Modelling waveguide-QED systems with time-delayed coherent feedback

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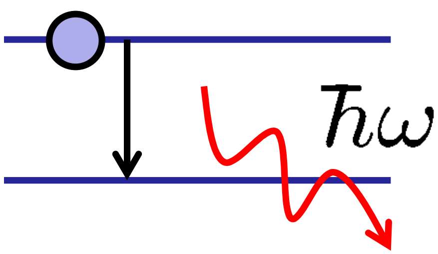
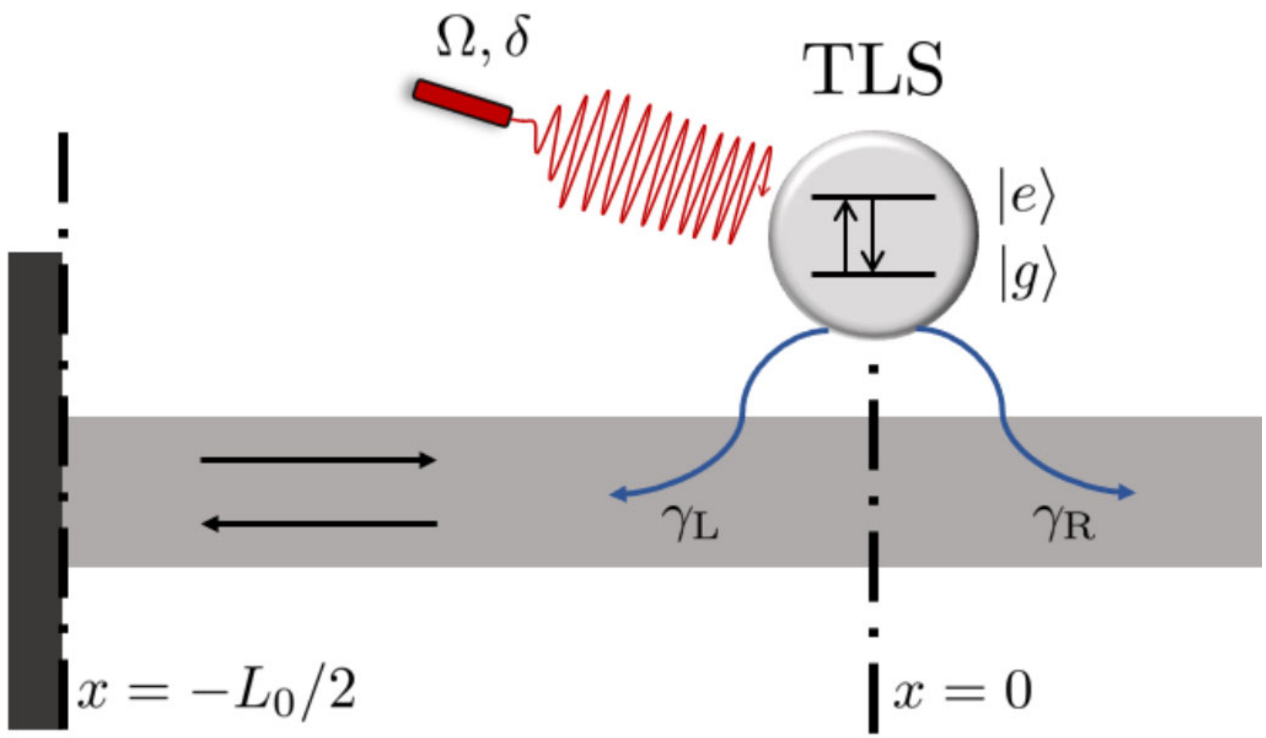




Riken Seminar, Nov 10, 2022

Big thanks – two excellent PhD Students

Quantum Trajectories (QTs)

Gavin Crowder

Sofia Regidor

Matrix Product States (MPS)

Funding: Natural Sciences and Engineering Research Council of Canada (NSERC), Canadian Foundation for Innovation (CFI), and National Research Council (NRC)

Franco for kind invitation, JSPS for funding, and Howard Carmichael and Lora Ramunno for collaborations

Waveguide-QED and feedback - motifs

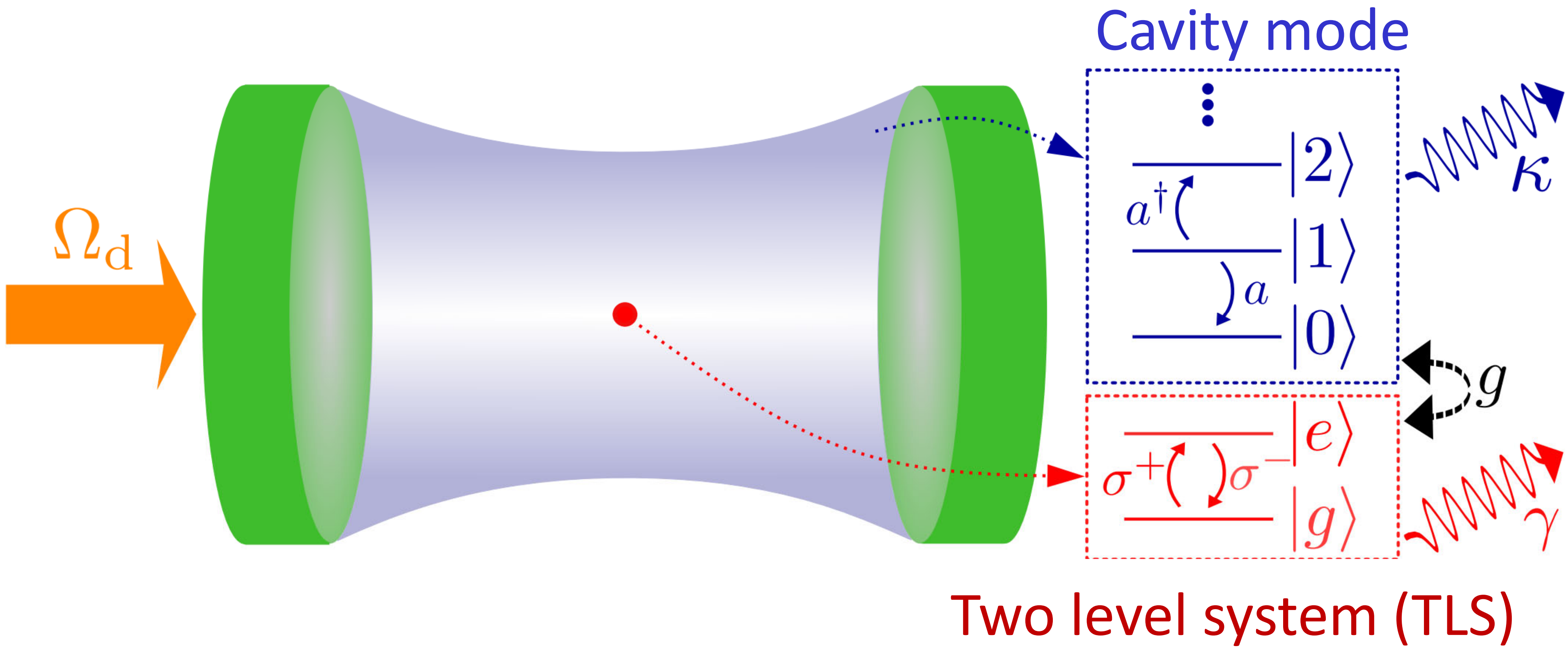
Waveguide systems lead to much richer dynamics than simple cavity systems

Photons can be correlated (and bound) on chip

Rich many-body physics problems can be simulated with chains of quantum emitters and waveguides

Test-bed for exploring effects of coherent feedback and quantum noise on *nonlinear quantum dynamics*

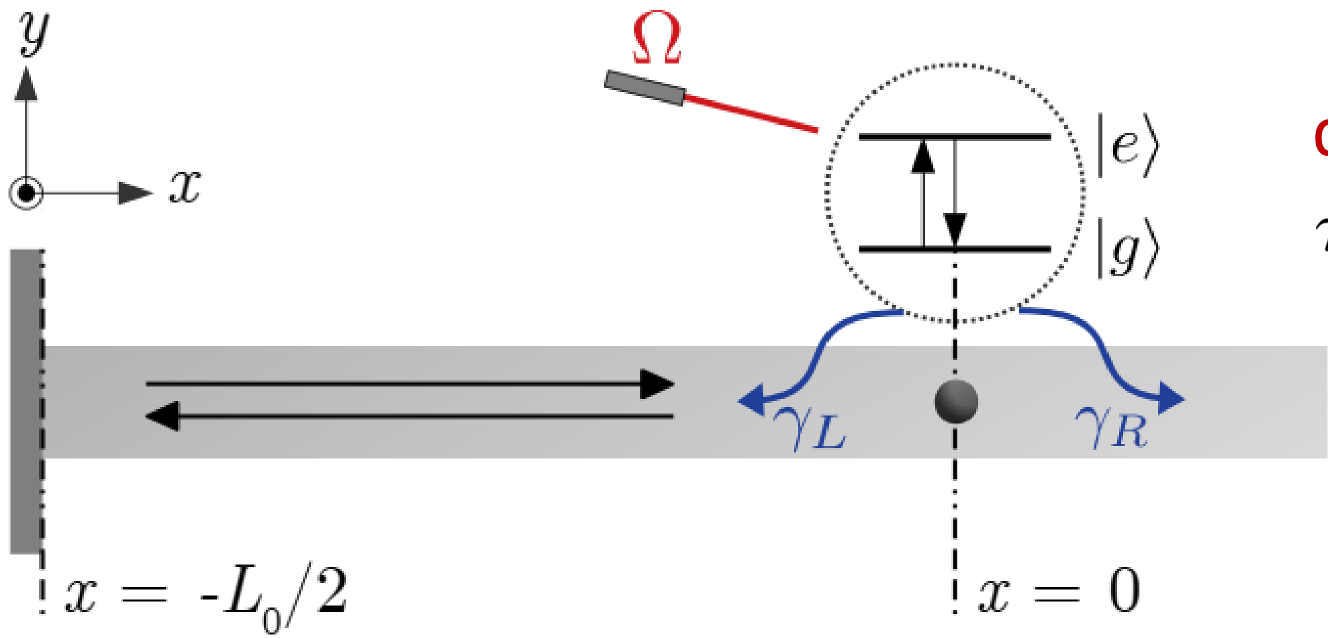
Simple ("baby") cavity-QED



$$H_{\rm JC} = \omega_c a^\dagger a + \omega_0 \sigma^+ \sigma^- + g(a^\dagger \sigma^- + a\sigma^+)$$

Complimented with Markovian decay channels (Lindbladians)

Waveguide-QED with *time-delayed* feedback



delay time

$\tau = L_0/c_\omega$

$$H_{\mathsf{W-QED}} = \sum_{\alpha=L,R} \int_{-\infty}^{\infty} d\omega \omega b_\alpha^\dagger(\omega) b_\alpha(\omega) + \omega_0 \sigma^+ \sigma^-$$
$$+ \int_{-\infty}^{\infty} d\omega \left[\left(\sqrt{\frac{\gamma_{\mathsf{L}}}{2\pi}} \sigma^+ b_{\mathsf{L}}(\omega) + \sqrt{\frac{\gamma_{\mathsf{R}}}{2\pi}} e^{i(\phi + \omega\tau)} \sigma^+ b_{\mathsf{R}}(\omega) \right) + \mathsf{H.c.} \right]$$

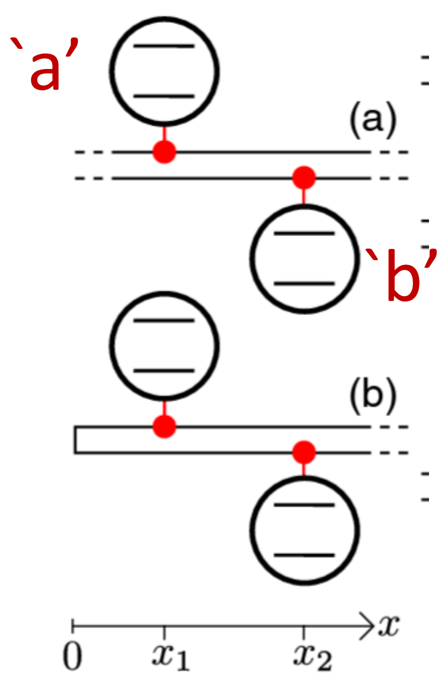
Coupling rates now have an inherent delay time, resulting in a round trip memory (non-Markovian decay)

Lots of great waveguide-QED from Riken: example using "giant atoms"

PHYSICAL REVIEW LETTERS **120,** 140404 (2018)

Decoherence-Free Interaction between Giant Atoms in Waveguide Quantum Electrodynamics

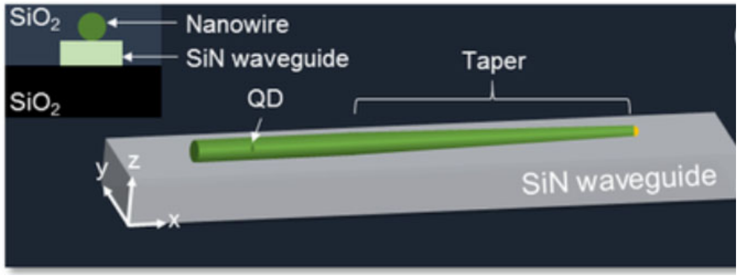
Anton Frisk Kockum,[1,*] Göran Johansson,[2] and Franco Nori[1,3]
[1]*Theoretical Quantum Physics Laboratory, RIKEN, Saitama 351-0198, Japan*
ent of Microtechnology and Nanoscience (MC2), Chalmers University of Technology, SE-412 96 Gothenburg, Sweden
[3]*Physics Department, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA*



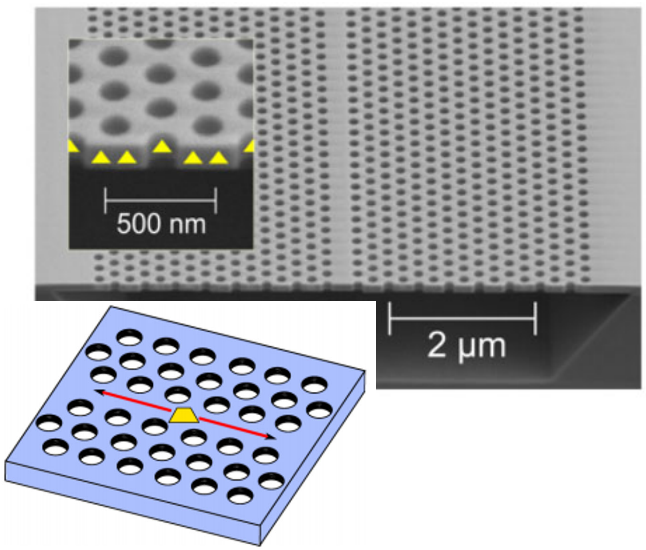
$$\dot{\rho} = -i\left[\omega'_a \frac{\sigma_z^a}{2} + \omega'_b \frac{\sigma_z^b}{2} + g(\sigma_-^a \sigma_+^b + \sigma_+^a \sigma_-^b), \rho\right] + \Gamma_a \mathcal{D}[\sigma_-^a]\rho + \Gamma_b \mathcal{D}[\sigma_-^b]\rho + \Gamma_{\text{coll}}\left[\left(\sigma_-^a \rho \sigma_+^b - \frac{1}{2}\{\sigma_+^a \sigma_-^b, \rho\}\right) + \text{H.c.}\right], \quad (1)$$

Engineered phase interference effects, but interactions are instantaneous (Markovian)

Example experimental systems

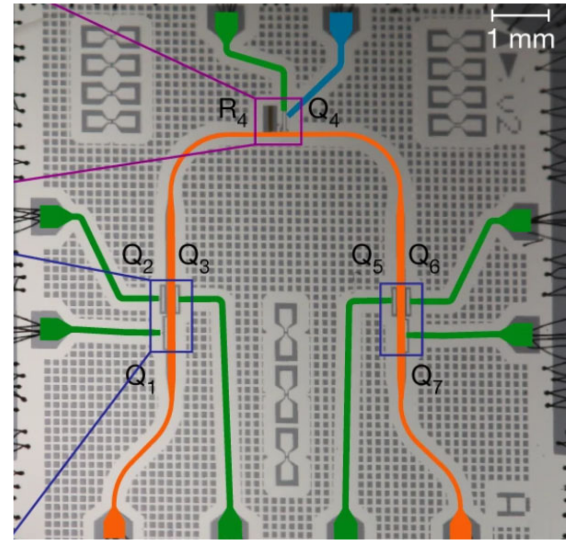


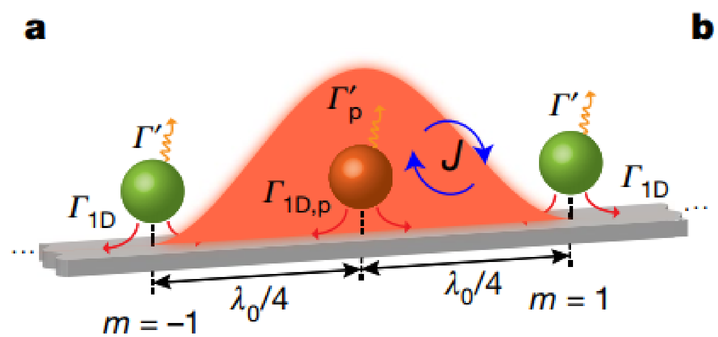
(Dalacu et al, Advanced Q. Tech, 2019)



(Sapienzo et al, Nature 2010)

Nanophotonic systems (integrated quantum dots (QDs) with waveguides)





Circuit QED (Mirhosseini et al, Nature, 2019)

Models of time-delayed quantum feedback

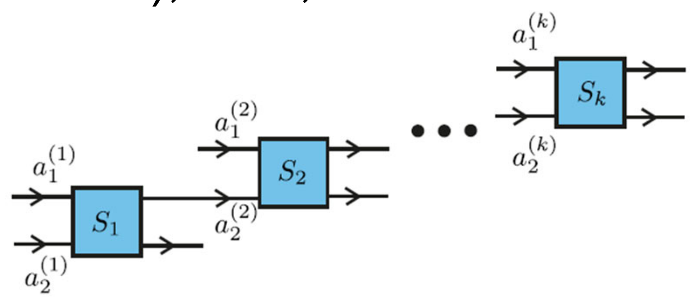
Analytic approaches in linear response and some nonlinear

Easy, problem limited

Hughes, PRL, 2007; Carmele et al, PRL, 2013; Crowder, Carmichael, Hughes, PRA (2020); Sinha et al (Solano), PRL, 2020

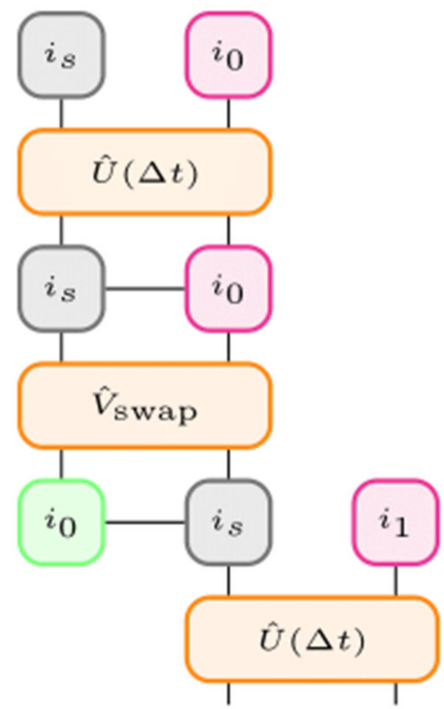
Fictitious cascading systems

Grimsmo, Phys. Rev. Lett., 2015.



Complex, not so practical, numerically limited

Matrix product states (Zoller), TEMPO, …



Powerful, complex, difficult to implement, can be overkill

e.g., Regidor, Crowder, Carmichael and Hughes, Phys. Rev. Res., 2021.

Quick summary of MPS

Quantum state for a 1D spin-chain with *N* spins

$$|\psi\rangle = \sum_{i_1,\ldots,i_N}^{d} c_{i_1,\ldots,i_N} \, |i_1, \ldots, i_N\rangle$$

Singular Value Decomposition (SVD) : Any matrix can be factorized, decomposing it into 3 new matrices

$$M = USV^{\dagger}$$

S contains the Schmidt coefficients in descendent order

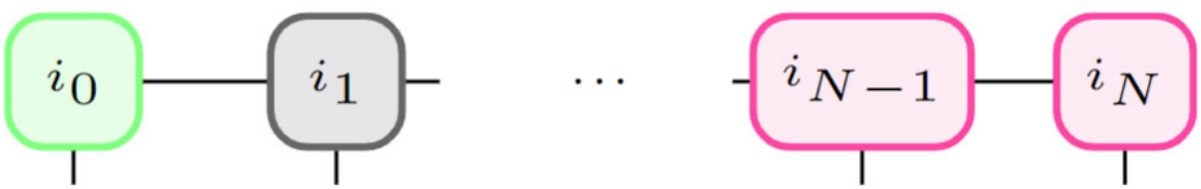
Schollwöck, U., 2011, Annals of Physics

Quick summary of MPS

After several SVDs, general expression for MPS follows:

$$\boxed{|\psi\rangle = \sum_{i_1 \ldots i_N} A^{i_1}_{a_1} A^{i_2}_{a_1,a_2} \ldots A^{i_{N-1}}_{a_{N-2},a_{N-1}} A^{i_N}_{a_{N-1}} |i_1 \ldots i_N\rangle}$$

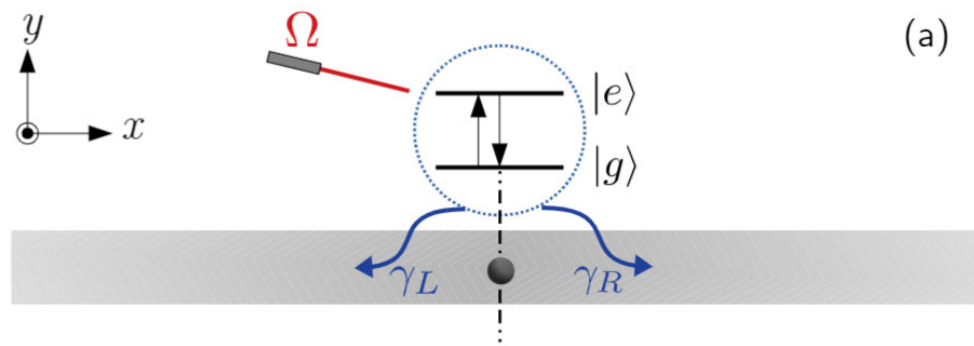
Reduced Hilbert space by keeping relevant parts. Note is there is no unique way of performing a SVD



McCulloch, I. P., 2007, J. of Stat. Mech: Theory and Experiment

Single TLS (qubit) in a infinite waveguide

The creation and annihilation operators for photons in time,



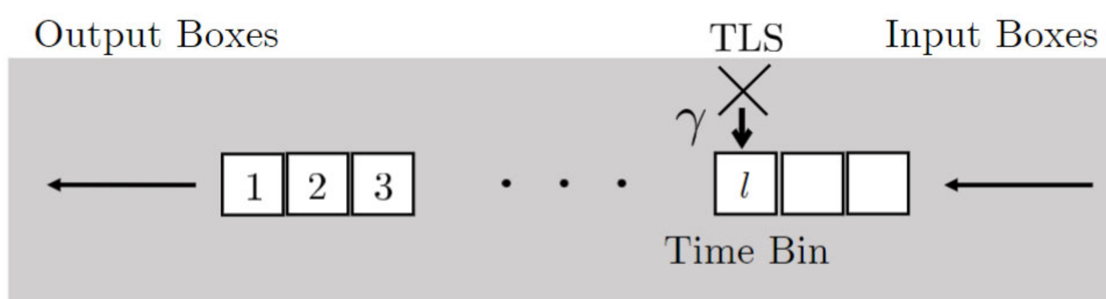
$$b_\alpha(t) = \frac{1}{\sqrt{2\pi}} \int d\omega b_\alpha(\omega) e^{-i(\omega-\omega_0)t}$$

Quantum noise operators (for the “time bins”),

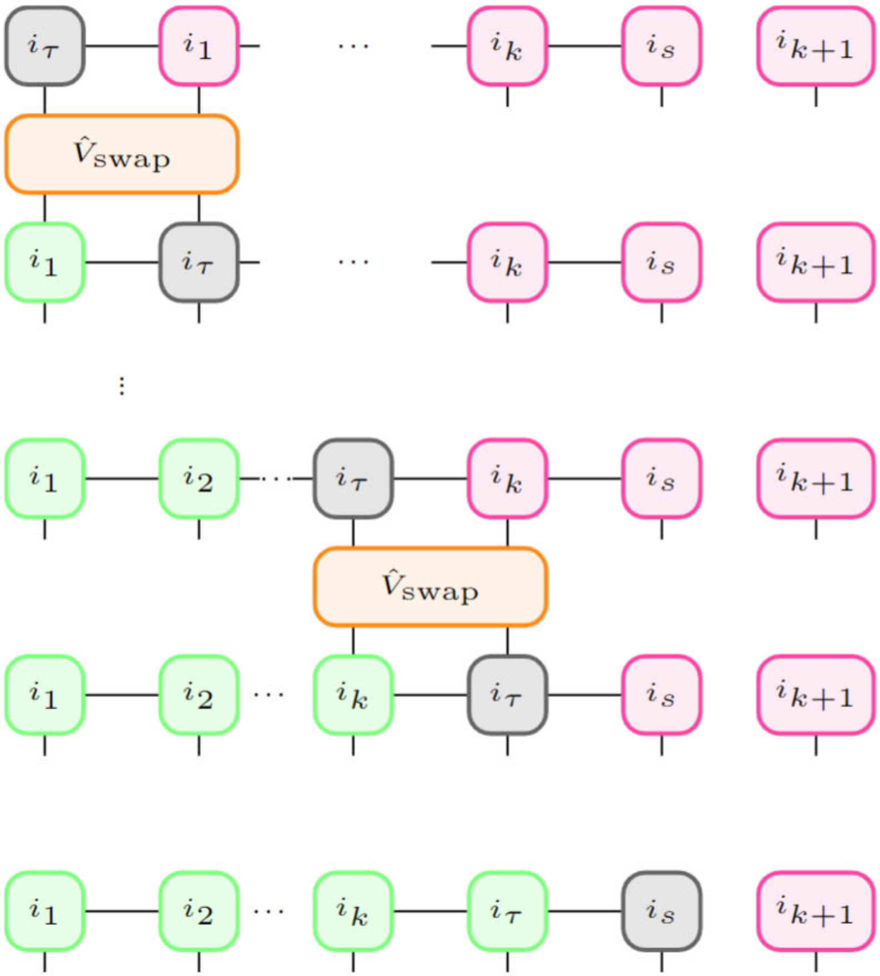
$$\Delta B_\alpha^{(\dagger)}(t_k) = \int_{t_k}^{t_{k+1}} dt' b_\alpha^{(\dagger)}(t') \qquad \left[\Delta B_\alpha(t_k), \Delta B_{\alpha'}^\dagger(t_{k'})\right] = \Delta t \delta_{k,k'} \delta_{\alpha,\alpha'}$$

A time-discrete number basis is created

$$|i_k^\alpha\rangle = \frac{(\Delta B_\alpha^\dagger(t_k))^{i_k^\alpha}}{\sqrt{i_k^\alpha!(\Delta t)^{i_k^\alpha}}} |vac\rangle$$

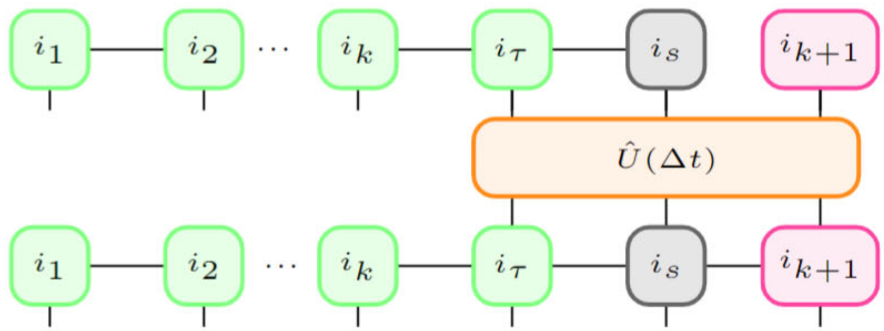


Numerical implementation is complex



See Regidor, Crowder, Carmichael, Hughes, PRR, 2021

Non-Markov



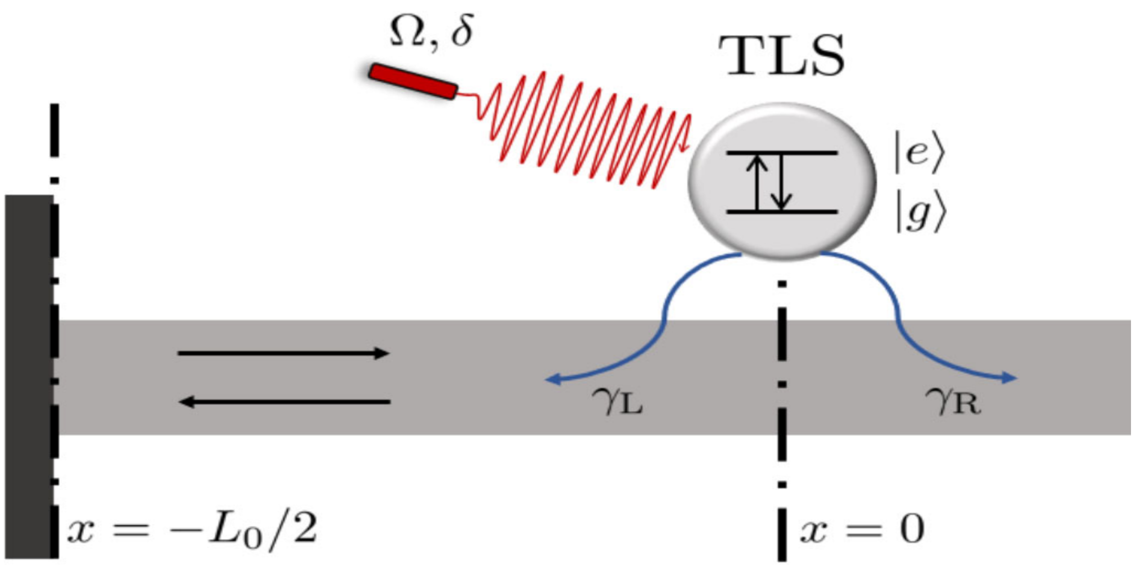
Becomes a series of tensor manipulations, SVDs, and tensor contractions, best visualized diagrammatically for each problem

Outline

Quantum trajectory (QT) theory

QT theory applied to waveguide QED with feedback

Selected examples for few photon dynamics



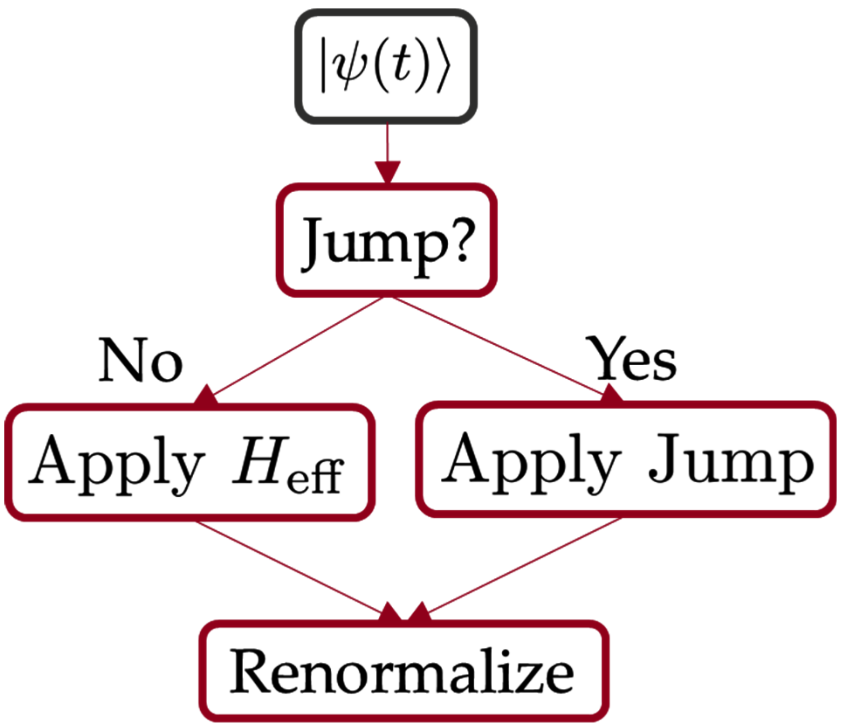
Quantum Trajectory Theory

Initial condition and evolve individual trajectories

Time steps forward of length δt

Stochastically introduce quantum jumps to system, e.g.

- Out of system decay
- Pure dephasing (of qubit)



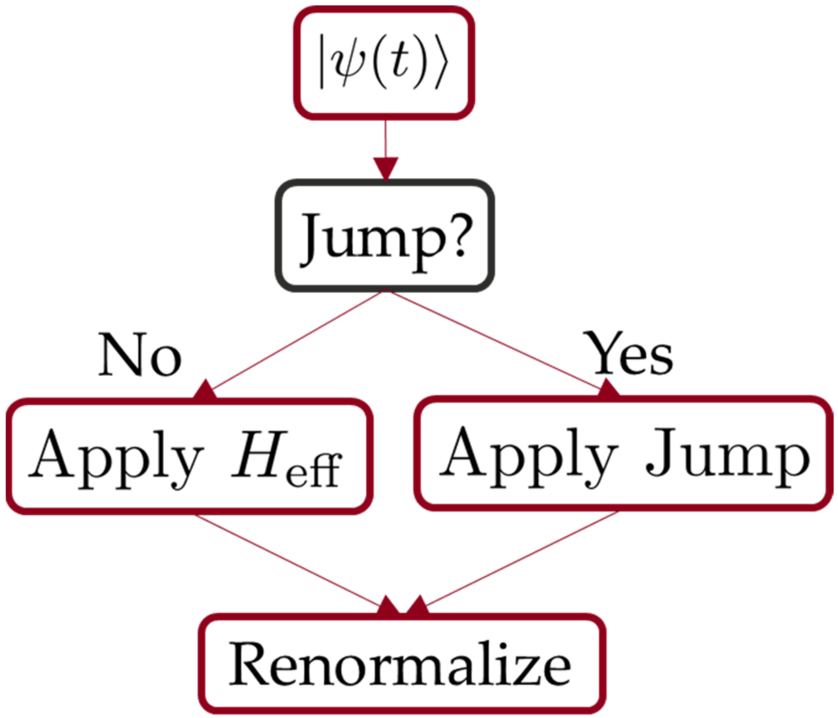
Dalibard, Castin, and Mølmer, Phys. Rev. Lett., 1992.; Tian and Carmichael, Phys. Rev. A, 1992.; Dum, Zoller, and Ritsch, Phys. Rev. A, 1992.

Quantum Trajectory Theory

Calculating the probability of a jump:

$$\delta p = \sum_m \delta p_m$$

$$\delta p_m = \delta t \langle \psi(t) | C_m^\dagger C_m | \psi(t) \rangle$$



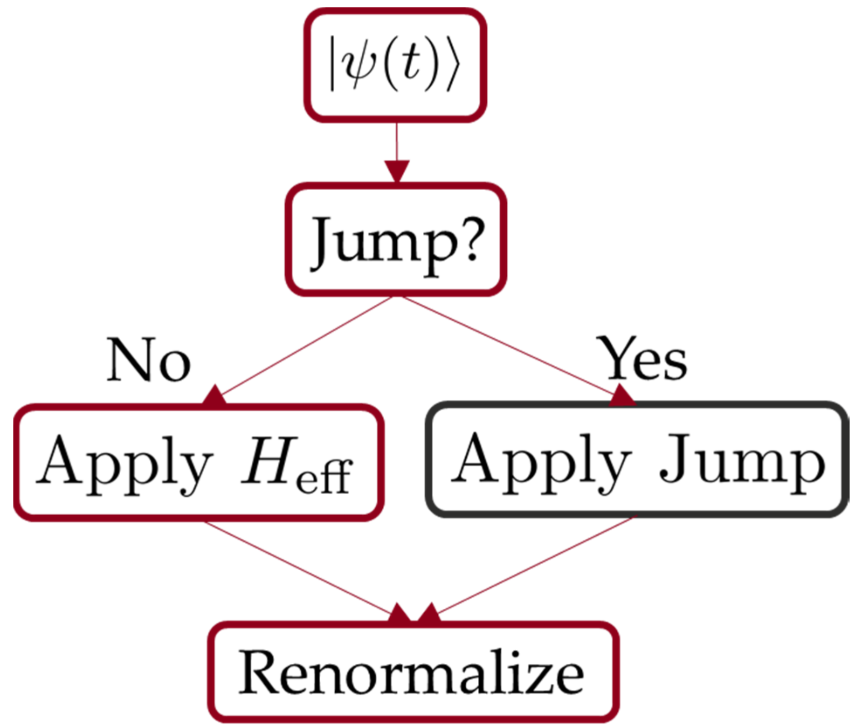
Quantum Trajectory Theory

A quantum jump occurs

Choose the responsible jump operator by the relative probability

If operator C_m occurs, evolve as:

$$|\psi'(t+\delta t)\rangle = C_m|\psi(t)\rangle$$



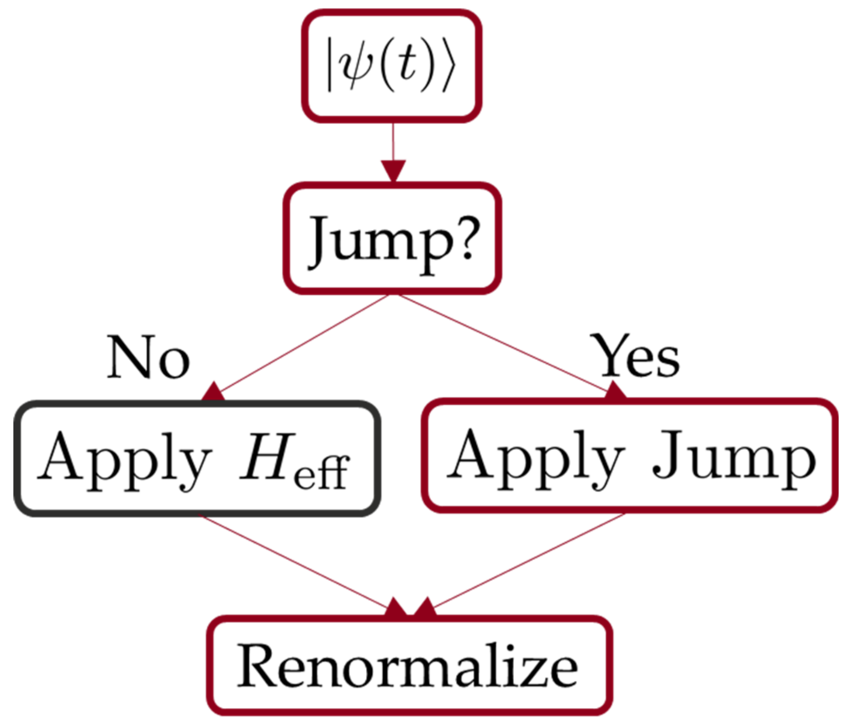
Quantum Trajectory Theory

A quantum jump does not occur

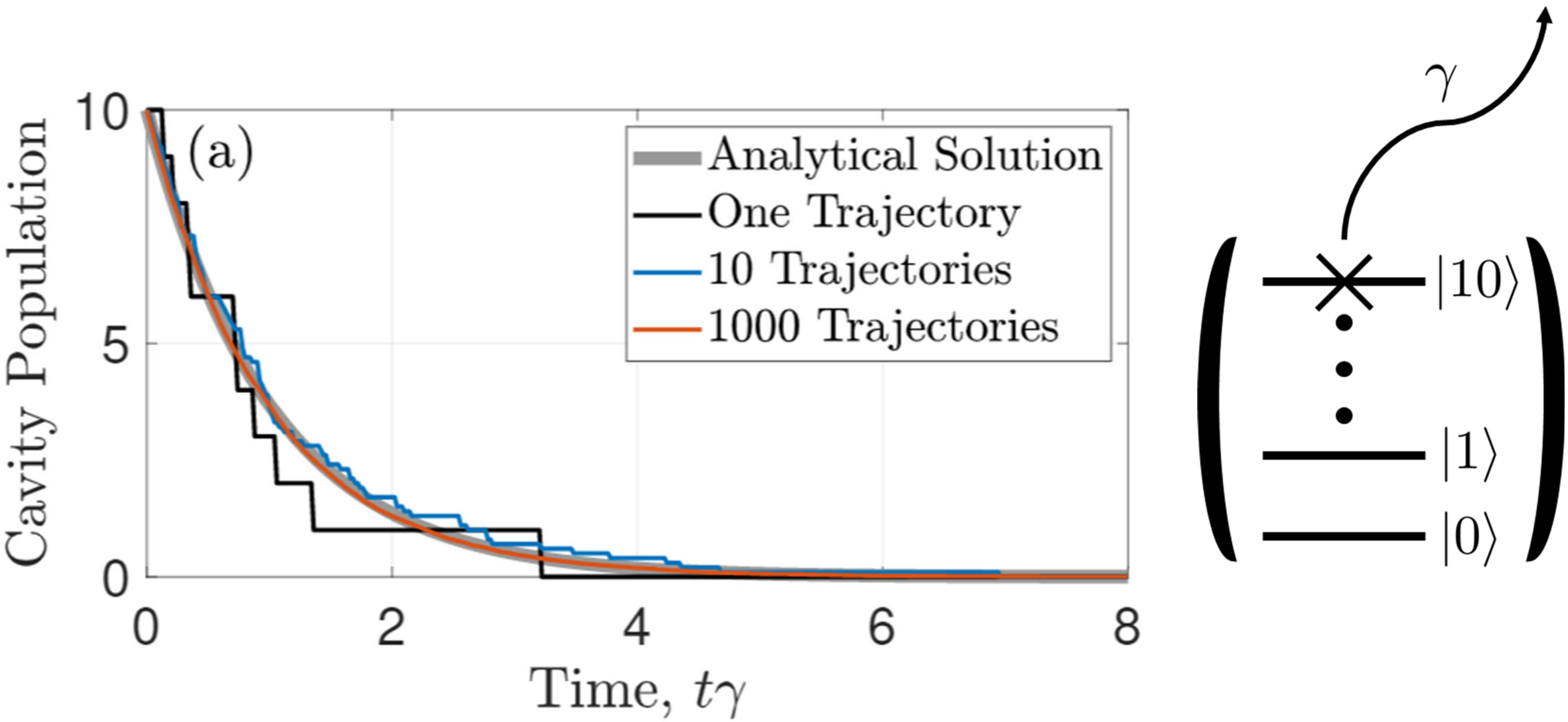
Evolve as:

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}H_{\text{eff}}|\psi(t)\rangle$$

$$H_{\text{eff}} = H - \frac{i\hbar}{2}\sum_m C_m^\dagger C_m$$



Example cavity decay (with 10 photons)



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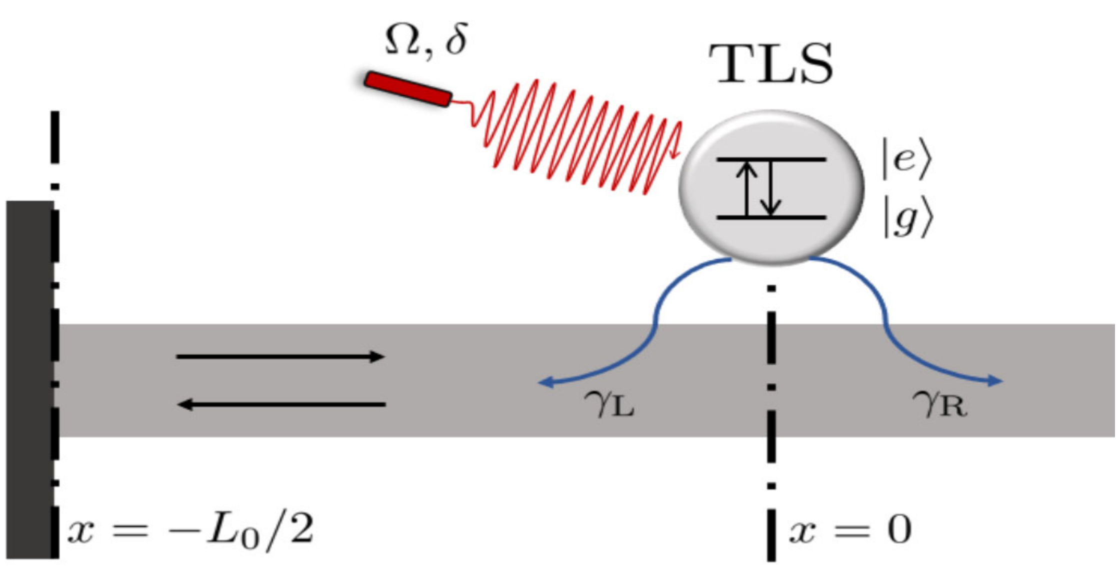
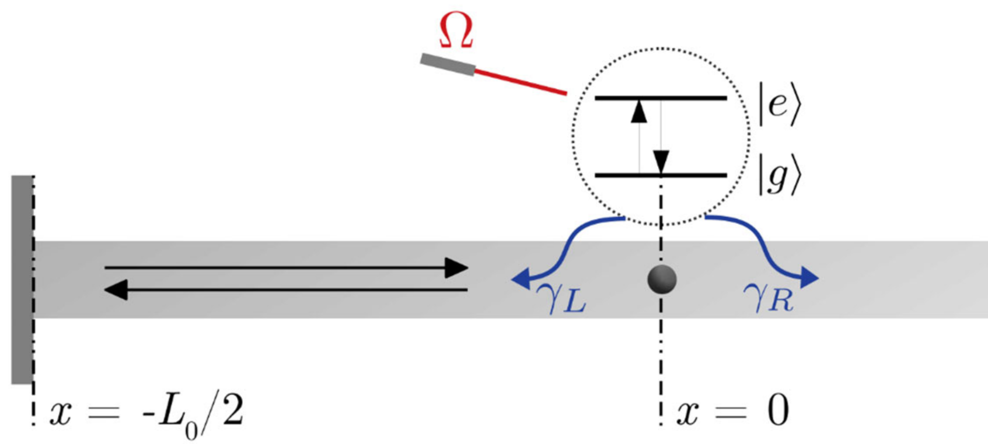


Quantum-trajectory discretized waveguide



Full Hamiltonian

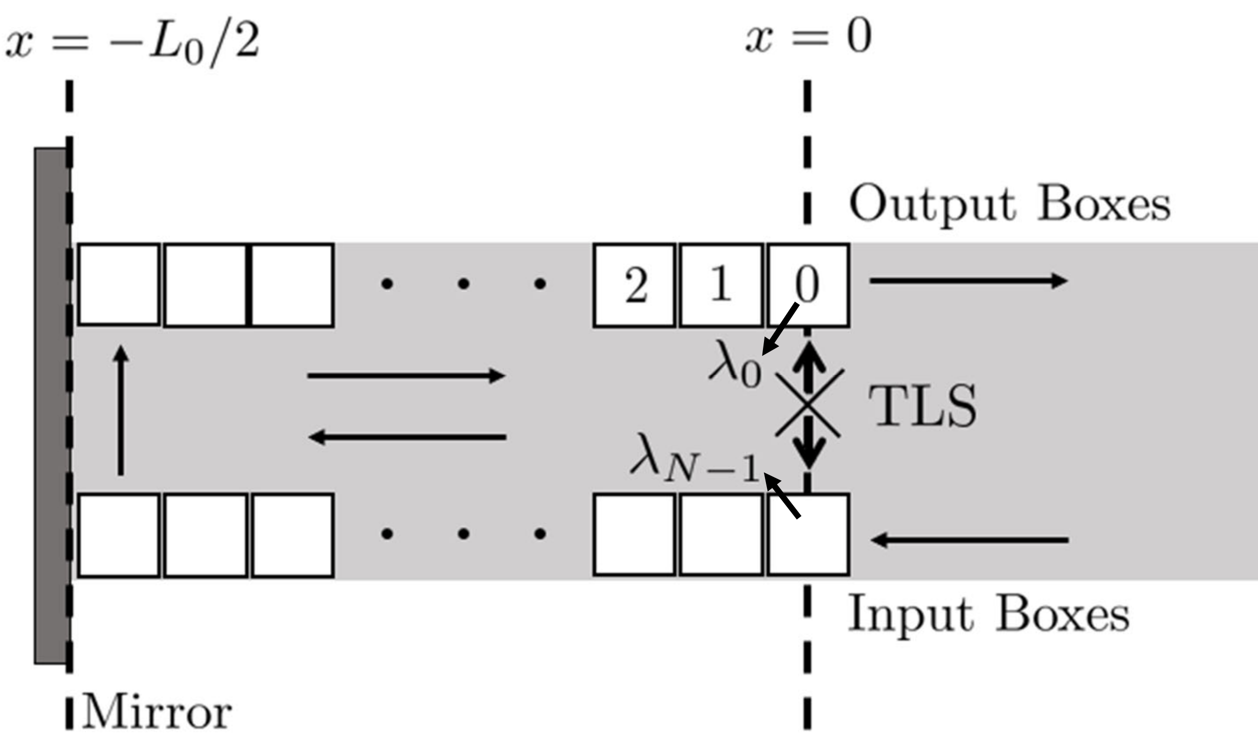
$$H = H_S + H_W + H_{\text{int}}$$

Combines QT theory with a collisional model for the waveguide (spatially discretized)

Continuously monitors the output down the waveguide (end) for photons to reduce Hilbert space

Quantum-trajectory discretized waveguide

Regidor, Crowder et al, PRR, 2021
Whalen, PRA, 2019



Input boxes enter empty

Output boxes are "measured" for a photon

The system is then projected into the measured state

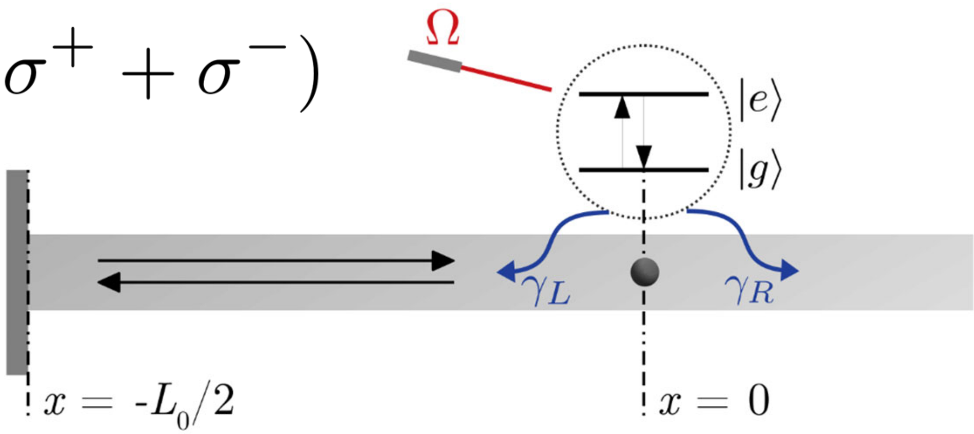
Discrete operators:

$$H_W = \sum_{k=0}^{N-1} \omega_k b_k^\dagger b_k$$

$$B_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_k e^{i\omega_k n \Delta t}$$

Quantum-trajectory discretized waveguide

$$H_S = \delta\sigma^+\sigma^- + \Omega(\sigma^+ + \sigma^-)$$



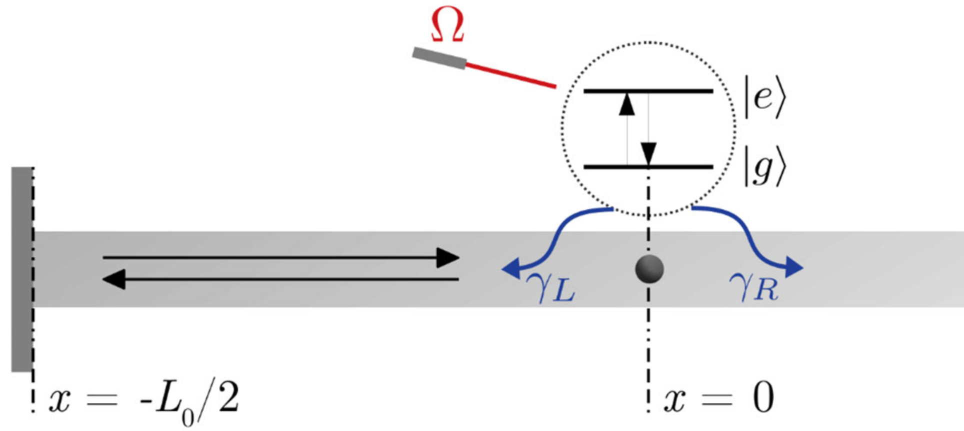
$$H = H_S + \boxed{H_W} + H_{\text{int}}$$

Evolution of Spatial Operators (in waveguide):

$$U_W^\dagger(\Delta t) B_n U_W(\Delta t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_k e^{i\omega_k (n-1)\Delta t} = B_{n-1}$$

Quantum-trajectory discretized waveguide

$$H = H_S + H_W + \boxed{H_{\text{int}}}$$



Interaction Hamiltonian:

Coupling to Box 0

$$\lambda_0 = \sqrt{\gamma_R}\sqrt{N/\tau}$$

$$H_{\text{int}} = \boxed{\left(\lambda_0 B_0^\dagger \sigma^- + H.a.\right)} + \boxed{\left(\lambda_{N-1} B_{N-1}^\dagger \sigma^- + H.a.\right)}$$

$$\lambda_{N-1} = e^{i\phi}\sqrt{\gamma_L}\sqrt{N/\tau}$$

Coupling to Box $N-1$

Observables

Population of TLS

$$n_a = \langle\psi|\sigma^+\sigma^-|\psi\rangle$$

Population of single waveguide-bin photon (small)

$$N_{B_j} = \langle\psi|N_j^\dagger N_j|\psi\rangle$$

Photon-bin photon flux rate

$$n_{B_j} = \langle\psi|N_j^\dagger N_j|\psi\rangle\,/\Delta t$$

Spectra and Correlation Functions

Spectra (e.g., with CW laser driving)

$$S_{\rm incoh}^{\rm out}(\omega) = \int_0^\infty d\tau' e^{i(\omega-\omega_{\rm L})\tau'} \left[\langle B_0^\dagger(\tau') B_0(0)\rangle_{\rm ss} - \langle B_0^\dagger(0)\rangle_{\rm ss} \langle B_0(0)\rangle_{\rm ss}\right]$$

Two-time correlation function (normalized)

$$[g_{\rm out}^{(2)}(\tau')]_{\rm ss} = \frac{\langle B_0^\dagger(0) B_0^\dagger(\tau') B_0(\tau') B_0(0)\rangle_{\rm ss}}{\langle B_0^\dagger B_0\rangle_{\rm ss}^2}$$

Also more direct ways of simulating measurands with QTs

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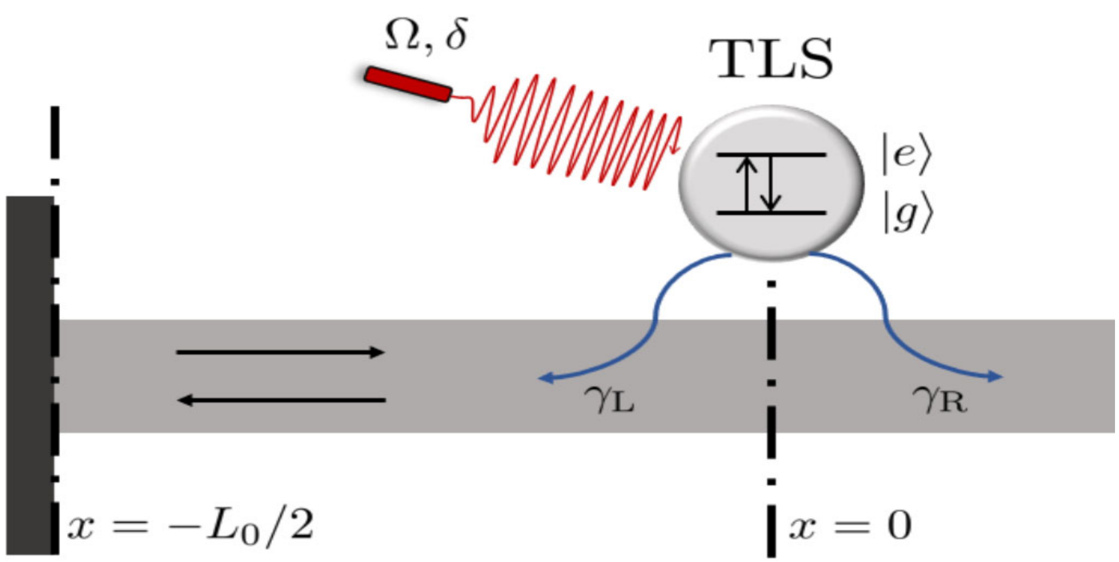
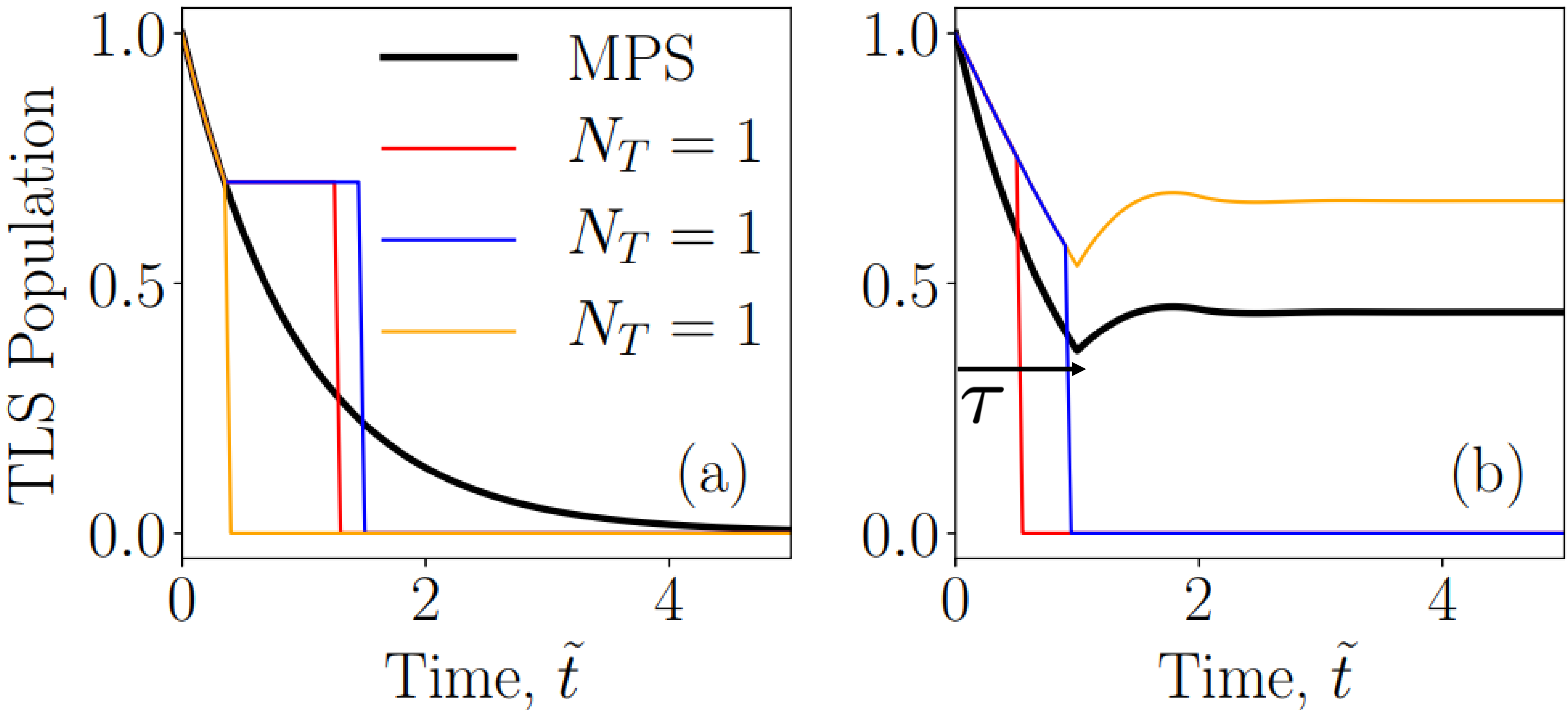


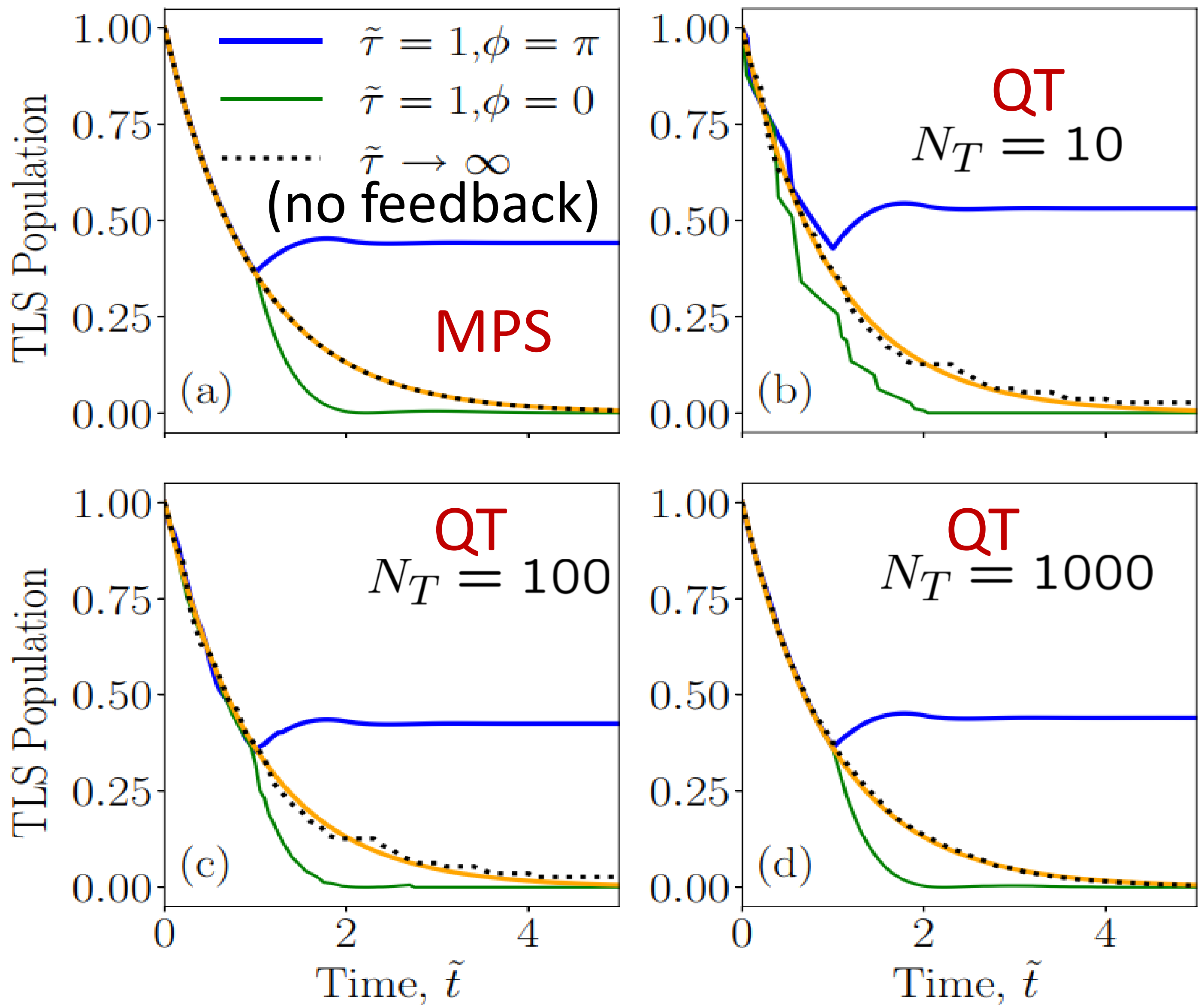
Vacuum dynamics – single trajectories

No feedback With feedback



Delay time and mirror phase: $\tau = \gamma^{-1}, \phi = \pi$

Vacuum dynamics: QTs vs MPS



Average over 10 trajectories

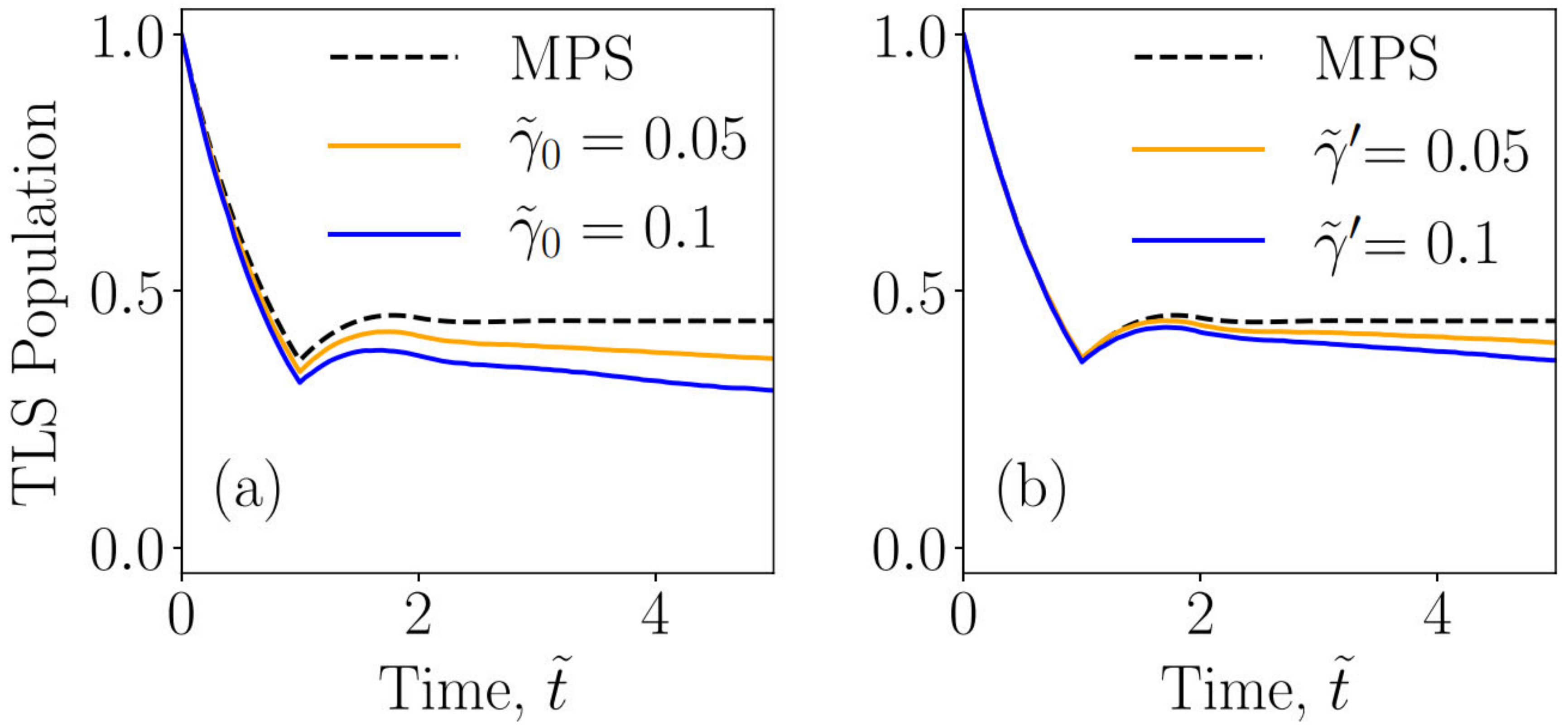
Population trapping

Regidor et al, Phys Rev Res, 2021 (recovers Carmele et al, PRL, 2013)

Role of off-chip decay and pure dephasing (these are not so easy to add to MPS!)

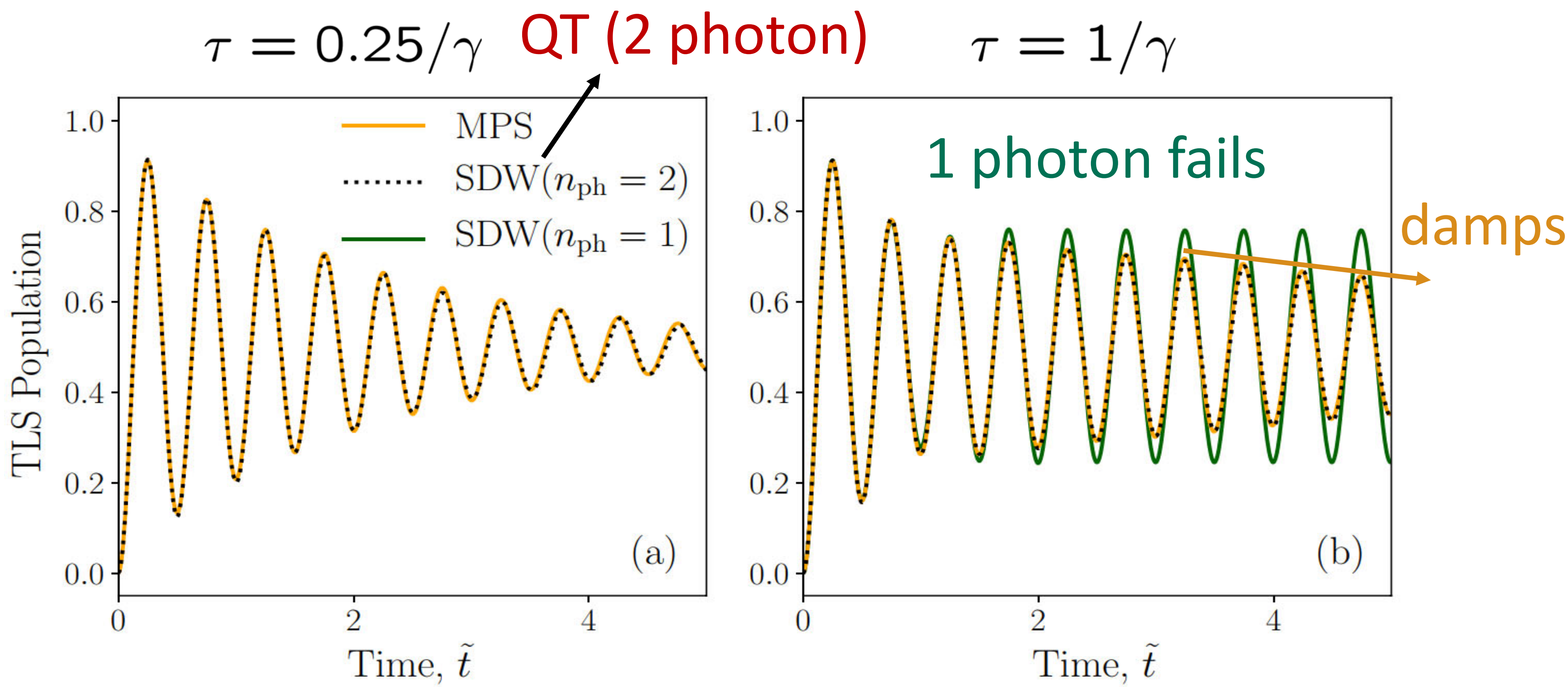
With off-chip decay

With pure dephasing



⇨ A perfect population trapped state is no longer possible

Nonlinear dynamics with and without feedback: benchmark with MPS



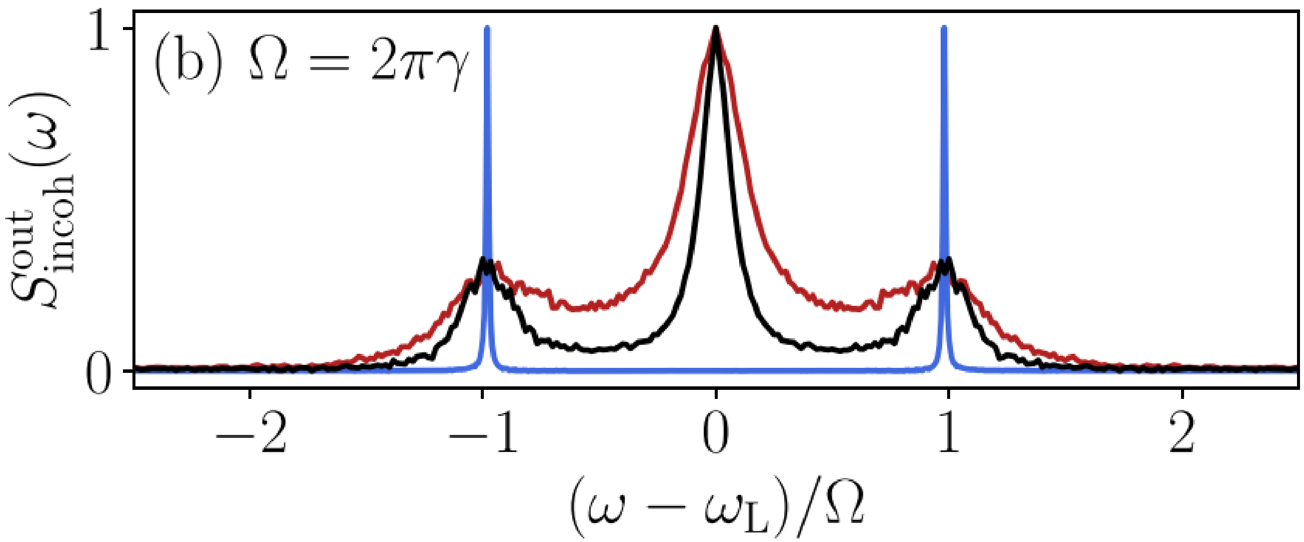
$\phi = \pi$

CW pump field, Rabi Frequency: $\Omega = 2\pi\gamma$

Two photon truncation is excellent (even for very long delays)

Regidor, Crowder et al, Phys Rev Res, 2021

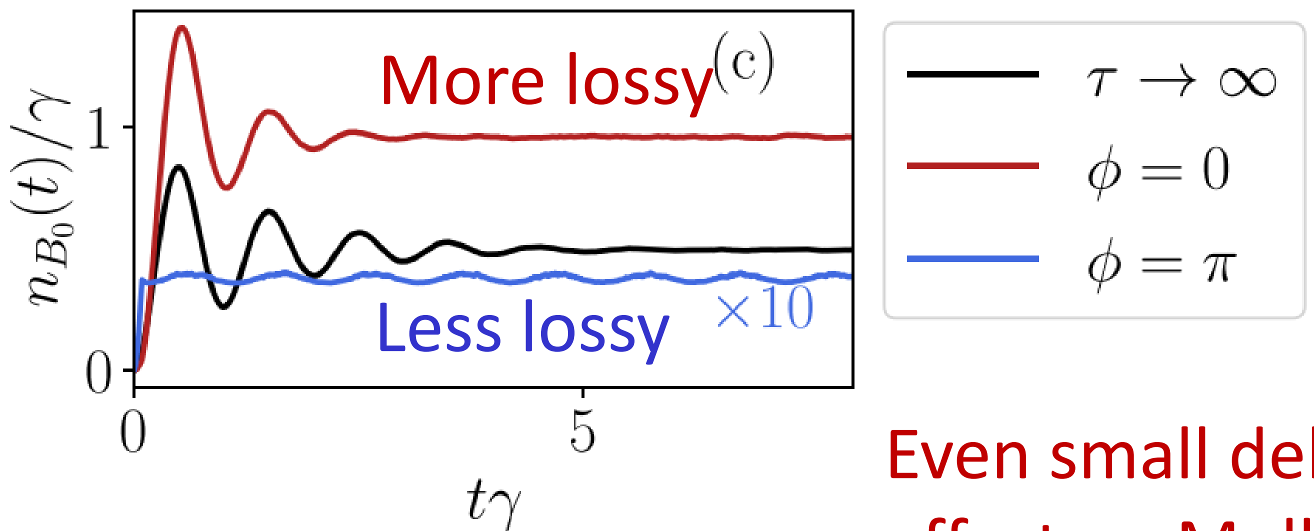
Influence of feedback on CW spectra



CW pumping

20,000 QTs

Mollow triplet becomes a doublet



No feedback

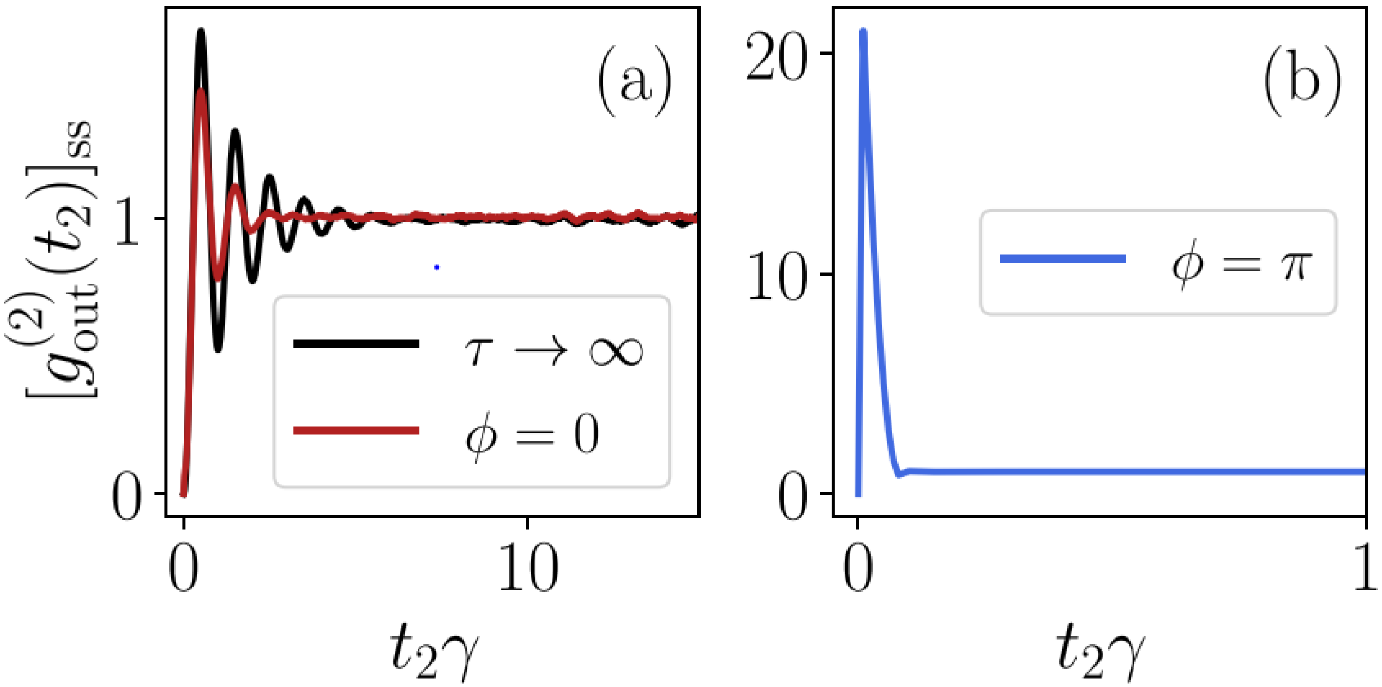
$\} \ \tau = 0.1\gamma$

Even small delays have significant effect on Mollow triplet

Corresponding g2 correlations

Sub-Poissonian

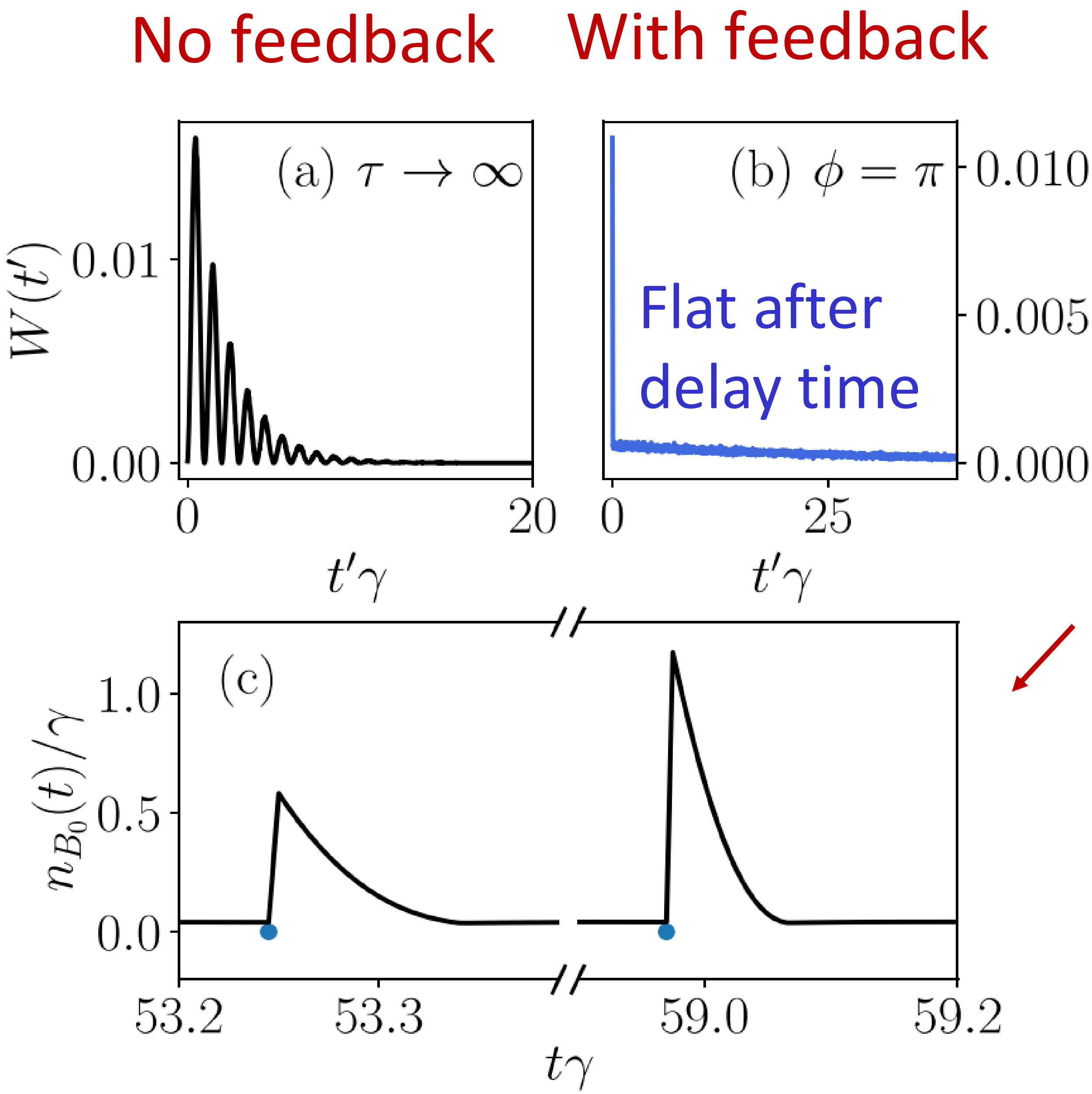
Super-Poissonian
(trapping case)



Feedback here causes photons to "bunch" (double clicks)

Crowder, Ramunno, Hughes, Phys Rev A, 2022

Waiting-time distribution functions

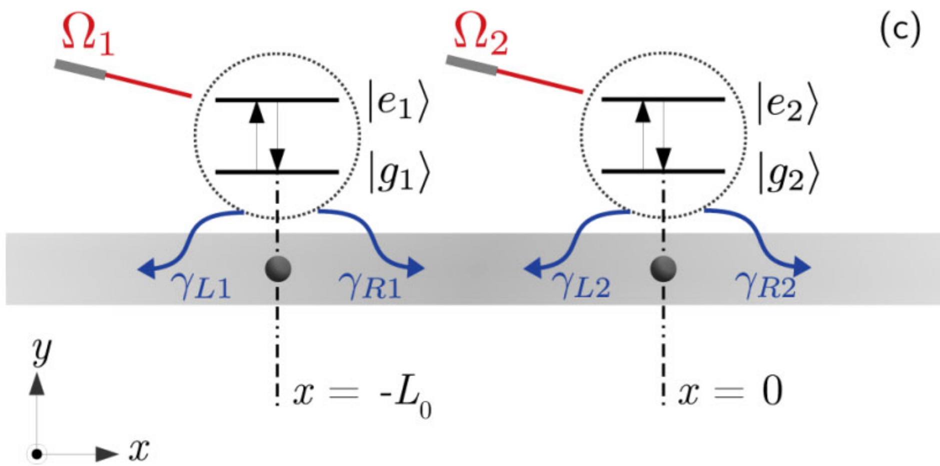


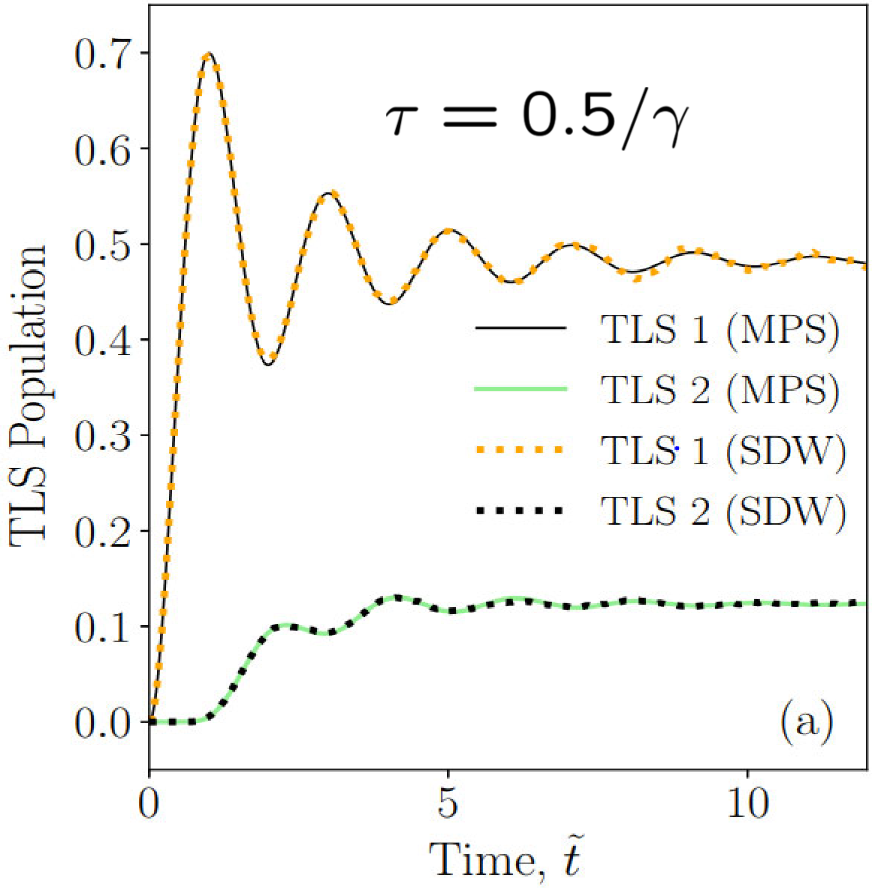
Example jump events during a QT run, showing an increase of output flux immediately after the jumps

Two coupled qubits in infinite waveguide

Pump qubit 1

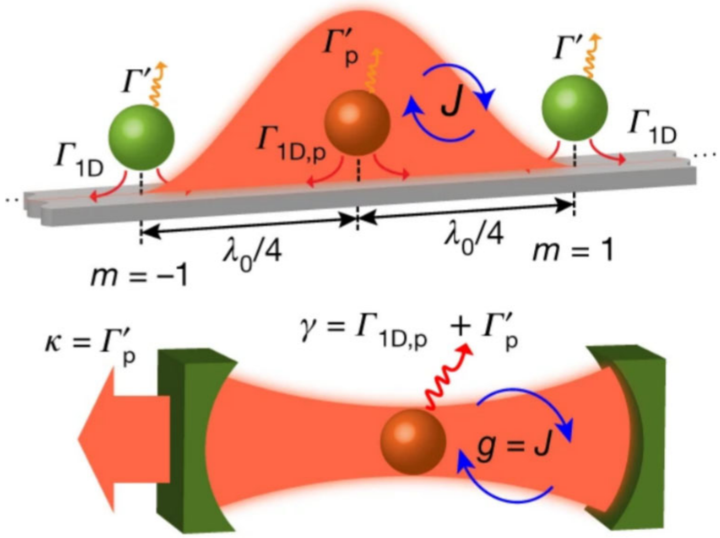
$\Omega_1 = 0.5\pi\gamma$

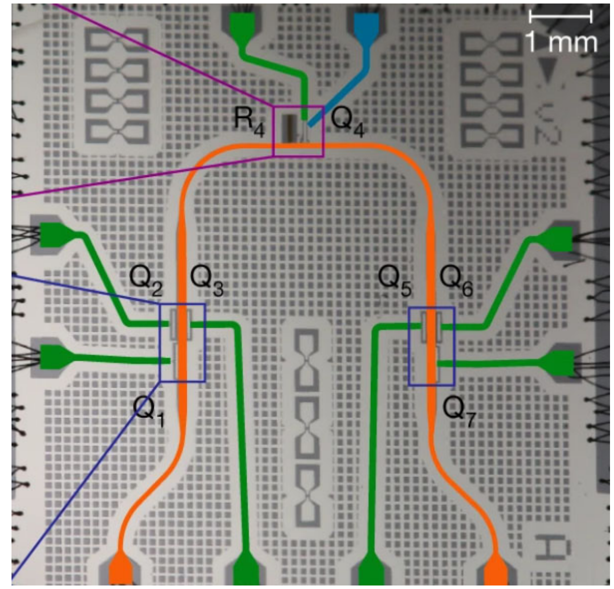




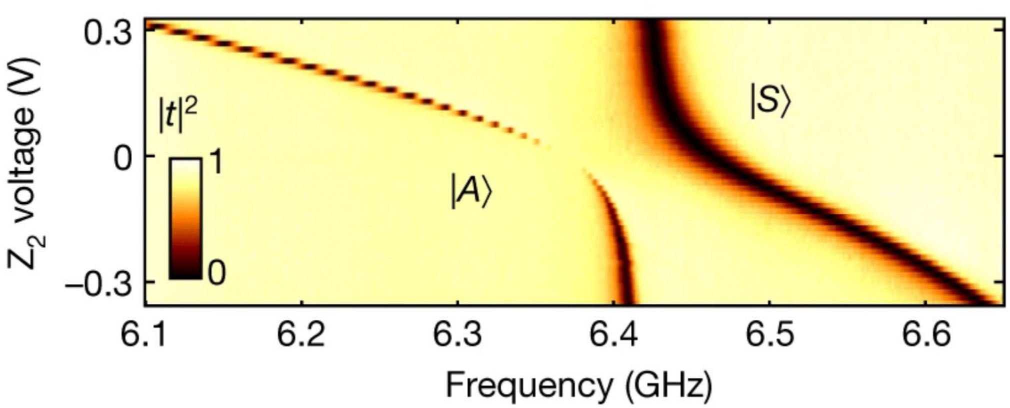
Again, we see an excellent agreement between QTs (average over 3000) with MPS, in a very complex nonlinear regime. Can also add in mirror feedback.

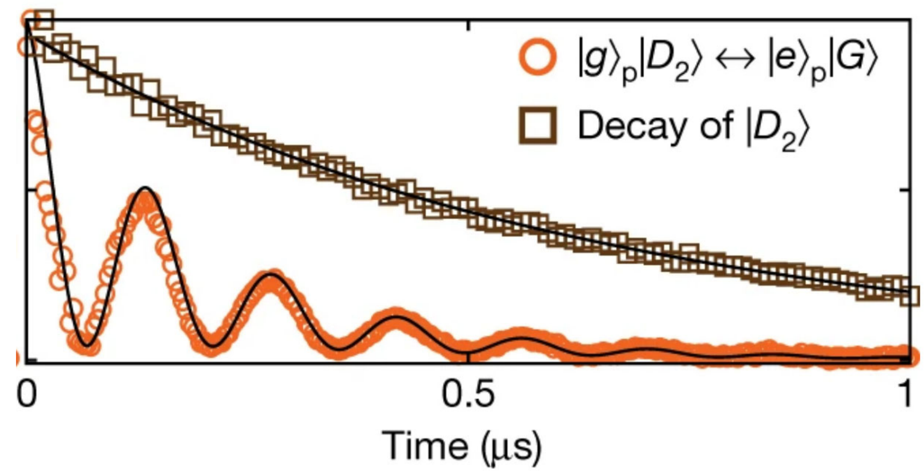
Three coupled qubits with mirror qubits





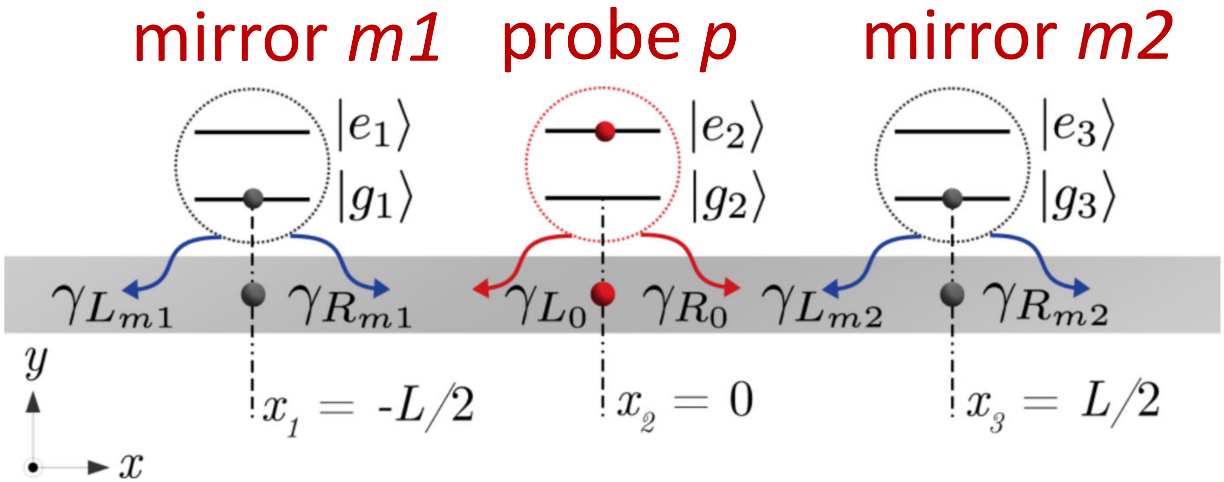
Vacuum Rabi oscillations with atom-like mirrors – **real data**





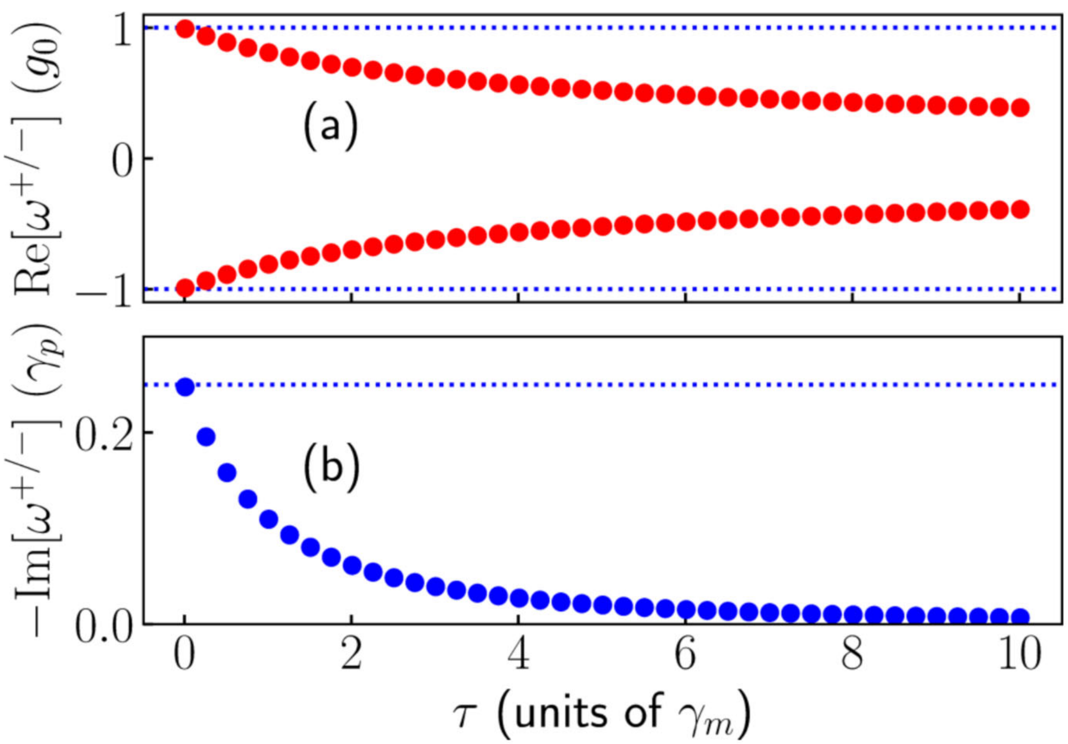
Mirhosseini et al, Nature, 2019 (basically Markovian)

Three qubits in deep non-Markov regime



"Ultra-giant atoms"?

$$g_0 = \sqrt{2\gamma_m\gamma_p}/2 \quad \gamma_m = 10\gamma_p$$

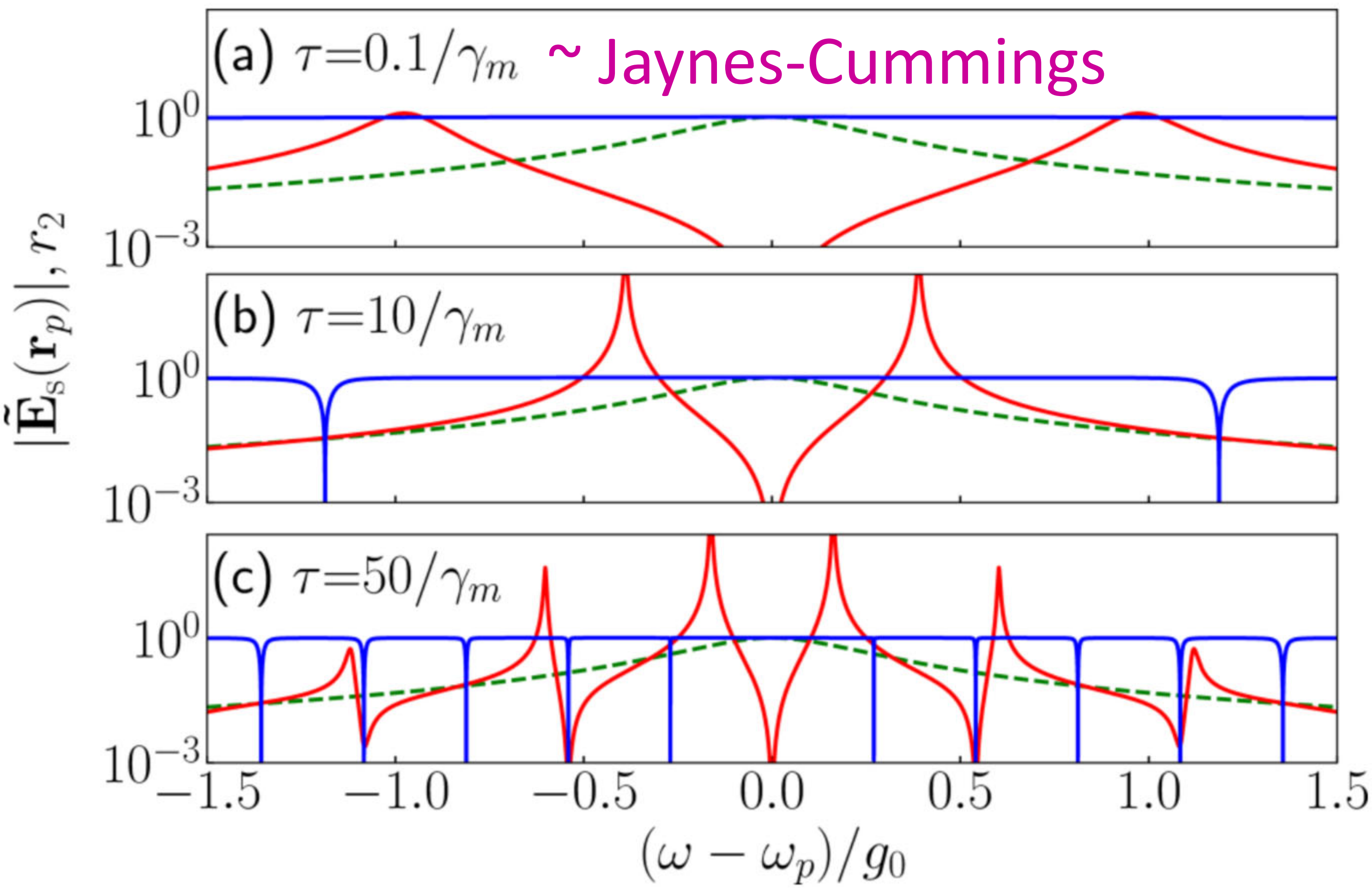


Main two complex poles versus time delay

Losses are substantially reduced!

Regidor, SH, PRA (2021)

Exact scattered field solution (linear)



Increasing delay time

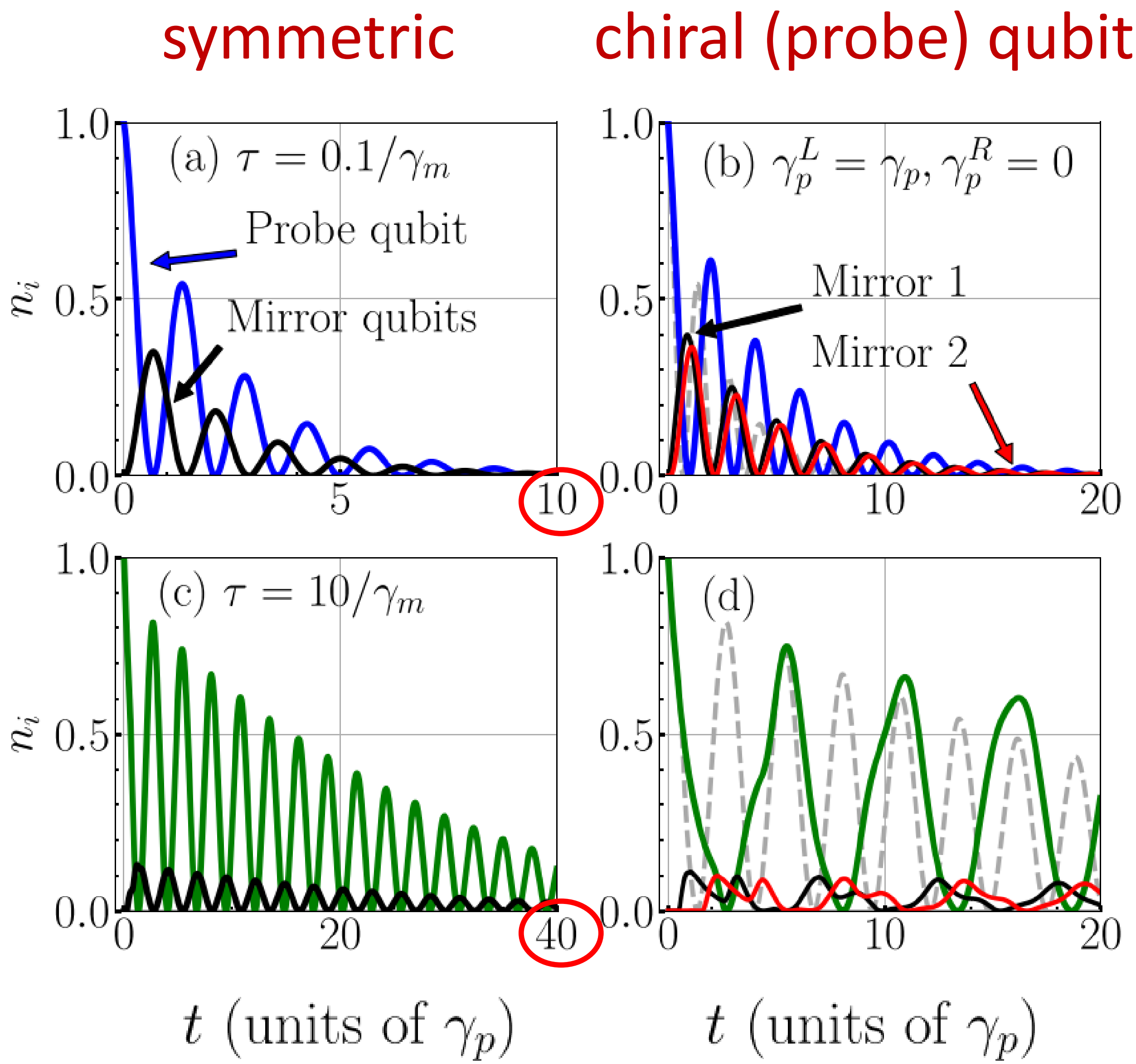
$$\tau = L/c_\omega$$

Blue is reflection of effective cavity

Red is response at center qubit

Regidor, SH, PRA (2021)

Example quantum dynamics (1 excitation)



$\tau = 0.1/\gamma$

Short delay time

$\tau = 10/\gamma$

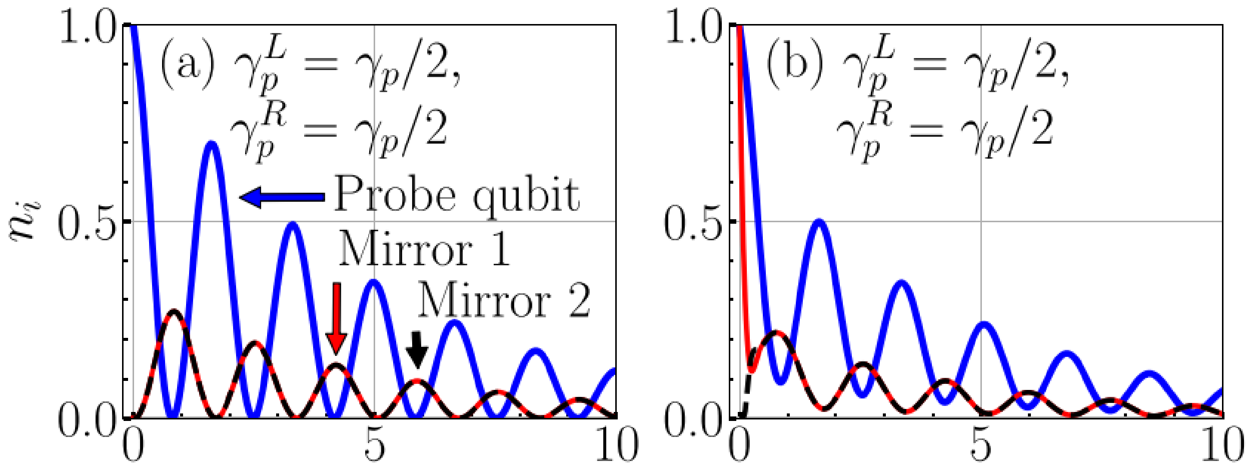
Long delay time

Regidor, SH, PRA (2021)

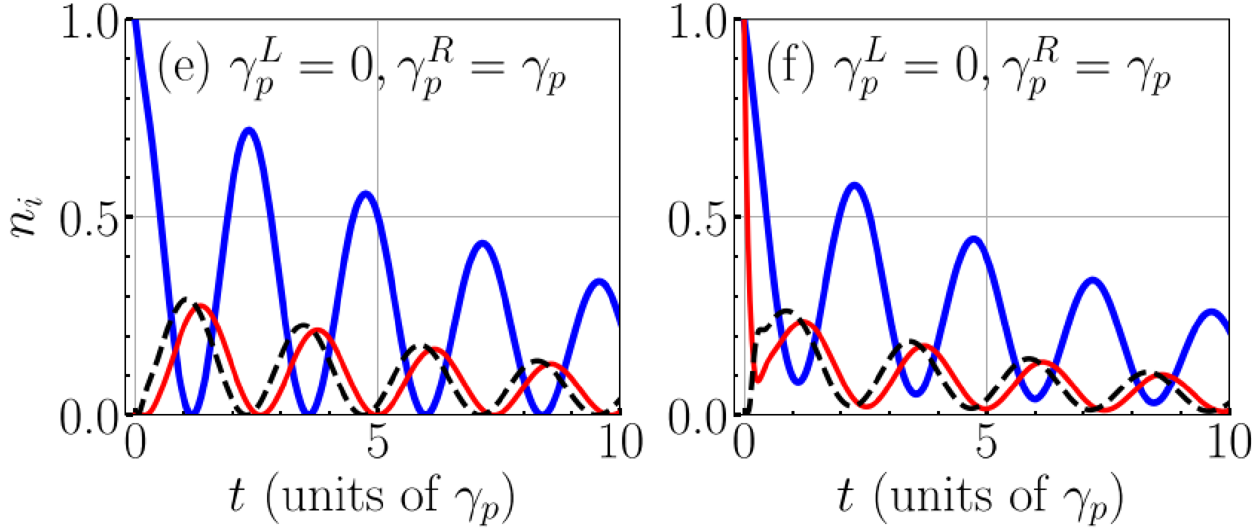
Example quantum dynamics (2 excitations)

1 excitation **2 excitations**

symmetric probe qubit



chiral (probe) qubit



Nonlinear pumping is more complex, see Regidor, SH, arXiv (2022)

Summary

Introduced a waveguide QT approach to simulate few photon dynamics for a system with coherent feedback

QT method is easy to implement numerically, compared to MPS, and is just as accurate at the few photon level

Yields unique insight into the stochastic dynamics and functions like the waiting time distribution function

Method easily includes additional scattering channels and perfectly parallelizable

MPS has an advantage of scaling better with multiple qubits and more waveguide photons