Peer Review File

Interplay between disorder and topology in Thouless pumping on a superconducting quantum processor

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
In this manuscript, the authors describe the use of a new quantum simulating platform of coupled superconducting qubits to study disorder effect on a topological pump. They simulate both randomness of the hoppings and of site potentials of a 1D time-dependent Rice-Mele model. This subject is interesting and topical, as shown by several recent experimental and theoretical studies along these lines. While similar studies have been conducted with cold-atoms simulators (e.g. Ref. [19]), this is the first time that a solid-state platform is used in this context. As such, this work is interesting.

However, the authors faced a major difficulty in this task, which is not clearly stated in the manuscript. This difficulty lies in the necessity to have a stable simulator over a time period long enough to study accurately the adiabatic regime of a topological pump. This is illustrated on the Figure 2b of this manuscript, displaying the displacement of an initial wavepacket over a period of the drive.
Such a figure is most welcome, and should be systematically presented in experimental studies of topological pumping. For a topological pump, this displacement is expected to be quantized when the initial wavepacket is prepared in a Wannier state. However topological pumping requires the dynamics to be adiabatic, i.e. slow enough. Any finite frequency of the drive will ultimately lead to a departure from the adiabatic dynamics on long timescales. However for a slow enough drive, this departure occurs on a sufficiently long times such that topological pumping can be experimentally measured on intermediate time scales. As expected, for short enough periods T of the drive, the authors observe a departure from the quantized displacement. The quantized displacement is reached for T=650ns. We would expect a plateau for this quantized displacement for larger periods. This is not what is observed for large T ! The displacement is no longer quantized, and is reduced for larger T. We suspect that this platform is then limited by the dephasing time T2=826ns of the qubit, see Table S1 of the Supp. Mat. Hence without any disorder, this platform is on the verge of realizing a topological pump, being limited on one side by non-adiabatic dynamics of the pump and on the other side by the dephasing mechanisms of the qubit. It appears to be a very unstable topological pump. This instability is observed on the Fig.2a where a departure from a topological pumping is observed after 3 periods of drive for the clean case. Thus we expect that a study of the effects of disorder on this platform will overemphasize its consequences. Indeed, this is demonstrated by the discrepancy between the expected quantized rate in Fig. 4b and the experimental results which show no expected quantized pumping. This is our major criticism of this experimental study, which severely limits its potential impact.

Besides this, we have found the results of Fig. 3, focused on the effects of a site potential disorder, to be convincing, but very close to those obtained in Ref. 19 on a cold-atom simulator, with identical protocols. However, we do not expect the pumping corresponding to the Fig.3e to be topological since its quantization (which is not demonstrated in this figure) would require fine tuned Hamiltonian parameters to precisely combine the effects of the trivial inner and outer trajectories. Indeed, a quantized pumping can occur for parametric pumping and not only for topological pumping. For the reasons mentioned above, the results of Fig.4, which were more original and central for this study, turn out to be disappointing, with no sign of a quantized rate of displacement or topological pumping induced by quasi-periodic hopping disorder in Fig. 4b.

For the above reasons, we believe that the platform developed by the authors is extremely interesting, but that the present study of topological pumping was not successful enough to warrant a publication in a broad audience journal such as Nature Communications.

Besides, we also noted that the writing of this manuscript could also be slightly improved. Let us mention a few points worth

improving:
- The first sentence of the abstract « Thouless pumping […] represents the quantized charge pumped during an adiabatic cyclic evolution. »
- On l.25, the sentence « Thus is bridges the quantized conductance and the topological invariant […]»
- In the definition of the model (1), the authors should state that Delta(t) and delta(t) are periodic function of period T, which is not defined but used in eq. (2).
- The precise way that delta x is experimentally measured (e.g. in Fig. 3b, 4a, 4b, etc) is unclear : over how many periods, what averaging is performed, etc.
- A discussion on how close to a Wannier is the initial state would be useful.
- What do we learn from the instantaneous energy spectra shown in Fig. 2d and 3f?
- On Fig 1c-d, the representation of on-site disorder by a spin is confusing.
- On l.163-165, how can a Chern number be defined for a gapless cycle?
- On l. 206, is the phase beta_j identical on all sites j or does it take a random value from site to site? If beta_j is not constant, then the disorder is random rather than quasi-periodic.
- The section on Floquet engineering in the Methods is hardly understandable independently of the Supplementary Materials.
- On l.315, what does the sentence "all qubit probabilities are corrected to eliminate the measurement errors" mean?
- In Fig. S4, titles would help the reading.

Reviewer #2

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Reviewer #3

(Remarks to the Author)
This paper describes observation of Thouless charge pumping in a (relatively) large superconducting quantum processor. From a condensed matter perspective Thouless pumping is interesting as it is one of the simplest examples of topological phenomena. Moreover it emerges from periodically driven systems, making highly controllable quantum platforms, such as their Chuang-tzu superconducting quantum process, an ideal candidate to study such phenomena. The authors study topological pumping induced from both (1) on-site potential disorder and (2) hopping disorder. (2) being the more novel result.

Overall, the authors' work is impressive, but before recommending publication, there are some items I would like the authors to address.

Please add more details about how noise affects the experiment. Currently there are only standard reports of, for example, T1 and T2. However, for example, in Fig 2a, beyond 3T the C1 and C4 curves begin to deviate the numerical results. Why? I presume this has to do with decoherence, but there is no comment on why these deviations occur at long times. Moreover, T2* = ~800 ns. This about as long as the period times T (for example T = 650 ns in Fig 2b), shouldn't going to times of a few T (such as in Fig 2a) result in significant dephasing? Also, the numerics are noiseless numerics I assume? Perhaps this can be specified such that it is more clear that experiment agrees well with noiseless numerics rather than numerics that take hardware noise into consideration.

In the last sentence of the caption of Fig. 1, is there a typo? It reads: "... are staggered with one (up) large and one down (small) due to the staggered RM Hamiltonian." Should this instead read: "... are staggered with one up (large) and one down (small) due to the staggered RM Hamiltonian."

In the first paragraph of the "Pumping with hopping disorder" section, it reads: "... the decay of \delta x obeys a distinct law from the on-site disorder case…". I see that there is a citation, but think it would help with understanding to mention what the distinct laws actually are for each case (on-site vs hopping disorder) and include appropriate citations. I wasn't really sure what this was supposed to mean or what "law" really meant when reading through.

In the last sentence of the same paragraph discussed in item 3 (lines 192/193): I found the phrasing "... making the adiabatic conditions hardly saturated." Can you clarify this?

In Fig 4b, I don't see a clear difference for \delta x when inside vs outside the blue dashed line (topological index curve). Should I? Why don't I? Where is the evidence for pumping with the quasi-periodic disorder coming from in the data? Please clarify this.

In the last sentence of the conclusion (lines 230-233), this feels vague and unjustified. I recommend removing this last sentence, unless it can be sufficiently justified. Using quantum devices with disorder (different on-site energies/qubit frequencies) is nothing new.

In the experimental setup, it says: "All qubit probabilities are corrected to eliminate measurement errors." What does this mean? Does "qubit probabilities" mean "probability of measuring the 1 state? And how are they corrected? Please clarify this.

Reviewer #4

(Remarks to the Author)
In this manuscript, the authors experimentally demonstrate topological pumping of excitations in their superconducting quantum devices. The authors experiment topological pumping under controllable on-site and hopping disorder. Especially, the authors study topological pumping under quasi-periodic hopping disorder, which has not been experimentally realized before. The authors leverage parametric flux modulation of frequency-tunable transmon qubits for Floquet engineering, in order to achieve the required precise and dynamical controllability in the Rice-Mele (RM) model under study.

The authors first prepare a single-excitation initial state in the middle of the 1-dimensional chain of the superconducting qubits. Then, the hopping strengths and and on-site potentials are dynamically controlled by Floquet engineering with a trajectory in the parameter space, slowly enough in order to satisfy adiabaticity conditions. The resulting transport of the excitation after such time evolution is estimated by measuring the center-of-mass (CoM) of the excitations. They observe quantized displacement of CoM depending on the choice of pumping cycle period and winding number of the trajectory of the parameters, demonstrating the quantized pumping. Importantly, the authors claim observation of quantized pumping persisting under sufficiently small disorder in on-site energies and topological pumping induced by quasi-periodic disorder in hopping rates.

On the whole, the manuscript is well written and presents the essential details of the author’s work. The experiments conducted in this work are nicely designed for the related theory. This work shows good advances in control of an array of superconducting qubits and their utility in analog quantum simulation. Below are the reviewer’s comments on the manuscript.

What distinguishes this manuscript from existing experimental work on the RM model is the observation of hopping disorder induced topological pumping. However, this main point does not seem to be sufficiently supported from the main text. In the main text section “Pumping with hopping disorder”, the authors should show cleaner data than the data shown in Fig.4b, in order to support the claim that they observe topological pumping induced by quasi-periodic disorder.
First, the change in δx does not seem significant over the presented range of Wp. (c.f. changes in Fig.3a or Fig.4a. are much more significant) Second, it is unclear if observation of non-zero δx itself can imply observation of topological pumping. For example, in Fig. 2b, shorter pumping period could also induce non-zero δx but it apparently doesn’t satisfy the adiabatic condition and therefore cannot be used as evidence of “nontrivial pumping”. Additionally, the small change in δx does not seem significantly correlated with the topological index.
Furthermore, interpretation of experimental data as well as comparison with simulation is not sufficiently given in the main text, while the explanation for simulated data itself is given. As the experimental observation of topological pumping induced by quasi-periodic disorder is one of the authors’ major claimed novelties, cleaner data and interpretation of experimental results should be given.

From the title and the introduction of the manuscript, one of the main achievements the authors wish to highlight is that they simulated topological pumping with controllable disorder and observation of disorder-induced quantized pumping, as theoretically predicted and experimentally demonstrated by related works. [Wauters et al., PRL, 2019, Cerjan et al., Light Sci Appl, 2020, and Nakajima et al., Nature Physics, 2021, each Ref 23, 21, and 19 in the manuscript]

In order to strength the claim of similar observation in this work, I suggest providing more quantitative comparison and explanation on the comparison between the CoM (center-of-mass) shift δx and the numerically estimated charge pumped per cycle ΔQ being made in the main text section “Pumping in the presence of on-site disorder” and the main text Fig. 3. There are two main points that may have room for improvement for better explanation. First, there is a relatively fast decrease in δx in Fig.3a compared to the tendency in ΔQ given in Fig.3c in the range 0 MHz < V/2π < 10 MHz, dropping steadily down from 1.95 to ~1.5. Second, there is an unexpectedly large δx in Fig.3b compared to the tendency in ΔQ given in Fig.3e, in the range 0 MHz < V/2π < 7 MHz. It would be better grounded if the authors provide a more quantitative acceptance criteria for quantized pumping, or explanation for deviations and potential sources for them.

There is room for improvement in making connections between the data being discussed and the claim in the paragraph “Next, we experimentally investigate topological pumping… hardly saturated” in the main text section “Pumping with hopping disorder”. In this paragraph, comparison between δx in Fig.3a (random on-site disorder) and Fig.4a (random hopping disorder) are made, and it is mentioned that the two curves are similar. Still, the main claim of this paragraph is that “it is difficult to observe quantized pumping under random hopping disorder”. This argument seems to be made based on theoretical predictions that the two curves are supposed to obey different decay laws and possess different susceptibility against non-adiabatic evolution. As an experimental work, I suggest making sufficient connections to experimental data to back up this argument.

Although this work shows analog quantum simulation on a highly controllable large scale (41 superconducting qubits) quantum processor, the demonstrated capability does not imply significant technological advancement. First, the experiments are still limited to single-excitation subspace, which is easily simulated classically. Second, the provided data for disorder-induced pumping (Fig. 3 and Fig. 4) does not seem to require all 41 qubits, as it can be inferred by the small observed CoM displacement compared to the system size and small hopping rates between sites. At any given time before the decoherence effect shows up, the system remains in a relatively simple few-body entangled or even product state. The second point may be resolved if the authors provide experimental data for δx per multiple pumping cycles (>> 1 cycles) is provided.

In addition, here are several minor points:

In the main text section “Topological invariant and topological pumping”, the authors mention the “careful” choice of pumping cycle period T = 650 ns, as a point to provide maximum δx. However, a more detailed explanation would be helpful to understand such choice, as the explanation for required adiabaticity in quantized pumping seems to imply that any period that satisfies the adiabaticity condition (i.e., longer than 650 ns) should provide the maximum δx.
If the smaller δx for large T is limited by decoherence , the experimental results should be better compared with the theoretical / numerical predictions if the effect of decoherence is taken into account. This comment is applicable to other observations as well. For example, the duration of the experimental pulse sequences after state preparation and before readout is not negligible when compared with the coherence times of the qubits. I recommend post-selection for single-excitation states if it has not been applied to the presented data.

Although it may not be consistent with the title and main claims of the manuscript, I recommend giving more recognition in double-excitation topological pumping experiments in the main text highlighting such capability. which is currently only discussed in the supplementary. Although given the parameter regime it may still be classically simulatable, it would allude to the potential capability of the proposed scheme for studying topological pumping or other many-body physics phenomena under significantly large onsite energy that may not be easily accessible with classical simulations and other experimental platforms.

State preparation and readout processes may need more detailed descriptions. Based on the main text, an example sketch of a single measurement sequence may be inferred to be the following: the system is first prepared in a single-qubit single-excitation state when the qubits are detuned and no parametric drive is applied. Then, parametric flux modulation drives are turned on to bring the system to the initial point of a pumping sequence. After completing the pumping sequence, the parametric drives are turned off, and the qubits are brought back to the frequencies where readout is followed. It would be more informative to provide such pulse sequences, and discuss how the turning-on / turning-off part of the parametric drives are performed (e.g., adiabatically or instantaneously) and how they affect state preparation and measurement outcome. I recommend adding discussion of these in supplementary information or Methods section.

Related to Comments 1 - 3, the comparisons being made in the main text Fig.3 and Fig.4 would be easier if the x-axes of Fig.3a-e and Fig.4a-c are provided in a normalized scale such as $V/\Delta 0$ and $Wp/\delta 0$, and if the ranges are matched. Note that the location of peaks in Fig.3b would be compared easier to the peaks of Fig.3e if Fig.3b includes 30 MHz < $V/2\pi$ < 40 MHz.

Fig.3f needs explanation for distinction between faint blue lines and solid blue lines.

It would be informative to explicitly mention the validity of the hard-core boson approximation. One possibility may be to provide $|U/g_{j,j+1}|$. (please find [Yan et al., Science, 2019, Ref 42 of the manuscript])

In summary, this work shows good progress in superconducting circuit-based quantum simulation, Floquet engineering, and analog quantum simulation under controlled disorder. However, the experiments shown in this manuscript require improvements in data and interpretation with quantitative comparisons and analysis, in order to support several of the authors’ major claims. Therefore, I am hesitant to recommend publication in Nature Communications. The current form of the manuscript is more suitable for publication in npj Quantum Information, unless the above mentioned points are properly addressed.

Reviewer #5

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
I have read in great details the long response of the authors to the three referees and the modifications of their manuscript.
I appreciate the efforts that the authors devoted to answer in details the various comments of the referees, including mine.
I all now convinced by their answer that the decoherence time of their coupled qubit is much longer than I initially understood.
The various corrections that the authors provided, in particular to the figures, definitely improved the clarity of the manuscript.
However, in the end I have mixed feelings about the recommendation of publication of this manuscript in Nature

Communications.
On one hand, the experimental platform is very impressive, and I have the impression that the experiments are at the state of the art with a solid state simulation platform for topological pumping. On the other hand, as the authors acknowledge themselves in the answer to the referees, this platform fail to demonstrate a quantized pumping induced by disorder but come as close as currently possible to do so, as exemplified in their Fig. R1-4 of their response.
This achievement is not as impressive as expected, in particular given the topological nature of the underlying phenomenon. However I have the impression that it could possibly warrant a publication in Nature Communications.

Reviewer #3

(Remarks to the Author)
I have reviewed the authors' thorough responses and recommend the manuscript for publication.

Reviewer #4

(Remarks to the Author)
After carefully examining the response letter and the updated manuscript, I believe that the authors' revisions and their responses to the review comments have significantly improved the paper. I appreciate their attentiveness to the feedback and the increased clarity present throughout the manuscript. The revisions made in light of the previous comments are persuasive and have successfully addressed many of the concerns raised in the initial review.

Below we have outlined our responses to the revised manuscript and provided suggestions for further improvements. If these points are adequately addressed, I recommend the paper for publication in Nature Communications.

1. The authors attempt to strengthen their claim of topological pumping induced by quasi-periodic hopping disorder by presenting the emergence of a quantized plateau under adiabatic conditions using numerical simulation. Additionally, they show agreement between simulation and experimental results when these conditions are not met.

However, I am concerned that even with the additional explanation the data does not imply experimental demonstration of disorder-induced topological pumping. Although the data is consistent with the simulation under non-adiabatic regime (short evolution period), this does not mean the agreement would extend to adiabatic conditions (longer evolution period). Furthermore, it is still not clear if the consistency at weak adiabaticity is coincidental unless the authors rule out the influence of other factors, including discrepancy between the initial and Wannier states, challenges in precise Hamiltonian control, qubit frequency fluctuations, accidental qubit swaps due to Hamiltonian specifics or accidental resonances to TLSs or untracked modes, etc. Additionally, the existing comparison does not extend beyond $W_p/\delta_0'' > 3$. In short, the current experimental data are still ambiguous and not clean enough to support the claim of experimental realization of quasi-periodic hopping induced topological pumping.

Generally, the claim or agreement with the underlying model would be more convincing if the authors provide experimental evidence of the emergence of a quantized plateau over several increasing evolution periods. This is similar to the necessary demonstration of quantum phase transition at increasing system sizes. Due to realistic experimental constraints, I understand that this may not be achievable with the current capabilities of their platform.

If this suggestion is not realizable, I suggest weakening their claim about the experimental realization of topological pumping induced by quasi-periodic hopping disorder. An example might be to tone down the claim to “observation of signatures consistent with topological pumping induced by quasi-periodic hopping disorder under insufficient adiabaticity”.

2. The revised manuscript is titled "Reciprocity in Disorder and Topology of Thouless Pumping on a Superconducting Quantum Processor." However, the use of "Reciprocity" is unclear. It may misleadingly suggest a sort of symmetry observed during the experiment, which is not relevant to the manuscript's content. A more suitable title might be “Interplay Between Disorder and Topology in Thouless Pumping … ” which would better reflect the focus of the manuscript.

In addition, here is a minor point:

The authors attribute the discrepancies observed in their experiments to a small overlap (~1%) in the initial state from the ideal Wannier state. This is an intriguing observation and seems to explain many of the unexpected discrepancies in their results. I suggest that the authors further discuss how close the assumed initial state is to the actual prepared state in their measurements. It would be clearer if the authors either utilize measured initial states to calculate deviation from the Wannier state, or incorporate some key coherent error sources into their model of the prepared state, such as imperfect rotation in X gate. In addition, while incoherent errors may not impact the dynamics significantly, it would be also helpful to understand the impacts of thermal population or ground state preparation fidelity.

In conclusion, the manuscript is well-written and organized, though there are a few areas that could be improved. It makes a significant contribution to experimentally demonstrating topological pumping with controllable disorder using superconducting qubits. If the highlighted issues are sufficiently addressed, I recommend this manuscript for publication in Nature Communications.

Reviewer #5

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

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Report of Referee #1 – NCOMMS-24-13506-T

(1-1) Reviewer #1 wrote that:

In this manuscript, the authors describe the use of a new quantum simulating platform of coupled superconducting qubits to study disorder effect on a topological pump. They simulate both randomness of the hoppings and of site potentials of a 1D time-dependent Rice-Mele model. This subject is interesting and topical, as shown by several recent experimental and theoretical studies along these lines. While similar studies have been conducted with cold-atoms simulators (e.g. Ref. [19]), this is the first time that a solid-state platform is used in this context. As such, this work is interesting.

Our response:

We would like to thank the Reviewer for reviewing our manuscript, the accurate summary, and the recognition of the novelty of our manuscript. We will respond to comments in the report point by point as follows.

(1-2) Reviewer #1 wrote that:

However, the authors faced a major difficulty in this task, which is not clearly stated in the manuscript. This difficulty lies in the necessity to have a stable simulator over a time period long enough to study accurately the adiabatic regime of a topological pump. This is illustrated on the Figure 2b of this manuscript, displaying the displacement of an initial wavepacket over a period of the drive. Such a figure is most welcome, and should be systematically presented in experimental studies of topological pumping.

For a topological pump, this displacement is expected to be quantized when the initial wavepacket is prepared in a Wannier state. However topological pumping requires the dynamics to be adiabatic, i.e. slow enough. Any finite frequency of the drive will ultimately lead to a departure from the adiabatic dynamics on long timescales. However for a slow enough drive, this departure occurs on a sufficiently long times such that topological pumping can be experimentally measured on intermediate time scales. As expected, for short enough periods T of the drive, the authors observe a departure from the quantized displacement.

The quantized displacement is reached for T=650 ns. We would expect a plateau for this quantized displacement for larger periods. This is not what is observed for large T ! The displacement is no longer quantized, and is reduced for larger T. We suspect that this platform is then limited by the dephasing time T2=826ns of the qubit, see Table S1 of the Supp. Mat. Hence without any disorder, this platform is on the verge of realizing a topological pump, being limited on one side by non-adiabatic dynamics of the pump and on the other side by the dephasing mechanisms of the qubit. It appears to be a very unstable topological pump. This instability is observed on the Fig. 2a where a departure from a topological pumping is observed after 3 periods of drive for the clean case. Thus we expect that a study of the effects of disorder on this platform will overemphasize its consequences. Indeed, this is demonstrated by the discrepancy between the expected quantized rate in Fig. 4b and the experimental results which show no expected quantized pumping. This is our major criticism of this experimental study, which severely limits its potential impact.

Our response:

We thank the Reviewer for pointing out this issue. We perform the experiments over a longer evolution time than the qubit dephasing time, and the results (as shown in Fig. 2 of the main text) are obtained by evolving the Rice-Mele Hamiltonian up to 2,600 ns. Indeed, the stability of the device plays a key role in simulating topological pumping in the adiabatic regime. Here, we would like to emphasize that the effective T_2 is much longer than the one obtained by single-qubit measurements and does not effect our experimental results within the evolution time. Moreover, we will demonstrate that the discrepancy between the expected quantized rate in Fig. 4b and the experimental results is due to the choice of a single-excitation state as the initial state, which is slightly different from an exact Wannier state.

The dephasing time, measured via a Ramsey sequence, is affected by the temporal fluctuation in the qubit transition frequency [Krantz2019, Kwon2021], or equivalently, the flux noise for frequency-tunable transmon qubits. For each individual qubit at its idle point with other qubits being detuned far away, the Ramsey dephasing time T_2^* is measured to be an average value of 826 ns. However, the effective dephasing time for an interacting system is proven in Refs. [Dewes2012, Song2019, Guo2021] to be much longer than the one measured individually. Due to the fact that the eigenenergies of many-body systems depend weakly on the magnetic flux on each qubit [Xu2018], the collective dynamical process also protects the coherence [Li2018]. Especially, the resonant energy swap experiments have been performed [Xu2018, Guo2021] to quantify the T_2 of interacting systems, and the effective dephasing time has been obtained by fitting the experimental results, according to the results derived from the Lindblad master equation. For T_2^*~1.5 μs by the single-qubit measurement at idle points, the fitting T_2 reaches a value ~20 μs. Although the resonant energy swap experiment is different from the time evolution of topological pumping, it can provide a simple and intuitive way to show the suppression of dephasing when involving interactions.

Using experimental parameters, we numerically calculate the time evolution of a 12-qubit chain without and with dephasing, respectively. We find that our experimental results are consistent with the numerical results with an effective dephasing time T_2, which is much longer than our average T_2^*~826 ns at the idle point. The time evolution without dephasing and with dephasing of T_2~5.5 μs are shown in Figs. R1-1(A) and R1-1(B), respectively. The corresponding results of center-of-mass (CoMs) as a function of time are plotted in Fig. R1-1(C). We conclude that only when the cycle number is large, i.e., not less than three pumping periods $t \geq 3T$ with T = 650 ns, δx for the dephasing case deviates from the one for the closed system case.

For results demonstrated in Figs. 4,5 of the main text, which are more central in our work, the system evolves within 1,500 ns, and we have calibrated the states with a conserved particle number to avoid the influence of energy relaxation. Hence, we conclude that the limited dephasing time does not affect the experimental results within the experimental evolution time.

In addition, in Fig. 2b of our manuscript, we show the results of δx as a function of the pumping period, T, by choosing the initial state as a single-excitation state. We observe that δx oscillates for a long period even without considering decoherence [red curve in Fig. R1-2(A)]. Indeed, the appearance of a quantized plateau is predicted theoretically,

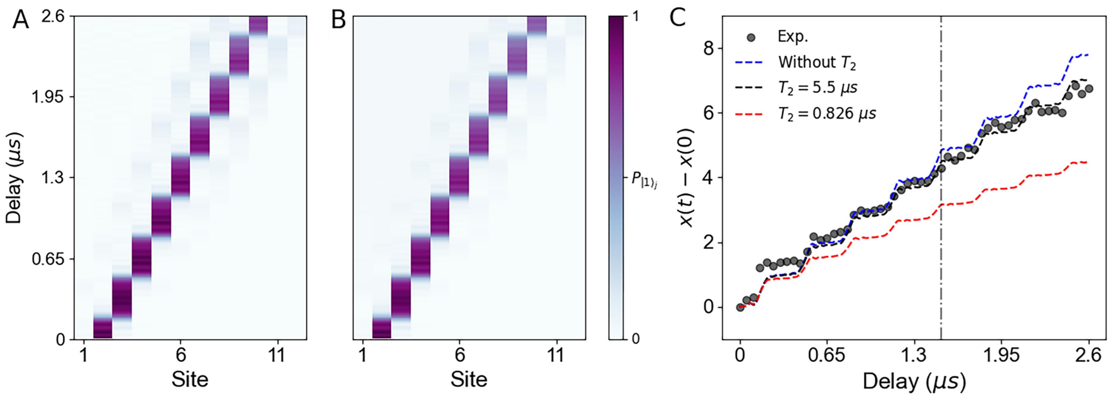

Figure R1-1: Numerical results for topological pumping without (A) dephasing and with (B) dephasing with $T_2 = 5.5\ \mu s$. The CoMs extracted from the evolution data without dephasing, and with dephasing for $T_2 = 5.5\ \mu s$ and $0.826\ \mu s$ are plotted in (C). The black circles show the experimental result of topological pumping for the trajectory $\mathcal{C}_4$, also in Fig. 2a of the manuscript.

when T is larger than 650 ns. We consider that the discrepancy is due to the fact that the initial state is not an exact Wannier state. To verify that, we numerically simulate the evolution, when choosing the initial state as a maximally localized Wannier state (MLWS) [Marzari2012], and calculate δx versus T [the blue curve in Fig. R1-2(A)]. The details about MLWS are discussed in the "*Maximally localized Wannier state*" Section of the revised Supplementary Materials.

In addition, we calculate ΔQ versus T, and find that ΔQ is almost equal to δx, when choosing a Wannier initial state. Furthermore, we calculate δx as a function of T for these two different initial states for longer periods, as shown in Fig. R1-2(B). We choose the y-axis as $(2-\delta x)$ and take the logarithmic coordinates of y. As noted by the Reviewer, we observe that "any finite frequency of the drive will ultimately lead to a departure from the adiabatic dynamics on long timescales" as shown in Ref. [Privitera2018], for the Wannier initial state case.

Even with the discrepancy, the single-excitation initial state is still a good approximation to the Wannier state with a small hopping strength [Ke2017, Cerjan2020, Fedorova2020]. The wavefunctions of the MLWSs, localized at the edge and the bulk, are plotted in Fig. R1-3. The overlap between the MLWSs and the single-particle state is almost

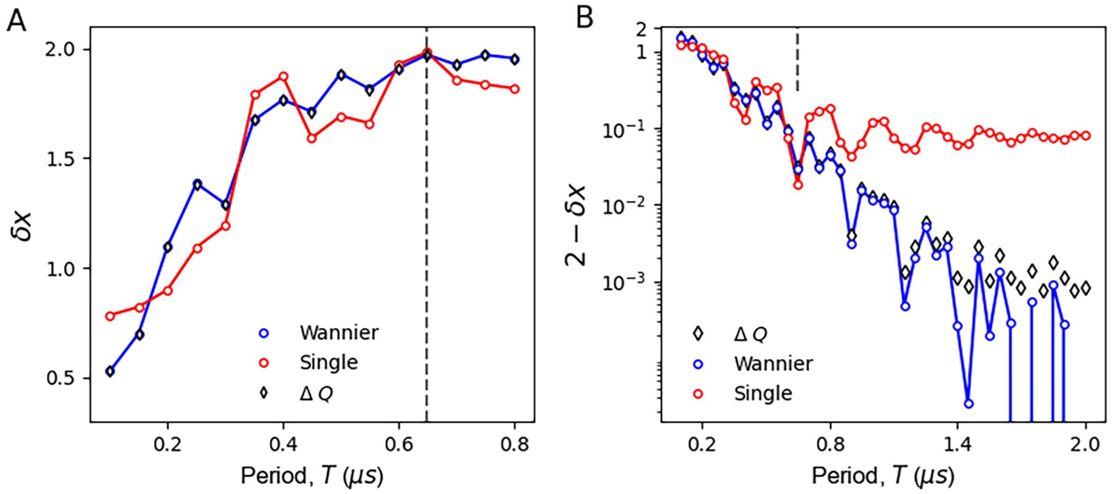

Figure R1-2: The δx and ΔQ versus the period T. Numerical results of δx (A) and ($2-\delta x$) (B) versus the period T, when preparing the initial state as a single-excitation state and a Wannier state, respectively.

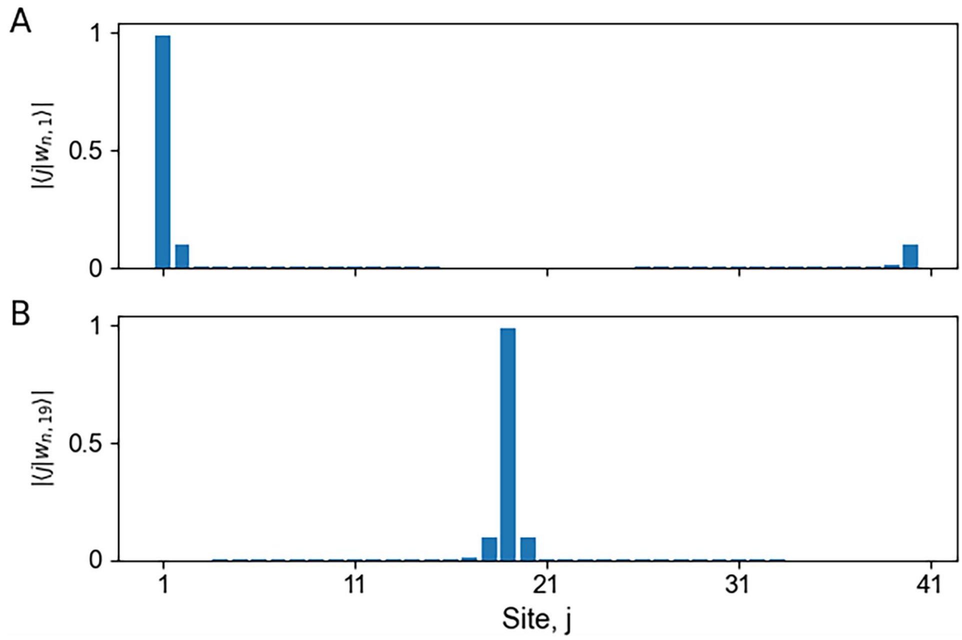

Figure R1-3: Wavefunctions of the maximally localized Wannier state of the Rice-Mele model localized at the left edge (A) and the bulk (B).

one. Note that the quantum fidelity between the MLWS and the single-excitation state is over 0.99 for our experimental parameters. Therefore, we conclude that the slight oscillation of δx (the Reviewer mistook it as as a reduction), observed in Fig. 2b of our manuscript, is due to the choice of a single-excitation state as the initial state and verified by the numerical simulation, which is not caused by dephasing.

In the revised Supplementary Materials, we have added more discussions about the Wannier states and effects of decoherence in the "*Effects of decoherence*" section. In the manuscript, we have added a dashed curve in Fig. 2b, which shows δx versus T when the initial state is chosen as a Wannier state, and added

> *"The dashed curve shows the numerical results of δx as a function of T, when the initial state is an exact Wannier state"*

in the caption of Fig. 2 and in the main text

> *"Here, the slight oscillation of δx for T > 650 ns originates from the difference between the single-excitation state and the exact Wannier state."*

REFERENCE:

[Krantz2019] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, *A quantum engineer's guide to superconducting qubits,* Appl. Phys. Rev. **6**, 021318 (2019).

[kwon2021] S. Kwon, A. Tomonaga, G. Lakshmi Bhai, S. J. Devitt, and J.-S. Tsai, *Gate-based superconducting quantum computing*, J. Appl. Phys. **129**, 041102 (2021).

[Dewes2012] A. Dewes, *et al. Characterization of a two-transmon processor with individual single-shot qubit readout*, Phys. Rev. Lett. **108**, 057002 (2012).

[Song2019] C. Song, *et al.*, *Generation of multicomponent atomic Schrodinger cat states of up to 20 qubits*, Science **365**, 574–577 (2019).

[Guo2021] Q. Guo, C. Cheng, Z.-H. Sun, Z. Song, H. Li, Z. Wang, W. Ren, H. Dong, D. Zheng, Y.-R. Zhang, R. Mondaini, H. Fan, and H. Wang, *Observation of energy-resolved many-body localization*, Nat. Phys. **17**, 234 (2021).

[Xu2018] K. Xu, *et al.*, *Emulating many-body localization with a superconducting quantum processor*, Phys. Rev. Lett. **120**, 050507 (2018).

[Li2018] X. Li, *et al.*, *Perfect quantum state transfer in a superconduct-*

ing qubit chain with parametrically tunable couplings, Phys. Rev. Appl. **10**, 054009 (2018).

[Marzari2012] N. Marzari, A. A. Mostofi, J. R. Yates, I. Souza, and D. Vanderbilt, *Maximally localized Wannier functions: Theory and applications*, Rev. Mod. Phys. **84**, 1419 (2012).

[Privitera2018] L. Privitera, A. Russomanno, R. Citro, and G. E. Santoro, *Nonadiabatic breaking of topological pumping*, Phys. Rev. Lett. **120**, 106601 (2018).

[Ke2017] Y. Ke, X. Qin, Y. S. Kivshar, and C. Lee, *Multiparticle Wannier states and Thouless pumping of interacting bosons*, Phys. Rev. A **95**, 063630 (2017).

[Cerjan2020] A. Cerjan, M. Wang, S. Huang, K. P. Chen, and M. C. Rechtsman, *Thouless pumping in disordered photonic systems*, Light Sci. Appl. **9**, 178 (2020).

[Fedorova2020] Z. Fedorova, H. Qiu, S. Linden, and J. Kroha, *Observation of topological transport quantization by dissipation in fast Thouless pumps*, Nat. Commun. **11**, 3758 (2020).

(1-3) Reviewer #1 wrote that:

Besides this, we have found the results of Fig. 3, focused on the effects of a site potential disorder, to be convincing, but very close to those obtained in Ref. 19 on a cold-atom simulator, with identical protocols. However, we do not expect the pumping corresponding to the Fig. 3e to be topological since its quantization (which is not demonstrated in this figure) would require fine tuned Hamiltonian parameters to precisely combine the effects of the trivial inner and outer trajectories. Indeed, a quantized pumping can occur for parametric pumping and not only for topological pumping. For the reasons mentioned above, the results of Fig.4, which were more original and central for this study, turn out to be disappointing, with no sign of a quantized rate of displacement or topological pumping induced by quasi-periodic hopping disorder in Fig. 4b.

Our response:

We thank the Reviewer for this comment. In our revised manuscript, we have added a discussion about pumping for double-loop trajectory with random on-site disorder:

"However, a quantized disorder-induced pump can hardly be realized, since quantized transport requires trajectory parameters to be finely tuned to combine the effects of the trivial inner and outer trajectories [20]".

Due to the limited decoherence time, the evolution period is chosen as T = 1,400 ns, leading to a non-zero δx even in the clean limit. Even for short periods, a clear increase with an increasing quasi-periodic hopping disorder strength is non-trivial. We have added original data in the "*Additional Experimental Data*" Section of the revised Supplementary Materials. For the quasi-periodic hopping disorder case, δx = 0.3632, $W_p/2\pi$ = 0 MHz, and δx reaches the maximum 0.9852, when $W_p/2\pi$ = 2.1 MHz. The maximum difference is over 0.6.

We numerically calculate ΔQ as a function of the evolution period T. The quantized plateau can be observed when T is long enough, i.e.,$T \geq 100$ μs, as shown in Fig. R1-4. However, the region of the peaks observed in our experiments, indicated by the Chern number, remains for different periods. In addition, the region complies where the range that the bandgap reopens, as shown in Fig. R1-5.

In experiments, to observe the theoretically predicted quantized plateau is now too challenging either under the state-of-art realistic conditions [König2007, Roth2009] or on a quantum simulation platform [Meier2018, Lin2022]. Our results, as a dynamic version of topological Anderson insulators (TAIs), provide similar experimental evidences in addition to the TAIs results obtained in atomic wires [Meier2018]. More discussions on this topic can be found in the Response (4-2) to the comment from Reviewer #4. We believe that the current results are sufficient to support the evidence of "*the competition and interplay between Thouless pumping and disorder*" in the elusive topics.

REFERENCE:

[Stützer2018] S. Stützer, *et. al*, *Photonic topological Anderson insulators*, Nature **560**, 461 (2018).

[Roth2009] A. Roth, C. Brüne, H. Buhmann, L. W. Molenkamp, J. Maciejko, X.-L. Qi, and S.-C. Zhang, *Nonlocal transport in the quantum spin Hall state*, Science **325**, 294 (2009).

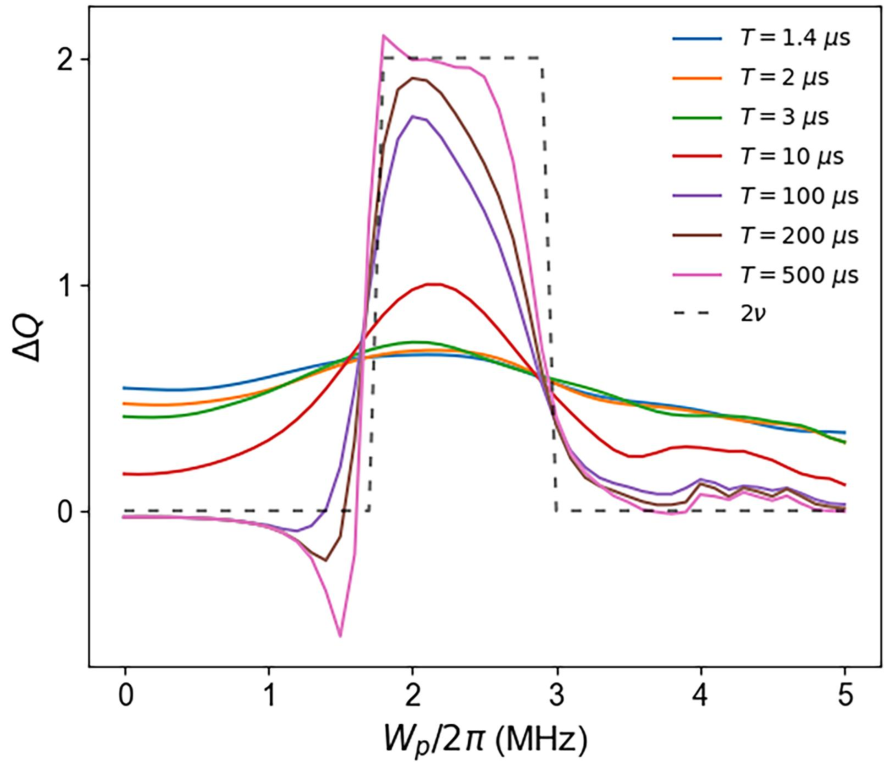

Figure R1-4: The charge pumped per cycle ΔQ versus disorder strength W_p for different evolution periods T. Here,

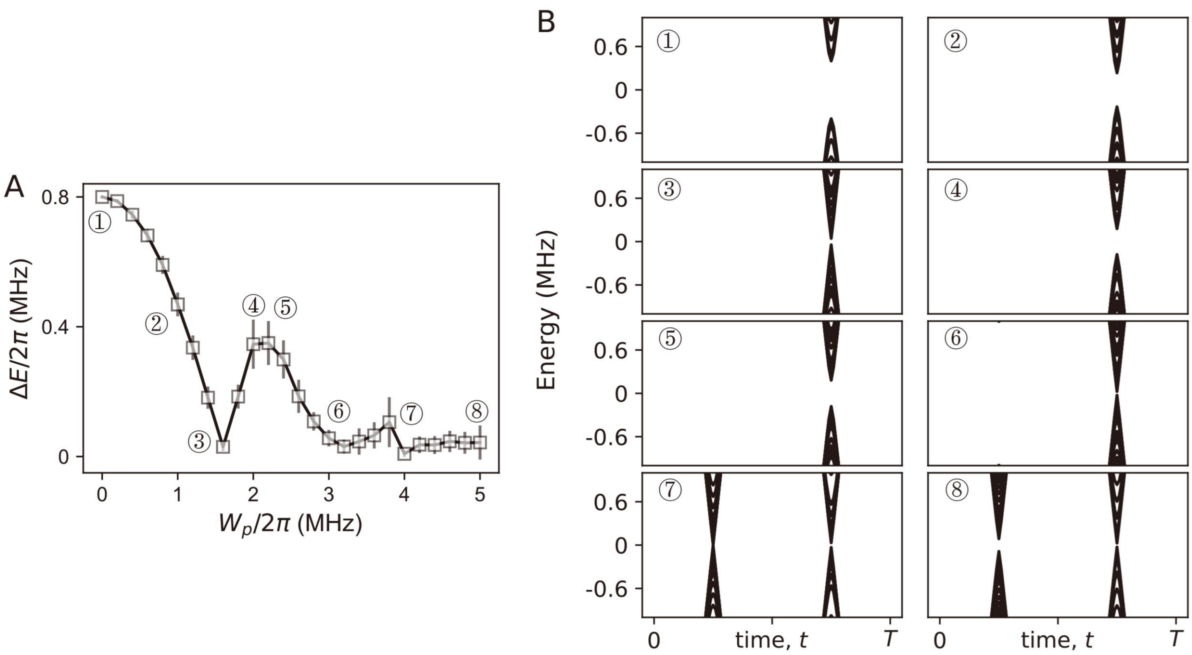

Figure R1-5: Instantaneous energy spectra of the bulk under quasi-periodic disorder. (A) The bandgap, ΔE, as a function of disorder strength W_p. (B) Energy spectrum under disorder for several different disorder strengths. The results are averaged over 100 disorder configurations.

[Meier2018] E. J. Meier, F. A. An, A. Dauphin, M. Maffei, P. Massignan, T. L. Hughes, and B. Gadway, *Observation of the topological Anderson insulator in disordered atomic wires*, Science **362**, 929 (2018).

[Lin2022] Q. Lin, T. Li, L. Xiao, K. Wang, W. Yi, and P. Xue, *Observation of non-Hermitian topological Anderson insulator in quantum dynamics*, Nat. Commun. **13**, 3229 (2022).

(1-4) Reviewer #1 wrote that:

For the above reasons, we believe that the platform developed by the authors is extremely interesting, but that the present study of topological pumping was not successful enough to warrant a publication in a broad audience journal such as Nature Communications.

Our response:

We believe that we have addressed the comments, in particular, the stability of our device, and the effects of dephasing. With our reply and the revision of our manuscript, we believe our work is suitable for publication in *Nature Communications.*

(1-5) Reviewer #1 wrote that:

Besides, we also noted that the writing of this manuscript could also be slightly improved. Let us mention a few points worth improving:
- The first sentence of the abstract « Thouless pumping […] represents the quantized charge pumped during an adiabatic cyclic evolution. »

Our response:

We thank the Reviewer for the comment. We have revised the abstract as

"*Topological phases are robust against weak perturbations, but break down when disorder becomes sufficiently strong. However, moderate disorder can also induce topologically nontrivial phases. Thouless pumping, as a* (1+1)D *counterpart of the integer quantum Hall effect, is one of the simplest manifestations of topology.*"

(1-6) Reviewer #1 wrote that:

- On l.25, the sentence « Thus is bridges the quantized conductance and the topological invariant […]»

Our response:

We thank the Reviewer for the comment. We have revised the sentence to

"*Thouless pumping, as a dynamical version of the integer quantum Hall effect (IQHE) [5], bridges the quantized conductance and the topological invariant, known as the Chern number of the occupied energy bands [1, 6].*"

(1-7) Reviewer #1 wrote that:

- In the definition of the model (1), the authors should state that Delta(t) and delta(t) are periodic function of period T, which is not defined but used in eq. (2).

Our response:

We thank the Reviewer for the comment. We have added that

"*..., and $\Delta(t)$ and $\delta(t)$ are periodic with period T*",

when introducing the definition of $\Delta(t)$ and $\delta(t)$.

(1-8) Reviewer #1 wrote that:

- The precise way that delta x is experimentally measured (e.g. in Fig. 3b, 4a, 4b, etc) is unclear : over how many periods, what averaging is performed, etc.

Our response:

We thank the Reviewer for the comment. We have added the precise number of cycles and counts of disorder configurations in the caption for each figure in our revised manuscript.

(1-9) Reviewer #1 wrote that:

- A discussion on how close to a Wannier is the initial state would be useful.

Our response:

We thank the Reviewer for the comment. We have replied to this question in detail in the Response (1-2). The fidelity between the initial single-excitation state and a Wannier state is over 0.99.

We also discussed in detail the Wannier state in the "*Maximally localized Wannier state*" Section of the revised Supplementary Materials. Using the single-excitation state to approximate the Wannier state leads to a slight deviation between δx measured in experiments and ΔQ predicted in theory, see Fig. 2b and also our Response (1-2). To clarify this issue, we have added in the revised manuscript that

> "*The initial state is prepared as a single-excitation state, having an overlap of over 0.99 with the exact Wannier state [29],*"

(1-10) Reviewer #1 wrote that:

- What do we learn from the instantaneous energy spectra shown in Fig. 2d and 3f?

Our response:

We thank Reviewer #1 for raising this question. The main reasons for showing energy spectra are to help potential readers outside the condensed matter physics community to better understand the picture of physics during topological pumping.

We can clearly distinguish between bulk bands and edge states from the instantaneous energy spectra. Hence, the bandgap is clearly shown in Figs. 2d,3f, corresponding to the adiabatic conditions required for our different pumping sequences.

In addition, if an edge state is initially occupied, it transverses to the bulk and further to the other side of the lattice when the trajectory encircles the origin of the (Δ,δ)-plane, while the phenomena does not occur, when the trajectory does not circle the gapless point [Cerjan2020].

REFERENCE:

[Cerjan2020] A. Cerjan, M. Wang, S. Huang, K. P. Chen, and M. C. Rechtsman, *Thouless pumping in disordered photonic systems*, Light Sci. Appl. **9**, 178 (2020).

(1-11) Reviewer #1 wrote that:

- On Fig 1c-d, the representation of on-site disorder by a spin is confusing.

Our response:

We thank Reviewer #1 for this comment.

Figure 1c represents qubits under on-site disorder. Figure 1d represents qubits under hopping disorder by using spin pairs with uncertain coupling. To better represent the qubits under on-site disorder and hopping disorder, <u>we have revised the schematic diagrams in Fig. 1c,d.</u>

(1-12) Reviewer #1 wrote that:

- On l.163-165, how can a Chern number be defined for a gapless cycle?

Our response:

We thank the Reviewer for the comment.

We agree that Chern number cannot be defined for a gapless cycle. We meant to say that quantized pumping does not exist in this case. Thus, have revised the corresponding sentence as

> "*the inner loop cannot encircle the whole gapless regime and no topological pumping phenomenon occurs*."

and afterwards

> "*As the disorder strength increases further to $V/\Delta_0>2.5$, pumping becomes trivial, since no topological pumping exists for the outer loop.*"

(1-13) Reviewer #1 wrote that:

- On l. 206, is the phase beta_j identical on all sites j or does it take a random value from site to site? If beta_j is not constant, then the disorder is random rather than quasi-periodic.

Our response:

We thank Reviewer #1 for pointing out this typo. We agree with the Reviewer that to apply a quasi-periodic disorder, β_j is constant for one single disorder configuration [Li2023]. If not, this kind of disorder will be random. Thus, we have removed the subscript *j* from the symbol in the revised manuscript.

Reference:

[Li2023] X.-G. Li, *et al.*, *Mapping a topology-disorder phase diagram with a quantum simulator*, arXiv:2301.12138.

(1-14) Reviewer #1 wrote that:

- The section on Floquet engineering in the Methods is hardly understandable independently of the Supplementary Materials.

Our response:

We thank the Reviewer for the comment. We have revised the content about Floquet engineering in the Methods. We have added more detailed discussions as:

"To realize the high-frequency expansion, the modulation frequency should be higher than the simulated frequency regime for fulfilling the adiabatic condition, and the effective Hamiltonian contains a series of frequency bands. The Nyquist condition requires that the variation range of the difference between two neighbor on-site potentials should be lower than half the modulation frequency $\mu/2$. This can avoid any overlap between different frequency bands, resulting in an effective simulation of the target time-evolved Hamiltonian under the rotating wave approximation."

(1-15) Reviewer #1 wrote that:

- On l.315, what does the sentence "all qubit probabilities are corrected to eliminate the measurement errors" mean?

Our response:

We thank the Reviewer for the comment.

In our experiments, we measured the fidelities for all qubits to perform the real-time correction of the readout errors to construct the fidelity matrix. The qubit probabilities can be corrected from the measured probabilities by the inverse of the fidelity matrix. Moreover, we calibrate for single-excitation or double-excitation states, to mitigate the effect of the energy relaxation, due to the conservation of the total photon number of the disordered Rice-Mele Hamiltonian.

During the experiment, we measured the fidelities for all qubits to perform the real-time correction of the readout errors [Zheng2017, Xiang2023]. The fidelity F_0 (F_1) for |0**>** (|1**>**) state is defined as the conditional probability of measuring the qubit at |0**>** (|1**>**) when it is prepared at |0**>** (|1**>**). We can construct the fidelity matrix for the *j*-th qubit as

$$\boldsymbol{F}_j = \begin{bmatrix} F_{0,j} & 1 - F_{1,j} \\ 1 - F_{0,j} & F_{1,j} \end{bmatrix}.$$

According to the Bayesian formula, the qubit probabilities $\boldsymbol{P}_j$, written as a column vector, can be corrected from the measured probabilities $\boldsymbol{P}_j'$ by multiplying the inverse of the fidelity matrix

$$\boldsymbol{P}_{\boldsymbol{j}} = \boldsymbol{F}_j^{-1} \boldsymbol{P}_j'.$$

More discussions also can be found in the Response (3-8) to Reviewer #3.

(1-16) Reviewer #1 wrote that:

- In Fig. S4, titles would help the reading.

Our response:

We thank Reviewer #1 for the suggestion. We agree with the Reviewer and have added titles and more detailed annotations in Figs. S4,S5 in the revised Supplementary Materials.

Report of Referee #3 – NCOMMS-24-13506-T

(3-1) Reviewer #3 wrote that:

This paper describes observation of Thouless charge pumping in a (relatively) large superconducting quantum processor. From a condensed matter perspective Thouless pumping is interesting as it is one of the simplest examples of topological phenomena. Moreover it emerges from periodically driven systems, making highly controllable quantum platforms, such as their Chuang-tzu superconducting quantum process, an ideal candidate to study such phenomena. The authors study topological pumping induced from both (1) on-site potential disorder and (2) hopping disorder. (2) being the more novel result.

Overall, the authors' work is impressive, but before recommending publication, there are some items I would like the authors to address.

Our response:

We thank Reviewer #3 for the recognition of the correctness, the quality, and the impact of our manuscript. According to the comments raised by the Reviewer, we have revised the manuscript and improved the presentation of our results.

(3-2) Reviewer #3 wrote that:

Please add more details about how noise affects the experiment. Currently there are only standard reports of, for example, T1 and T2. However, for example, in Fig 2a, beyond 3T the C1 and C4 curves begin to deviate the numerical results. Why? I presume this has to do with decoherence, but there is no comment on why these deviations occur at long times. Moreover, T2* = ~800 ns. This about as long as the period times T (for example T = 650 ns in Fig 2b), shouldn't going to times of a few T (such as in Fig 2a) result in significant dephasing? Also, the numerics are noiseless numerics I assume? Perhaps this can be specified such that it is more clear that experiment agrees well with noiseless numerics rather than numerics that take hardware noise into consideration.

Our response:

We thank Reviewer #3 for this suggestion. We agree with the

Reviewer that more details about the effects of decoherence will help to better demonstrate our experimental results.

We numerically simulate topological pumping with dephasing, using the Lindblad master equation for a 12-qubit chain. Although the average dephasing time is measured as 826 ns by applying single-qubit Ramsey sequences, the interactions between qubits lead to a longer effective dephasing time [Xu2018, Guo2021]. To verify that, a resonant energy swap experiment has been performed in Ref. [Guo2021]. The single-qubit dephasing time is about 1.5 *μ*s, while the dephasing time of the interacting system is about 20 *μ*s, which is obtained by fitting the results derived from the Lindblad master equation.

As shown in Fig. R3-1(C), we find that our experimental results correspond to the case when T_2 ~ 5.5 *μs*. However, the effect of dephasing is negligible, when the evolution time is less than 1,500 ns.

More discussions about dephasing can also be found in the Response (1-2) to Reviewer #1. We have added more details about the effects of decoherence in the "*Effects of decoherence*" Section of the revised Supplementary Materials. In the manuscript, we have added a dashed curve in Fig. 2b, which shows *δx* versus *T* when the initial state is chosen as a Wannier state, and added

> *"The dashed curve shows the numerical results of δx as a function of T, when the initial state is an exact Wannier state"*

in the caption of Fig. 2 and in the main text

> *"Here, the slight oscillation of δx for T > 650 ns originates from the difference between the single-excitation state and the exact Wannier state."*

REFERENCE:

[Xu2018] K. Xu, *et al.*, *Emulating many-body localization with a superconducting quantum processor*, Phys. Rev. Lett. **120**, 050507 (2018).

[Guo2021] Q. Guo, C. Cheng, Z.-H. Sun, Z. Song, H. Li, Z. Wang, W. Ren, H. Dong, D. Zheng, Y.-R. Zhang, R. Mondaini, H. Fan, and H. Wang, *Observation of energy-resolved many-body localization*, Nat. Phys. **17**, 234 (2021).

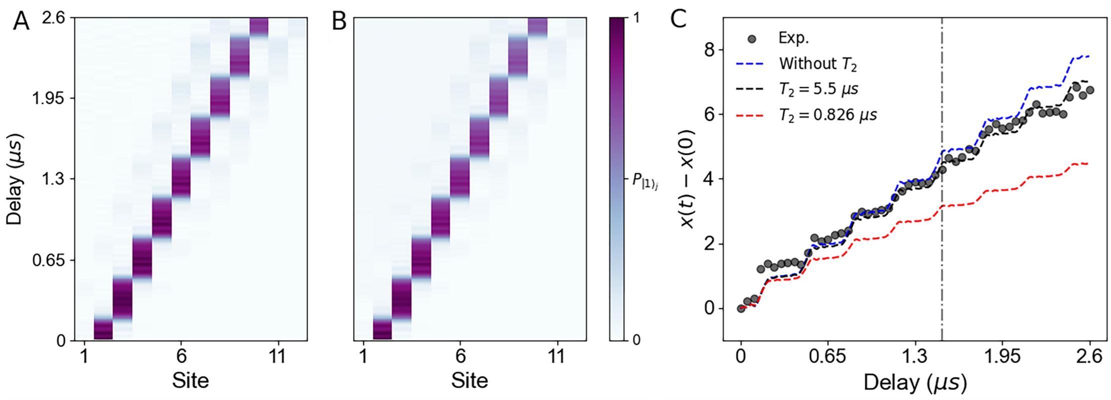

Figure R3-1: Numerical results for topological pumping without (A) dephasing and with (B) dephasing with $T_2 =$ 5.5 μs. The CoMs extracted from the evolution data without dephasing, and with dephasing for $T_2 =$ 5.5 μs and 0.826 μs are plotted in (C). The black circles show the experimental result of topological pumping for the trajectory $\mathcal{C}_4$, also in Fig. 2a of the manuscript.

(3-3) Reviewer #3 wrote that:

In the last sentence of the caption of Fig. 1, is there a typo? It reads: "... are staggered with one (up) large and one down (small) due to the staggered RM Hamiltonian." Should this instead read: "... are staggered with one up (large) and one down (small) due to the staggered RM Hamiltonian."

Our response:

We thank Reviewer #3 for pointing out this issue. We have revised the typos.

(3-4) Reviewer #3 wrote that:

In the first paragraph of the "Pumping with hopping disorder" section, it reads: "... the decay of \delta x obeys a distinct law from the on-site disorder case…". I see that there is a citation, but think it would help with understanding to mention what the distinct laws actually are for each case (on-site vs hopping disorder) and include appropriate citations. I wasn't really sure what this was supposed to mean or what "law" really meant when reading through.

Our response:

We thank Reviewer #3 for this comment. When the on-site disorder strength increases, the system undergoes a transition from the metallic phase with delocalized eigenstates to the insulator phase with exponentially localized eigenstates [Evers2008]:

$$|\psi(r)|^2 \sim \exp(-|r - r_0|/\xi),$$

where ξ denotes the localized length. The phenomenon is known as Anderson localization. For an integrable 1D system, any random disorder induces Anderson localization.

When only hopping random disorder is introduced, the localization length of zero-energy state is infinite [Theodorou1976]. Nevertheless, the state is considered to be localized, since the mean values of the transmission coefficient approach zero in the thermodynamic limit [Fleishman1977, Soukoulis1981]. Especially, the geometric and harmonic mean values behave as $\exp(-\gamma L^{1/2})$, with a chain length L, and the arithmetic mean value follows a power law $L^{-\delta}$ with $\delta \cong 0.5$, in comparison with all three mean values behaving as $\exp(-\gamma L)$ under on-site disorder.

In the "*Localization in the Rice-Mele model with disorder*" Section of the Supplementary Materials, we have added more discussions on the decay laws of eigenstates of system for the case under random on-site disorder and random hopping disorder, respectively.

REFERENCE:

[Evers2008] F. Evers and A. D. Mirlin, *Anderson transitions*, Rev. Mod. Phys. **80**, 1355 (2008).

[Theodorou1976] G. Theodorou and M. H. Cohen, *Extended states in a one-demensional system with off-diagonal disorder,* Phys. Rev. B **13**, 4597 (1976).

[Fleishman1977] L. Fleishman and D. C. Licciardello, *Fluctuations and localization in one dimension*, J. Phys. C: Solid State Phys. **10**, L125 (1977).

[Soukoulis1981] C. M. Soukoulis and E. N. Economou, *Off-diagonal disorder in one-dimensional systems*, Phys. Rev. B **24**, 5698 (1981).

(3-5) Reviewer #3 wrote that:

In the last sentence of the same paragraph discussed in item 3 (lines 192/193): I found the phrasing "... making the adiabatic conditions hardly saturated." Can you clarify this?

Our response:

We thank Reviewer #3 for this comment. The conclusion that "*quantized pumping with uniform random hopping disorder is extremely difficult to realize, due to the rapid close of the band gap*" can be found in the "*Topological pumping under random disorders*" Section D in the Supplementary Materials of Ref. [Wu2022]. Here, by writing this sentence, we would like to explain why we do not tend to experimentally investigate the pumping induced by introducing random hopping disorder. We misused the word "Thus". Moreover, we would like to emphasize that this is not the main result of our work.

To avoid any confusion, we have removed this sentence in the revised manuscript. The new sentence reads:

> "*Moreover, as the gap would reopen, applying quasi-periodic hopping disorder may intrinsically induce topological pumping, which can hardly be realized by introducing random hopping disorder [29, 35].*"

More discussions can be found in the Response (4-4) to Reviewer #4.

REFERENCE:

[Wu2022] Y.-P. Wu, L.-Z. Tang, G.-Q. Zhang, and D.-W. Zhang, *Quantized topological Anderson-Thouless pump*, Phys. Rev. A **106**, L051301 (2022).

(3-6) Reviewer #3 wrote that:

In Fig 4b, I don't see a clear difference for \delta x when inside vs outside the blue dashed line (topological index curve). Should I? Why don't I? Where is the evidence for pumping with the quasi-periodic disorder coming from in the data? Please clarify this.

Our response:

We thank Reviewer #3 for pointing out this issue. There is a difference for δx as shown in Fig. 5b of the revised manuscript, but this difference is not very obvious, because limitation of the pumping period due to the limited decoherence time.

In the "*Additional Experimental Data*" Section of the revised Supplementary Materials, we add the original data of all results. For the quasi-periodic hopping disorder case, δx = 0.3632, $W_p/2\pi$ = 0 MHz, and δx reaches the maximum 0.9852, when $W_p/2\pi$ = 2.1 MHz. The maximum difference is over 0.6.

We numerically calculate ΔQ versus the period T, as shown in Fig. R3-2. The quantized plateau can be observed when T is long enough, i.e., $T \geq 100$ μs, which is also identified by the Chern number (the dashed curve in Fig. R3-2). Even with shorter periods, the parameter region of peaks observed in experiments can be observed for different periods. Moreover, the region is also in accordance with the range that the bandgap reopens (Fig. R3-3). Considering that the experimental observations are inevitably affected by decoherence, we carefully choose the evolution period as T = 1,400 ns, leading to a non-zero δx even in the clean limit. More detailed discussions can be found in the Response (4-2) to Reviewer #4.

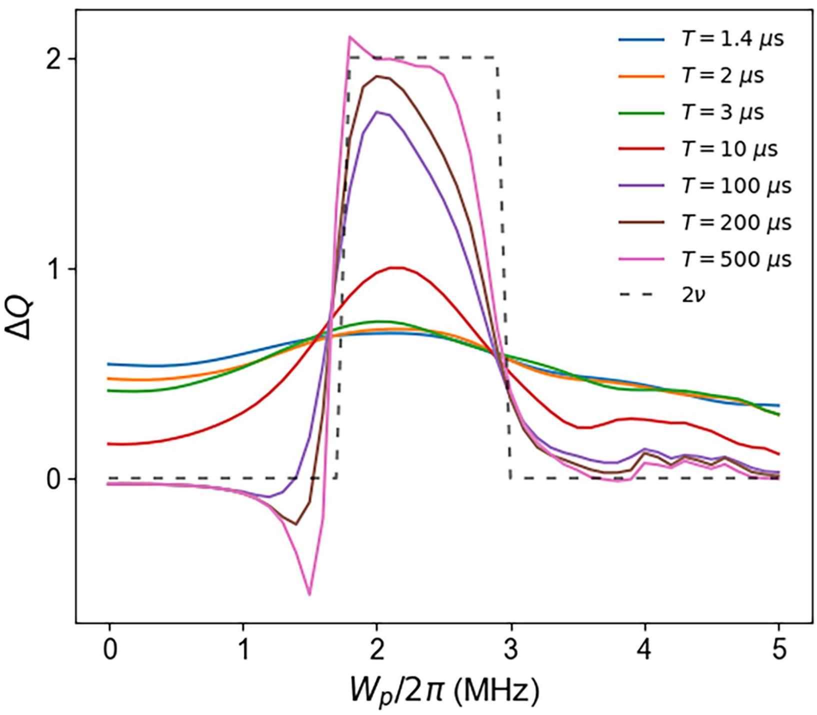

Figure R3-2: The ΔQ versus disorder strength W_p for different evolution periods T.

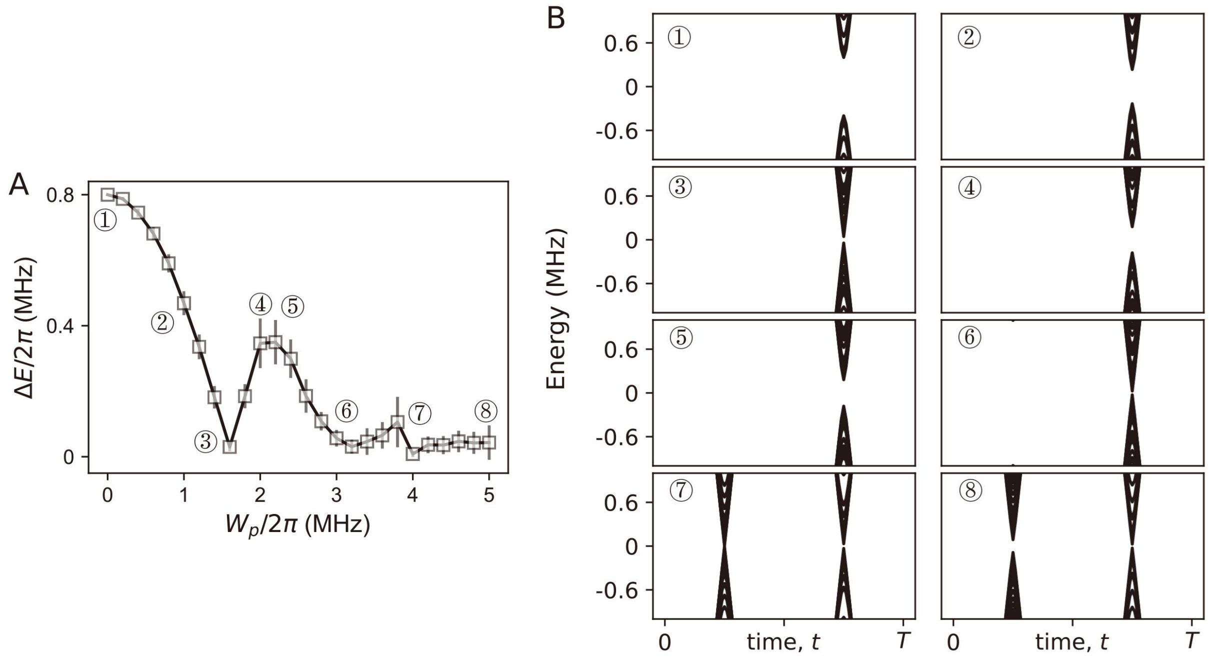

Figure R3-3: Instantaneous energy spectra of the bulk under quasi-periodic disorder. (A) The bandgap, ΔE, as a function of disorder strength W_p. (B) Energy spectrum under disorder for several different disorder strengths. The results are averaged over 100 disorder configurations.

(3-7) Reviewer #3 wrote that:

In the last sentence of the conclusion (lines 230-233), this feels vague and unjustified. I recommend removing this last sentence, unless it can be sufficiently justified. Using quantum devices with disorder (different on-site energies/qubit frequencies) is nothing new.

Our response:

We thank Reviewer #3 for this suggestion. We have removed this sentence in the revised manuscript.

(3-8) Reviewer #3 wrote that:

In the experimental setup, it says: "All qubit probabilities are corrected to eliminate measurement errors." What does this mean? Does "qubit probabilities" mean "probability of measuring the 1 state? And how are they corrected? Please clarify this

Our response:

We thank Reviewer #3 for this comment. Due to |U/J| ~ 29 >> 1 for our device, the system behaves like a hard-core bosonic system [Yan2019]. As a result, the probabilities occupied by higher energy levels can be neglected, and the “qubit probability” refers to the “probability of measuring the |1**>** state”.

During the experiment, we measured the fidelities for all qubits to perform the real-time correction of the readout errors [Zheng2017, Xiang2023]. The fidelity F_0 (F_1) for |0**>** (|1**>**) state is defined as the conditional probability of measuring the qubit at |0**>** (|1**>**) when it is prepared at |0**>** (|1**>**). We can construct the fidelity matrix for the j-th qubit as

$$\boldsymbol{F}_j = \begin{bmatrix} F_{0,j} & 1 - F_{1,j} \\ 1 - F_{0,j} & F_{1,j} \end{bmatrix}.$$

According to the Bayesian formula, the qubit probabilities $\boldsymbol{P}_j$, written as a column vector, can be corrected from the measured probabilities $\boldsymbol{P}_j$’ by multiplying the inverse of the fidelity matrix

$$\boldsymbol{P}_{\boldsymbol{j}} = \boldsymbol{F}_j^{-1}\boldsymbol{P}'_j.$$

In addition, since the Rice-Mele Hamiltonian converses the total excitation number of the initial state, we can calibrate for single-excitation or double-excitation states, to mitigate the effect of energy relaxation [Guo2021], which is also recommended by Reviewer #4.

REFERENCE:

[Yan2019] Z. Yan, *et al.*, *Strongly correlated quantum walks with a 12-qubit superconducting processor*, Science **364**, 753 (2019).

[Zheng2017] Y. Zheng, *et al.*, *Solving systems of linear equations with a superconducting quantum processor*, Phys. Rev. Lett. **118**, 210504 (2017).

[Xiang2023] Z.-C. Xiang, *et al.*, *Simulating quantum Hall effects on a superconducting quantum processor*, Nat. Commun. **14**, 5433 (2023).

[Guo2021] Q. Guo, *et al.*, *Observation of energy-resolved many-body localization*, Nat. Phys. **17**, 234 (2021).

Report of Referee #4 – NCOMMS-24-13506-T

(4-1) Reviewer #4 wrote that:

In this manuscript, the authors experimentally demonstrate topological pumping of excitations in their superconducting quantum devices. The authors experiment topological pumping under controllable on-site and hopping disorder. Especially, the authors study topological pumping under quasi-periodic hopping disorder, which has not been experimentally realized before. The authors leverage parametric flux modulation of frequency-tunable transmon qubits for Floquet engineering, in order to achieve the required precise and dynamical controllability in the Rice-Mele (RM) model under study.

The authors first prepare a single-excitation initial state in the middle of the 1-dimensional chain of the superconducting qubits. Then, the hopping strengths and and on-site potentials are dynamically controlled by Floquet engineering with a trajectory in the parameter space, slowly enough in order to satisfy adiabaticity conditions. The resulting transport of the excitation after such time evolution is estimated by measuring the center-of-mass (CoM) of the excitations. They observe quantized displacement of CoM depending on the choice of pumping cycle period and winding number of the trajectory of the parameters, demonstrating the quantized pumping. Importantly, the authors claim observation of quantized pumping persisting under sufficiently small disorder in on-site energies and topological pumping induced by quasi-periodic disorder in hopping rates.

On the whole, the manuscript is well written and presents the essential details of the author's work. The experiments conducted in this work are nicely designed for the related theory. This work shows good advances in control of an array of superconducting qubits and their utility in analog quantum simulation. Below are the reviewer's comments on the manuscript.

Our response:

We thank Reviewer #4 for the careful review of our manuscript, the recognition of the novelty of our results, and the comments that helped to improve our manuscript. We are enclosing a new version of our manuscript revised according to all comments.

(4-2) Reviewer #4 wrote that:

What distinguishes this manuscript from existing experimental work on the RM model is the observation of hopping disorder induced topological pumping. However, this main point does not seem to be sufficiently supported from the main text. In the main text section "Pumping with hopping disorder", the authors should show cleaner data than the data shown in Fig.4b, in order to support the claim that they observe topological pumping induced by quasi-periodic disorder.

First, the change in δx does not seem significant over the presented range of Wp. (c.f. changes in Fig.3a or Fig.4a. are much more significant) Second, it is unclear if observation of non-zero δx itself can imply observation of topological pumping. For example, in Fig. 2b, shorter pumping period could also induce non-zero δx but it apparently doesn't satisfy the adiabatic condition and therefore cannot be used as evidence of "nontrivial pumping". Additionally, the small change in δx does not seem significantly correlated with the topological index.

Furthermore, interpretation of experimental data as well as comparison with simulation is not sufficiently given in the main text, while the explanation for simulated data itself is given. As the experimental observation of topological pumping induced by quasi-periodic disorder is one of the authors' major claimed novelties, cleaner data and interpretation of experimental results should be given.

Our response:

We thank Reviewer #4 for the comment.

In the "*Additional Experimental Data*" Section of our revised Supplementary Materials, we have added the original data of all curves plotted in the main text. For the quasi-periodic hopping disorder case, δx = 0.3632, $W_p/2\pi$ = 0 MHz, and δx reaches the maximum 0.9852, when $W_p/2\pi$ = 2.1 MHz. The maximum difference is over 0.6, which is indeed a clear rise.

We agree with Reviewer #4 that when the adiabatic condition is not fully saturated with a period being not long enough, we cannot obtain a very definitive evidence of "nontrivial" topological pumping, by observing a clear rise of δx, relating to the Chern number. Only with a very long pumping period, we can clearly observe quantized topological pumping. However, a direct measurement of the topological index is extremely challenging under state-of-the-art realistic conditions, e.g., the limited decoherence time and the finite chain length. It is similar as the observation of quantum spin Hall insulators [König2007, Roth2009], in which the measured conductance is not very quantized, either.

Moreover, current experimental observations of the topological Anderson phase provide neither qualitative results (by spectra or transport) [Stützer2018, Liu2020, Dai2024] nor the corresponding quantities apart from the one under ideal conditions [Meier2018, Lin2022].

Nevertheless, these results are in accordance with the theoretical predictions under constrained conditions. For example, the results of topological Anderson insulator (TAI) phase obtained in atomic wires, as shown in Fig. R4-1, are similar to our results.

Furthermore, fast Thouless pumping has also been studied both theoretically and experimentally [Fedorova2020], which can demonstrate novel physics out of the adiabatic conditions. Thus, we believe that the results shown in Fig. 5b of the manuscript represents one of the most accessible measurements on pumping, induced by quasi-periodic disorder, and Anderson-Thouless pumping as shown in Ref. [Wu2022], across a variety of quantum simulating platform at the current stage.

In addition, we numerically calculate ΔQ versus the quasi-periodic hopping disorder strength for different evolution periods T, as shown in

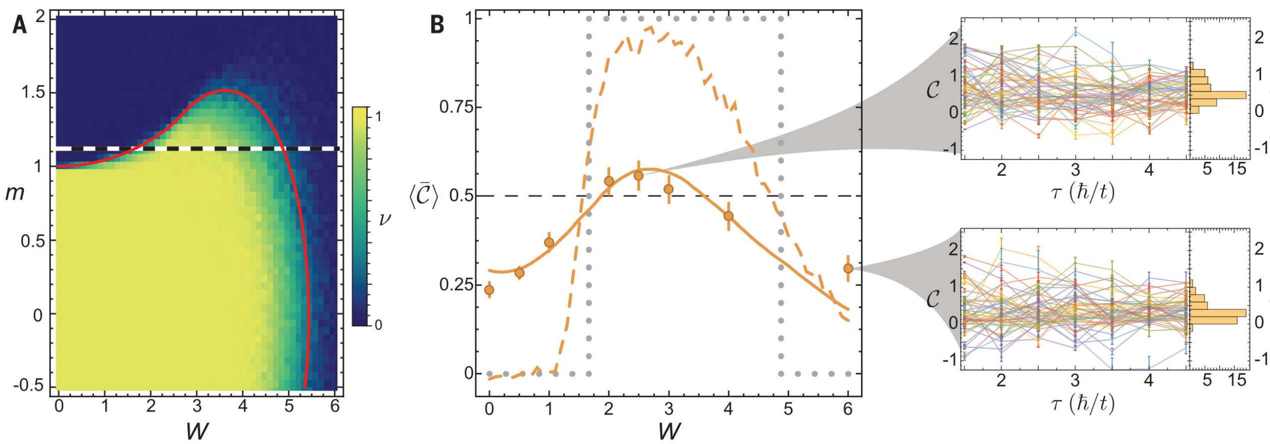

Figure R4-1: Observation of the TAI phase, which is from Ref. [Meier2018].

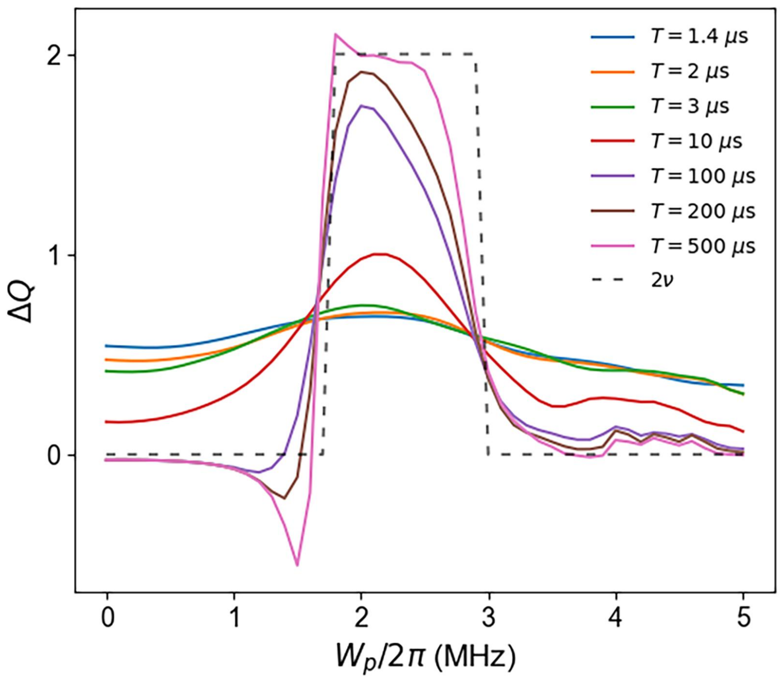

Figure R4-2: Emergence of the quantized plateau. The dashed curve denotes the Chern number.

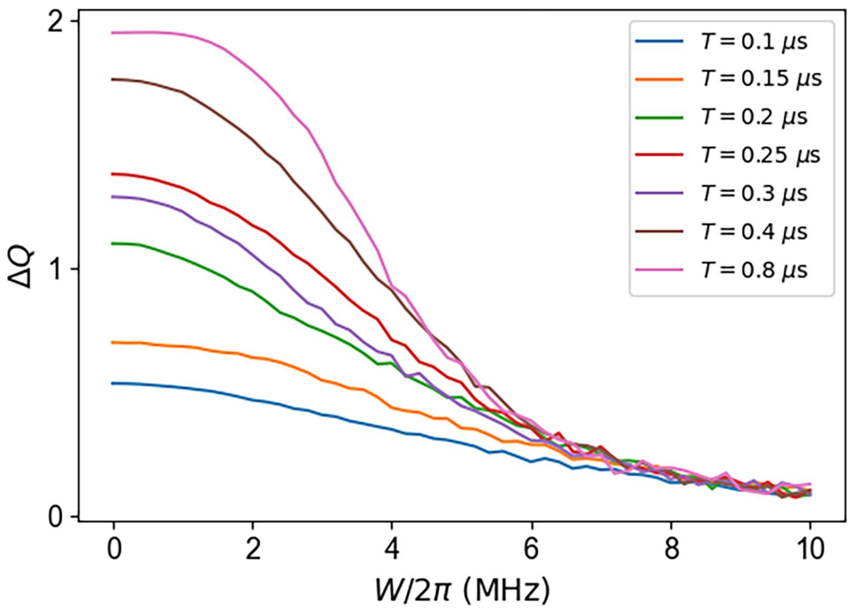

Figure R4-3: ΔQ versus the random hopping disorder strength for the pumping trajectory $\mathcal{C}_4$ defined in the manuscript. The tendency of descending retains for shorter periods. The results are obtained from the average over 200 disorder configurations.

Fig. R4-2. For T =500 μs, a clear quantized plateau can be observed as: ΔQ = 0 when the disorder strength $W_p/2\pi$ is less than 1 MHz; ΔQ = 2 when 2 MHz < $W_p/2\pi$ < 2.5 MHz; ΔQ = 0 for $W_p/2\pi$ > 3.5 MHz.The dashed curve denotes the Chern number, obtained by numerical methods [Fukui2005, Zhang2013]. Although the maximum of ΔQ decreases as T decreases, the peak still implies the existence of the topological nontrivial parameter region indicated by the Chern number.

In addition, this tendency remains as the period shortens. Specifically, we calculate ΔQ versus the random hopping disorder strength, for the pumping trajectory $\mathcal{C}_4$ defined in the manuscript. The numerical results in Fig. R4-3 demonstrate an expected decrease of ΔQ as the disorder strength increases even for short periods. Hence, the rise of δx compared with the clean case is over 0.6, which is non-trivial in this regard.

Another probe of pumping under quasi-periodic hopping disorder is that the bandgap reopens in the intermediate region. We numerically calculated the instantaneous energy spectrum, which can also be observed by using dynamical spectroscopic measurement techniques [Roushan2017, Xiang2023, Shi2023PRL, Wang2024]. The results in Fig. R4-4A demonstrates that the bandgap reopens in the same region as indicated by the Chern number. The detailed spectrum for different disorder strengths is shown in Fig. R4-4B. The slight rise of the bandgap near $W_p/2\pi$ = 4 MHz is due to the effect of the finite chain length, which is flat in the thermodynamic limit. We also discuss the behavior of the bandgap in the "*Single-loop pumping induced by hopping disorder*" Section of the revised Supplementary Materials. In the revised manuscript, we have added extra numerical results of δx, when T is much longer for the quasi-periodic case, and written that:

> *"Theoretically, with an extremely long evolution period, e.g., 20 μs and 80 μs as shown in Fig. 5b, non-adiabatic effects can be suppressed. "*

In summary, we believe that the current results can support our claim on the competition and interplay between Thouless pumping and disorder for these reasons:

First, a direct observation of quantized pumping in the state-of-

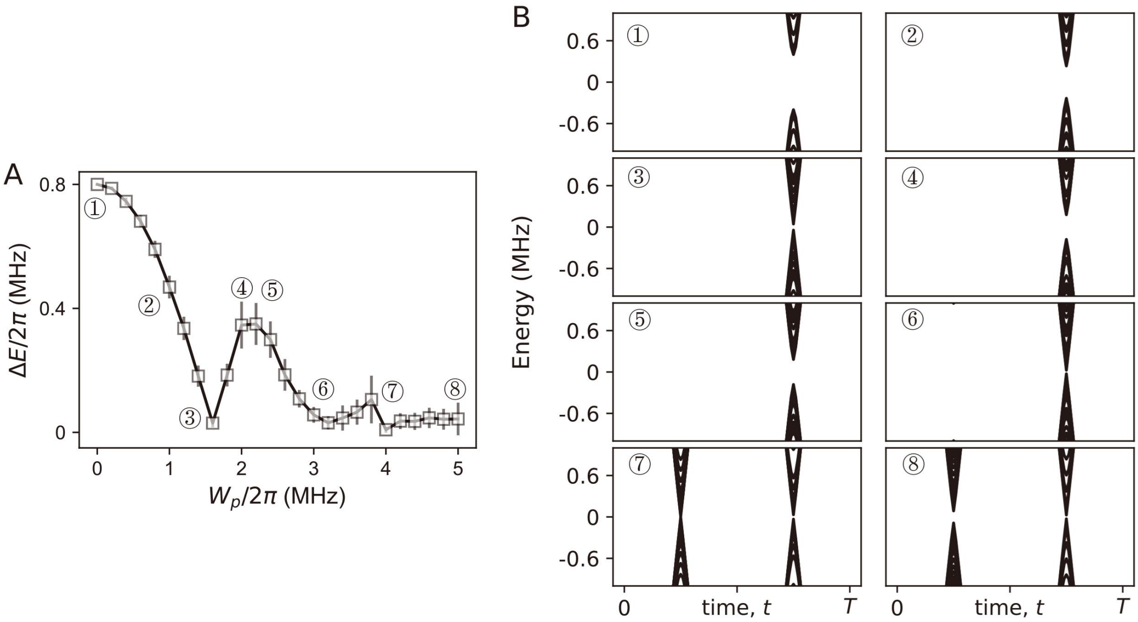

Figure R4-4: Instantaneous energy spectra of the bulk under quasi-periodic disorder. (A) The bandgap, ΔE, as a function of disorder strength W_p. (B) Energy spectrum under disorder for several different disorder strengths. The results are averaged over 100 disorder configurations.

the-art realistic conditions is difficult, due to the limited decoherence time. Our observation as shown in Fig. 5b is similar to the experimental results of topological Anderson insulators [Meier2018, Lin2022], which are also limited by the finite evolution time.

Second, a relatively clear quantized pumping could be observed for a long period T >100 μs. However, the parameter region of peaks of ΔQ, matches the theoretical region predicted by the Chern number, even for a relatively short pumping period.

Last, the region of peaks is in accordance with the range where the bandgap reopens, implying the occurrence of topological band pumping [Wauters2019, Cerjan2020, Wu2022].

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(4-3) Reviewer #4 wrote that:

From the title and the introduction of the manuscript, one of the main achievements the authors wish to highlight is that they simulated topological pumping with controllable disorder and observation of disorder-induced quantized pumping, as theoretically predicted and experimentally demonstrated by related works. [Wauters et al., PRL, 2019, Cerjan et al., Light Sci Appl, 2020, and Nakajima et al., Nature Physics, 2021, each Ref 23, 21, and 19 in the manuscript]
In order to strength the claim of similar observation in this work, I suggest providing more quantitative comparison and explanation on the comparison between the CoM (center-of-mass) shift δx and the numerically estimated charge pumped per cycle ΔQ being made in the main text section “Pumping in the presence of on-site disorder” and the main text Fig. 3. There are two main points that may have room for improvement for better explanation. First, there is a relatively fast decrease in δx in Fig.3a compared to the tendency in ΔQ given in Fig.3c in the range 0 MHz < $V/2\pi$ < 10 MHz, dropping steadily down from 1.95 to ~1.5. Second, there is an unexpectedly large δx in Fig.3b compared to the tendency in ΔQ given in Fig.3e, in the range 0 MHz < $V/2\pi$ < 7 MHz. It would be better grounded if the authors provide a more quantitative acceptance criteria for quantized pumping, or explanation

for deviations and potential sources for them.

Our response:

We thank Reviewer #4 for this significant suggestion.

In our manuscript, to characterize the topological properties, we calculated the charge pumped per cycle

$$\Delta Q = d\int_0^T dt\langle\psi(t)|J(t)|\psi(t)\rangle,$$

where $|\psi(t)>$ is the time evolved state initially as the ground state of the system at half-filling. This quantity was also used in related theoretical works [Wauters2019, Wu2022]. The displacement of center-of-mass (CoM), δx, is the quantity of primary experimental interest.

In the clean limit, ΔQ is equal to δx, when the initial state is prepared as an exact Wannier state. Both ΔQ and δx are quantized in the adiabatic limit. The detailed discussion about the Wannier state can also be found in the Response (1-2) to Reviewer #1. In our experiments, the initial state is prepared as a single-excitation state, leading to the results observed in Fig. 2b of the manuscript.

In the presence of on-site disorder, the lattice momentum is no longer conserved, and the Chern number of the disordered system is well-defined only when the bandgap remains open [Cerjan2020, Wu2022]. *The calculations about bandgaps are shown in the "Pumping under disorder" section of the supplementary materials.* Since our system can be regarded as a hard-core bosonic system [Yan2019, Shi2023PRL], the ground state at half-filling is the Slater determinant of eigenstates of the lower band. The final ΔQ can be obtained by averaging over the quantity for each eigenstate of the band, and the ΔQ under disorder is averaged with different disorder configurations, when considering different initial eigenstates due to the existence of disorder. Therefore, we believe that ΔQ is still equal to δx if the initial state is chosen as a Wannier state. The Wannier state is relevant to the choice of disorder samples:

$$|w_{n,j}\rangle = \frac{1}{\sqrt{N}}\sum_m e^{i\varphi_m}|\psi_{n,m}\rangle$$

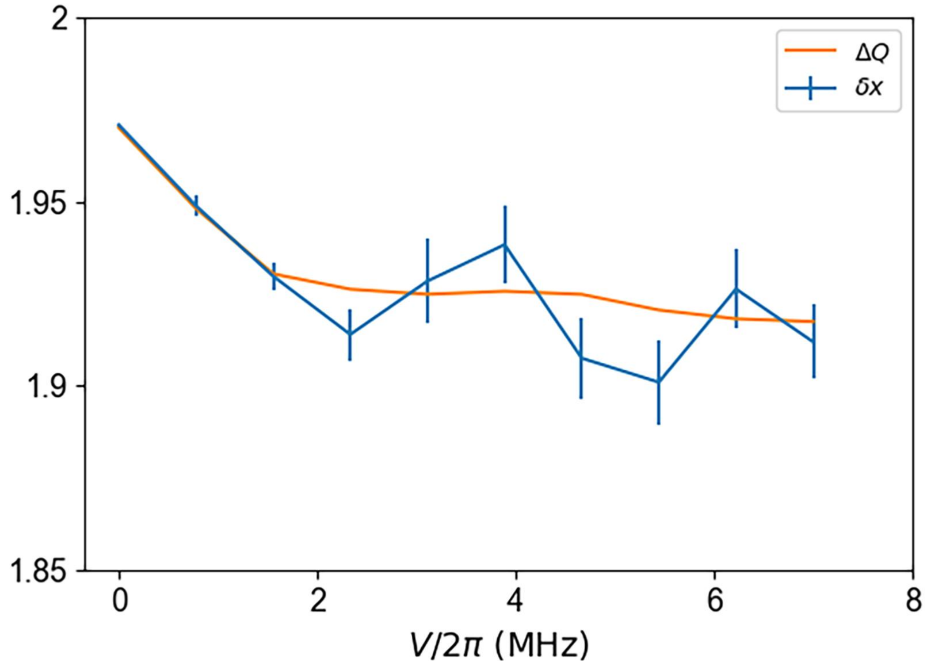

Figure R4-5: Numerical results of δx and ΔQ under weak on-site disorder.

where $|\psi_{n,m}>$ is obtained by diagonalizing the Hamiltonian with disorder, N = 20 for our device is the state number in one single band, φ_m is the non-unique phase determined by the maximally localized property [Marzari2012]. The generalized Wannier state is well-defined, when the bandgap remains open. Numerically, we calculate δx, when choosing the generalized Wannier state and ΔQ with weak on-site disorder. The results are shown in Fig. R4-5 and comply with our statements.

The experiments with disorder were performed by preparing the initial state as a single-excitation state. We conclude that the discrepancy, noted by the Reviewer, also originates from the single-excitation initial state. Furthermore, we define another quantity $\Delta Q'$ as

$$\Delta Q' = d\int_0^T dt\langle\lambda(t)|J(t)|\lambda(t)\rangle$$

where $|\lambda(t)>$ is the time evolved state initially with a single-excitation state. Using our experimental parameters, we numerically calculate ΔQ and $\Delta Q'$ as the on-site disorder strength increases, as shown in Fig. R4-6. We find that $\Delta Q'$ slightly drops for weak-disorder 0 MHz < $V/2\pi$< 10 MHz, which is in accordance with our observation in Fig. 3a of our manuscript. Although δx decreases in the weak disorder regime, there

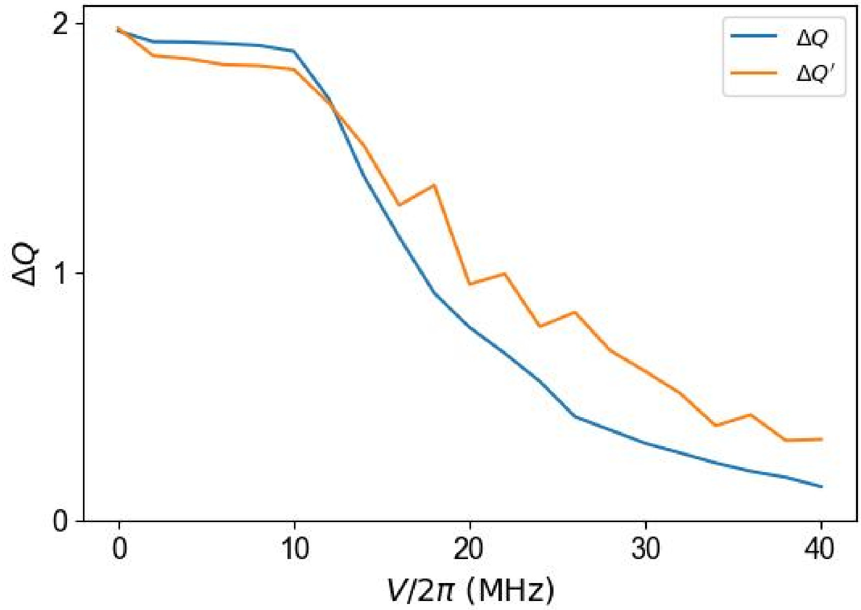

Figure R4-6: Numerical results of ΔQ and ΔQ' under on-site disorder based on our experimental setup parameters.

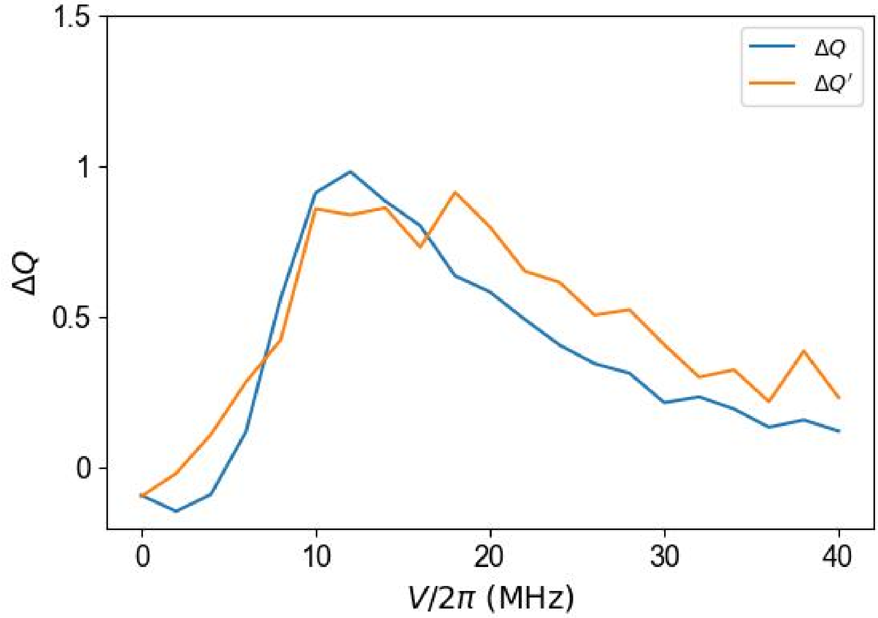

Figure R4-7: Numerical results of ΔQ and ΔQ' for the double-loop pumping trajectory under on-site disorder, with our experimental parameters.

exists a clear change of slope when 10 MHz < $V/2\pi$ < 20 MHz, which indicates that the topological phase transitions occur.

Similarly, we calculated ΔQ and $\Delta Q'$ for the double-loop trajectory, as shown in Fig. R4-7. We find $\Delta Q'$ is larger than ΔQ in the region of 0 MHz < $V/2\pi$ < 7 MHz, which verifies our experimental observation as shown in Fig. 3b. The topological index can be integer when the opposite Chern numbers for inner and outer loops cancel each other [Nakajima2021, Wu2022]. In other words, quantized pumping can be observed easier, if the radius of the outer pumping loop is much larger than the inner one, which are limited by the tunable range of experimental parameters.

Overall, these unexpected deviations can all be well explained by the numerical simulations. In the revised manuscript, we have added:

> *"The slight reduction of ΔQ when $V/\Delta_0 \lesssim 1$ results from the use of a single-excitation initial state instead of an exact Wannier state."*

when discussing the breakdown of pumping under random on-site disorder, and

> *"The increase of δx in the region $0 \lesssim V/\Delta_0 \lesssim 0.7$ is also due to the discrepancy between the single-excitation and Wannier initial states."*

when discussing the double-loop pumping trajectory with random on-site disorder.

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[Wu2022] Y.-P. Wu, L.-Z. Tang, G.-Q. Zhang, and D.-W. Zhang, *Quantized topological Anderson-Thouless pump*, Phys. Rev. A **106**, L051301 (2022).

(4-4) Reviewer #4 wrote that:

There is room for improvement in making connections between the data being discussed and the claim in the paragraph "Next, we experimentally investigate topological pumping… hardly saturated" in the main text section "Pumping with hopping disorder". In this paragraph, comparison between δx in Fig.3a (random on-site disorder) and Fig.4a (random hopping disorder) are made, and it is mentioned that the two curves are similar. Still, the main claim of this paragraph is that "it is difficult to observe quantized pumping under random hopping disorder". This argument seems to be made based on theoretical predictions that the two curves are supposed to obey different decay laws and possess different susceptibility against non-adiabatic evolution. As an experimental work, I suggest making sufficient connections to experimental data to back up this argument.

Our response:

We thank Reviewer #4 for pointing out this issue.

In fact, the conclusion of "it is difficult to observe quantized pumping under random hopping disorder" is not the major claim in our work. This results can be found in the "*D. Topological pumping under random disorders*" Section of the Supplementary Materials in Ref. [Wu2022].

In fact, we misused the word "Thus". That argument was made based on the fact that "small or moderate random disorder induces energy gap closing before moving the gapless point inside the pump loop", and the Chern number is non-integer. Here, by writing this sentence, we only would like to state the reason why we do not tend to experimentally investigate the pumping interplaying with random hopping disorder.

In Fig. R4-8, we show the numerical results in Ref. [Wu2022]. For the sequence that encircles the gapless point, as plotted in Fig. R4-8(a), the Chern number C and the pumped charge ΔQ remains 1 under weak disorder and start to decrease under strong disorder, which complies with our experimental observation shown in Fig. 5a of our manuscript.

For the sequence that does not encircle the gapless point, as plotted in Fig. R4-8b, C and ΔQ approach non-integer values, as the disorder strength increases, due to the fact that moderate random disorder induces bandgap closing before the gapless point moves inside the pumping loop. The absence of quantized pumping with random disorder is further corroborated by checking the Berry phase γ

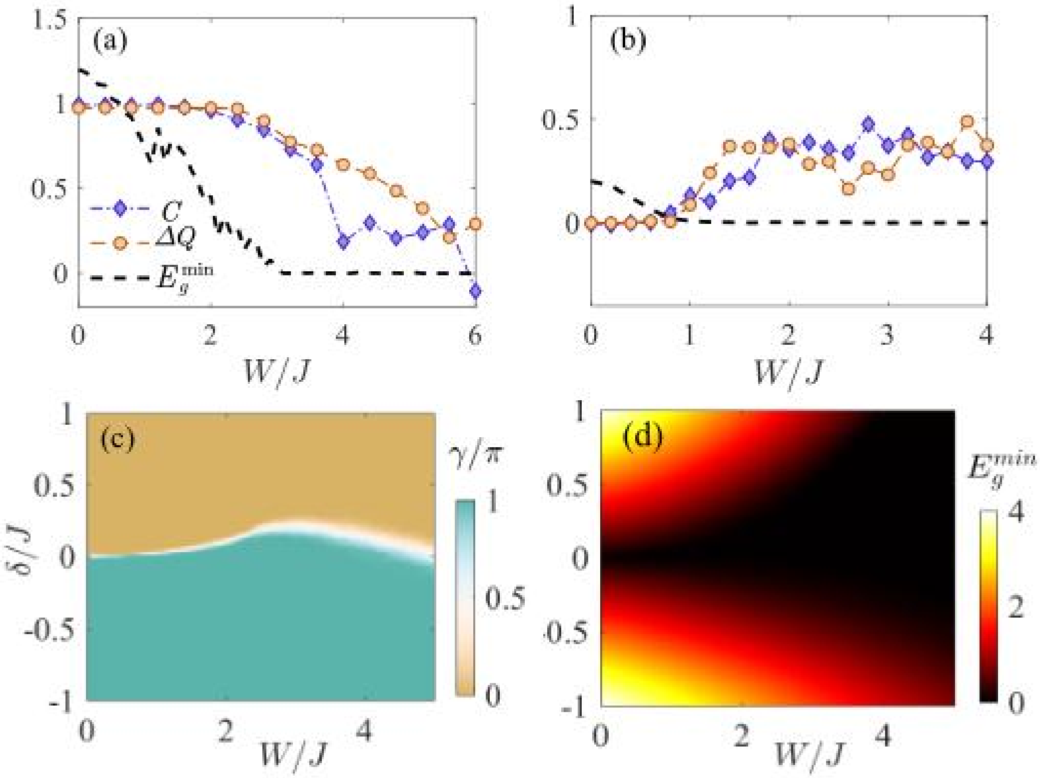

Figure R4-8: Numerical results of the Chern number *C*, pumped charge ΔQ and bandgap E_g^{min} under random hopping disorder for the sequence that encircles (a) and does not encircle (b) the gapless point. (c) The Berry phase *γ*, and (d) E_g^{min} as functions of and random disorder strength *W* with fixed Δ = 0. This figure is adapted from Ref. [Wu2022].

and E_g^{min} for different pump loops in a larger parameter space, as shown in Fig. R4-8(c,d).

In our experiments, quantized pumping under random hopping disorder can hardly be observed even under adiabatic conditions. Numerical results for even longer periods are shown in the "*Single-loop pumping induced by hopping disorder*" Section of the Supplementary Materials.

In the revised manuscript, to avoid any confusions, we have removed the corresponding description about the claim and added that

"*Moreover, as the gap reopens, applying quasi-periodic hopping disorder may intrinsically induce topological pumping,* *which can hardly be realized by introducing random hopping disorder* [29, 35]",

REFERENCE:

[Wu2022] Y.-P. Wu, L.-Z. Tang, G.-Q. Zhang, and D.-W. Zhang, *Quantized topological Anderson-Thouless pump*, Phys. Rev. A **106**, L051301 (2022).

(4-5) Reviewer #4 wrote that:

Although this work shows analog quantum simulation on a highly controllable large scale (41 superconducting qubits) quantum processor, the demonstrated capability does not imply significant technological advancement. First, the experiments are still limited to single-excitation subspace, which is easily simulated classically. Second, the provided data for disorder-induced pumping (Fig. 3 and Fig. 4) does not seem to require all 41 qubits, as it can be inferred by the small observed CoM displacement compared to the system size and small hopping rates between sites. At any given time before the decoherence effect shows up, the system remains in a relatively simple few-body entangled or even product state. The second point may be resolved if the authors provide experimental data for δx per multiple pumping cycles (>> 1 cycles) is provided.

Our response:

We thank Reviewer #4 for this comment.

In this work, we study the competition and interplay between disorder and topology in Thouless pumping, which can be investigated by observing the single excitation of qubits, as an approximation to the Wannier state with a high fidelity. Other experiments on Thouless pumping that have been reported so far were conducted under similar circumstances, such as observations in ultracold atoms [Lohse2016, Nakajima2016, Nakajima2021], and photonic systems [Cerjan2020]. We explored novel physics in comparison to previous works. Although we focus on pumping with quasi-periodic hopping disorder, we emphasize that the breakdown of pumping under random hopping disorder is also non-trivial:

First, the precise programming of both on-site potentials and hopping strengths, which provides an opportunity to study different topological physics under different kinds of disorder, is one of the advantages of superconducting circuits, in comparison with other platforms.

Second, the breakdown of quantized pumping with random on-site disorder is regarded as a manifestation of a delocalization–localization transition of Floquet eigenstates [Wauters2019, Nakajima2021]. However, it is not clear whether this conclusion is still valid for the random-hopping case.

Third, with random-hopping disorder, the eigenstates of systems tend to be localized but obey a different law [Theodorou1976, Fleishman1977, Soukoulis1981] from the usual Anderson localization (exponential law) under on-site disorder [Evers2008]. More details are discussed in the revised Supplementary Materials and can be found in the Response (3-4) to Reviewer #3.

Hence, we believe that our experimental study on pumping under random hopping disorder will attract more attention in this field. In addition, as mentioned by Reviewer #4 below, we also study Thouless pumping by preparing double-excitation states (new figure added as Fig. 3 in the revised manuscript), corresponding to which two bands are initially occupied, which is not involved in previous experiments.

Even with single-excitation initial states, the multi-qubit measurement technique is necessary, because the crosstalk and residue interaction are inevitable in experiments. Therefore, to observe results that are predicted theoretically, the necessary technique is almost the same as the one of multi-qubit experiments. Moreover, from a technical perspective, different from previous quantum simulation experiments using superconducting qubits with Floquet engineering [Cai2019, Zhao2022, Shi2023PRL], we developed an advanced Floquet engineering technique for adiabatic dynamical systems by simulating a time-dependent Hamiltonian. This Floquet engineering technique for adiabatic systems has already been validated during the multi-excitation experiment in this work. In comparison with the recent experiments using couplers [Yan2018], which induce extra unexpected leakage [Xu2020], the advantage of Floquet engineering is obvious: More qubits can be fabricated on the same chip size with much fewer control lines (couplers are equivalent to qubits from a fabrication perspective) and

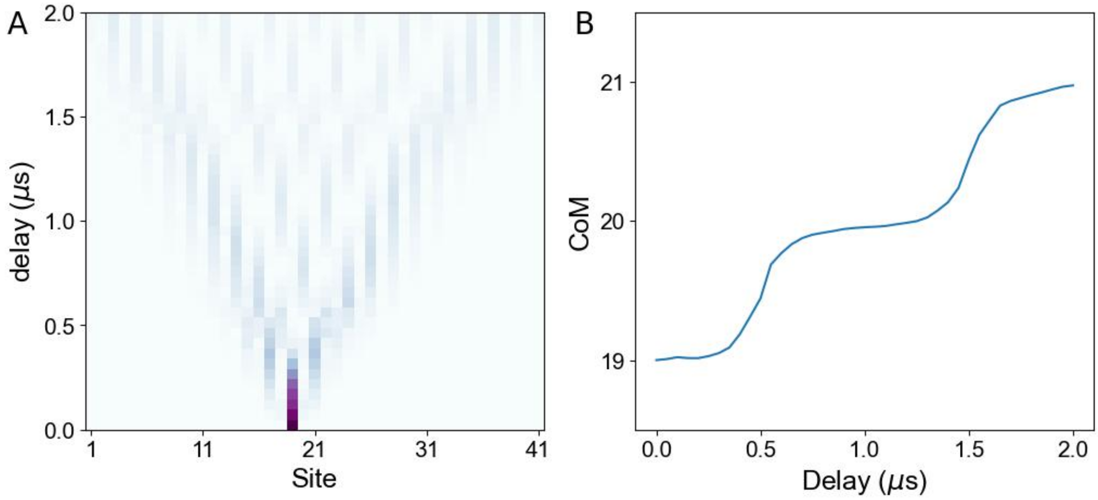

Fig. R4-8: Topological pumping for the trajectory $\mathcal{C}_4$, with $\Delta_0/2\pi$ = 12 MHz, $\delta_0/2\pi$ = 2 MHz, $J/2\pi$ = 3 MHz, and T = 2 μs. The adiabatic time evolution is shown in (A), and the corresponding CoMs extracted from (A) are shown in (B).

less energy level leakage.

Moreover, the displacement of CoM, δx, is calculated as the weighted average of positions by readout probability. The time evolution is diffusive dependent on pumping parameters, such as the numerical numerical results of topological pumping for the trajectory $\mathcal{C}_4$, but with different parameters (see Fig. R4-8). Therefore, a small δx does not necessarily mean that this experiment can be achieved on a small-scale quantum processor.

In addition, for the pumping trajectory $\mathcal{C}_4$, we calculate the entanglement propagation of two neighbor qubits in a 16-qubit chain numerically, as shown in Fig. R4-9 by considering the concurrence [Amico2008]. We find the entanglement between qubit pairs. The entanglement propagation can be described by the coherent interference of quasi-particle modes of the collective behavior of the system [Yan2019].

Furthermore, while the results demonstrated by our devices behave like non-interacting fermions in the hard-core limit [Yan2019], the processor cannot be completely described by a hard-core bosonic model; in particular, when involving decoherence and higher energy

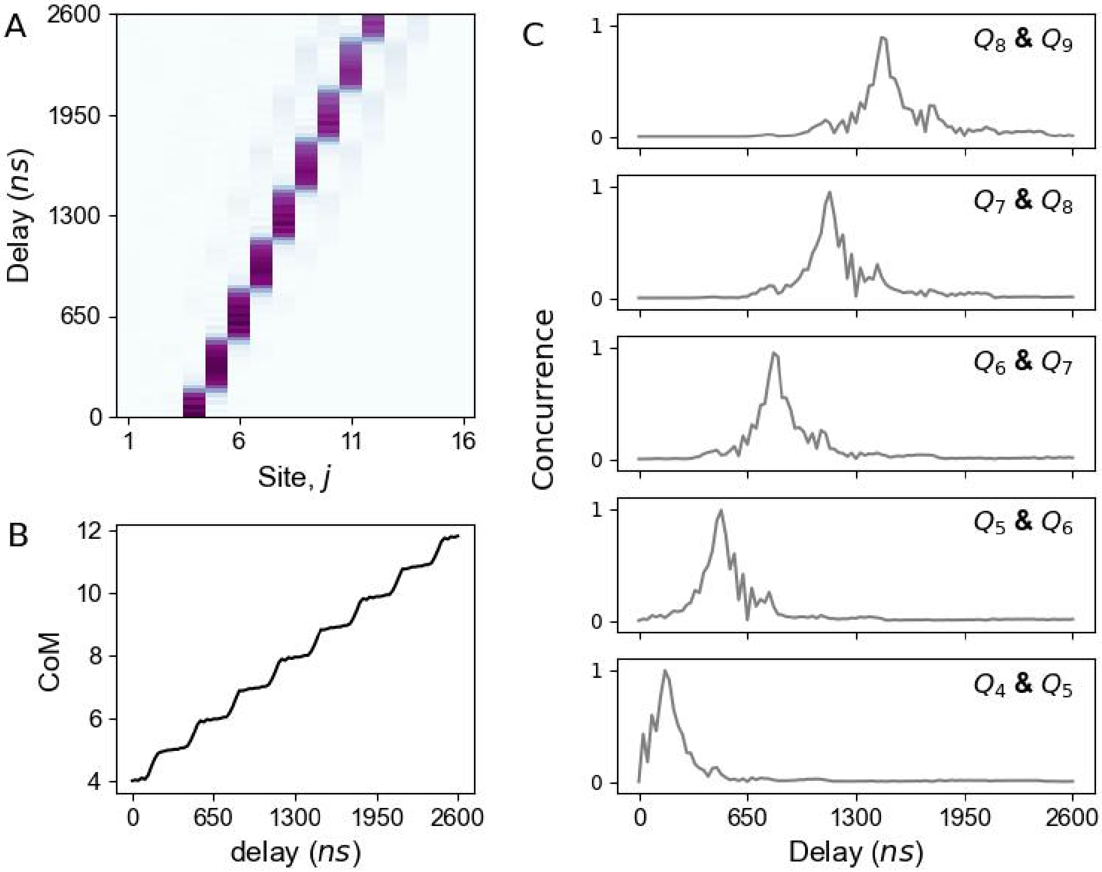

Fig R4-9: (A) Evolutions of qubits excitation probabilities during topological pumping. The numerical results of adiabatic time evolution of topological pumping for the trajectory $\mathcal{C}_4$ in Fig. 2c of our manuscript. (B) The CoMs extracted from (A). (C) Entanglement propagation with concurrence between neighboring qubits.

levels. The effects of non-desirable factors need to be considered to understand and evaluate experimental results.

Due to the limited decoherence time, which can be found in the Response (1-2) to Reviewer #1, it is still difficult to perform experiments over too many circles.

REFERENCE:

[Lohse2016] M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch, *A Thouless quantum pump with ultracold bosonic atoms in an optical superlattice*, Nat. Phys. **12**, 350 (2016).

[Nakajima2016] S. Nakajima, T. Tomita, S. Taie, T. Ichinose, H. Ozawa,

L. Wang, M. Troyer, and Y. Takahashi, *Topological Thouless pumping of ultracold fermions*, Nat. Phys. **12**, 296 (2016).

[Nakajima2021] S. Nakajima, N. Takei, K. Sakuma, Y. Kuno, P. Marra, and Y. Takahashi, *Competition and interplay between topology and quasi-periodic disorder in Thouless pumping of ultracold atoms*, Nat. Phys. **17**, 844 (2021).

[Cerjan2020] A. Cerjan, M. Wang, S. Huang, K. P. Chen, and M. C. Rechtsman, *Thouless pumping in disordered photonic systems*, Light Sci. Appl. **9**, 178 (2020).

[Wauters2019] M. M. Wauters, A. Russomanno, R. Citro, G. E. Santoro, and L. Privitera, Localization, *Topology, and Quantized Transport in Disordered Floquet Systems*, Phys. Rev. Lett. **123**, 266601 (2019).

[Theodorou1976] G. Theodorou and M. H. Cohen, *Extended states in a one-demensional system with off-diagonal disorder,* Phys. Rev. B **13**, 4597 (1976).

[Fleishman1977] L. Fleishman and D. C. Licciardello, *Fluctuations and localization in one dimension*, J. Phys. C: Solid State Phys. **10**, L125 (1977).

[Soukoulis1981] C. M. Soukoulis and E. N. Economou, *Off-diagonal disorder in one-dimensional systems*, Phys. Rev. B **24**, 5698 (1981).

[Cai2019] W. Cai, J. Han, F. Mei, Y. Xu, Y. Ma, X. Li, H. Wang, Y. Song, Z.-Y. Xue, Z.-q. Yin, S. Jia, and L. Sun, *Observation of Topological Magnon Insulator States in a Superconducting Circuit*, Phys. Rev. Lett. **123**, 080501 (2019).

[Zhao2022] S. K. Zhao, *et al.*, *Probing Operator Spreading via Floquet Engineering in a Superconducting Circuit*, Phys. Rev. Lett. **129**, 160602 (2022).

[Shi2023PRL] Y.-H. Shi, *et al.*, *Quantum Simulation of Topological Zero Modes on a 41-Qubit Superconducting Processor*, Phys. Rev. Lett. **131**, 080401 (2023).

[Xu2020] Y. Xu, J. Chu, J. Yuan, J. Qiu, Y. Zhou, L. Zhang, X. Tan, Y. Yu, S. Liu, J. Li, F. Yan, and D. Yu, *High-fidelity, high-scalability two-qubit gate scheme for superconducting qubits*, Phys. Rev. Lett. **125**, 240503 (2020).

[Amico2008] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, *Entanglement in many-body systems*, Rev. Mod. Phys. **80**, 517 (2008).

[Yan2019] Z. Yan, *et al.*, *Strongly correlated quantum walks with a 12-qubit superconducting processor*, Science **364**, 753 (2019).

(4-6) Reviewer #4 wrote that:

In addition, here are several minor points:

In the main text section "Topological invariant and topological pumping", the authors mention the "careful" choice of pumping cycle period T = 650 ns, as a point to provide maximum δx. However, a more detailed explanation would be helpful to understand such choice, as the explanation for required adiabaticity in quantized pumping seems to imply that any period that satisfies the adiabaticity condition (i.e., longer than 650 ns) should provide the maximum δx.

If the smaller δx for large T is limited by decoherence, the experimental results should be better compared with the theoretical / numerical predictions if the effect of decoherence is taken into account. This comment is applicable to other observations as well. For example, the duration of the experimental pulse sequences after state preparation and before readout is not negligible when compared with the coherence times of the qubits. I recommend post-selection for single-excitation states if it has not been applied to the presented data.

Our response:

We thank Reviewer #4 for this comment. The quantized plateau could be observed, when T is long enough to satisfy the adiabatic conditions. We numerically calculate δx and ΔQ for different periods, as shown in Fig. R4-10(A). The quantized plateau of δx could be observed for $T > 650$ ns, when the initial state is an exact Wannier state. However, δx oscillates $T > 650$ ns, when the initial state is prepared as a single-excitation state.

In Fig. R4-10(B), we plot $(2-\delta x)$ for longer periods, showing that any finite frequency of the drive will ultimately lead to a departure from the adiabatic dynamics on long timescales, as demonstrated in Ref. [Privitera2018]. Hence, we believe that the slight reduction of δx, observed in Fig. 2b of our main text, mainly results from the choice of

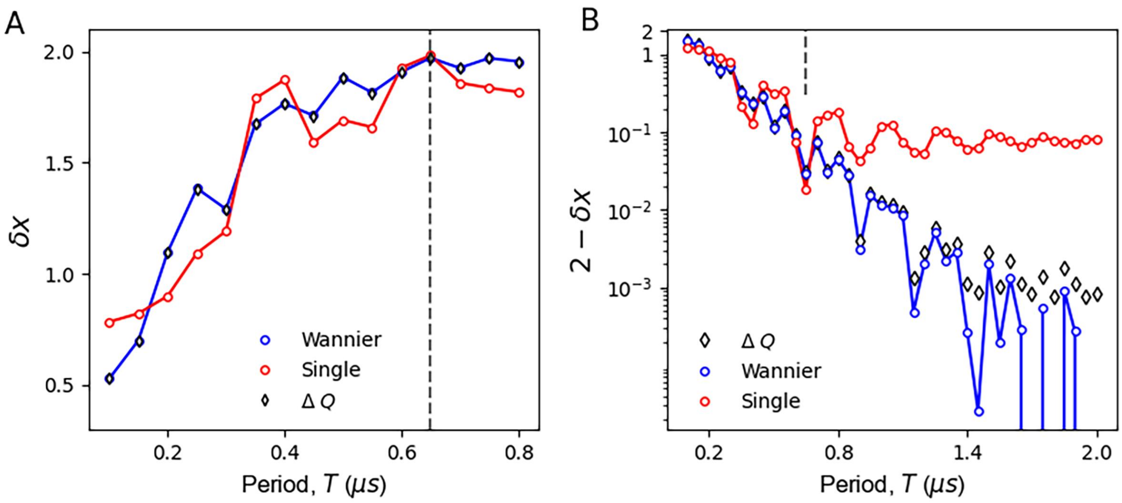

Figure R4-10: The δx and ΔQ versus periods. Numerical results of δx (A) and ($2-\delta x$) (B) versus the period T, when preparing the initial state as a single-excitation state and a Wannier state, respectively.

the single-excitation state and is possible even without decoherence.

More discussions about the effects of decoherence can also be found in the Response (1-2) to Reviewer #1. We conclude that our system can be well regarded as a closed system within the experimental timescale of less than 1,500 ns. The data presented in the manuscript have been calibrated for single and double excitation states.

REFERENCE:

[Privitera2018] L. Privitera, A. Russomanno, R. Citro, and G. E. Santoro, *Nonadiabatic breaking of topological pumping*, Phys. Rev. Lett. **120**, 106601 (2018).

(4-7) Reviewer #4 wrote that:

Although it may not be consistent with the title and main claims of the manuscript, I recommend giving more recognition in double-excitation topological pumping experiments in the main text highlighting such capability. which is currently only discussed in the supplementary. Although given the parameter regime it may still be classically simulatable, it would allude to the potential capability of the proposed scheme for studying topological pumping or other many-body physics phenomena under significantly large onsite energy that may not be easily accessible with classical simulations and other experimental platforms.

Our response:

We thank Reviewer #4 for this suggestion. We have added more discussions in the main text about topological pumping with double-excitation initial states and experimental results as shown in Fig. 3 of the manuscript. In addition, in the main text, we have added this:

"In addition, we experimentally monitor the double-excitation pumps for different trajectories [29], which are shown in Fig. 3. The experimental results are similar to the single-excitation cases, as the system is in the hard-core limit [43]. Since the pumps of excitations initially prepared at odd and even sites have opposite winding numbers [46], no quantized pumping is observed for the topologically nontrivial pumping trajectory C_4, when the parity of the initial excitation sites is different (Fig. 3d)."

(4-8) Reviewer #4 wrote that:

State preparation and readout processes may need more detailed descriptions. Based on the main text, an example sketch of a single measurement sequence may be inferred to be the following: the system is first prepared in a single-qubit single-excitation state when the qubits are detuned and no parametric drive is applied. Then, parametric flux modulation drives are turned on to bring the system to the initial point of a pumping sequence. After completing the pumping sequence, the parametric drives are turned off, and the qubits are brought back to the frequencies where readout is followed. It would be more informative to provide such pulse sequences, and discuss how the turning-on / turning-off part of the parametric drives are performed (e.g., adiabatically or instantaneously) and how they affect state preparation and measurement outcome. I recommend adding discussion of these in supplementary information or Methods section.

Our response:

We thank Reviewer #4 for the comment. The experimental sequence, as shown in Fig. R4-11, is similar to the quantum walks (QWs) protocol [Yan2019, Gong2021]. With all 41 superconducting qubits being initialized at their idle points, which are carefully arranged to minimize the unexpected crosstalk errors and residue interaction, we prepared the localized state using an X gate. By using the derivative removal by adiabatic gate (DRAG) theory [Motzoi2009], the X gate pulse is optimized to minimize the leakage to higher energy levels, achieving the mean gate fidelity of 99.2%. Then, all qubits are applied parametric flux modulation to engineer the Rice-Mele Hamiltonian, for the specific pumping protocol with a time *t*. After turning off the parametric driving, the qubits are tuned back to their idle points. Then, the probabilities of |0**>** and |1**>** can be read out simultaneously for all qubits using the transmission lines coupled to readout resonators.

For a single qubit, spontaneous radiation can be ignored in a time scale much shorter than the qubit energy relaxation time. For interacting systems, if the rising and falling edges of the pulse are much shorter compared with the coupling strength, undesired state tunneling can be almost avoided, according to the Laudau-Zener transition [Ivakhnenko2023]. In our device, the duration of the rising or falling edges of the parametric pulse, which are usually determined by the sampling rate of our arbitrary waveform generator (AWG), does not exceed 2 ns. The rate of change during the turning-on and turning-off part is more than 50 times of the coupling strength. Therefore, the effect of the rapid switches between the idle points and the work points, which occur before and after topological pumping, is negligible for both the initial-state preparation and the final-state measurement. <u>In the revised manuscript, we have added more discussions on pulse sequence at the “Experimental setup” Section in the Methods.</u>

REFERENCE:

[Yan2019] Z. Yan, *et al.*, *Strongly correlated quantum walks with a 12-qubit superconducting processor*, Science **364**, 753 (2019).

[Gong2021] M. Gong, *et al.*, *Quantum walks on a programmable two-dimensional 62-qubit superconducting processor*, Science **372**, 948 (2021)

[Motzoi2009] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm, *Simple pulses for elimination of leakage in weakly nonlinear qubits*, Phys. Rev. Lett. **103**, 110501 (2009).

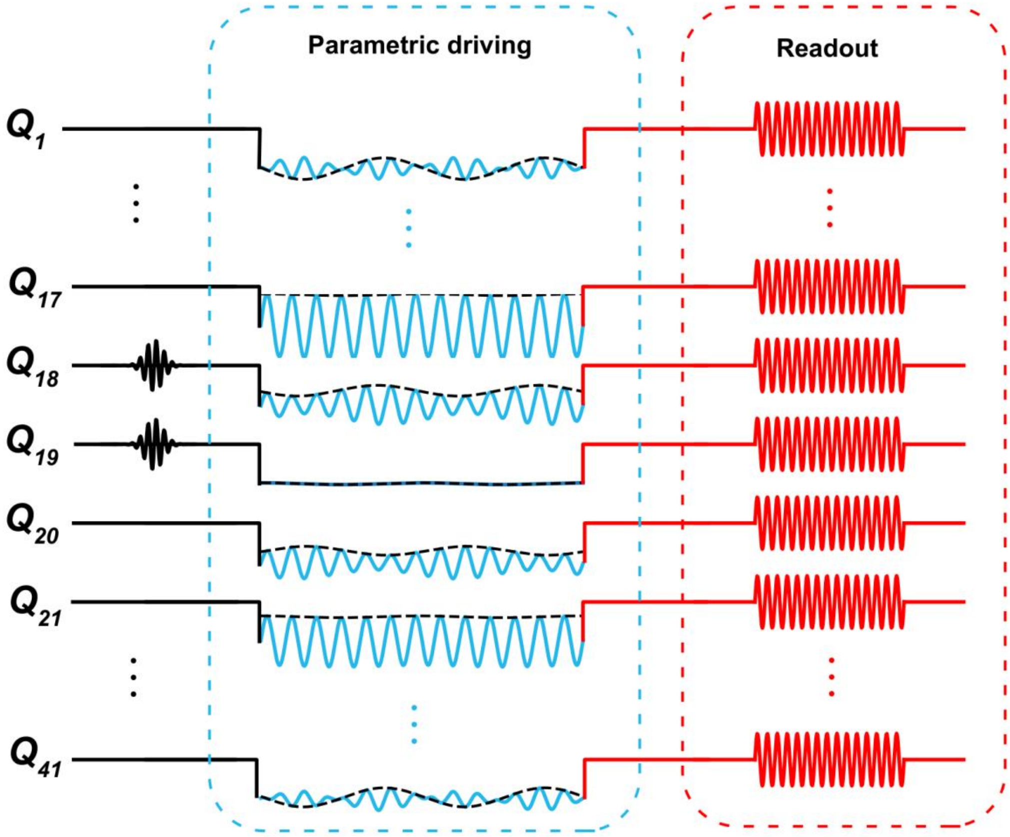

Figure R4-11: Pulse sequence for topological pumping with double excitations.

[Ivakhnenko2023] O. V. Ivakhnenko, S. N. Shevchenko, and F. Nori, *Nonadiabatic Landau-Zener-Stückelberg-Majorana transitions, dynamics, and interference*, Phys. Rep. **995**, 1 (2023).

(4-9) Reviewer #4 wrote that:

Related to Comments 1 - 3, the comparisons being made in the main text Fig.3 and Fig.4 would be easier if the x-axes of Fig.3a-e and Fig.4a-c are provided in a normalized scale such as $V/\Delta 0$ and $Wp/\delta 0$, and if the ranges are matched. Note that the location of peaks in Fig.3b would be compared easier to the peaks of Fig.3e if Fig.3b includes 30 MHz < $V/2\pi$ < 40 MHz.

Our response:

We thank Reviewer #4 for this kind suggestion. We have revised the ticks of the corresponding figures in Fig. 4a,b and Fig. 5a,b as the normalized scale.

(4-10) Reviewer #4 wrote that:

Fig.3f needs explanation for distinction between faint blue lines and solid blue lines.

Our response:

We thank the Reviewer for pointing out this issue. We plot the instantaneous energy spectrum for the trajectory C_{dl} in Fig. 4f in the new version of our manuscript. The spectrum, composed of two bulk bands and edge states, is obtained by diagonalizing the Rice-Mele Hamiltonian under open boundary conditions. The shades of color represent the density of states. The color of the edge states is very light, because this part is composed of a single curve, while the bulk band is dark for the high density of states. We have added

" Darker colors imply higher state density."

in the caption of Fig. 4f of the revised manuscript. In addition, we have changed the color of the edge state as red.

(4-11) Reviewer #4 wrote that:

It would be informative to explicitly mention the validity of the hard-core boson approximation. One possibility may be to provide |U/gj,j+1|. (please find [Yan et al., Science, 2019, Ref 42 of the manuscript])

Our response

We thank the Reviewer for this suggestion. As demonstrated in Tab. S1 of Supplementary Materials, the qubit-qubit coupling $g_{j,j+1}/2\pi$ has a mean value of 7.11 MHz, and the on-site nonlinear (attractive) interaction $U/2\pi$ has a value of −208 MHz. Hence, our 1D bosonic system with $|U/g_{j,j+1}|$ ~ 29 >> 1 can be regarded as non-interacting spinless fermions [Yan2019]. We have added that

"Since the average anharmonicity is $U/2\pi$ ~ −208 MHz, with a ratio $|U/g|$ ~ 29 >> 1, our processor can be regarded as a hard-core bosonic system [43].

in the "*Experimental setup*" Section in the Methods of our revised manuscript.

REFERENCE:

[Yan2019] Z. Yan, *et al.*, *Strongly correlated quantum walks with a 12-qubit superconducting processor*, Science **364**, 753 (2019).

(4-12) Reviewer #4 wrote that:

In summary, this work shows good progress in superconducting circuit-based quantum simulation, Floquet engineering, and analog quantum simulation under controlled disorder. However, the experiments shown in this manuscript require improvements in data and interpretation with quantitative comparisons and analysis, in order to support several of the authors' major claims. Therefore, I am hesitant to recommend publication in Nature Communications. The current form of the manuscript is more suitable for publication in npj Quantum Information, unless the above mentioned points are properly addressed.

Our response:

We thank the constructive suggestions and useful comments from Reviewer #4, which help a lot to improve the quality and impact of our manuscript. We have carefully revised the manuscript according to all comments from Reviewer #4. We believe that the current version of our manuscript with these revisions have now met the high standard of *Nature Communications*.

List of changes to the manuscript

1. Various minor typos have been fixed, and other text modifications prompted by the Reviewers.
2. Prompted by Reviewer #1 remarks, we revised schematic diagram of the Rice-Mele model in Fig. 1 c-d, and added titles in Fig. S5.
3. Prompted by Reviewer #4 remarks, we added the results of pumping with double-excitation initial state as Fig. 3.
4. We added detailed discussion on the effects of decoherence to our experiments in supplementary materials. In particular, the effective dephasing time T_2 is longer than T_2^* obtained by single-qubit measurement.
5. We added detailed discussion on the Wannier state, and how close to the initial state prepared in our experiments. And we added the numerical result of δx versus T when the initial state prepared as the Wannier state in Fig. 2b.
6. Prompted by Reviewer #1 remarks, we added

    "However, a quantized disorder-induced pump can hardly be realized, since quantized transport requires trajectory parameters to be finely tuned to combine the effects of the trivial inner and outer trajectories [20].",

    when discussing pumping induced by the double-loop trajectory.
7. We removed the corresponding content about the adiabatic conditions of pumping with random hopping disorder.
8. We added extra numerical results of δx when T is much longer for the quasi-periodic case in Fig. 5b.
9. Prompted by the Reviewer #4 remarks, we added the ratio of on-site interaction U to coupling strength g about 29 in Methods.
10. We added more contents about Floquet engineering in Methods.
11. We added more discussion about the pulse sequence in Methods.
12. We revised the abstract, introduction and outlook according to suggestions of all Reviewers.

REVIEW COMMENTS

Reviewer #1(Remarks to the Author)

I have read in great details the long response of the authors to the three referees and the modifications of their manuscript. I appreciate the efforts that the authors devoted to answer in details the various comments of the referees, including mine. I all now convinced by their answer that the decoherence time of their coupled qubit is much longer than I initially understood. The various corrections that the authors provided, in particular to the figures, definitely improved the clarity of the manuscript. However, in the end I have mixed feelings about the recommendation of publication of this manuscript in *Nature Communications*. On one hand, the experimental platform is very impressive, and I have the impression that the experiments are at the state of the art with a solid-state simulation platform for topological pumping. On the other hand, as the authors acknowledge themselves in the answer to the referees, this platform fail to demonstrate a quantized pumping induced by disorder but come as close as currently possible to do so, as exemplified in their Fig. R1-4 of their response. This achievement is not as impressive as expected, in particular given the topological nature of the underlying phenomenon. However, I have the impression that it could possibly warrant a publication in *Nature Communications*.

Reviewer #3 (Remarks to the Author):

I have reviewed the authors' thorough responses and recommend the manuscript for publication.

Reviewer #4 (Remarks to the Author):

After carefully examining the response letter and the updated manuscript, I believe that the authors' revisions and their responses to the review comments have significantly improved the paper. I appreciate their attentiveness to the feedback and the increased clarity present throughout the manuscript. The revisions made in light of the previous comments are persuasive and have successfully addressed many of the concerns raised in the initial review.

Below we have outlined our responses to the revised manuscript and provided suggestions for further improvements. If these points are adequately addressed, I recommend the paper for publication in *Nature Communications*.

1. The authors attempt to strengthen their claim of topological pumping induced by quasi-periodic hopping disorder by presenting the emergence of a quantized plateau under adiabatic conditions using numerical simulation. Additionally, they show agreement between simulation and experimental results when these conditions are not met.

However, I am concerned that even with the additional explanation the data does not imply experimental demonstration of disorder-induced topological pumping.

Although the data is consistent with the simulation under non-adiabatic regime (short evolution period), this does not mean the agreement would extend to adiabatic conditions (longer evolution period). Furthermore, it is still not clear if the consistency at weak adiabaticity is coincidental unless the authors rule out the influence of other factors, including discrepancy between the initial and Wannier states, challenges in precise Hamiltonian control, qubit frequency fluctuations, accidental qubit swaps due to Hamiltonian specifics or accidental resonances to TLSs or untracked modes, etc. Additionally, the existing comparison does not extend beyond Wp/δ0'' > 3. In short, the current experimental data are still ambiguous and not clean enough to support the claim of experimental realization of quasi-periodic hopping induced topological pumping.

Generally, the claim or agreement with the underlying model would be more convincing if the authors provide experimental evidence of the emergence of a quantized plateau over several increasing evolution periods. This is similar to the necessary demonstration of quantum phase transition at increasing system sizes. Due to realistic experimental constraints, I understand that this may not be achievable with the current capabilities of their platform.

If this suggestion is not realizable, I suggest weakening their claim about the experimental realization of topological pumping induced by quasi-periodic hopping disorder. An example might be to tone down the claim to "observation of signatures consistent with topological pumping induced by quasi-periodic hopping disorder under insufficient adiabaticity".

2. The revised manuscript is titled "Reciprocity in Disorder and Topology of Thouless Pumping on a Superconducting Quantum Processor." However, the use of "Reciprocity" is unclear. It may misleadingly suggest a sort of symmetry observed during the experiment, which is not relevant to the manuscript's content. A more suitable title might be "Interplay Between Disorder and Topology in Thouless Pumping … " which would better reflect the focus of the manuscript.

In addition, here is a minor point:

The authors attribute the discrepancies observed in their experiments to a small overlap (~1%) in the initial state from the ideal Wannier state. This is an intriguing observation and seems to explain many of the unexpected discrepancies in their results. I suggest that the authors further discuss how close the assumed initial state is to the actual prepared state in their measurements. It would be clearer if the authors either utilize measured initial states to calculate deviation from the Wannier state, or incorporate some key coherent error sources into their model of the prepared state, such as imperfect rotation in X gate. In addition, while incoherent errors may not impact the dynamics significantly, it would be also helpful to understand the impacts of thermal population or ground state preparation fidelity.

In conclusion, the manuscript is well-written and organized, though there are a few areas that could be improved. It makes a significant contribution to experimentally demonstrating topological pumping with controllable disorder using superconducting qubits. If the highlighted issues are sufficiently addressed, I recommend this manuscript for publication in *Nature Communications.*

REVIEW COMMENTS

Reviewer #1(Remarks to the Author)

I have read in great details the long response of the authors to the three referees and the modifications of their manuscript. I appreciate the efforts that the authors devoted to answer in details the various comments of the referees, including mine. I all now convinced by their answer that the decoherence time of their coupled qubit is much longer than I initially understood. The various corrections that the authors provided, in particular to the figures, definitely improved the clarity of the manuscript. However, in the end I have mixed feelings about the recommendation of publication of this manuscript in Nature Communications. On one hand, the experimental platform is very impressive, and I have the impression that the experiments are at the state of the art with a solid-state simulation platform for topological pumping. On the other hand, as the authors acknowledge themselves in the answer to the referees, this platform fail to demonstrate a quantized pumping induced by disorder but come as close as currently possible to do so, as exemplified in their Fig. R1-4 of their response. This achievement is not as impressive as expected, in particular given the topological nature of the underlying phenomenon. However, I have the impression that it could possibly warrant a publication in *Nature Communications.*

Our response

We thank Reviewer #1 for the recommendation. Due to the limited decoherence time, the ideal quantized topological pumping can hardly be observed in the state-of-art solid-state quantum simulators. We still believe that the results in Fig. 5b demonstrate reasonable and instructive experimental observations of nontrivial pumping induced by quasi-periodic hopping disorder across a variety of quantum simulation platforms. Based on the constructive suggestions from all Reviewers, we believe the current version of our manuscript now meets the high standard of *Nature Communications*.

Reviewer #3 (Remarks to the Author):

I have reviewed the authors' thorough responses and recommend the manuscript for publication.

Our response

We thank the Reviewer #3 for recommending our work for publication in *Nature Communications.*

Reviewer #4 (Remarks to the Author):

(4-1) Reviewer #4 commented

After carefully examining the response letter and the updated manuscript, I believe that the authors' revisions and their responses to the review comments have significantly improved the paper. I appreciate their attentiveness to the feedback and the increased clarity present throughout the manuscript. The revisions made in light of the previous comments are persuasive and have successfully addressed many of the concerns raised in the initial review.

Below we have outlined our responses to the revised manuscript and provided suggestions for further improvements. If these points are adequately addressed, I recommend the paper for publication in *Nature Communications*.

1. The authors attempt to strengthen their claim of topological pumping induced by quasi-periodic hopping disorder by presenting the emergence of a quantized plateau under adiabatic conditions using numerical simulation. Additionally, they show agreement between simulation and experimental results when these conditions are not met.

However, I am concerned that even with the additional explanation the data does not imply experimental demonstration of disorder-induced topological pumping. Although the data is consistent with the simulation under non-adiabatic regime (short evolution period), this does not mean the agreement would extend to adiabatic conditions (longer evolution period). Furthermore, it is still not clear if the consistency at weak adiabaticity is coincidental unless the authors rule out the influence of other factors, including discrepancy between the initial and Wannier states, challenges in precise Hamiltonian control, qubit frequency fluctuations, accidental qubit swaps due to Hamiltonian specifics or accidental resonances to TLSs or untracked modes, etc. Additionally, the existing comparison does not extend beyond Wp/δ0'' > 3. In short, the current experimental data are still ambiguous and not clean enough to support the claim of experimental realization of quasi-periodic hopping induced topological pumping.

Generally, the claim or agreement with the underlying model would be more convincing if the authors provide experimental evidence of the emergence of a quantized plateau over several increasing evolution periods. This is similar to the necessary demonstration of quantum phase transition at increasing system sizes. Due to realistic experimental constraints, I understand that this may not be achievable with the current capabilities of their platform.

If this suggestion is not realizable, I suggest weakening their claim about the experimental realization of topological pumping induced by quasi-periodic hopping disorder. An example might be to tone down the claim to “observation of signatures consistent with topological pumping induced by quasi-periodic hopping disorder under insufficient adiabaticity”.

Our response

We thank Reviewer #4 for the reasonable comments. However, we would like to demonstrate that other factors listed by Reviewer #4 could not influence our experimental results, so the consistency at weak adiabaticity of observed signatures is predictable in the clean limit. First, the experimental results can be well explained by numerical simulations. Second, due to the topological nature of pumping, the theoretically predicted results can be observed in a larger parameter region, as shown in Fig. 5c of the manuscript.

We numerically calculate the displacement of CoM δx versus the quasi-periodic hopping disorder strength W_p, when initial state is prepared as a single-excitation state or a Wannier state. The results are shown in Fig. R4-1. We find that the parameter region of peaks is still in accordance with the one indicated by the Chern number (the dotted curve in Fig. R4-1).

In addition, to preclude the influence of frequency fluctuations and accidental TLSs swaps, we routinely monitor the readout stability and the environment around both the qubits idle points and working points. The data of the relaxation time near qubits working points are shown in Fig. R4-2, and the readout fidelity stability is shown

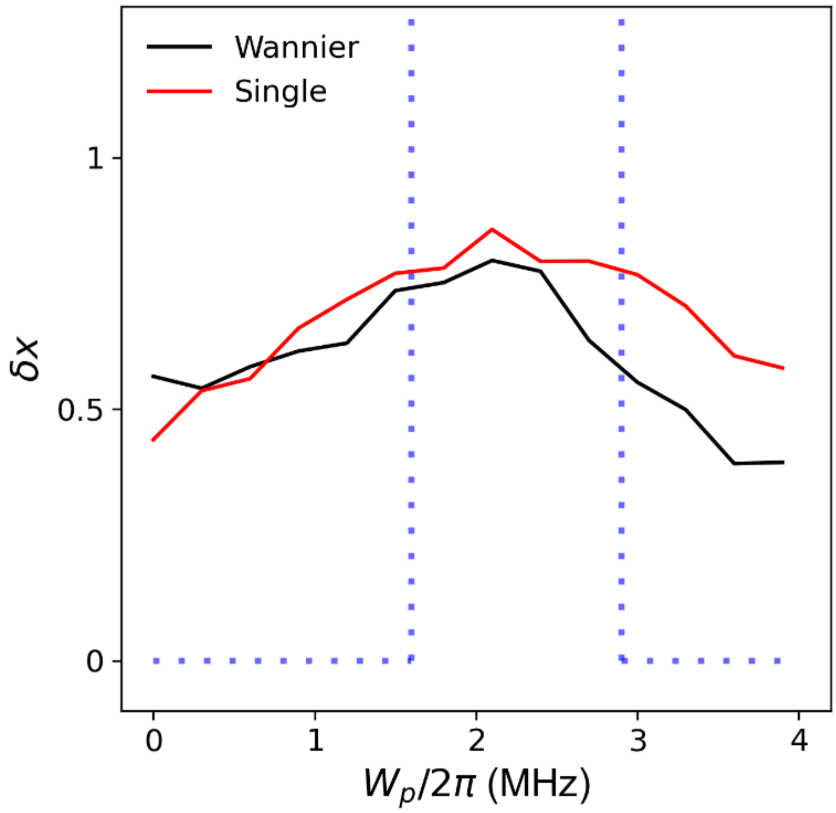

Figure R4-1: The displacement of CoM δx versus the quasi-periodic hopping disorder strength W_p, when the initial state is prepared as a single-excitation state or a Wannier state.

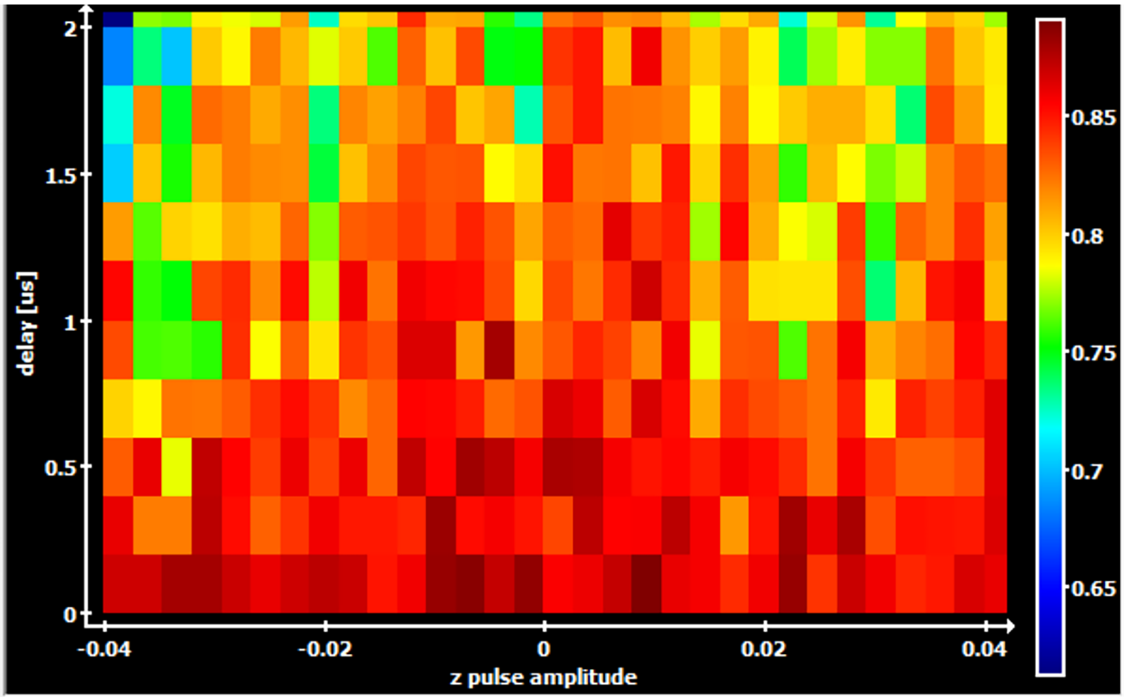

Figure R4-2: The data of the relaxation time near qubits working points.

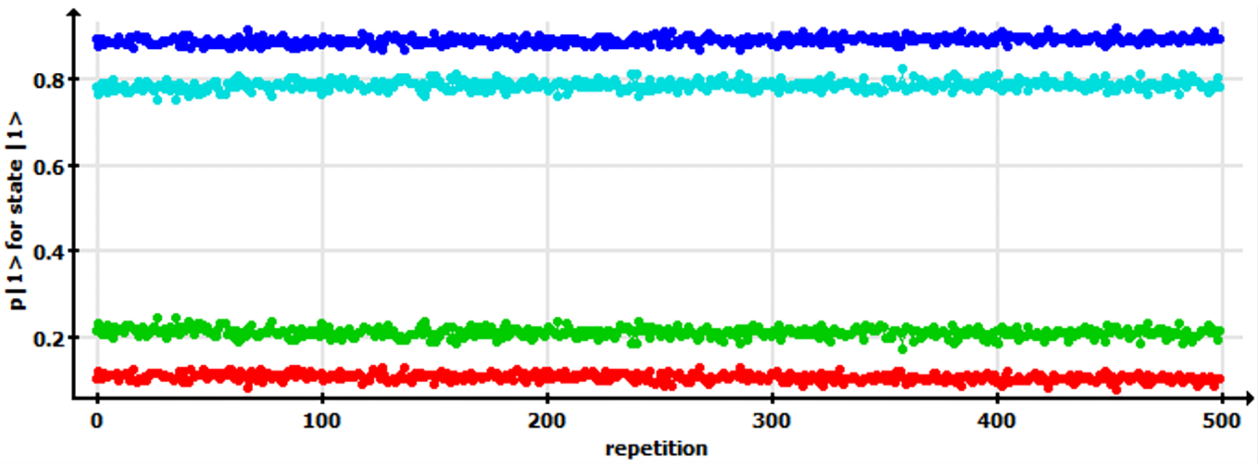

Figure R4-2: The data of the readout fidelity stability monitor.

in Fig. R4-3. The precision of the Hamiltonian control is verified with the consistence between experimental and numerical results.

In the revised manuscript, we have added in Conclusion that:

"Though under insufficient adiabaticity, we demonstrate the observation of signatures consistent with topological pumping induced by quasi-periodic hopping disorder, which leads to nonzero δx in the clean limit."

(4-2) Reviewer #4 commented

2. The revised manuscript is titled "Reciprocity in Disorder and Topology of

Thouless Pumping on a Superconducting Quantum Processor." However, the use of "Reciprocity" is unclear. It may misleadingly suggest a sort of symmetry observed during the experiment, which is not relevant to the manuscript's content. A more suitable title might be “Interplay Between Disorder and Topology in Thouless Pumping … ” which would better reflect the focus of the manuscript.

Our response

We thank Reviewer #4 for this suggestion and have revised the title as:

“*Interplay between disorder and topology in Thouless Pumping on a superconducting quantum processor* “.

(4-3) Reviewer #4 commented

In addition, here is a minor point:

The authors attribute the discrepancies observed in their experiments to a small overlap (~1%) in the initial state from the ideal Wannier state. This is an intriguing observation and seems to explain many of the unexpected discrepancies in their results. I suggest that the authors further discuss how close the assumed initial state is to the actual prepared state in their measurements. It would be clearer if the authors either utilize measured initial states to calculate deviation from the Wannier state, or incorporate some key coherent error sources into their model of the prepared state, such as imperfect rotation in X gate. In addition, while incoherent errors may not impact the dynamics significantly, it would be also helpful to understand the impacts of thermal population or ground state preparation fidelity.

Our response

We thank Reviewer #4 for the comment. As shown in the “Experimental setup” Section, the average single-gate fidelity is ~99.2%, by using the derivative reduction by adiabatic gate (DRAG) pulsing to avoid the unexpected leakage [Motzoi2009, Krantz2019].

To evaluate the effect of initial state errors, we consider the qubit’s third energy level and numerically simulate the process of initial state preparation by applying a Gaussian-like DRAG pulse with a form:

$$f(t) = \Omega(1 - i\eta \frac{t-t_c}{\alpha\sigma^2})e^{-\frac{(t-t_c)^2}{2\sigma^2}} e^{i\omega t},$$

where $\alpha/2\pi = -0.2$ GHz is the anharmonicity of qubit, Ω, σ, and t_c are the amplitude, standard deviation, and the center of the gaussian pulse, respectively. The duration of pulse is chosen as $4\sqrt{2\ln 2}\,\sigma = 60$ ns. With these parameters, we obtain the state with errors as $|\psi\rangle = -0.054|0\rangle + (0.998 - 0.03\mathrm{i})|1\rangle$, and the X gate fidelity is 99.8%. Next, we numerically calculate the adiabatic evolution of topological pumping with the initial state $|\psi\rangle$, for a 12-qubit chain. The displacements of CoMs as a function

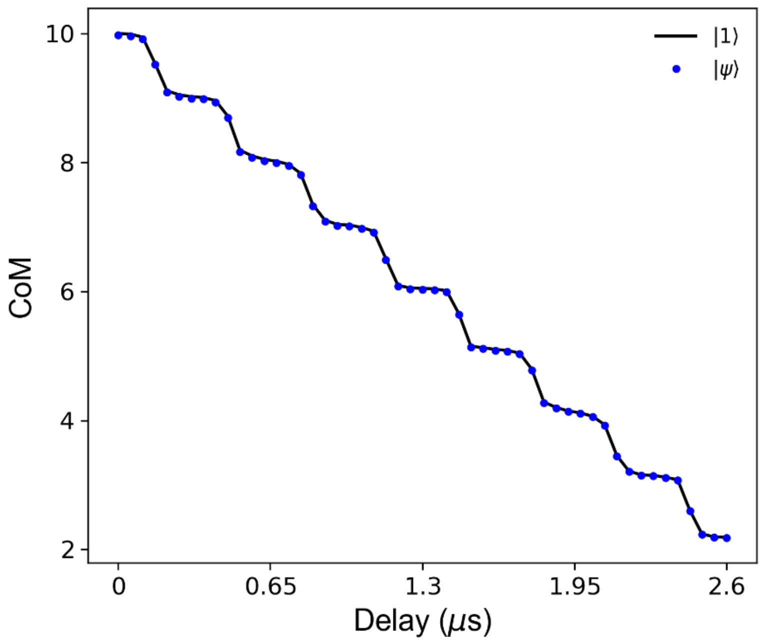

Figure 4-4: The CoMs versus time when the initial state is prepared as $|1\rangle$ and $|\psi\rangle$, respectively. The state $|\psi\rangle$ is obtained by simulating the preparation of X gate.

of time are shown in Fig. R4-4. We find that the result for the case with $|\psi\rangle$ agrees well with the case with the single-excitation state $|1\rangle$. Therefore, we conclude that the errors in preparing the initial state do not affect main results in our experiments.

Reference

[Motzoi2009] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm, Simple Pulses for Elimination of Leakage in Weakly Nonlinear Qubits, Phys. Rev. Lett. **103**, 110501 (2009)

[Krantz2019] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, A quantum engineer's guide to superconducting qubits, Appl. Phys. Rev. **6**, 021318 (2019)

(4-4) Reviewer #4 commented

In conclusion, the manuscript is well-written and organized, though there are a few areas that could be improved. It makes a significant contribution to experimentally demonstrating topological pumping with controllable disorder using superconducting qubits. If the highlighted issues are sufficiently addressed, I recommend this manuscript for publication in *Nature Communications.*

Our response

We thank Reviewer #4 for the constructive suggestions. We have carefully revised the manuscript according to all comments from Reviewers that help to improve the quality and impact of our manuscript. We believe that the current version of our manuscript with these revisions is now suitable for publication in *Nature Communications*.