprocal phase and its potential relevance for practical applications. We fully agree with this perspective.

In the revised manuscript, we have expanded the discussion to emphasize how the proposed nonreciprocal control not only enables unidirectional quantum steering but also contributes to the significant robustness of purely quantum synchronization against thermal noise and random fabrication of practical devices. It may be advantageous for the implementation of purely quantum effects in noisy or engineered quantum environments.

"By these two methods on elaborating the dynamical stability, we have demonstrated that all the used parameter values work in the stable zone. We highlight the enhanced stability in the nonreciprocal phase and its potential relevance for practical applications. Specifically, it not only enables one-way quantum manipulation but also contributes to the significant robustness of purely quantum effects against thermal noise and random fabrication of practical devices. These findings may be advantageous for the implementation of purely quantum behaviors in noisy or engineered quantum environments." Please see the second paragraph of the right column of page 3 in the revised main text, and the third paragraph of Sec. IV-D of page 14 of the revised Supplemental Materials.

## COMMENT C7

*CONS:*

*1) it is difficult to tell how generic the obtained results really are, and hence judge their potential impact to a wider field.*

## OUR REPLY TO COMMENT C7

We appreciate Referee C's concern regarding the generality and broader relevance of our results. To highlight the broad applicability and universality of our study, we have extended the discussion to show that our central findings and physical model are not confined to a specific unidirectional quantum effect (i.e., nonreciprocal quantum synchronization), but describe a more general class of unidirectional quantum phenomena (e.g., nonreciprocal quantum entanglement and one-way topological phonon transfer), as shown in Figs. R1, R2, and R3. In particular, we emphasize that while the primary focus of our work is on fundamental aspects of nonreciprocal quantum synchronization and one-way nonclassical correlations, our results have potential implications for unidirectional quantum information processing, particularly in the context of quantum entanglement distribution, quantum sensing, and the design of robust quantum networks. The nonreciprocal quantum-synchronization-induced correlations could serve as a key quantum resource for stabilizing quantum coherence across distributed systems, even in the presence of noise or disorder, as shown in Tab. IV.

**Firstly, we detailedly elaborate how generic the obtained results really are:**

(i) Our proposed physical framework can be naturally extended to the study of nonreciprocal quantum entanglement between two mechanical oscillators [see Figs. R1(a) and R2]. This project is being done by us in Ref. [Deng-Gao Lai, Adam Miranowicz, and Franco Nori, Nonreciprocal Mechanical Quantum Entanglement, in preparation (2025)].

Quantum-entangled vibrations serve as a key resource for quantum information processing and quantum memory [R93]. More broadly, quantum entanglement, manifesting as nonclassical correlations between spatially separated systems, is central to a wide range of quantum technologies, including quantum precision metrology, quantum secure communication, and quantum computation [R22]. So far, efficiently high quantum entanglement between photons and a variety of quantum systems, including atoms [R23–R29], trapped ions [R30, R31], quantum dots [R32], and superconducting qubits [R33–R35] has been realized in quantum platforms ranging from microscopic to macroscopic scales [R36, R37], laying the foundation for connecting remote, long-lived quantum memories in emerging quantum networks [R38–R41].

Our method can be used to generate nonreciprocal phonon-phonon (mechanical) entanglement by harnessing the synergy of the Sagnac and magnon-Kerr effects, which gives rise to an anomalous Sagnac-Fizeau shift and an exceptional magnon-Kerr-induced exceptional transition, respectively. Remarkably, this mechanism exhibits inherent immunity against fabrication imperfections and thermal noise in realistic devices. Specifically, two phonon modes are entangled in a chosen direction of the pump laser (magnetic field) but separable in the other, yielding a unique nonreciprocal quantum phonon-phonon entanglement, as shown in Fig. R1(a) and Fig. R2. Unlike previous proposals naturally restricted to small-mass (small-dissipation) and/or low-noise regimes, our approach overcomes these limitations, owing to the improvement in the resonator resilience. This work lays the groundwork for generating robust nonreciprocal quantum resources from fragile quantum correlations.

(ii) Our approach is readily extendable to the exploration of nonreciprocal topological phononics and photonics [see Figs. R1(b), R2, and R3]. This project is being done by us in Ref. [Deng-Gao Lai, Adam Miranowicz, and Franco Nori, Nonreciprocal Topological Phononics, in preparation (2025)].

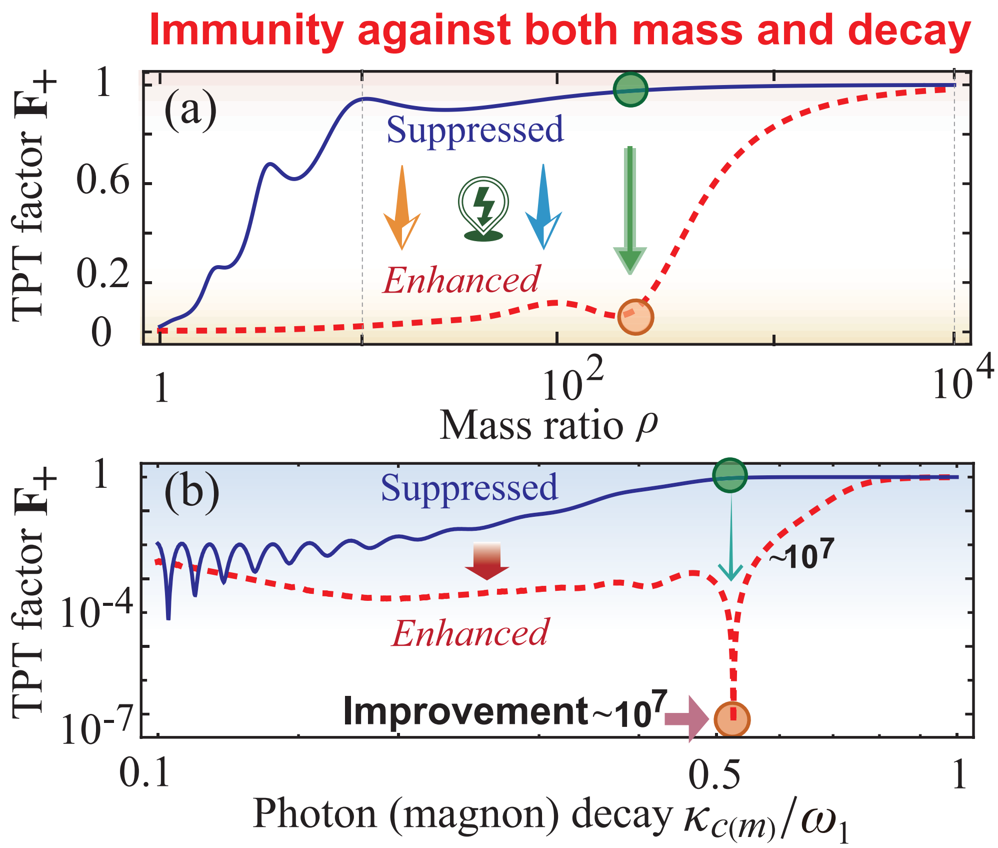


FIG. R3. Nonreciprocal topological phononics robust against random fabrication imperfections. TPT quality factor $\mathbf{F}_+$ versus (a) the mass ratio $\rho$ and (b) cavity (magnon) decay rates $\kappa_{c(m)}$, in both the standard method (solid curves) and our approach (dashed curves).

Losses and masses, which are ubiquitous in nature, are related to nontrivial topologies resulting from the presence of non-Hermitian degeneracies [R42–R59]. With increasing losses and/or masses in conventional systems, topological responses are generally deteriorated or even fully destroyed, yielding various imperfection-sensitive topological phenomena, such as mode exchange [R60], resonance trapping [R61], and singular topology [R62] in parameter landscapes. Their proper description requires a departure from conventional physical models, inherently sensitive to losses and masses, to *unconventional* devices with an immunity against these nature imperfections. Providing robustness in topological responses against these detrimental random fabrication imperfections (e.g., large masses and/or losses) in practical devices is highly desirable, because it can exhibit a dramatic effect on topological systems, resulting in nontrivial physics with even more counterintuitive topological features. We note that topological nature has been harnessed to shield quantum resources from impurities and disorders *rather than* masses and losses of realistic setups [R63–R72].

Very recently, nonreciprocal topology engendered by imposing topological operations encircling an exceptional point (EP) has been attracting enormous attention, with fascinating and counterintuitive effects, such as nonreciprocal TPT [R73–R83], non-adiabatic jumps [R84, R85], and chiral phase accumulation [R86, R87], which have already been experimentally observed. These topological behaviors, however, are inherently constrained by a strict dependence on the direction of adiabatic EP-enclosing control loops [R73, R74]. This characteristics leads to a giant *suppression* effect of both topological response and its nonreciprocity. We are aware of the significance of these major challenges, where nonreciprocal topology not only strictly depends on the EP-encircling direction but also is extremely fragile to decoherence and fidelity degradation in the large-mass (large-decay) regimes [R73–R87]. It is ultimately required to exploit a new unidirectional topology independent of the EP-winding direction and immunize topological resources against random fabrication imperfections of practical devices.

Building on our proposed method, we can tailor an extraordinary nonreciprocal TPT in a well-controlled manner [see Fig. R1(b) and Fig. R2], revealing its counterintuitive *independence* of the EP-encirclement direction and *immunity* against the inherent imperfections of quantum devices, as shown in Fig. R3. This is achieved by the synergy of topological operations and the magnon-Kerr effect, resulting in turning detrimental imperfections into benefits. We find that applying the magnetic field along one chosen direction leads to TPT, while injecting it along the other does not, enabling a fundamentally different nonreciprocal topology, which is completely independent of the EP-enclosing direction. These findings are otherwise likely unattainable via conventional approaches [R73–R87].

In stark contrast to previous schemes [R73–R87], where as decay rates and/or masses of quantum devices are increased, topological performances are always decreased or even completely destroyed, our approach, surprisingly, demonstrates that the performances can be dramatically revived and increased despite of increasing losses and/or masses, as shown in Fig. R3. *This loss-induced and/or mass-induced recovery of the topological performance is in contrast to the expectation that the standard topological performance would decrease with increasing losses and/or masses and is a direct manifestation of our method.* Note that the resulting topological behavior differs from all prior progresses [R73–R87], mainly because we are *not interested in* reducing the detriments of random fabrication imperfections of practical devices, but in *converting these detriments into benefits*.

Our approach enables immunizing all topological responses against device fabrication imperfections, without the necessity of employing any high-cost, low-loss materials or noise filters [R88, R89]. This overthrows the consensus that poor intrinsic factors and rugged extrinsic environment suppress the preparation of such extremely fragile topological resources. In a broader view, our work sheds new light on mapping a general path towards achieving a profoundly different nonreciprocal topology, independent of both EP-winding direction and random fabrication imperfections of practical devices. In a word, we believe these extended analyses clarify the robustness and generality of our findings, and their potential impact beyond the specific context originally presented.

To clarify this point, we have added a dedicated section entitled "Noteworthiness, Significance, and Advantages" in both the revised main text and the Supplementary Materials. Please see the last paragraph of page 6 and the first three paragraphs of page 7 in the revised main text, as well as Sec. I of the revised Supplemental Materials.

**Next, we detailedly show the potential impact of our work to a wider field:**

To highlight the potential applications and advantages of our findings, we have added a new section entitled "Discussions and conclusions" to the revised main text, including the following explanations. The demonstrated nonreciprocal control of quantum synchronization in this platform may find use in unidirectional quantum information processing, where synchronized quantum systems can serve as robust building blocks for distributed quantum networks [R18, R19, R90–R93]. The tunable magnon-Kerr nonlinearity effect and its resilience to random fabrication imperfections of practical devices suggest possible applications in quantum sensing and quantum signal transduction, especially in noisy or imperfect environments. Our robustness analysis provides insights relevant for the design of scalable chiral quantum networks, where fabrication-induced imperfections are inevitable. These findings may also contribute to future developments in nonreciprocal quantum sensing architectures that exploit collective dynamics for enhanced quantum precision [R18, R19, R90–R93].

In particular, our nonreciprocal quantum synchronization framework unlocks multiple exciting opportunities for application across quantum technologies. Including: (i) Nonreciprocal quantum information processing.—The resulting nonreciprocal quantum synchronization enables a controllable unidirectional flow of quantum correlations (quantum information), which can be harnessed for unidirected quantum signal routing in phononic or hybrid quantum networks, where thermal robustness and coherence preservation are essential [R18, R90]. (ii) Nonreciprocal quantum state engineering.—Our scheme offers a tunable, nonlinearity-engineered route to stabilize phase-locked mechanical states. This can be employed to prepare non-classical mechanical states, which is good for nonreciprocal quantum sensing or interface protocols between mechanical and optical (magnonic) degrees of freedom [R19, R91]. (iii) Quantum transduction architectures.—In hybrid quantum systems where mechanical resonators serve as intermediaries between disparate platforms (e.g., microwave-to-optical conversion), nonreciprocal quantum synchronization could enable efficient and noise-robust temporal alignment across subsystems [R92, R93]. (iv) Fundamental studies of irreversibility.—The intrinsic unidirectionality in the quantum synchronization dynamics constitutes a controlled setting for investigating microscopic origins of irreversibility and entropy production in open quantum systems, thus offering insights relevant to nonreciprocal quantum thermodynamics [R18].

We believe these points illustrate the broader utility of our findings, for both potential quantum technologies and fundamental physics.

Please see the second paragraph of the left column on page 8 in the revised main text, and Sec. VIII of the revised Supplemental Materials.

## COMMENT C8

*2) the paper is not clearly written: it is too dense, there are too many abbreviations, it is not written for people not familiar with (all) the topics of the manuscript. I mean, the paper combines many different fields (quantum synchronization, nonreciprocal interactions, hybrid quantum systems), which makes it really interesting - potentially. Yet, the authors do not try at all to make it accessible to an interdisciplinary audience. Concretely, the choice (and importance) of the chosen system in not really emphasized or explained, Sagnac and Kerr are supposed to be well known (and are not explained), quantum synchronization is neither introduced nor explained.*

## OUR REPLY TO COMMENT C8

We thank Referee C for this thoughtful and constructive critique. We fully agree that clarity and accessibility are essential, particularly for a journal with an interdisciplinary audience. In response, we have undertaken a substantial revision of the manuscript with the goal of improving its readability and broadening its accessibility.

Specifically, we have:

(i) Rewritten the Introduction to provide a more intuitive overview of the key concepts involved, including clear and concise descriptions of quantum synchronization, nonreciprocal interactions, and hybrid quantum systems. In particular, we have replaced the previous abbreviation "QS" with the full term "quantum synchronization" throughout the main text and Supplementary Materials for clarity and consistency. To improve clarity and avoid potential ambiguity, we have also replaced the abbreviations "$\mathcal{LD}$" and "$\mathcal{RD}$" with the more descriptive terms "left direction" and "right direction", respectively, throughout both the main text and Supplementary Materials.

(ii) Added a new paragraph motivating the choice of the proposed system and clarifying its physical significance and experimental relevance. Please see the new section entitled "Experimental realization" in the revised main text, and the first paragraph of Sec. III-B on page 5 in the revised Supplemental Materials.

(iii) Introduced brief, self-contained physical explanations of the Sagnac effect, the magnon-Kerr effect, and quantum synchronization, emphasizing their role in the physical model and avoiding assumptions of prior familiarity. Specifically, to ensure accessibility for a broad interdisciplinary audience, we provide brief introductions to the Sagnac effect, the Kerr nonlinearity, and quantum synchronization in the article as follows:

(a) "The Sagnac effect arises in rotating reference frames, where counterpropagating waves traveling along a closed loop accumulate a relative phase shift proportional to the rotation rate [R1]. This relativistic interference phenomenon underpins modern gyroscopes and enables directional sensitivity in photonic and phononic systems." Please see the first two sentences of the last paragraph of page 2 in the revised main text, and the first paragraph of Sec. IV-C on page 12 in the revised Supplemental Materials.

(b) "The magnon-Kerr effect refers to a nonlinear frequency shift of magnon modes induced by magnon-magnon interactions in a magnetically ordered material [R4]. This self-phase modulation leads to intensity-dependent magnon dynamics, analogous to the optical-Kerr effect, enabling tunable nonlinearity in hybrid quantum systems. Arising from magnetocrystalline anisotropy, this nonlinear interaction enables tunable magnon dynamics and facilitates nonperturbative phenomena such as bistability and nonreciprocal signal propagation." Please see the third and fourth sentences of the last paragraph of page 2 and the first paragraph on page 3 in the revised main text, and the first paragraph of Sec. IV-B on page 11 in the revised Supplemental Materials.

(c) "Quantum synchronization describes the emergence of phase or frequency locking between interacting quantum systems, despite intrinsic quantum fluctuations [R3]. It extends classical synchronization into the quantum regime, revealing nontrivial correlations in quantum dynamics of coupled oscillators, spins, or fields. Unlike its classical counterpart, quantum synchronization manifests through correlations in quantum observables and is constrained by quantum noise and noncommutativity, offering a route to controlling collective quantum dynamics. Please see the first paragraph of the right column on page 1 of the revised main text, and the first paragraph of Sec. V on page 14 of the revised Supplemental Materials.

(iv) Minimized the use of abbreviations throughout and defined all terms upon first appearance.

(v) Streamlined technical passages and reorganized the structure of the main text to improve flow and accessibility.

We hope these revisions make the manuscript more approachable to readers representing an interdisciplinary audience from a wide range of backgrounds, while preserving the technical depth necessary for specialists.

## COMMENT C9

*3) the discussion of the quantum synchronization parameter on page 3 almost follows verbatim that of [67].*

## OUR REPLY TO COMMENT C9

We thank Referee C for pointing out this issue. Our intention was to provide a concise and self-contained summary of the quantum synchronization parameter as introduced in Ref. [R3] [PRL 111, 103605 (2013)], which is foundational to our subsequent analysis. We agree that the overlap in wording was too close and could raise concerns regarding originality.

(i) "In this work, we focus on the coupled continuous-variable quantum systems based on cavity opto-magno-mechanics. Currently, in cavity optomechanics, the diagnostic $\mathcal{S}_{\mathcal{Q}}$ [R3] is generally used to describe the measure of quantum synchronization in continuous-variable systems. While Referee C has expressed some concerns regarding the quantum synchronization measurement, it is undeniable that this method remains **the most widely recognized and commonly used** in cavity opto-magno-mechanical continuous-variable systems. This measurement method develops a consistent and quantitative theory of synchronization for continuous variable systems evolving in the quantum regime. Specifically, the quantum-synchronization measure $\mathcal{S}_{\mathcal{Q}}$ of continuous variable systems has been introduced by extrapolating it from notions of complete synchronization that is introduced for classical models. Note that our method on the quantum-synchronization measure $\mathcal{S}_{\mathcal{Q}}$ in continuous-variable systems is based on this well-known work [R3], which introduces and characterizes the measure quantifying the level of quantum synchronization of coupled continuous variable systems. This measure enables the extension of synchronization concepts into the quantum domain, and the Heisenberg principle sets a universal limitation to complete synchronization." Please see the last paragraph on page 8 in the revised

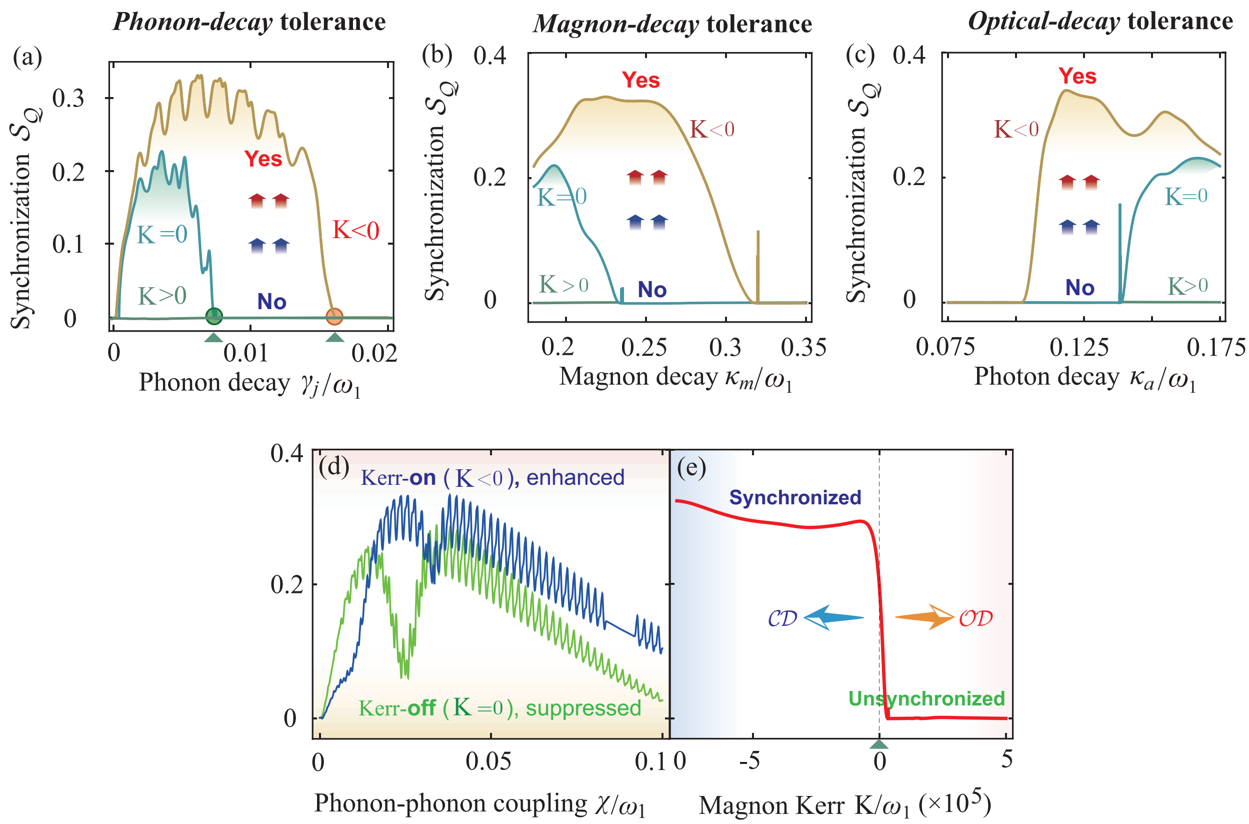


FIG. R4. (a) Quantum synchronization measure $\mathcal{S}_Q$ versus the mechanical decay rate $\gamma_j$ when the magnon-Kerr strength $\mathrm{K} = 0, < 0, > 0$. (b) Quantum synchronization measure $\mathcal{S}_Q$ versus the magnetic decay rate $\kappa_m$ when the magnon-Kerr strength $\mathrm{K} = 0, < 0, > 0$. (c) Quantum synchronization measure $\mathcal{S}_Q$ versus the optical decay rate $\kappa_m$ when the magnetic-kerr strength $\mathrm{K} = 0, < 0, > 0$. (d) Suppression of quantum synchronization due to thermal noise in both standard reciprocal [R3] and our nonreciprocal cases, and counterintuitive quantum synchronization revival resulted from the magnon-Kerr-nonlinearity induced compensation. Quantum synchronization measure $\mathcal{S}_Q$ versus the thermal phonon numbers $\bar{n}_j$, in both magnon-Kerr-off ($\mathrm{K} = 0$, reciprocal) and magnon-Kerr-on ($\mathrm{K} < 0$, nonreciprocal) regimes. Surprisingly, quantum synchronization is enhanced in the magnon-Kerr-on regime, reaching almost (or even transcending) that as in an ideal device without thermal noise ($\bar{n}_j = 0$). (e) Quantum synchronization measure $\mathcal{S}_Q$ versus the phonon-phonon coupling strength $\chi$, in the presence of the magnon-Kerr nonlinearity (i.e., $\mathrm{K}/\omega_1 \neq 0$), assuming that the externally injected magnetic field enters from the $\mathcal{CD}$ ($\mathrm{K}/\omega_1 = -2 \times 10^{-5}$) and $\mathcal{OD}$ ($\mathrm{K}/\omega_1 = 2 \times 10^{-5}$). (f) Quantum synchronization measure $\mathcal{S}_Q$ versus the magnon-Kerr nonlinearity strength K. Here we assume $\Delta_m/\omega_1 = -1$ and $\Delta_a/\omega_1 = -1$. The used parameter values are shown in Tab. I.

Supplemental Materials.

(ii) "In fact, the values of the quantum synchronization parameters are not constrained to strictly follow those reported in Ref. [R3], but remain highly effective over a wide range of parameter settings. To demonstrate this, we plot the quantum synchronization measure $\mathcal{S}_Q$ versus the decay rates $\kappa_m$ and $\gamma_j$ and thermal noise $\bar{n}_j$ when the magnon-Kerr strength $\mathrm{K} = 0, < 0, > 0$, as shown Fig. R4. Our results reveal that the effectiveness of the scheme extends well beyond the parameter choices in Ref. [R3], demonstrating robust performance across a wide parameter space." Please see Figs. S6, S7, and S8 of Sec. V-A and Sec. V-B of the revised Supplemental Materials, the last paragraph on page 3 and the first paragraph on page 4 of the revised main text, and Sec. II of page 3 of the revised Supplemental Materials.

In the revised main text, we have explicitly clarified and emphasized this point by adding the following sentence.

"While the values of the quantum synchronization parameters largely follow those reported in Ref. [R3], they remain highly effective across a broad range of parameter regimes." Please see the last sentence of the first paragraph on page 4 of the revised main text.

## COMMENT C10

*4) the captions are suboptimal (especially those of Figs. 2 and 3): there is absolutely no way to learn something about the results of the paper by simply reading the captions.*

## OUR REPLY TO COMMENT C10

We thank Referee C for this valuable suggestion. We fully agree that the original captions for Figs. 2 and 3 did not sufficiently convey the key findings. In response, we have thoroughly revised the figure captions to provide a clearer and more informative summary of the results, including a concise description of the physical significance and main trends.

In particular, inspired by Referee C's suggestion that readers should be able to grasp the main results from the figure captions alone, we added subheadings that are self-contained and effectively highlight the key insights of each figure.

The subheading of Fig. 1 reads: "Model and Sagnac-effect-induced nonreciprocal quantum synchronization".

The subheading of Fig. 2 reads: "Magnon-Kerr-induced nonreciprocal quantum synchronization".

The subheading of Fig. 3 reads: "Imperfection-immune quantum synchronization".

The subheading of Fig. 4 reads: "Symmetric and asymmetric couplings" and "Broad applicability and universality of our model".

The updated captions now highlight the physical significance of the results and guide the reader through the underlying results and conclusions. We believe these improvements enhance the clarity and accessibility of the manuscript.

## COMMENT C11

*5) the paragraph about the robustness against random fabrication is not understandable. What have $\rho$ and $K$ to do with that.*

## OUR REPLY TO COMMENT C11

We thank Referee C for pointing out the lack of understandability in the paragraph about the robustness of quantum synchronization against random fabrication imperfections of practical quantum devices. "In quantum physics, large mass and high dissipation are frequently manifestations of random fabrication imperfections in quantum devices. Deviations in etching depth, layer uniformity, or material composition can lead to increased inertial mass; while microscopic defects, impurities, and surface roughness introduce unwanted dissipation, as shown in Tab. III. Such imperfections can significantly impair quantum coherence and quantum control, highlighting the imperative for ultrahigh-precision shielding in scalable quantum technologies." We have added this paragraph to the revised main text. Please see the last paragraph of the right column of page 4 of the revised main text, and the first paragraph and Tab. II of Sec. V-B of page 17 in the revised Supplemental Materials.

TABLE III. **Classification of Large Mass and High Dissipation as Random Fabrication Imperfections in Quantum Physics**

| **Property** | **Random Fabrication Imperfection?** | **Scientific Justification** |
|---|---|---|
| Large mass | Yes | Often determined by design, but unintended variations in geometry, etching depth, or material deposition during fabrication can randomly increase the effective mass of quantum components. |
| High dissipation | Yes | Predominantly caused by uncontrollable factors such as microscopic defects, interface roughness, and residual impurities introduced stochastically during imperfect nanofabrication processes, leading to decoherence and energy loss. |

According to Referee C's insightful comment, we have revised the main text to more clearly explain the connection between the parameters $\rho$ and K and the robustness against random fabrication imperfections. Specifically, the parameter $\rho = m_j/m_0$ characterizes the ratio of the microsphere-resonator mass (see Tab. I), while the parameter K is the magnon-Kerr nonlinearity strength. By analyzing how the quantum-synchronization measure $\mathcal{S}_{\mathcal{Q}}$ varies with $\rho$ and K, we assess the resilience of quantum synchronization to fabrication-induced randomness, as shown in Fig. R5.

In particular, we provide a detailed clarification below:

Quantum synchronization of resonators in the regimes of large masses, large decays, and/or high temperatures is extremely challenging, because it requires an ultra-high optical power,

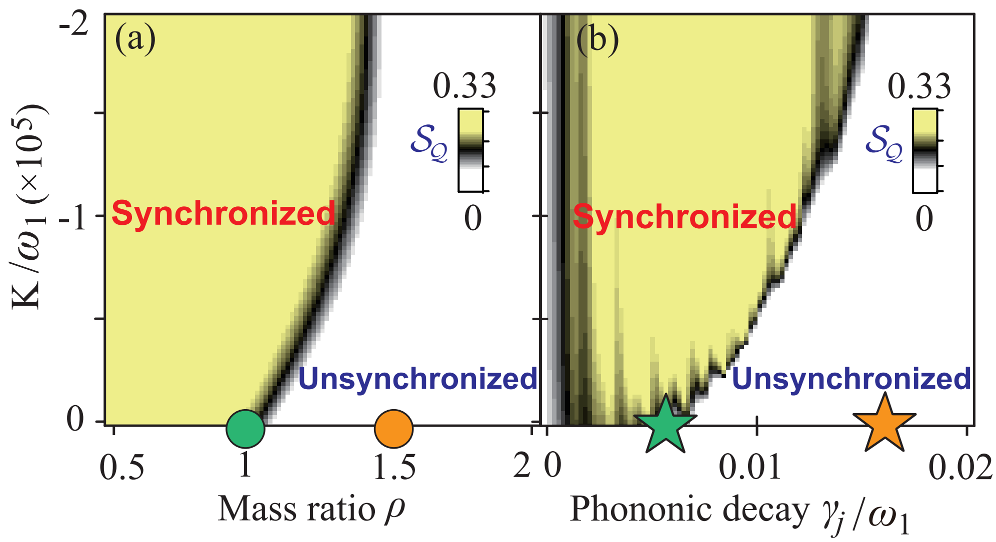


FIG. R5. (a) QS measure $\mathcal{S}_\mathcal{Q}$ versus the mass ratio $\rho = \rho_j = m_j/m_0$ and K, when $\Delta_m/\omega_1 = -1$ and $\Delta_a/\omega_1 = -1.005$. (b) $\mathcal{S}_\mathcal{Q}$ versus $\gamma_j$ and K, when $\Delta_m/\omega_1 = -1$ and $\Delta_a/\omega_1 = -1.005$.

which introduces extraneous excessive heating and intricate instabilities.

In recent decades, significant developments have been accomplished in the quantum synchronization of resonators in the *small-mass, low-damping, and low-noise* regimes, which have been widely reported both theoretically and experimentally, using cavity optomechanical platforms [R11]. However, these proposals and experiments still inherently suffer from the *large-mass, large-decay, and/or high-noise* limitations, which are a major challenge for the preparation of such extremely fragile quantum synchronization (see Fig. R5). The physical origin behind these obstacles is as follows:

*(i)* Quantum synchronization of resonators crucially depends on the strength of driving fields. Typically, a single-excitation coupling strength scales as $g = \eta x_{\rm ZPM}$, where $\eta$ quantifies the coupling strength to the resonator's position $x(t)$, and $x_{\rm ZPM}$ is the zero-point motion of the resonator in the trap, $x_{\rm ZPM} \sim \sqrt{\hbar/(2m\omega_m)}$, where $\omega_m$ is the center-of-mass oscillation frequency. For a large-mass resonator, the decrease in $x_{\rm ZPM}$ with increasing mass leads to a greatly reduced coupling strength, making quantum synchronization of resonators hard to achieve.

*(ii)* A large-decay and/or high-temperature resonator in the large-mass regime accelerates its intrinsic thermal motion, resulting in blocking efficient quantum synchronization of resonators.

*(iii)* For resonators in the regimes of a large mass and/or a high temperature, their quantum synchronization requires an ultra-high driving strength, which introduces extraneous excessive heating and intricate dynamical instabilities.

In this work, we propose how to overcome these obstacles and achieve quantum synchronization of resonators by *simply* employing the magnon-Kerr effect; and we reveal its exceptional synchronization properties otherwise unachievable in conventional devices. **Unlike previous schemes**, where quantum resources are generally deteriorated or even fully destroyed with increasing mass, decay, and/or noise of practical devices, our approach, surprisingly, shows that it is possible to directly *immunize* inherently fragile quantum synchronization against these detrimental factors [see Figs. 3(a) and 3(b)], without the need of utilizing any high-cost low-loss materials and noise filters at the expense of system's complexity [R88, R89] or any topological structures [R44, R46, R55, R58, R67, R68, R78].

In particular, we added the following explanation into the revised main text:

"Synchronizing resonators in the quantum regime crucially depends on the driving strengths. The coupling strength between a single excitation and the $j$th resonator scales as $g_j = \eta_j x_{\mathrm{ZPM},j}$, where $\eta_j$ quantifies the coupling strength to the resonator's position $x_j(t)$, and $x_{\mathrm{ZPM},j} \sim \sqrt{\hbar/(2m_j\omega_j)}$ is the zero-point motion of the resonator in a trap, with $m_j$ and $\omega_j$ being the resonator mass and the resonance frequency, respectively. For a large-mass resonator, a greatly reduced coupling strength $g_j$ results from the decrease in the zero-point motion $x_{\mathrm{ZPM},j}$ with increasing resonator mass $m_j$, and it makes quantum synchronization hard to achieve." Please see the first paragraph of the right column on page 2 of the revised main text, and Sec. V-D of the revised Supplemental Material.

In the revised version, we have added the following sentence to the main text:

"The reduction in quantum synchronization results from decreasing coupling strengths with increasing microsphere-resonator mass; while these diminished couplings can be considerably compensated via the magnon-Kerr nonlinearity, which improves both the magnon and photon numbers." Please see the second paragraph of the left column of page 5 in the revised main text.

In our analytical considerations, the masses of the two mechanical oscillators are in general different, as described by $m_1$ and $m_2$. However, for convenience, in our simulations, we consider the case where the masses of the two microsphere resonators are equal. Moreover, the masses, corresponding to the results of the other plots, are $m_1 = m_2 = 100$ ng, as given in Tab. I.

In the updated manuscript, we have emphasized these points and added the following sentences to the main text:

"Although the masses (decay rates) of the two microspheres generally differ, for simplicity, we assume equal masses (decay rates) in our simulations." Please see the caption of Fig. 3 on page 5 in the revised main text.

In particular, we have added Sec. V-D ("Effect of mass on quantum synchronization") to demonstrate this point in the revised Supplemental Material. Please see Sec. V-D of the revised Supplemental Material.

## COMMENT C12

*6) Potential applications are not addressed at all. What are these results good for (possibly)?*

## OUR REPLY TO COMMENT C12

| Application Area | Implications of Our Findings |
|---|---|
| **Nonreciprocal quantum information processing** | Enables unidirectional routing of quantum correlations in phononic or hybrid quantum networks, and enhances thermal robustness and quantum coherence preservation. |
| **Nonreciprocal quantum state engineering** | Provides a tunable Kerr-nonlinear mechanism to stabilize phase-locked mechanical states for preparing non-classical mechanical resources used in quantum sensing and quantum interface protocols. |
| **Quantum transduction architectures** | Facilitates noise-resilient quantum synchronization and temporal alignment across subsystems in hybrid quantum systems (e.g., microwave-to-optical converters), and improves quantum transduction efficiency. |
| **Nonreciprocal quantum sensing** | Robustness to fabrication imperfections enables practical quantum sensing platforms that operate reliably in noisy or imperfect environments. |
| **Chiral quantum networks** | Supports the design of scalable quantum networks resilient to disorder, with synchronized units functioning as robust quantum nodes. |
| **Fundamental studies of irreversibility** | Provides a testbed to explore entropy production and time-asymmetric quantum dynamics in open quantum systems, with applications in informing nonreciprocal quantum thermodynamics. |

TABLE IV. Potential applications and broader advantages of nonreciprocal quantum synchronization.

We thank Referee C for this important comment. While the primary focus of our work is on fundamental aspects of nonreciprocal quantum synchronization and one-way nonclassical correlations, our results have potential implications for unidirectional quantum information processing, particularly in the context of quantum entanglement distribution, quantum sensing, and the design of robust quantum networks [R18, R19, R90–R93]. The nonreciprocal quantum-synchronization-induced correlations could serve as a key quantum resource for stabilizing quantum coherence across distributed systems, even in the presence of noise or disorder [R18, R19, R90–R93], as shown in Tab. IV.

To highlight the potential applications and advantages of our findings, we have added a new section entitled "Discussions and conclusions" to the revised main text, including the following explanations.

"The demonstrated nonreciprocal control of quantum synchronization in this platform may find use in unidirectional quantum information processing, where synchronized quantum systems can serve as robust building blocks for distributed quantum networks [R18, R19, R90–R93]. The tunable magnon-Kerr nonlinearity effect and its resilience to random fabrication imperfections of practical devices suggest possible applications in quantum sensing and quantum signal transduction, especially in noisy or imperfect environments. Our robustness analysis provides insights relevant for the design of scalable chiral quantum networks, where fabrication-induced imperfections are inevitable. These findings may also contribute to future developments in nonreciprocal quantum sensing architectures that exploit collective dynamics for enhanced quantum precision [R18, R19, R90–R93].

In particular, our nonreciprocal quantum synchronization framework unlocks multiple exciting opportunities for application across quantum technologies. Including:

(i) Nonreciprocal quantum information processing.—The resulting nonreciprocal quantum synchronization enables a controllable unidirectional flow of quantum correlations (quantum information), which can be harnessed for unidirected quantum signal routing in phononic or hybrid quantum networks, where thermal robustness and coherence preservation are essential [R18, R90].

(ii) Nonreciprocal quantum state engineering.—Our scheme offers a tunable, nonlinearity-engineered route to stabilize phase-locked mechanical states. This can be employed to prepare non-classical mechanical states, which is good for nonreciprocal quantum sensing or interface protocols between mechanical and optical (magnonic) degrees of freedom [R19, R91].

(iii) Quantum transduction architectures.—In hybrid quantum systems where mechanical resonators serve as intermediaries between disparate platforms (e.g., microwave-to-optical conversion), nonreciprocal quantum synchronization could enable efficient and noise-robust temporal alignment across subsystems [R92, R93].

(iv) Fundamental studies of irreversibility.—The intrinsic unidirectionality in the quantum synchronization dynamics constitutes a controlled setting for investigating microscopic origins of irreversibility and entropy production in open quantum systems, thus offering insights relevant to nonreciprocal quantum thermodynamics [R18]."

We believe these points illustrate the broader utility of our findings, for both potential quantum technologies and fundamental physics.

Please see the second paragraph of the left column on page 8 in the revised main text, and Sec. VIII of the revised Supplemental Materials.

## COMMENT C13

***To conclude, I feel that the manuscript is potentially publishable in Nature Communications in view of the importance and novelty of the obtained results.*** *However, in my judgement, the authors have done a bad job at presenting them in a clear and accessible way. I therefore invite them to address the above comments.*

## OUR REPLY TO COMMENT C13

We thank Referee C for the encouraging assessment regarding the importance and novelty of our results, and especially for acknowledging that our manuscript is potentially publishable in Nature Communications. We also appreciate the candid feedback concerning the presentation.

In response, we have carefully revised the manuscript to improve clarity, structure, and accessibility. This includes rewriting key paragraphs for greater conceptual transparency, enhancing figure captions to better convey the main findings, and reorganizing parts of the text to improve logical flow. We trust that these changes address Referee C's concerns and help communicate the significance of our results more effectively.

We sincerely appreciate Referee C's thorough review of our manuscript and their valuable comments and suggestions. Incorporating these valuable feedback has significantly improved the clarity and impact of our work. We now believe our manuscript meets the standards for publication in Nature Communications.

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[R1] S. Maayani, R. Dahan, Y. Kligerman, E. Moses, A. U. Hassan, H. Jing, F. Nori, D. N. Christodoulides, and T. Carmon, Flying couplers above spinning resonators generate irreversible refraction, Nature (London) **558**, 569 (2018).

[R2] R. Huang, A. Miranowicz, J.-Q. Liao, F. Nori, and H. Jing, Nonreciprocal Photon Blockade, Phys. Rev. Lett. **121**, 153601 (2018).

[R3] A. Mari, A. Farace, N. Didier, V. Giovannetti, and R. Fazio, Measures of Quantum Synchronization in Continuous Variable Systems, Phys. Rev. Lett. **111**, 103605 (2013).

[R4] Y.-P. Wang, G.-Q. Zhang, D. Zhang, T.-F. Li, C.-M. Hu, and J. Q. You, Bistability of Cavity Magnon Polaritons, Phys. Rev. Lett. **120**, 057202 (2018).

[R5] R.-C. Shen, Y.-P. Wang, J. Li, S.-Y. Zhu, G. S. Agarwal, and J. Q. You, Long-Time Memory and Ternary Logic Gate Using a Multistable Cavity Magnonic System, Phys. Rev. Lett. **127**, 183202 (2021).

[R6] J. Qian, J. Li, S.-Y. Zhu, J. Q. You, and Y.-P. Wang, Probing PT-Symmetry Breaking of Non-Hermitian Topological Photonic States via Strong Photon-Magnon Coupling, Phys. Rev. Lett. **132**, 156901 (2024).

[R7] J. Li, S.-Y. Zhu, and G. S. Agarwal, Magnon-Photon-Phonon Entanglement in Cavity Magnomechanics, Phys. Rev. Lett. **121**, 203601 (2018).

[R8] Z. Shen, G.-T. Xu, M. Zhang, Y.-L. Zhang, Y. Wang, C.-Z. Chai, C.-L. Zou, G.-C. Guo, and C.-H. Dong, Coherent Coupling between Phonons, Magnons, and Photons, Phys. Rev. Lett. **129**, 243601 (2022).

[R9] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Cavity magnomechanics, Sci. Adv. **2**, e1501286 (2016).

[R10] M. F. Colombano, G. Arregui, F. Bonell, N. E. Capuj, E. Chavez-Angel, A. Pitanti, S. O. Valenzuela, C. SotomayorTorres, D. Navarro-Urrios, and M. V. Costache, Ferromagnetic Resonance Assisted Optomechanical Magnetometer, Phys. Rev. Lett. **125**, 147201 (2020).

[R11] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod.

Phys. **86**, 1391 (2014).

[R12] C. Dong, V. Fiore, M. C. Kuzyk, and H. Wang, Optomechanical dark mode, Science **338**, 1609 (2012).

[R13] X. Liu, J. Hu, Z.-F. Li, X. Li, P.-Y. Li, P.-J. Liang, Z.-Q. Zhou, C.-F. Li, and G.-C. Guo, Heralded entanglement distribution between two absorptive quantum memories, Nature (London) **594**, 41 (2021).

[R14] N. Ofek, A. Petrenko, R. Heeres, P. Reinhold, Z. Leghtas, B. Vlastakis, Y. Liu, L. Frunzio, S. Girvin, L. Jiang et al., Extending the lifetime of a quantum bit with error correction in superconducting circuits, Nature (London) **536**, 441 (2016).

[R15] M. Mirhosseini, A. Sipahigil, M. Kalaee, and O. Painter, Superconducting qubit to optical photon transduction, Nature (London) **588**, 599 (2020).

[R16] S. Klingler, V. Amin, S. Geprägs, K. Ganzhorn, H. MaierFlaig, M. Althammer, H. Huebl, R. Gross, R. D. McMichael, M. D. Stiles, S. T. B. Goennenwein, and M. Weiler, Spin-Torque Excitation of Perpendicular Standing Spin Waves in Coupled YIG/Co Heterostructures, Phys. Rev. Lett. **120**, 127201 (2018).

[R17] S. Maity, B. Pingault, G. Joe, M. Chalupnik, D. Assumpção, E. Cornell, L. Shao, and M. Lončar, Mechanical Control of a Single Nuclear Spin, Phys. Rev. X **12**, 011056 (2022).

[R18] E. Chitambar and G. Gour, Quantum resource theories, Rev. Mod. Phys. **91**, 025001(2019).

[R19] R. Uola, A. C. S. Costa, H. C. Nguyen, and O. Gühne, Quantum steering, Rev. Mod. Phys. **92**, 015001 (2020).

[R20] T. Nadolny, C. Bruder, and M. Brunelli, Nonreciprocal synchronization of active quantum spins, Phys. Rev. X **15**, 011010 (2025).

[R21] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).

[R22] F. J. Duarte and T. Taylor, *Quantum Entanglement Engineering and Applications* (IOP Press, London, 2021).

[R23] J. M. Raimond, M. Brune, and S. Haroche, Manipulating quantum entanglement with atoms and photons in a cavity, Rev. Mod. Phys. **73**, 565 (2001).

[R24] J. Volz, M. Weber, D. Schlenk, W. Rosenfeld, J. Vrana, K. Saucke, C. Kurtsiefer, and H. Weinfurter, Observation of entanglement of a single photon with a trapped atom, Phys. Rev. Lett. **96**, 030404 (2006).

[R25] T. Wilk, S. C. Webster, A. Kuhn, and G. Rempe, Single-atom single-photon quantum interface, Science **317**, 488 (2007).

[R26] Z.-S. Yuan, Y.-A. Chen, B. Zhao, S. Chen, J. Schmiedmayer, and J.-W. Pan, Experimental demonstration of a BDCZ quantum repeater node, Nature (London) **454**, 1098 (2008).

[R27] D. N. Matsukevich, T. Chanelière, M. Bhattacharya, S.-Y. Lan, S. D. Jenkins, T. A. B. Kennedy, and A. Kuzmich, Entanglement of a photon and a collective atomic excitation, Phys. Rev. Lett. **95**, 040405 (2005).

[R28] J. F. Sherson, H. Krauter, R. K. Olsson, B. Julsgaard, K. Hammerer, I. Cirac, and E. S. Polzik, Quantum teleportation between light and matter, Nature (London) **443**, 557 (2006).

[R29] W. Qin, A. Miranowicz, P.-B. Li, X.-Y. Lü, J. Q. You, and F. Nori, Exponentially Enhanced Light-Matter Interaction, Cooperativities, and Steady-State Entanglement Using Parametric Amplification, Phys. Rev. Lett. **120**, 093601 (2018).

[R30] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Quantum dynamics of single trapped ions, Rev. Mod. Phys. **75**, 281 (2003).

[R31] B. B. Blinov, D. L. Moehring, L. M. Duan, and C. Monroe, Observation of entanglement between a single trapped atom and a single photon, Nature (London) **428**, 153 (2004).

[R32] W. B. Gao, P. Fallahi, E. Togan, J. Miguel-Sanchez, and A. Imamoğlu, Observation of entanglement between a quantum dot spin and a single photon, Nature (London) **491**, 426 (2012).

[R33] J. Q. You and F. Nori, Atomic physics and quantum optics using superconducting circuits, Nature (London) **474**, 589 (2011).

[R34] X. Gu, A. F. Kockum, A. Miranowicz, Y.-x. Liu, and F. Nori, Microwave photonics with superconducting quantum circuits, Phys. Rep. **718**, 1 (2017).

[R35] Z. L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, Rev. Mod. Phys. **85**, 623 (2013).

[R36] J. D. Jost, J. P. Home, J. M. Amini, D. Hanneke, R. Ozeri, C. Langer, J. J. Bollinger, D. Leibfried, and D. J. Wineland, Entangled mechanical oscillators, Nature (London) **459**, 683 (2009).

[R37] B. Hensen *et al.,* Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres, Nature (London) **526**, 682 (2015).

[R38] H. J. Kimble, The quantum internet, Nature (London) **453**, 1023 (2008).

[R39] C. W. Chou, H. D. Riedmatten, D. Felinto, S. V. Polyakov, S. J. van Enk, and H. J. Kimble, Measurement-induced entanglement for excitation stored in remote atomic ensembles, Nature (London) **438**, 828 (2005).

[R40] D. L. Moehring, P. Maunz, S. Olmschenk, K. C. Younge, D. N. Matsukevich, L.-M. Duan, and C. Monroe, Entanglement of single-atom quantum bits at a distance, Nature (London) **449**, 68 (2007).

[R41] A. H. Safavi-Naeini, D. V. Thourhout, R. Baets, and R. V. Laer, Controlling phonons and photons at the wavelength scale: integrated photonics meets integrated phononics, Optica **6**, 213 (2019).

[R42] T. Kato, *Perturbation Theory for Linear Operators* (Springer, Berlin 2013).

[R43] W. D. Heiss, Phases of wave functions and level repulsion, Eur. Phys. J. D **7**, 1 (1999).

[R44] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Non-Hermitian physics and $\mathcal{PT}$ symmetry, Nat. Phys. **14**, 11 (2018).

[R45] M. Parto, Y. G. N. Liu, B. Bahari, M. Khajavikhan, and D. N. Christodoulides, Non-Hermitian and topological photonics: optics at an exceptional point, Nanophotonics **10**, 403 (2021).

[R46] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Beam dynamics in $\mathcal{PT}$ symmetric optical lattices, Phys. Rev. Lett. **100**, 103904 (2008).

[R47] S. Klaiman, U. Günther and N. Moiseyev, Visualization of branch points in $\mathcal{PT}$-symmetric waveguides, Phys. Rev. Lett. **101**, 080402 (2008).

[R48] M. C. Zheng, D. N. Christodoulides, R. Fleischmann, and T. Kottos, $\mathcal{PT}$ optical lattices and universality in beam dynamics, Phys. Rev. A **82**, 010103 (2010).

[R49] C. M. Bender and S. Boettcher, Real Spectra in NonHermitian Hamiltonians Having $\mathcal{PT}$ Symmetry, Phys. Rev. Lett. **80**, 5243 (1998).

[R50] C. M. Bender, Making sense of non-Hermitian Hamiltonians, Rep. Prog. Phys. **70**, 947 (2007).

[R51] A. B. Khanikaev, S. H. Mousavi, W. K. Tse, M. Kargarian, A. H. MacDonald, and G. Shvets, Photonic topological insulators, Nat. Mater. **12**, 233 (2013).

[R52] H. Jing, Ş. K. Özdemir, X.-Y. Lü, J. Zhang, L. Yang, and F. Nori, $\mathcal{PT}$-Symmetric Phonon Laser, Phys. Rev. Lett. **113**, 053604 (2014).

[R53] Ş. K. Özdemir, S. Rotter, F. Nori, and L. Yang, Parity-time symmetry and exceptional points in photonics, Nat. Mater. **18**, 783 (2019).

[R54] F. P. D. Pile, Gaining with loss, Nat. Photon. **11**, 742 (2017).

[R55] C. Chen, L. Jin, and R.-B. Liu, Sensitivity of parameter estimation near the exceptional point of a non-Hermitian system, New J. Phys. **21**, 083002 (2019).

[R56] L. Pickup, H. Sigurdsson, J. Ruostekoski, and P. G. Lagoudakis, Synthetic band-structure engineering in polariton crystals with non-Hermitian topological phases, Nat. Commun. **11**, 4431 (2020).

[R57] E. J. Bergholtz, J. C. Budich, and F. K. Kunst, Exceptional topology of non-Hermitian systems, Rev. Mod. Phys. **93**, 015005 (2021).

[R58] J. M. P. Nair, D. Mukhopadhyay, and G. S. Agarwal, Enhanced Sensing of Weak Anharmonicities through Coherences in Dissipatively Coupled Anti-$\mathcal{PT}$ Symmetric Systems, Phys. Rev. Lett. **126**, 180401 (2021).

[R59] J. del Pino, J. J. Slim, and E. Verhagen, Non-Hermitian chiral phononics through optomechanically induced squeezing, Nature (London) **606**, 82 (2022).

[R60] C. Dembowski *et al.*, Phys. Rev. Lett. **86**, 787-790 (2001).

[R61] E. Persson, I. Rotter, H. Stöckmann, M. Barth, Phys. Rev. Lett. **85**, 2478-2481 (2000).

[R62] S. B. Lee *et al.*, Phys. Rev. Lett. **103**, 134101 (2009).

[R63] T. Kitagawa *et al.*, Observation of topologically protected bound states in photonic quantum walks, Nat. Commun. **3**, 882 (2012).

[R64] S. Malzard, C. Poli, and H. Schomerus, Topologically Protected Defect States in Open Photonic Systems with Non-Hermitian Charge-Conjugation and Parity-Time Symmetry, Phys. Rev. Lett. **115**, 200402 (2015).

[R65] F. Cardano *et al.*, Statistical moments of quantum-walk dynamics reveal topological quantum transitions, Nat. Commun. **7**, 11439 (2016).

[R66] M. C. Rechtsman *et al.*, Topological protection of photonic path entanglement, Optica **3**, 925 (2016).

[R67] S. Mittal, V. V. Orre, and M. Hafezi, Topologically robust transport of entangled photons in a 2D photonic system, Opt. Express **24**, 15631 (2016).

[R68] M. A. Gorlach, A. N. Poddubny, Topological edge states of bound photon pairs, Phys. Rev. A **95**, 053866 (2017).

[R69] S. Barik *et al.*, A topological quantum optics interface, Science **359**, 666 (2018).

[R70] A. Blanco-Redondo, B. Bell, D. Oren, B. J. Eggleton, and M. Segev, Topological protection of biphoton states, Science **362**, 568 (2018).

[R71] S. Mittal, E. A. Goldschmidt, and M. Hafezi, A topological source of quantum light, Nature (London) **561**, 502 (2018).

[R72] S. Mittal, V. V. Orre, E. A. Goldschmidt, and M. Hafezi, Tunable quantum interference using a topological source of indistinguishable photon pairs, Nat. Photonics **15**, 542 (2021).

[R73] H. Xu, D. Mason, L. Jiang, and J. G. E. Harris, Topological energy transfer in an optomechanical system with exceptional points, Nature (London) **537**, 80 (2016).

[R74] J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, and S. Rotter, Dynamically encircling an exceptional point for asymmetric mode switching, Nature (London) **537**, 76 (2016).

[R75] H. Ren, T. Shah, H. Pfeifer, C. Brendel, V. Peano, F. Marquardt, and O. Painter, Topological phonon transport in an optomechanical system, Nat. Comm. **13**, 3476 (2022).

[R76] M. Abbasi, W. Chen, M. Naghiloo, Y. N. Joglekar, and K. W. Murch, Topological Quantum

State Control through Exceptional-Point Proximity, Phys. Rev. Lett. **128**, 160401 (2022).

[R77] A. U. Hassan, B. Zhen, M. Soljačić, M. Khajavikhan, and D. N. Christodoulides, Dynamically encircling exceptional points: exact evolution and polarization state conversion, Phys. Rev. Lett. **118**, 093002 (2017).

[R78] H. Wang, S. Assawaworrarit, S. Fan, Dynamics for encircling an exceptional point in a nonlinear non-Hermitian system, Opt. Lett. **44**, 638 (2019).

[R79] A. U. Hassan, G. L. Galmiche, G. Harari, P. LiKamWa, M. Khajavikhan, M. Segev, and D. N. Christodoulides, Chiral state conversion without encircling an exceptional point, Phys. Rev. A **96**, 052129 (2017).

[R80] Q. Zhong, M. Khajavikhan, D. N. Christodoulides, and R. El-Ganainy, Winding around non-Hermitian singularities, Nat. Commun. **9**, 4808 (2018).

[R81] J. Feilhauer, A. Schumer, J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, N. Moiseyev, and S. Rotter, Encircling exceptional points as a non-Hermitian extension of rapid adiabatic passage, Phys. Rev. A **102**, 040201(R) (2020).

[R82] H. Nasari, G. Lopez-Galmiche, H. E. Lopez-Aviles, A. Schumer, A. U. Hassan, Q. Zhong, S. Rotter, P. LiKamWa, D. N. Christodoulides, and M. Khajavikhan, Observation of chiral state transfer without encircling an exceptional point, Nature (London) **605**, 256 (2022).

[R83] I. I. Arkhipov, A. Miranowicz, F. Minganti, Ş. K. Özdemir, and F. Nori, Dynamically crossing diabolic points while encircling exceptional curves: A programmable symmetric-asymmetric multimode switch, Nat. Commun. **14**, 2076 (2023).

[R84] Y. Choi, C. Hahn, J. W. Yoon, S. H. Song, and P. Berini, Extremely broadband, on-chip optical nonreciprocity enabled by mimicking nonlinear anti-adiabatic quantum jumps near exceptional points, Nat. Commun. **8**, 14154 (2017).

[R85] J. W. Yoon *et al.,* Time-asymmetric loop around an exceptional point over the full optical communications band, Nature (London) **562**, 86 (2018).

[R86] R. Uzdin, A. Mailybaev, and N. Moiseyev, On the observability and asymmetry of adiabatic state flips generated by exceptional points, J. Phys. A **44**, 435302 (2011).

[R87] E.-M. Graefe, A. A. Mailybaev, and N. Moiseyev, Breakdown of adiabatic transfer of light in waveguides in the presence of absorption, Phys. Rev. A **88**, 033842 (2013).

[R88] A. Boltasseva, H. A. Atwater, Low-loss plasmonic metamaterials, Science **331**, 290 (2011).

[R89] S. M. Kuo, D. R. Morgan, Active noise control: A tutorial review, Proc. IEEE **87**, 943 (1999).

[R90] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. **84**, 621 (2012).

[R91] A. I. Lvovsky and M. G. Raymer, Continuous-variable optical quantum-state tomography, Rev. Mod. Phys. **81**, 299 (2009).

[R92] K. Azuma, S. E. Economou, D. Elkouss, P. Hilaire, L. Jiang, H.-K. Lo, and I. Tzitrin, Quantum repeaters: From quantum networks to the quantum internet, Rev. Mod. Phys. **95**, 045006 (2023).

[R93] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).

***** REPORT OF REFEREE A – NCOMMS-25-04564A *****

## COMMENT A1

*In the revised manuscript, the concerns that I raised in my previous report have been addressed.*

## OUR REPLY TO COMMENT A1

We sincerely thank Referee A for recommending the publication of our study in Nature Communications.

## COMMENT A2

*Shouldn't the new phrase in the caption of Fig. 1 read "The phonon-phonon coupling chi originates from direct physical contact between a spinning silica microsphere and a COUNTER-rotating YIG sphere, both maintained at constant angular velocities Omega and* $-\Omega$*"? I am still somewhat skeptical about the experimental realization of a direct contact between two rotating silica and YIG spheres at angular velocities of* 6 *kHz but leave this for the community to judge.*

## OUR REPLY TO COMMENT A2

We apologize for any confusion resulting from the unclear description of the new phrase in the caption of Fig. 1.

Inspired by Referee A's insightful comment, we have revised the phrase in the caption of Fig. 1 as: "The phonon-phonon coupling $\chi$ originates from direct physical contact between a spinning silica microsphere and a counter-rotating YIG sphere, both maintained at constant angular velocities $\Omega$ and $-\Omega$."

## COMMENT A3

*A last remark about the new section "Discussions and conclusions": the authors refer repeatedly to RMP 91, 025001 (2019) [Ref. 80] in the context of quantum synchronization and coupled quantum oscillators. However, Ref. 80 is a review about quantum resource theories and does not address quantum synchronization. The same remark applies to Section F and Ref. S31 of the Supplementary Material.*

### OUR REPLY TO COMMENT A3

According to Referee A's helpful comment, we have removed this reference [i.e., RMP 91, 025001 (2019)] throughout the main text and Supplementary Material.

### COMMENT A4

*The revised manuscript is substantially improved and ready for publication.*

### OUR REPLY TO COMMENT A4

We sincerely thank Referee A for recommending the publication of our study in Nature Communications.

## *** REPORT OF REFEREE B – NCOMMS-25-04564A ***

### COMMENT B1

*The authors have fully addressed all my comments. The revised manuscript has also been significantly improved. I can recommend a publication now.*

### OUR REPLY TO COMMENT B1

We are grateful to Referee B for recommending our work for publication in Nature Communications.

## *** REPORT OF REFEREE C – NCOMMS-25-04564A***

### COMMENT C1

*The authors have clarified all the issues that I have raised to my satisfaction. I therefore recommend publication in Nature Communications.*

### OUR REPLY TO COMMENT C1

We thank Referee C for recommending the publication of our work in Nature Communications.

Quantum nonreciprocity is of interest in fundamental physics and many important applications. It has been widely studied in optical systems. Recently, it is explored in phononic systems, in particular, the optomechanical resonators. By spinning resonators, this manuscript theoretically investigates nonreciprocal quantum synchronization of two mechanical modes and their one-way quantum squeezing. The model is clear and reasonable. The results may reveal new physics in phonons. The manuscript is well organized and written in English. Before recommendation of publication, I have some comments for the authors:

1. As claimed by the authors, the phononic coupling is created via a direct physical contact between the spinning silica microsphere and the YIG sphere. How can a physical contact be made between a moving (spinning) part and a static part, if the YIG sphere is not spinning?

2. How to drive the mechanical mode and the magnon mode? Is it easy in experiment?

3. I can't find the definition of $m_0$.

4. As shown in Fig. 4(a), the two-mode squeezing is very weak. What limits the squeezing degree? Can the squeezing be improved?

5. What is the difference between nonreciprocal quantum synchronization and quantum steering?