Quantum Circuit Complexity Reveals Hidden Quantum Phases

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Quantum circuit complexity QCC

Hidden Orders in the XXZ Spin Chain Are Revealed by Innovative Quantum Machine Learning Techniques

With a notable demonstration involving the well-known bond-alternating XXZ spin chain, recent pioneering research has successfully used the power of **quantum circuit complexity** to uncover the elusive patterns of topological order within complicated quantum systems. A group led by Yanming Che from the University of Michigan, Clemens Gneiting and Franco Nori from RIKEN, and Xiaoguang Wang from Zhejiang Sci-Tech University developed this novel technique, which provides a potent and comprehensible way to comprehend and categorize quantum phases of matter.

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The Enduring Challenge of Topological Order

In contemporary physics, determining the topological phases of matter is a difficult task, especially when working with quantum systems that interact intensely. Topological phases embed their defining features in global, non-local aspects, including distributed quantum entanglement, in contrast to traditional phases that may be characterized by local order parameters. The intricacies of these systems frequently cause traditional approaches to go down.

Although supervised machine learning has demonstrated potential in the classification of topological phases, it requires pre-labeled data and previous knowledge, both of which are often unavailable in practical settings. On the other hand, unsupervised machine learning provides a way to find information without these labels, but there is still a lack of a solid, understandable, and broadly applicable theory for topological quantum order.

With symmetry-protected band topologies (which are characterized by short-range entanglement), prior unsupervised kernel-based techniques that rely on path-finding algorithms or spectral gap closing have shown some success. However, they face significant challenges when used in more intricate, highly interacting systems where entanglement is a prominent feature. These methods may be inconclusive due to the undecidability of the spectral gap for generic quantum many-body Hamiltonians.

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Quantum Circuit Complexity: A New Metric for Quantum Similarity

The study presents a novel idea **quantum circuit complexity** (QCC) to overcome these obstacles. The smallest cost of changing one quantum state into another is measured by QCC. The researchers suggest that the smallest quantum circuit cost needed to create a target quantum states from a reference state provides a theoretically ideal solution for unsupervised learning of topological order. This is based on Kolmogorov complexity, a measure of the informational distance between data strings. The transformation between two topologically equivalent quantum states is regarded as "algorithmically trivial or cheap" due to the fact that they share entanglement patterns and underlying structures.

Although it is frequently impossible to calculate accurate QCC, the team has developed two crucial theorems that connect this abstract idea to real-world uses. These theorems relate QCC to easily quantifiable phenomena, such as entanglement generation and fidelity changes. Because of this relationship, useful similarity metrics called kernels may be created and easily incorporated into machine learning systems.

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1. **Fidelity-Based Kernel:** This kernel makes use of the Uhlmann-Jozsa fidelity between subsystems of constant size and reduced density matrices. Theorem 1 shows that QCC has a direct relationship with total fidelity variation by providing upper bounds on the Bures distance and the Quantum Fisher Complexity (QFC). For geometrically local quantum circuits, the sum of the Bures distances of reduced density matrices on tiny, constant-sized subsystems approximates the lower bound of QCC. This implies that the degree to which the local attributes of quantum states (represented by reduced density matrices) are comparable to one another can be used to evaluate how similar they are. Interestingly, this fidelity-based kernel can be viewed as a specific example of the shadow kernel, a method proposed in prior

studies.

2. **Entanglement-Based Kernel:** This kernel concentrates on entanglement profiles and is motivated by Theorem 2, which asserts that the sum of absolute changes in entanglement entropy across different cuts of the system lower bounded QCC for a geometrically local quantum circuit. This method is especially useful when entanglement profiles can be effectively approximated, as in the case of experimental measurements using traditional shadow tomography or tensor-network simulations such as DMRG. More precise information on multi-scale quantum entanglement is captured by the more rigorous entanglement-based kernel.

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The Bond-Alternating XXZ Spin Chain: A Benchmark for Validation

Numerical investigations on various quantum models, most notably the bond-alternating XXZ spin-1/2 chain, provided a strong demonstration of the effectiveness of these novel kernels.

The researchers used both the fidelity-based and entanglement-based kernels, using DMRG (Density Matrix Renormalization Group) computations to identify the ground states and extract features such as fidelities and entanglements.

The outcomes were striking: three separate clusters were successfully created from the unsupervised learning when it was submitted to the diffusion map algorithm for nonlinear embedding into a two-dimensional space. The three recognized quantum phases of the model trivial, symmetry-broken, and topological are precisely matched by these clusters. For the XXZ spin chain, this clustering is exactly in agreement with independent computations of topological invariants. With the fidelity-based kernel concentrating on randomly sampled two-body reduced density matrices, the study specifically sampled qubits.

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Impact and Future Trajectories

The XXZ spin chain's effective implementation demonstrates how these new kernel techniques outperform current ones in terms of performance, interpretability, and clarity. Although the ultimate objective is to reveal topological order with non-local quantum entanglement, the XXZ results validate the efficacy of the approaches even for symmetry-protected topological orders with short-range entanglement and symmetry-broken phases. Specifically, the entanglement-based kernel showed improved resilience to noise and offered a more profound understanding of the basic connection between topological phases and long-range entanglement.

Importantly, new theoretical developments in effectively measuring many-body entropies imply that it may be possible to compute these entanglement-based kernels from experimental data. This creates intriguing opportunities to directly apply these techniques to states created on near-term quantum computing hardware.

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In the future, the researchers hope to extend these kernels to low sample complexity supervised learning tasks and investigate their use in more complicated situations such gapless systems, entanglement transitions, and mixed-state topological order. This strategy has great promise for creating even more potent instruments to examine and work with intricate quantum systems when combined with other quantum machine learning approaches, such as parametrized quantum circuits and reinforcement learning for quantum state generation.

This study fosters a deeper interaction between machine learning, quantum complexity, quantum parameter estimation, and quantum computation, and represents a major step towards a more interpretable and generalizable theory of unsupervised machine learning for topological order.

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