

Delayed-response quantum back action in nanoelectromechanical systemsS. N. Shevchenko,^{1,2,3} D. G. Rubanov,² and Franco Nori^{3,4}¹*B. Verkin Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine*²*V. Karazin Kharkov National University, Kharkov, Ukraine*³*CEMS, RIKEN, Saitama 351-0198, Japan*⁴*Physics Department, University of Michigan, Ann Arbor, Michigan 48109-1040, USA*

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We present a semiclassical theory for the delayed response of a quantum dot (QD) to oscillations of a coupled nanomechanical resonator (NR). We prove that the back action of the QD changes both the resonant frequency and the quality factor of the NR. An increase or decrease in the quality factor of the NR corresponds to either an enhancement or damping of the oscillations, which can also be interpreted as Sisyphus amplification or cooling of the NR by the QD.

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I. INTRODUCTION

An important model hybrid system is a resonator coupled to a mesoscopic normal or superconducting system [1]. In many cases, the resonator, which can be electrical or nanomechanical, is slow and can be described classically. This implies the relation $\hbar\omega_0 < k_B T$ between its resonant frequency ω_0 and the temperature T . In contrast to this, the characteristic energy of a mesoscopic quantum subsystem is usually larger than $k_B T$. In this case, the resonator and the quantum subsystem evolve on different time scales. Adjustment should be made if, in addition, there is a slow component in the evolution of the quantum system. One such situation takes place [2,3] if the Rabi oscillations are induced with a frequency $\Omega_R \sim \omega_0$, resulting in an effective energy exchange between the subsystems.

Another interesting situation occurs when the relaxation of the quantum subsystem is so slow that its characteristic time T_1 is of the order of the resonator's period $T_0 = 2\pi/\omega_0$, which is a realistic assumption for quantum dots [4]. Then the delayed response of the quantum subsystem to the resonator's perturbation implies that the resonator is influenced by both the in-phase and out-of-phase forces [5–8]. The out-of-phase force can damp or amplify the resonator oscillations [9]. Such effects can be described as a decrease or increase in the number of photons in the resonator, which relates to lasing and cooling [10–15].

Alternatively, the slow evolution of a quantum subsystem subject to a periodic driving by a resonator with a significant probability of relaxation can be described in terms of periodic *Sisyphus-type processes*. This was studied for an electric resonator coupled to a superconducting qubit [16–19]. In such systems, the electric resonator performs Sisyphus-type work by slowly driving a qubit along a continuously ascending (or descending) trajectory in energy space, while the cyclic Sisyphus destiny is completed by resonant excitation on one side of the trajectory and relaxation on the other [17]. Our aim in this paper is to study an analogous process for a typical nanoelectromechanical system [20,21], which consists of a nanomechanical resonator (NR) coupled to a single-electron transistor or a quantum dot (QD) [22–26]. This study is partly motivated by the experiments in Refs. [16,27].

A straightforward approach for describing a slow classical resonator coupled to a fast quantum subsystem is a

fully quantum description of the coalesced system [2,16]. Arguably, a more intuitively clear procedure assumes a delayed response of the quantum subsystem to the resonator driving. The effectiveness of this *delayed-response method* has been confirmed in different contexts [5–7,9,28,29]. In particular, the observation of Sisyphus cooling and amplification of an electrical *LC* circuit by a flux qubit [16] can be described by solving the master equation of the coalesced system [2,16]; the delayed-response method performs equally well in describing such a system [28,30]. In both cases, successful fitting of the experimental results yields a similar value for the key delay parameter, $\omega_0 T_1 \approx 1$, close to the optimal value for Sisyphus cooling and amplification.

Accordingly, for a coupled slow classical resonator and a fast quantum subsystem, we will use a semiclassical theory within the framework of the delayed-response method. The resonator (here a NR) slowly drives the quantum subsystem (a QD, in our case), with the response of the latter at a time t determined by the driving parameters at some prior time $\tilde{t} = t - \tau$. We will show that this produces an out-of-phase force, with the resonator's oscillations amplified or attenuated by the back action of this force. While we leave the detailed discussions for the Appendixes, in the rest of the paper we consider in detail the delayed response of the QD to the oscillations of the coupled NR. The presentation is organized in such a way that the approach could be straightforwardly adapted to other similar systems, where a slowly driven system is coupled to a fast quantum system, whose back action is delayed by the (possibly slow) relaxation process.

II. SEMICLASSICAL THEORY FOR THE COUPLED QUANTUM DOT AND NANOMECHANICAL RESONATOR SYSTEM**A. Model**

A schematic diagram for a coupled QD-NR system, analogous to a feasible experimental setup [27,31], is shown in Fig. 1. Here, the essential element is the island or quantum dot (QD). It is characterized by the total capacitance $C_\Sigma = C_1 + C_2 + C_g + C_{NR}$, average number of excessive electrons $\langle n \rangle$, and the island's potential V_I . The QD is biased by the gate voltage V_g and the voltage V_{NR} applied via the capacitance

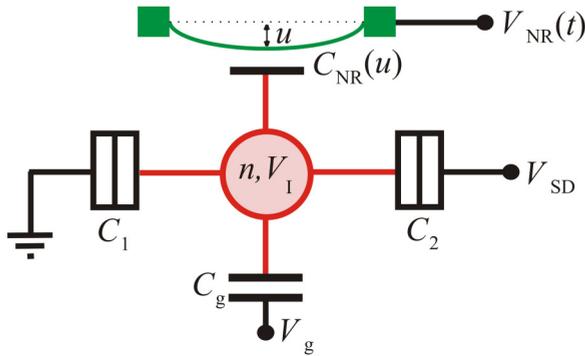


FIG. 1. (Color online) Schematic diagram of a system composed of a nanomechanical resonator (green) electrostatically coupled to a quantum dot (red). The source (left) and drain (right) electrodes of the QD are biased by the voltage V_{SD} ; the QD state is controlled by the gate voltage V_g . The NR is actuated by the voltage $V_{NR}(t) = V_{NR} + V_A \sin \omega_0 t$. The coupling between the NR and the QD is characterized by the displacement-dependent capacitance $C_{NR}(u)$.

$C_{NR}(u)$, one of the plates of which is able to perform mechanical oscillations. This is the NR, and its displacement u is related to the current through the QD.

Consider the mechanical resonator as a beam with mass m , elasticity k_0 , and damping factor λ_0 (which is assumed to be small). The oscillator has an eigenfrequency $\omega_0 = \sqrt{k_0/m}$ and quality factor $Q_0 = m\omega_0/\lambda_0$. The oscillator is assumed to be driven by the probe periodic force $F_p \sin \omega_0 t$ but its state is also influenced by the quantum subsystem, QD, through the force F_q . This external nonlinear force F_q is taken to depend only on the position variable u and its derivative, $F_q = F_q(u, \dot{u})$. Accordingly, the displacement u is the solution of the equation of motion [20]

$$m\ddot{u} + \frac{m\omega_0}{Q_0}\dot{u} + m\omega_0^2 u = F_q(u, \dot{u}) + F_p \sin \omega_0 t. \quad (1)$$

In general, for small oscillations

$$F_q(u, \dot{u}) \approx F_{q0} + \frac{\partial F_q}{\partial u} u + \frac{\partial F_q}{\partial \dot{u}} \dot{u}. \quad (2)$$

It follows that the second term above shifts the elasticity coefficient $k_0 = m\omega_0^2$ and the resonant frequency ω_0 to the effective frequency ω_{eff} ,

$$\omega_{\text{eff}}^2 = \omega_0^2 - \frac{1}{m} \frac{\partial F_q}{\partial u}, \quad (3)$$

while the third term changes the damping factor $\lambda_0 = m\omega_0/Q_0$, producing an effective quality factor Q_{eff} satisfying

$$\frac{1}{Q_{\text{eff}}} = \frac{1}{Q_0} - \frac{1}{m\omega_0} \frac{\partial F_q}{\partial \dot{u}}. \quad (4)$$

From these results, the expressions for the small frequency shift ($\Delta\omega \ll \omega_0$) and the quality factor shift ($\Delta Q \ll Q_0$) become

$$\Delta\omega \equiv \omega_{\text{eff}} - \omega_0 \approx -\frac{1}{2m\omega_0} \frac{\partial F_q}{\partial u}, \quad (5)$$

$$\Delta Q \equiv Q_{\text{eff}} - Q_0 \approx \frac{Q_0^2}{m\omega_0} \frac{\partial F_q}{\partial \dot{u}}. \quad (6)$$

There are various possible scenarios under which this back-action shift of the quality factor ΔQ becomes nontrivial. For example, the dependence $F_q = F_q(\dot{u})$ could originate from external forces, as is the case in Ref. [32]. Alternatively, nontrivial ΔQ also results when there is a lag in the back action. Here we consider this latter case in detail.

B. Lagged back action

If all the characteristic times of the QD are much faster than those of the NR, then its back action is characterized by $F_q = F_q(u)$ and no changes in Q are expected. However, in the next approximation, the QD sees the dependence $u = u(t)$ and we have $F_q = F_q(u, \dot{u})$. An illustrative way to describe this is by phenomenologically introducing a delayed time dependence in the QD response to the influence of the NR. This key assumption is discussed in detail in Appendix A. The delayed-response method can be formulated as follows.

We assume that without back action the force is linear in the NR displacement,

$$F_q = F_{q0} + \Xi u. \quad (7)$$

Then the delayed time dependence is characterized by replacing $t \rightarrow \tilde{t} = t - \tau$. Here τ stands for the characteristic time, which in our case describes the delay needed for changes in $C_{NR}(u)$ to affect the current I in the QD. There are two possible origins of the delayed response. The first relates to the tunneling rate Γ , with a delay time between the in- and out-tunneling events known as the Wigner-Smith time, $\tau \sim 1/\Gamma$ [33–35]. The second origin of the delayed response is when the upper-level occupation is created by any means, and the relaxation from it to the ground state has a delay $\tau \sim T_1$ [4]. This latter case is considered in detail in Appendix A.

The delayed-response assumption means that the back action of the QD is described by the displacement which defined the position of the NR some time ago: $F_q(t) = F_q[u(t - \tau)]$. For the induced NR oscillations, $u(t) = v \cos(\omega_0 t + \delta)$, we then have

$$u(t - \tau) = v \mathcal{C} \cos(\omega_0 t + \delta) + v \mathcal{S} \sin(\omega_0 t + \delta) \quad (8)$$

with $\mathcal{C} = \cos(\omega_0 \tau)$ and $\mathcal{S} = \sin(\omega_0 \tau)$. So, the back action of the quantum dot produces the dependence on \dot{u} , $F_q = F_q(u, \dot{u})$, in the form

$$F_q(t) = F_{q0} + \Xi [Cu(t) - \omega_0^{-1} \mathcal{S}\dot{u}(t)]. \quad (9)$$

This together with Eqs. (2), (5), and (6) provides expressions for the effective frequency and the quality factor shifts:

$$\frac{\Delta\omega}{\omega_0} = -\frac{\mathcal{C}}{2m\omega_0^2} \Xi, \quad (10)$$

$$\frac{\Delta Q}{Q_0} = -\frac{\mathcal{S}Q_0}{m\omega_0^2} \Xi. \quad (11)$$

From these, it follows that the quality factor changes ΔQ are directly related to the changes in the frequency shift $\Delta\omega$, i.e., $\Delta Q \propto \Delta\omega$. Moreover, their ratio quantifies the delay

measure $\omega_0\tau$

$$\tan(\omega_0\tau) = \frac{1}{2Q_0} \frac{\Delta Q/Q_0}{\Delta\omega/\omega_0}. \quad (12)$$

Note that if the changes of the quality factor ΔQ are not small, one should use Eq. (4) instead of Eq. (6). In any case, the quality factor changes can be termed as the ‘‘Sisyphus’’ addition to the quality factor [18] as follows:

$$\frac{1}{Q_{\text{eff}}} = \frac{1}{Q_0} + \frac{1}{Q_{\text{Sis}}}, \quad Q_{\text{Sis}}^{-1} = \frac{S}{m\omega_0^2} \Xi. \quad (13)$$

Positive values of Q_{Sis} give rise to damping, while negative values result in amplification, which is the precursor of lasing [16]. Here a special case is when $Q_{\text{Sis}} \rightarrow -Q_0$: this corresponds to the theoretical lasing limit [18,36], in which the regime of self-sustaining oscillations is realized.

The delayed response can also be related to the work done on the resonator by the quantum system, QD [7,30]. The respective energy transfer during one period is given by

$$W = \oint du F_q = \int_0^{2\pi/\omega_0} dt F_q \frac{du}{dt} = -S\pi v^2 \Xi, \quad (14)$$

which is proportional to the quality factor changes:

$$\frac{W}{W_0} = \frac{\Delta Q}{Q_0}, \quad (15)$$

where the normalizing factor is $W_0 = \pi m\omega_0^2 v^2 / Q_0$. Note that for the driven resonant oscillations $v = F_p Q_0 / m\omega_0^2$. Therefore, the positive or negative shift in the quality factor, i.e., the amplification or damping of the NR oscillations, is related to the respective work done by the QD. Similar processes have been described as Sisyphus amplification and cooling of the NR [16,17]. For further discussion see also Appendixes B and C. Note also that such periodic processes are similar to quantum thermodynamic cycles, which can be used as quantum heat engines [16,37,38].

C. Quantum dot response

Let us now explicitly define the back-action force F_q for the system presented in Fig. 1. It is assumed that the mechanical frequency ω_0 is much smaller than the QD tunneling rate Γ , hence the NR sees the QD charge averaged over many stochastic tunneling events [39]. Supposing this, the averaged QD charge is given by

$$e \langle n \rangle = C_\Sigma V_1 + e n_g, \quad (16)$$

$$n_g = -\frac{1}{e} [C_2 V_{\text{SD}} + C_g V_g + C_{\text{NR}} V_{\text{NR}}(t)]. \quad (17)$$

It follows that $V_1 = e(\langle n \rangle - n_g) / C_\Sigma$. Here it is assumed that the NR is biased by a dc plus an ac voltage: $V_{\text{NR}}(t) = V_{\text{NR}} + V_A \sin \omega_0 t$. Then the electrostatic force becomes

$$\begin{aligned} F &= \frac{\partial}{\partial u} \frac{C_{\text{NR}}(u) [V_{\text{NR}}(t) - V_1(u)]^2}{2} \\ &\approx \frac{1}{2} \frac{\partial}{\partial u} C_{\text{NR}}(u) \{ V_{\text{NR}}^2 + 2V_{\text{NR}} [V_A \sin \omega_0 t - V_1(u)] \}. \end{aligned} \quad (18)$$

Expanding as a Taylor series to second order we obtain

$$\begin{aligned} C_{\text{NR}}(u) &\approx C_{\text{NR}}(0) + \left. \frac{dC_{\text{NR}}}{du} \right|_0 u + \left. \frac{d^2 C_{\text{NR}}}{du^2} \right|_0 \frac{u^2}{2} \\ &\equiv C_{\text{NR}} \left(1 + \frac{u}{\xi} + \frac{u^2}{2\lambda} \right), \end{aligned} \quad (19)$$

and similarly for $\langle n \rangle$ and n_g . The second term in the right-hand side of Eq. (18) results in the periodical driving, $F_p \sin \omega_0 t$, with $F_p = V_A V_{\text{NR}} C_{\text{NR}} / \xi$. Then keeping only the terms defined by the QD state, we obtain Eq. (7) with

$$\Xi = \frac{2E_C}{\xi^2} n_{\text{NR}}^3 \left(\frac{d^2 \langle n \rangle}{dn_g^2} + \frac{2\alpha}{n_{\text{NR}}} \frac{d \langle n \rangle}{dn_g} + \frac{\langle n \rangle - n_g}{n_{\text{NR}}^2} \frac{\xi^2}{\lambda} \right). \quad (20)$$

Here $E_C = e^2 / 2C_\Sigma$, $\alpha = 1 + \xi^2 / 2\lambda$, and $n_{\text{NR}} = -C_{\text{NR}} V_{\text{NR}} / e$. For estimations it is useful to note that for the plane-parallel capacitor with distance $d + u$ between the plates $\xi = -d$, $\lambda = d^2 / 2$, and $\alpha = 2$.

We note in passing that the same results as Eqs. (7) and (20) can be obtained in terms of the quantum capacitance [40–42] by introducing the effective capacitance

$$C_{\text{eff}} = \partial Q_{\text{NR}} / \partial V_{\text{NR}} = C_{\text{geom}} + C_q. \quad (21)$$

The effective capacitance consists of the irrelevant geometric component and the quantum capacitance,

$$C_q = -\frac{eC_{\text{NR}}}{C_\Sigma} \frac{\partial \langle n \rangle}{\partial V_{\text{NR}}}. \quad (22)$$

The force F_q is now given in terms of the effective capacitance as

$$F_q = \frac{\partial}{\partial u} \frac{C_{\text{eff}} V_{\text{NR}}^2}{2}. \quad (23)$$

By expanding $C_{\text{NR}}(u)$ and $\langle n \rangle$ as series in u , we obtain Eqs. (7) and (20).

To proceed, we require the QD occupation probability $\langle n \rangle$, which depends on the gate voltage via n_g . This is related to the QD conductance, $G(V_g) = I / V_{\text{SD}}$, as follows [43,44]:

$$G = -\frac{1}{2} \Gamma C_\Sigma \frac{d \langle n \rangle}{dn_g}, \quad (24)$$

where I is the source-drain current and Γ is the tunneling rate. The conductance at low temperature is defined by the transmission, $G = G_0 \mathcal{T}$, with the transmission \mathcal{T} given by the Breit-Wigner formula [45]:

$$G = G_0 g \frac{(\hbar\Gamma)^2}{(\hbar\Gamma)^2 + [2E_C(n_g - n_g^{(0)})]^2} \equiv \frac{G_0 g}{1 + (\varepsilon_0/\Delta)^2}, \quad (25)$$

where the Lorentzian curve half width at half maximum and its center are defined by $\hbar\Gamma$ and $n_g^{(0)}$. The formula for the conductance is valid for small tunneling rates, for $E_C \gg \hbar\Gamma, k_B T$. Here, $\Gamma = (\Gamma_1 + \Gamma_2) / 2$ stands for the averaged tunneling rate into the left (Γ_1) and right (Γ_2) reservoirs; the factor $g = \Gamma_1 \Gamma_2 / \Gamma^2$ diminishes the conductance and the current if the rates are not equal. We have also defined here the tunneling amplitude Δ and the energy bias ε_0 , as follows:

$$\varepsilon_0 = 2E_C(n_g - n_g^{(0)}). \quad (26)$$

Then the expression for the source-drain current I reads

$$I = I_0 \frac{1}{1 + (\varepsilon_0/\Delta)^2}, \quad I_0 = V_{SD} G_0 g. \quad (27)$$

Combined together, Eqs. (20)–(25) define the effective quality factor shift (11). For illustration, we take $|n_{NR}| \gg 1$, leaving only the second term in Eq. (20), to obtain

$$\Delta Q = -Q_S \frac{\varepsilon_0/\Delta}{[1 + (\varepsilon_0/\Delta)^2]^2}, \quad (28)$$

$$Q_S = S \frac{16\pi Q_0^2 \Delta}{m\omega_0^2 \xi^2} \left(\frac{2E_C}{\Delta}\right)^3 n_{NR}^3.$$

Note that the theoretical lasing condition [18] is when $Q_{Sis} = -Q_0$ [see Eq. (13)] and this is fulfilled in our case when $\Xi(\varepsilon_0) = -m\omega_0^2/SQ_0$.

In addition, from Eqs. (24) and (25) we also obtain

$$\langle n \rangle = -\frac{2}{\Gamma C_\Sigma} \int G(n_g) dn_g = -\frac{1}{\pi} \arctan\left(\frac{\varepsilon_0}{\Delta}\right) + \frac{1}{2}. \quad (29)$$

Then, from the QD Hamiltonian [4,42],

$$H = -\frac{1}{2}(\varepsilon_0\sigma_z + \Delta\sigma_x), \quad (30)$$

we have

$$E_\pm = \pm \frac{1}{2} \sqrt{\Delta^2 + \varepsilon_0^2}. \quad (31)$$

The above results, Eqs. (27)–(31), are illustrated in Fig. 2 and discussed below.

III. DISCUSSION

Figure 2 graphically describes the interaction of the NR and the QD. The controllable parameter is the offset $\varepsilon_0 = 2E_C(n_g - n_g^{(0)})$, which can be influenced by both the gate voltage V_g and the NR displacement u .

The ground- and excited-state energy levels of the QD are plotted in Fig. 2(a), while the respective average excessive electron number $\langle n \rangle$ is shown in Fig. 2(b). If the gate voltage V_g is fixed, the evolution is described by the changes in the NR displacement u . Its influence on the QD is discussed in Appendix B. Here we concentrate on the back action.

Figure 2(c) shows the Lorentzian-shaped dependence of the current I through the QD, given by Eq. (27). We note that the dependence of the force F_q , which influences the NR, is similar. To demonstrate this, we find the expression of the displacement-dependent force from Eqs. (7) and (20): for the changes Δu we have $\Delta F_q = \Xi \Delta u$. Then, integrating this and assuming $|n_{NR}| \gg 1$, we obtain

$$F_q(u) = \frac{2E_C n_{NR}^2}{\xi} \left(\frac{d\langle n \rangle}{dn_g}\right) = F_0 \frac{1}{1 + [\varepsilon_0(u)/\Delta]^2}, \quad (32)$$

$$F_0 = \frac{8gE_C^2 n_{NR}^2}{\pi \xi \Delta}.$$

This means that $I/I_0 = F_q/F_0$, and Fig. 2(c) describes both the current I and the force F_q . We note that $\Delta n_g = n_{NR} \Delta u/\xi$, and thus Δn_g and Δu have opposite signs for negative V_{NR} .

Alongside the discussion in Appendix A, the closed oval trajectories in Fig. 2(c) indicate the essence of the delayed-response method of analysis of the NR-QD system; see also

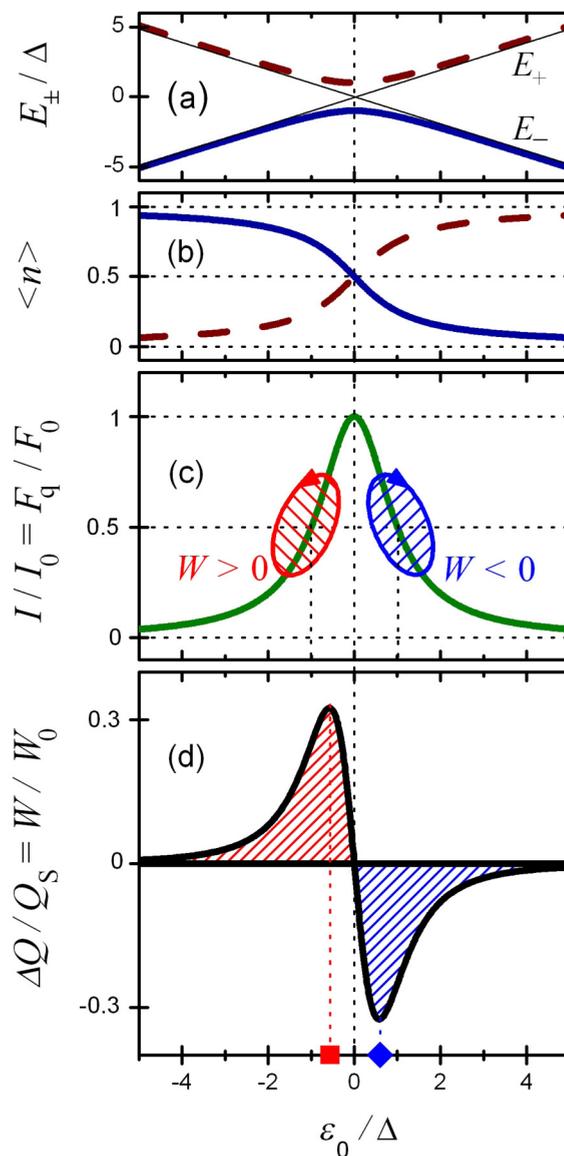


FIG. 2. (Color online) The gate-voltage offset ε_0 dependence of (a) the energy levels E_\pm/Δ ; (b) average electron number $\langle n \rangle$; (c) QD current I and the force F_q which influences the NR, where $I/I_0 = F_q/F_0$; and (d) the NR quality-factor changes ΔQ and the work W on the NR, where $\Delta Q/Q_S = W/W_0$. Closed trajectories in (c) describe the delayed value of the force, and the hatched areas give the work $W = \oint du F_q$. If there is no delay ($\tau \rightarrow 0$), then the red and blue hatched ovals in (c) merge with the solid green curve, which describes the adiabatic evolution. The source-drain current I in (c) is given by the Lorentzian (green) curve and the quality factor changes ΔQ in (d) are defined by its derivative, of which the maximum and the minimum are indicated by a square and a rhombus.

Ref. [20]. The periodic evolution of the NR displacement u results in the periodic sweeping the bias $\varepsilon_0(u)$ about its value at $u = 0$, defined by the gate voltage V_g . In Fig. 2(c) we demonstrate two such situations with $\varepsilon_0(u = 0) = \pm\Delta$ as examples. The points on the elliptical curves give the value of the force F_q for some previous time $\tilde{t} = t - \tau$. In particular, if there is no delay ($\tau \rightarrow 0$), these ovals in Fig. 2(c) shrink to

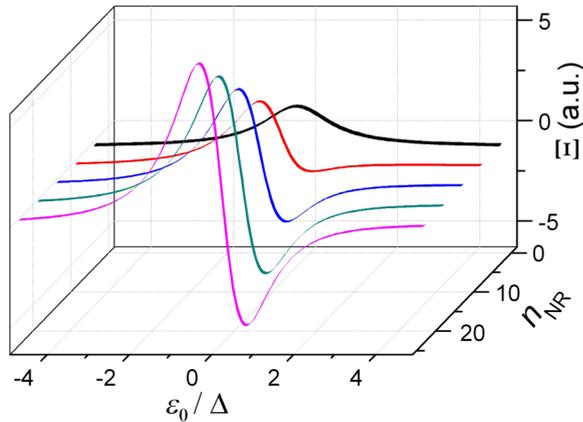


FIG. 3. (Color online) The bias ε_0 dependence of the response function Ξ for several values of the NR voltage, $n_{\text{NR}} = -C_{\text{NR}} V_{\text{NR}}/e$. For $n_{\text{NR}} \sim 1$ the response is described by a peak at $\varepsilon_0 = 0$, while for $|n_{\text{NR}}| \gg 1$ there are both an increase and a decrease of the function, which relates to the quality factor changes ΔQ , as demonstrated in Fig. 2(d).

the solid green curve, which describes the adiabatic evolution. In contrast to this, the back action with delay results in two types of trajectories, shown in Fig. 2(c), of which the nonzero area gives the work done by the drivings via the QD on the NR, $W = \oint du F_q \geq 0$; see Eq. (14). One can see from this geometric interpretation that the back-action effect is maximum when $\tau = T_0/4$, when the ovals tend to circles and the weight of the respective quadrature in Eq. (8) becomes maximal, at $S = 1$.

Finally, Fig. 2(d) displays the gate-voltage offset dependence of the quality factor changes ΔQ . We emphasize that, in agreement with Eqs. (10), (11), and (15), we have

$$\Delta Q \propto \Delta\omega \quad \text{and} \quad \Delta Q \propto W, \quad (33)$$

which means that Fig. 2(d) can also be interpreted (up to a normalizing factor) as the gate-voltage dependence of the frequency shift $\Delta\omega$ and the work W done by the QD on the NR.

In Fig. 3 the response function Ξ is plotted for several values of the NR voltage, $n_{\text{NR}} = -C_{\text{NR}} V_{\text{NR}}/e$. For this we used Eq. (20) without assuming $|n_{\text{NR}}| \gg 1$. Recall that the response function Ξ is the function which defines the quality factor changes, Eq. (11). Figure 3 demonstrates how for small values of n_{NR} the response is described by the first term in Eq. (20), while for large $|n_{\text{NR}}| \gg 1$, it is defined by the second term. In this way the first term describes only positive values of the response, while the second term can be both positive and negative and can result in respective changes of the quality factor; see also in Ref. [8].

IV. CONCLUSIONS

We have presented a quasiclassical theory for the “quantum dot—nanomechanical resonator” system using a phenomenological delayed-response method. This method is a useful and intuitive tool for the description of a coalesced system, where a slowly driven subsystem (resonator) is coupled to a quantum subsystem. The relaxation of the latter results in the delayed

back action. The advantage of this method over the use of a master equation is in the detachment of the dynamics of the two subsystems. The delayed response is included via the simple substitution $t \rightarrow \tilde{t} = t - \tau$. This means that the back-action force F_q is time delayed via the displacement u by the characteristic relaxation time τ : $F_q(t) = F_q[u(t - \tau)]$.

Our theory describes the increase and decrease of the NR quality factor due to the phase-shifted back-action force. This can be interpreted as Sisyphus cooling and amplification of the NR oscillations. This approach can be useful for the description and interpretation of experiments, such as those in Refs. [16,27].

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APPENDIX A: JUSTIFICATION OF THE DELAYED-RESPONSE METHOD

Here we present the justification for the delayed-response method, which was formulated in the Introduction and applied afterwards. Consider the force, which influences a resonator, to be exponentially decaying,

$$F_q(t) = F_{q0} + [F_q(t_0) - F_{q0}] \exp\left(-\frac{t - t_0}{T_1}\right). \quad (A1)$$

This is given at the initial moment, $t = t_0$, by $F_q(t_0)$ and tends to an equilibrium value F_{q0} with increasing time. The force enters the right-hand side of the resonator motion equation, Eq. (1). Consider the case of small retardation parameter,

$$\omega_0 T_1 \ll 1, \quad (A2)$$

which means that the relaxation happens fast in respect to the resonator period $T_0 = 2\pi/\omega_0$. It is then reasonable to average Eq. (1) during the time interval $\Delta t \sim T_1$. According to the assumption, during this interval one can neglect the changes in the resonator evolution, leaving the left-hand side of Eq. (1) unaffected. Next we assume the linear displacement dependence

$$F_q(t) - F_{q0} = \Xi u(t), \quad \Xi = \left. \frac{dF_q}{du} \right|_{u=0}, \quad (A3)$$

and obtain for the averaged force

$$\begin{aligned} \overline{F_q}(t) &\equiv \frac{1}{\Delta t} \int_{t-\Delta t}^t dt' F_q(t') \\ &= F_{q0} + \frac{1}{\Delta t} \int_{t-\Delta t}^t dt' [F_q(t - \Delta t) - F_{q0}] e^{-[t' - (t - \Delta t)]/T_1} \\ &= F_{q0} + \Xi u(t - \Delta t) f\left(\frac{T_1}{\Delta t}\right), \end{aligned} \quad (A4)$$

where $f(x) = x(1 - e^{-1/x})$. Then, choosing $\Delta t = T_1$ and neglecting distinction of $f(1)$ from unity, one obtains that the delayed force enters the equation of motion of the resonator,

$$\overline{F}_q(t) = F_{q0} + \Xi u(t - T_1). \quad (\text{A5})$$

This justifies the delayed-response approximation and results in the velocity dependence of the force

$$\overline{F}_q(t) = F_q(u, \dot{u}), \quad (\text{A6})$$

as it was discussed in the main text; see Eqs. (8) and (9).

Here we note that our Eq. (A5) gives the result consistent with those used in Refs. [6,7]. For comparison we rewrite here the respective averaged forces in our notations:

$$[6]: \overline{F}_q(t) = \int_0^t dt' \frac{dF_q[u(t')]}{dt'} \left[1 - \exp\left(-\frac{t-t'}{T_1}\right) \right], \quad (\text{A7})$$

$$[7]: \overline{F}_q(t) = \frac{d\overline{F}_q}{dt} \int_{-\infty}^t dt' \exp\left(-\frac{t-t'}{T_1}\right) u(t'). \quad (\text{A8})$$

One can check that the three equations, Eqs. (A5), (A7), and (A8), result for the steady-state oscillations in the same Eq. (9) in the limiting case of Eq. (A2).

Consider now the origin of Eq. (A1) in our problem of the qubit-resonator system. The system is described by the equation for the resonator displacement $u(t)$, Eq. (1), plus the Bloch equations for the reduced qubit density matrix $\rho = \frac{1}{2}(1 + X\sigma_x + Y\sigma_y + Z\sigma_z)$ with the relaxation times $T_{1,2}$,

$$\begin{aligned} \dot{X} &= \left(\frac{\Delta E}{\hbar} + \frac{\varepsilon_0}{\Delta} \beta u \right) Y - \frac{X}{T_2}, \\ \dot{Y} &= - \left(\frac{\Delta E}{\hbar} + \frac{\varepsilon_0}{\Delta} \beta u \right) X - \beta u Z - \frac{Y}{T_2}, \\ \dot{Z} &= \beta u Y - \frac{Z - Z^{(0)}}{T_1}, \end{aligned} \quad (\text{A9})$$

where

$$\beta = \frac{2ECn_{\text{NR}}}{\hbar\xi} \frac{\Delta}{\Delta E}, \quad Z^{(0)} = \tanh \frac{\Delta E}{2k_B T}. \quad (\text{A10})$$

Here $Z^{(0)}$ corresponds to the equilibrium value at nonzero temperature T . Then, if the coupling β between the resonator and the qubit is small and/or the oscillations $u(t)$ are small, one can neglect the first term in the equation for $Z(t)$, Eq. (A9). This results in the exponential dependence, as in Eq. (A1).

To be more specific, consider a QD with the Hamiltonian written in the charge representation in the two-level approximation in Eq. (30). Relating the charge and eigenbases, we have

$$\langle n \rangle = P_- \langle n \rangle_- + P_+ \langle n \rangle_+ = \langle n \rangle_- + P_+ (\langle n \rangle_+ - \langle n \rangle_-), \quad (\text{A11})$$

where the level occupation probabilities are $P_{\pm} = \frac{1}{2}(1 \mp Z)$ and we defined the coefficients

$$\langle n \rangle_{\pm} = \frac{1}{2} \left(1 \pm \frac{\varepsilon_0}{\Delta E} \right), \quad (\text{A12})$$

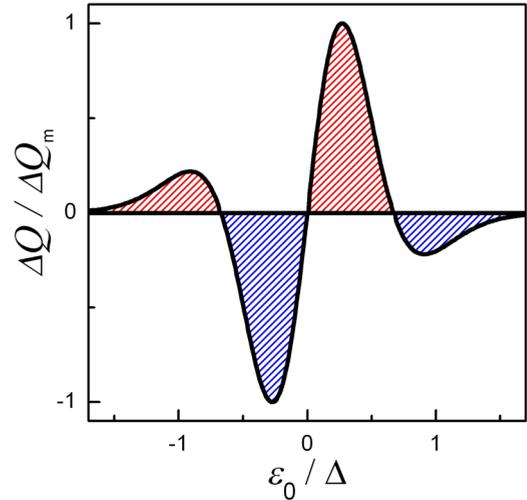


FIG. 4. (Color online) Normalized quality factor shift ΔQ as the function of the energy bias ε_0 calculated with Eq. (A14).

then $\langle n \rangle_+ - \langle n \rangle_- = \varepsilon_0/\Delta E$. We note that in the absence of excitation, $P_+ = 0$, we have $\langle n \rangle = \langle n \rangle_-$, which is in good agreement with the assumption of the Breit-Wigner tunneling; cf. Eqs. (25) and (29). In the other case, in thermal equilibrium, from Eq. (A11) we have [4]

$$\langle n \rangle = \frac{1}{2} - \frac{\varepsilon_0}{2\Delta E} \tanh \frac{\Delta E}{2k_B T}. \quad (\text{A13})$$

In this picture, the delayed response is related to the nonzero upper level occupation, which is the latter term in Eq. (A11), rather than to the ground-state average number $\langle n \rangle_-$. With this note, combining the equations above, we obtain the formula for the quality factor phase shift, which for $n_{\text{NR}} \gg 1$ reads

$$\Delta Q \approx -S \frac{4Q_0^2 E_C^3 n_{\text{NR}}^3}{m\omega_0^2 \xi^2} \frac{d^2}{d\varepsilon_0^2} \left[\frac{\varepsilon_0}{\Delta E} \left(1 - \tanh \frac{\Delta E}{2k_B T} \right) \right], \quad (\text{A14})$$

where the retardation parameter is defined by the relaxation time, $S = \sin(\omega_0 T_1)$.

We illustrate the result, Eq. (A14), in Fig. 4, where the quality factor shift ΔQ is normalized to its maximal value ΔQ_m and is plotted as the function of the energy bias $\varepsilon_0 = \varepsilon_0(V_g)$ for $k_B T = 0.1\Delta$. The figure demonstrates the amplification and attenuation of the NR oscillations. These can be interpreted in terms of the Sisyphus cycles, which we detail below. Here we emphasize that the important feature of the process is the double-amplification-attenuation structure, demonstrated in Fig. 4. This may be useful in analyzing the experimental results such as those detailed in Ref. [27].

APPENDIX B: SISYPHUS CYCLES FOR THE NANOELECTROMECHANICAL SYSTEM

In the main text we were principally interested in the back-action effect. In particular, Fig. 2(c) shows the work over the NR during one period. Here we consider this evolution as seen by the QD. For this goal, in Fig. 5 we consider the average excessive electron number $\langle n \rangle$ versus the bias ε_0 . These are the same curves as in Fig. 2(b), plotted with Eq. (A11), where

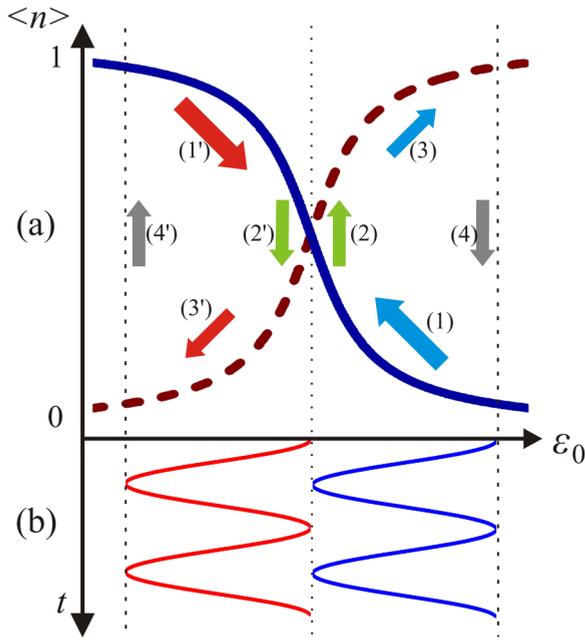


FIG. 5. (Color online) (a) Average excessive electron number on the QD $\langle n \rangle$ as a function of the bias ϵ_0 and (b) changes of the bias due to the periodic evolution of the NR. The right and left halves of the graph illustrate the cycles in which resonator changes the average number of electrons from 0 to 1 and vice versa. See text for a detailed description of these cycles.

the solid line corresponds to the ground state and the dashed one to the excited state. We consider slow periodic changes of the NR displacement, which correspond to changing the bias; see Fig. 5(b). Note that for illustration we consider the number of electrons and not the energy levels since we have an open driven system in which energy changes of one subsystem should not be equal to minus the energy changes in another subsystem.

In the right and left halves of Fig. 5 we consider two cases of positive and negative offsets. The amplitude of the oscillations is chosen to be twice the offset, so that the resonator drives the two-level system (TLS) between the point of energy level quasi-intersection (at $\epsilon_0 = 0$) and the point removed from it; see also Figs. 2(a) and 2(b). We assume that the region where the energy levels are curved (i.e., experience avoided-level crossing) plays the role of a 50:50 beam splitter. This means that after going out of this region, the TLS levels are equally populated. Here we assume that the characteristic relaxation time T_1 is longer than the time of passing this region. Moreover, we assume that it is of the order of the driving period, namely, $T_1 \sim T_0/4 = \pi/2\omega_0$.

Then, the overall dynamics of the TLS can be stroboscopically split in four intervals. Consider first the right part of Fig. 5. (1) The resonator drives the TLS uphill along the ground state, $\langle n \rangle$ changes from 0 to 1/2. (2) In the region of the avoided-level crossing, the two energy levels become equally populated; and then we monitor the upper-level evolution. (3) Again the resonator drives the TLS uphill, until it relaxes during the fourth evolution stage. In this cycle the resonator does work such that it changes $\langle n \rangle$ from 0 to 1, while the relaxation does vice versa. In contrast, in the inverted cycle, (1' - 4'), shown

in the left part of Fig. 5, the resonator does work changing $\langle n \rangle$ from 1 to 0.

The beam splitting can be created in several ways. (i) This can be created by means of nonadiabatic Landau-Zener transitions between the energy levels [46–50]. (ii) The 50:50-beam splitting can be created by resonantly driving the TLS as in Ref. [16]. (iii) Alternatively, the nonzero upper-level occupation can be created by the thermal excitation, which is essential when the temperature is comparable with the energy-level separation, as it was considered in the previous section.

APPENDIX C: SISYPHUS CYCLES DESCRIBED WITH THE DELAYED-RESPONSE THEORY

The equations for the source-drain current I and the changes of the NR quality factor ΔQ can be rewritten as follows:

$$\frac{I}{I_0} = \frac{1}{1 + (\epsilon_0/\Delta)^2}, \quad (C1)$$

$$\Delta Q \propto \mathcal{S} \left(I + a \frac{dI}{d\epsilon_0} \right), \quad (C2)$$

where $a = E_C n_{NR}/\alpha$. The former equation was illustrated in Fig. 2(c). The latter equation was illustrated in Fig. 3. The deep analogy with the Sisyphus cycles for the flux qubit- LC resonator system [16], mentioned earlier in the text, can be further justified by writing down analogous equations for this system. So, following Ref. [30], we consider now the driven flux qubit with the Hamiltonian

$$H = -\frac{\epsilon_0 + A \sin \omega_d t}{2} \sigma_z - \frac{\Delta}{2} \sigma_x. \quad (C3)$$

In this case, the averaged current in the flux qubit is [30] $I_{qb} = I_p \frac{\epsilon_0}{\Delta E} (2P_+ - 1)$, where I_p is the flux qubit persistent current and the averaged upper level occupation probability is given by the Lorentzian

$$P_+ = \frac{1}{2} \frac{1}{1 + (\delta\epsilon_0/\hbar\Omega)^2}, \quad (C4)$$

with $\delta\epsilon_0 = \epsilon_0 - \hbar\omega_d$ and $\hbar\Omega = \Delta A/2\hbar\omega_d$. Then for the changes of the quality factor ΔQ of the LC resonator one can obtain [30]

$$\Delta Q \propto \mathcal{S} \left(P_+ + b \frac{dP_+}{d\epsilon_0} \right), \quad (C5)$$

where $b = \Delta E^2 \epsilon_0/\Delta^2$. This equation is fully analogous to Eq. (C2); it is proportional to the lagging parameter \mathcal{S} (which is zero at $T_1 = 0$) and contains two competing terms: the Lorentzian and its derivative; the latter being the alteration of a peak and a dip. It is this latter term (when it is dominant) that describes the Sisyphus amplification and cooling, respectively [16].

APPENDIX D: DERIVATION OF EQ. (20)

The most essential appendixes are the other ones. This final appendix is more technical; here we present a more detailed

derivation of Eq. (20) in the main text, in addition to the theory in Sec. II C. There we considered the averaged QD charge given by the sum of the charges on the plates of the capacitors, which create the QD:

$$\begin{aligned} e\langle n \rangle &= \sum_i C_i (V_1 - V_i) = V_1 \sum_i C_i - \sum_i C_i V_i \\ &\equiv C_\Sigma V_1 + e n_g, \end{aligned} \quad (\text{D1})$$

where V_1 is the QD potential and V_i is the voltage applied to the i th capacitance C_i . Then the electrostatic force, Eq. (18), becomes

$$\begin{aligned} F &\approx \frac{1}{2} \frac{d}{du} C_{\text{NR}}(u) \{ V_{\text{NR}}^2 + 2V_{\text{NR}} [V_A \sin \omega_0 t - V_1(u)] \} \\ &\equiv F_q + F_p \sin \omega_0 t, \end{aligned} \quad (\text{D2})$$

where it was assumed that $V_{\text{NR}} \gg V_A, V_1$. Expanding as a Taylor series to second order we obtain Eq. (19). The second term in Eq. (D2) results in the periodic driving, $F_p \sin \omega_0 t$. Note that there is also an explicit time dependence in the third term, where $V_{\text{NR}}(t)$ also enters in V_1 (via n_g); then in addition to the second term there is a small term which can be neglected:

$$V_A \sin \omega_0 t - \frac{C_{\text{NR}}}{C_\Sigma} V_A \sin \omega_0 t \approx V_A \sin \omega_0 t, \quad (\text{D3})$$

assuming $C_{\text{NR}} \ll C_\Sigma$. Now we have

$$F_q = \frac{V_{\text{NR}}^2}{2} \frac{d}{du} C_{\text{NR}}(u) - \frac{e V_{\text{NR}}}{C_\Sigma} \frac{d}{du} C_{\text{NR}}(u) (\langle n \rangle - n_g). \quad (\text{D4})$$

The displacement dependence in C_Σ can be neglected for $C_{\text{NR}} \ll C_\Sigma$:

$$C_\Sigma(u) = C_\Sigma^{(0)} \left(1 + \frac{C_{\text{NR}}^{(0)}}{C_\Sigma^{(0)}} \frac{u}{\xi} \right) \approx C_\Sigma^{(0)} \equiv C_\Sigma. \quad (\text{D5})$$

Note that here and below, for brevity, we omit the superscript (0): $C_\Sigma^{(0)} = C_\Sigma(u=0) \equiv C_\Sigma$. Other values are expanded, making use of Eq. (19) and also neglecting the explicit time

dependence in n_g , as we noted above:

$$\begin{aligned} n_g(u) &= -\frac{1}{e} [C_2 V_{\text{SD}} + C_g V_g + C_{\text{NR}}(u) V_{\text{NR}}] \\ &\approx n_{g0} + n_{\text{NR}} \left(\frac{u}{\xi} + \frac{u^2}{2\lambda} \right), \end{aligned} \quad (\text{D6})$$

$$\langle n \rangle(u) \approx \langle n \rangle|_0 + \left. \frac{d\langle n \rangle}{du} \right|_0 u + \left. \frac{d^2\langle n \rangle}{du^2} \right|_0 \frac{u^2}{2}. \quad (\text{D7})$$

It is convenient to change the derivative from u to n_g , making use of Eq. (D6):

$$\left. \frac{d\langle n \rangle}{du} \right|_0 = \left. \frac{d\langle n \rangle}{dn_g} \frac{dn_g}{du} \right|_0 = \left. \frac{d\langle n \rangle}{dn_g} \frac{n_{\text{NR}}}{\xi} \right|_0; \quad (\text{D8})$$

$$\begin{aligned} \left. \frac{d^2\langle n \rangle}{du^2} \right|_0 &= \left. \left\{ \frac{d^2\langle n \rangle}{dn_g^2} \left(\frac{dn_g}{du} \right)^2 + \frac{d\langle n \rangle}{dn_g} \frac{d^2 n_g}{du^2} \right\} \right|_0 \\ &= \frac{d^2\langle n \rangle}{dn_g^2} \frac{n_{\text{NR}}^2}{\xi^2} + \frac{d\langle n \rangle}{dn_g} \frac{n_{\text{NR}}}{\lambda}. \end{aligned} \quad (\text{D9})$$

Now we can use these expansions in Eq. (D4). In what follows we are interested in terms linear in u , since displacement-independent terms (we name them F_{q0}) result only in a constant displacement of the resonator and do not influence the NR frequency and the quality factor:

$$F_q \approx F_{q0} + u \left\{ \frac{V_{\text{NR}}^2 C_{\text{NR}}}{2\lambda} - \frac{4E_C n_{\text{NR}}^2}{\xi^2} \alpha + \Xi|_0 \right\}, \quad (\text{D10})$$

where Ξ is given by Eq. (20)

The first two terms in the brackets in Eq. (D10) are of the form $\text{const} \times u$. This results in constant shifts in the frequency and quality factor, independent of the QD state. In contrast, the terms denoted by Ξ collect the QD-state dependent terms; these terms describe the impact of the QD charge variations, $\delta\langle n \rangle$, on the NR characteristics. In this way, when we obtained Eq. (20) in the main text, we meant “keeping only the terms defined by the QD state,” which assumed ignoring the impact of the first two terms in the brackets in Eq. (D10).

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