Entanglement distribution over quantum code-division multiple-access networks

Chang-long Zhu,1,2 Nan Yang,1,2 Yu-xi Liu,2,3 Franco Nori,4,5 and Jing Zhang1,2,*
1Department of Automation, Tsinghua University, Beijing 100084, People’s Republic of China
2Center for Quantum Information Science and Technology, TNList, Beijing 100084, People’s Republic of China
3Institute of Microelectronics, Tsinghua University, Beijing 100084, People’s Republic of China
4CEMS, RIKEN, Saitama 351-0198, Japan
5Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA
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We present a method for quantum entanglement distribution over a so-called code-division multiple-access network, in which two pairs of users share the same quantum channel to transmit information. The main idea of this method is to use different broadband chaotic phase shifts, generated by electro-optic modulators and chaotic Colpitts circuits, to encode the information-bearing quantum signals coming from different users and then recover the masked quantum signals at the receiver side by imposing opposite chaotic phase shifts. The chaotic phase shifts given to different pairs of users are almost uncorrelated due to the randomness of chaos and thus the quantum signals from different pairs of users can be distinguished even when they are sent via the same quantum channel. It is shown that two maximally entangled states can be generated between two pairs of users by our method mediated by bright coherent lights, which can be more easily implemented in experiments compared with single-photon lights. Our method is robust under the channel noises if only the decay rates of the information-bearing fields induced by the channel noises are not quite high. Our study opens up new perspectives for addressing and transmitting quantum information in future quantum networks.

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I. INTRODUCTION

With recent progress in various quantum systems such as ion-trap systems [1–3] and solid-state quantum circuits [4–6], it is now possible to discuss how to establish more efficient quantum networks or a so-called quantum internet [7]. Previous studies about quantum communication [8–10] and quantum cryptography [11,12] have shown that quantum network has great advantages to transfer classical or quantum information. However, how to best transfer information via quantum networks is still an open problem [13–23].

In transferring quantum information over a large-scale quantum network, a question that is asked is whether we can allow different pairs of users, who want to transmit information, to share the same channel [24–26]. This problem has been widely discussed in the field of classical communication [27,28]. In classical communication systems, such methods are called channel-access methods or multiple-access methods. There are mainly four different kinds of multiple-access methods [29]: the frequency-division multiple-access (FDMA) methods, the time-division multiple-access (TDMA) methods, the code-division multiple-access (CDMA) methods, and the orthogonal frequency-division multiple-access (OFDMA) methods. In FDMA methods, different frequency bands are assigned to different data streams, while in TDMA methods the users split their signals into pieces and transmit them at different time slots to share the same channel. The TDMA and FDMA methods work equally well and are the key techniques for the first-generation (analog) and the second-generation (digital) mobile networks. In CDMA methods, each pair of users shares the same channel and distinguishes each other by their own unique codes. It can be shown that the CDMA method can accommodate more bits per channel use, compared with the TDMA and FDMA [30] methods, and thus is used in third-generation mobile communication systems. However, the interference between different data streams will deteriorate the information rate of the CDMA method. Other competitive approaches are proposed including the OFDMA method, in which the available subcarriers are divided into several mutually orthogonal subchannels that are assigned to distinct users for simultaneous transmission. The OFDMA method is capable of avoiding the interference problem and thus provides better performance in classical digital communication.

Although the multiple-access problem has been widely studied in classical communication, it has been considered in quantum communication only recently due to the development of techniques for a scalable quantum network. The FDMA method, or, equivalently, the so-called wavelength-division multiple access method, has been used for quantum key distribution [31–36], in which classical information is transmitted over a quantum network, and the TDMA method has been used to generate large entangled cluster states [37]. However, whether more popular classical communication techniques such as the OFDMA [38,39] and CDMA [40–43] methods can be applied to quantum communication systems is still an interesting problem yet to be solved.

Recently, various protocols were proposed to extend the CDMA model to the quantum case [40–43] and there is evidence showing that the CDMA model can provide higher information rates for quantum communication compared with the FDMA model [40]. In Ref. [40], particular chaotic phase shifts, which work as a unique code in the CDMA model, are introduced to spread the information-bearing quantum signals in the frequency regime. Since the chaotic phase shifts introduced for different users are uncorrelated, the cryptographic quantum signals from different users are orthogonal and thus can be distinguished even when we transmit them via the same

*jing-zhang@mail.tsinghua.edu.cn
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(EOMs) and chaotic Colpitts oscillator circuits [44], which
used in our proposal by introducing electro-optic modulators
pairs of users sharing a single quantum channel. We also
entanglement can be controllably distributed between two
quantum information over a quantum CDMA network, which

broadened by two chaotic phase shifters, i.e., CPS1 and CPS2, at the
side, by which the transmitted quantum information can be
side and that at the receiver side can be exactly canceled, an auxiliary
classical channel between node 1 (node 2) and node 3 (node

FIG. 1. (Color online) Schematic diagram of the quantum
CDMA network by chaotic synchronization. The wave packets are
broaded by two chaotic phase shifters, i.e., CPS1 and CPS2, at the
senders, and then recovered by another two chaotic phase shifters,
i.e., CPS3 and CPS4, at the receiver side.

channel. The cryptographic quantum signals can be decoded
by introducing reversed chaotic phase shifts at the receiver
side, by which the transmitted quantum information can be
recovered coherently. The physical media used in Ref. [40]
to transmit quantum information are single-photon lights [42].

Different from the protocol in Ref. [40], instead of single-
photon lights we use bright coherent lights [43] to transmit
quantum information over a quantum CDMA network, which
is easier to realize in experiments. We find that quantum
entanglement can be controllably distributed between two
pairs of users sharing a single quantum channel. We also
present the particular design of the chaotic phase shifters
used in our proposal by introducing electro-optic modulators
(EOMs) and chaotic Colpitts oscillator circuits [44], which
are not clearly discussed in Ref. [40]. The Pecora-Carrol
synchronization technique [45,46] is introduced to generate the
reverse chaotic phase shifts at the receiver side. This
paper is organized as follows. In Sec. II we present the
general description of the quantum CDMA network we
use to transmit quantum information. In Sec. III we state
how to distribute maximally entangled quantum states over
the proposed quantum CDMA network mediated by bright
coherent lights. In Sec. IV we consider the nonideal case to
see how channel noise will affect our main results. In Sec. V
we summarize and present a forecast of future work.

II. QUANTUM CDMA NETWORK BY CHAOTIC
SYNCHRONIZATION

The main purpose of our work is to generate two maximally
entangled states between two pairs of nodes (one pair consists
of nodes 1 and 3 and the second pair consists of nodes 2 and
4) via a single quantum channel (see Fig. 1). The quantum
signals sent by nodes 1 and 2 are first encoded by two chaotic
phase shifters CPS1 and CPS2 and the two output beams are
combined by a 50:50 beam splitter and then transmitted via
a quantum channel. At the receiver side, this combined quantum
signal is divided into two branches by another 50:50 beam
splitter and sent to another two chaotic phase shifters CPS3
and CPS4 introduced to decode the information. The recovered
quantum signals are then sent to the two receiver nodes.

To understand the encoding and decoding processes of
our method, let us assume that the optical field entering the
i’th chaotic phase shifter is \( a_i \) (\( i = 1, 2, 3, 4 \)). The chaotic
phase shifter CPS1 (CPS2) induces an effective Hamiltonian
\( \delta_1(t)a_i^\dagger a_i \) \( [\delta_2(t)a_i^\dagger a_i^\dagger] \), where \( \delta_1(t) \) \( \delta_2(t) \) is a classical chaotic signal. It can be shown that CPS1 (CPS2) leads to phase-shift factor \( \exp[-i\theta_i(t)] \) \( \exp[i\theta_i(t)] \) for the optical field. At the receiver side, the chaotic phase shifter CPS3 (CPS4) induces the opposite Hamiltonian \( -\delta_1(t)a_i^\dagger a_i \) \( -[\delta_2(t)a_i^\dagger a_i^\dagger] \) by which a reversed phase-shift factor \( \exp[i\theta_i(t)] \) \( \exp[i\theta_i(t)] \) is introduced to decode the information-bearing signal masked
by the chaotic phase shift. Here \( \theta_i = \int_0^t \delta_i(t)dt, \ i = 1, 2 \). To ensure that the chaotic phase shift at the sender side and
that at the receiver side can be exactly canceled, an auxiliary
classical channel between node 1 (node 2) and node 3 (node
4) is introduced to synchronize the two chaotic phase shifters
[47,48] (see Fig. 1).

The whole information transmission process can be repre-
sented by the input-output relationship of the whole quantum
network from \( a_1, a_2 \) to \( a_3, a_4 \) (see Fig. 1). To derive it, we can see that the input-output response of the chaotic phase shifters
CPS1, \( i = 1, 2, 3, 4 \), can be written as

\[
\begin{align*}
a'_1 &= a_1 \exp[-i\theta_1(t)], \\
a'_2 &= a_2 \exp[-i\theta_2(t)], \\
a_3 &= a'_3 \exp[i\theta_3], \\
a_4 &= a'_4 \exp[i\theta_4].
\end{align*}
\]  

and the input-output response of the two beam splitters BS1
and BS2 can be written, respectively, as follows:

\[
\begin{align*}
a_5 &= \frac{1}{\sqrt{2}} a'_1 + \frac{1}{\sqrt{2}} a'_2, \\
a_6 &= \frac{1}{\sqrt{2}} a'_3 - \frac{1}{\sqrt{2}} a'_4, \\
a'_3 &= \frac{1}{\sqrt{2}} a_5 + \frac{1}{\sqrt{2}} a_6, \\
a'_4 &= \frac{1}{\sqrt{2}} a_5 - \frac{1}{\sqrt{2}} a_6.
\end{align*}
\]  

Hence, from Eqs. (1)-(3) we can obtain the input-output
relationship of the whole quantum network

\[
\begin{align*}
a_3 &= \frac{1}{2} a_1 + \frac{1}{2} a_2 \exp[i(\theta_3 - \theta_2)] + \frac{1}{\sqrt{2}} \exp[i\theta_3] a_6, \\
a_4 &= \frac{1}{2} a_2 + \frac{1}{2} a_1 \exp[i(\theta_3 - \theta_2)] - \frac{1}{\sqrt{2}} \exp[i\theta_3] a_6.
\end{align*}
\]  

For the chaotic phase shifts \( \theta_i(t), \ i = 1, 2 \), we should take
the average over these broadband random phases [49], by
which we have \( \exp[i\theta_i(t)] \approx \sqrt{M_i} \) [50–52], where

\[
M_i = \exp[-\pi \int_{o_i}^{o_{oi}} d\omega S_{\delta_i}(\omega)/\omega^2].
\]  

where \( S_{\delta_i}(\omega) \) is the power spectrum density of the signal \( \delta_i(t) \) and \( o_i \) and \( o_{oi} \) are the lower and upper bounds of the frequency
band of \( \delta_i(t) \), respectively. Equation (4) can then be reduced to

\[
\begin{align*}
a_3 &= \frac{1}{2} a_1 + \sqrt{\frac{M_1 M_2}{2}} a_2 + \sqrt{\frac{M_1}{2}} a_6, \\
a_4 &= \frac{1}{2} a_2 + \sqrt{\frac{M_1 M_2}{2}} a_1 - \sqrt{\frac{M_2}{2}} a_6.
\end{align*}
\]
The correction factor $M_i$ may become extremely small when $\delta_i(t)$ is induced by a chaotic signal that has a broadband frequency spectrum. Thus we have $a_3 \approx a_1/2$ and $a_4 \approx a_2/2$, which means that the quantum signal transmitted from node 1 to node 3 and the quantum signal transmitted from node 2 to node 4 can be totally decoupled from each other even though they are transmitted simultaneously on the same quantum channel. The mechanism of such a quantum multiple-access network is that the information-bearing fields transmitted on the quantum channel are broadened by the chaotic phase shifters in the frequency regime, which cannot be detected unless we can reduce the chaotic phase shifts and sharpen the quantum signal by chaotic synchronization. This idea is quite similar to the classical CDMA communication. That is why we call it the quantum CDMA network in Ref. [40].

Now let us consider the case where the quantum fields $a_1$ and $a_2$ are in the coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ and the field $a_{\text{QS}}$ is in a vacuum state. It can be easily checked from the input-output relationship given by Eq. (6) that the output fields $a_3$ and $a_4$ of the quantum network are in the coherent states

$$
|\alpha_3\rangle = |\frac{1}{2}\alpha_1 + \frac{1}{2}\sqrt{M_1M_2}\alpha_2\rangle \approx |\frac{1}{2}\alpha_1\rangle,
$$

$$
|\alpha_4\rangle = |\frac{1}{2}\alpha_2 + \frac{1}{2}\sqrt{M_1M_2}\alpha_1\rangle \approx |\frac{1}{2}\alpha_2\rangle.
$$

### III. QUANTUM ENTANGLEMENT DISTRIBUTION OVER THE QUANTUM CDMA NETWORK

Let us then consider how to distribute two-qubit quantum entanglement over the quantum CDMA network. In our proposal, the qubit states are stored in the dark states of four Λ-type three-level atoms in four optical cavities (see Fig. 2). What we want to do is generate a maximally entangled state between atom 1 (atom 2) and atom 3 (atom 4). Here we extend the strategy in Refs. [53,54] to generate such distributed entangled states by bright coherent lights. The Hamiltonian of the $i$th coupled atom-cavity system can be expressed as

$$
\tilde{H}_i^\text{QC} = \omega_0 a_i^\dagger a_i + \frac{\alpha_0}{2} \sigma^{(i)}_+ \sigma^{(i)}_- + g(a_i^\dagger \sigma^{(i)}_- + a_i \sigma^{(i)}_+),
$$

where $\omega_0$ and $a_i$ ($a_i^\dagger$) are the frequency and the annihilation (creation) operator of the cavity mode; $\alpha_0$, $\sigma^{(i)}_+$, and $\sigma^{(i)}_-$ are the frequency, the $z$-axis Pauli operator, and the ladder operators of the qubit; and $g$ is the coupling strength between the qubit and the cavity mode. Here, to simplify the discussion, we have assumed that the system parameters are the same for four-qubit-cavity systems. Under the dispersive-detuning condition $|\Delta| = |\omega_0 - \omega_1| \gg |\gamma|$, the Hamiltonian can be diagonalized and reexpressed in the interaction picture as [55]

$$
\tilde{H}_i^\text{QC} = \frac{g^2}{\Delta} a_i^\dagger a_i \sigma^{(i)}_z.
$$

In this paper we introduce four EOMs [56] acting as the chaotic phase shifters CPS. It is known that the refractive index of the electro-optic crystal in the EOM can be varied by changing the voltage $V(t)$ acting on it [see Fig. 3(a)]. Based on this effect, we let the information-bearing optical field pass through the EOM to obtain a phase shift $\beta$, which will be changed by varying the voltage $V(t)$ acting on it. This phase shift can be expressed as $\beta = \cos^n(\pi R/ed)V(t)$, where $\omega$ is the frequency of the injected light $n$ is the refractive index and the electro-optic coefficient of the electro-optic crystal in the EOM, $L$ and $d$ are the length and thickness of the EOM [see Fig. 3(a)], respectively, and $c$ is the velocity of light. Therefore, when the optical field transmits through the EOM, an interaction Hamiltonian $H_i = \delta_i(t)a_i^\dagger a_i = -(\hbar/\tau)\beta a_i^\dagger a_i$ [57] can be obtained, where $a_i^\dagger (a_i)$ is the annihilation (creation) operator of the injected field and $\tau$ is the optical round-trip time through the EOM. In the present system, each pair of EOMs is driven by two synchronized standard chaotic Colpitts oscillator circuits, as shown in Fig. 3(b), and the specific synchronized circuit is presented in Appendix A. We use the voltage $V_{c2}$ to drive one EOM at the sender side and the voltage $V_{c2}$ to drive another EOM at the receiver side, as shown in Fig. 3(b).

To show how the quantum entanglement is distributed over our quantum CDMA network, we assume that the $i$th atom is in a superposition state $|\psi_i\rangle = (|g_i\rangle + |e_i\rangle)/\sqrt{2}$ $(i = 1,2,3,4)$. The probe field entering cavity 1 (cavity 2) is a bright coherent light $|\alpha\rangle$ with average photon number $n = |\alpha|^2 \gg 1$. When the probe field comes out of cavity 1 (cavity 2) at time $\tau'$, the system composed of atom 1 and the probe field fed out of cavity 1 is in an entangled state

$$
e^{-i\tilde{H}_i^\text{QC}\tau'} |\psi_1\rangle|\alpha\rangle = \frac{1}{\sqrt{2}} (|g_1\rangle|ae^{-i\phi/2}\rangle + |e_1\rangle|ae^{i\phi/2}\rangle).
$$

Similarly, the system composed of atom 2 and the probe light fed out of cavity 2 is also in an entangled state

$$
e^{-i\tilde{H}_2^\text{QC}\tau'} |\psi_2\rangle|\alpha\rangle = \frac{1}{\sqrt{2}} (|g_2\rangle|ae^{-i\phi/2}\rangle + |e_2\rangle|ae^{i\phi/2}\rangle).
$$

Here $H_1^\text{QC}$ and $H_2^\text{QC}$ are the Hamiltonians given by Eq. (9) and $\phi = 2\pi c'\tau'/\Delta$ is the phase shift of the probe field induced by the qubit-cavity coupling. Thus, the system composed of atoms 1 and 2 and the two probe lights before entering our quantum CDMA network is in a separable state

$$
\frac{1}{2} \left( |g_1g_2\rangle |ae^{-i\phi/2}\rangle |ae^{-i\phi/2}\rangle + |e_1g_2\rangle |ae^{i\phi/2}\rangle |ae^{-i\phi/2}\rangle + |g_1e_2\rangle |ae^{-i\phi/2}\rangle |ae^{i\phi/2}\rangle + |e_1e_2\rangle |ae^{i\phi/2}\rangle |ae^{i\phi/2}\rangle \right).
$$

From Eq. (7), the system composed of atoms 1 and 2 and those two probe fields that enter cavities 3 and 4 is in the state

$$
|\Phi\rangle = \left( \frac{1}{\sqrt{2}} |g_1\rangle \left| \frac{1}{2} ae^{-i\phi/2} \right\rangle + \frac{1}{\sqrt{2}} |e_1\rangle \left| \frac{1}{2} ae^{i\phi/2} \right\rangle \right) \times \left( \frac{1}{\sqrt{2}} |g_2\rangle \left| \frac{1}{2} ae^{-i\phi/2} \right\rangle + \frac{1}{\sqrt{2}} |e_2\rangle \left| \frac{1}{2} ae^{i\phi/2} \right\rangle \right).
$$

Here we have omitted those $\sqrt{M_1M_2}$ terms since the factors $M_1$ and $M_2$ are negligibly small in the chaotic regime. After transmitting over the quantum CDMA network, the probe fields $a_3$ and $a_4$ interact with atoms 3 and 4 and the interaction times are both $\tau'$. Thus, the state of the total system composed of four atoms and the optical fields fed out of the quantum
FIG. 2. (Color online) Schematic diagram of the entanglement distribution over a quantum multiple-access network. The bright coherent lights are sent to two cavities 1 and 2 in which the optical fields interact with atoms 1 and 2, respectively. After that, the two output beams transmit through two EOMs, i.e., EOM1 and EOM2, and are broadened in the frequency domain. The two output beams are then combined by an beam splitter and transmitted through a single quantum channel. At the receiver side, the combined optical field is split into two branches transmit through two EOMs, i.e., EOM3 and EOM4, Since the chaotic circuits driving EOM1 (EOM2) and EOM3 (EOM4) are synchronized, the chaotic phases introduced at the sender side can be compensated at the receiver side and thus the quantum signals transmitted can be recovered. The recovered quantum signals are then stored in the dark states of atoms 3 and 4 and homodyne detections are preformed for the output fields to postselect the maximally entangled states.

The network is

\[ e^{-i(n_1^0 + n_2^0)} \langle \Phi \rangle \frac{1}{2} (|g_3\rangle + |e_3\rangle)(|g_4\rangle + |e_4\rangle) \]

\[ = \left( \frac{1}{\sqrt{2}} |\Psi_{13}^+\rangle \frac{1}{2} |\alpha\rangle + \frac{1}{2} |g_1g_3\rangle \frac{1}{2} |\alpha e^{-i\phi}\rangle \right) \]

\[ + \frac{1}{2} |e_1e_3\rangle \frac{1}{2} |\alpha e^{i\phi}\rangle \right) \times \left( \frac{1}{\sqrt{2}} |\Psi_{24}^+\rangle \frac{1}{2} |\alpha\rangle + \frac{1}{2} |g_2g_4\rangle \frac{1}{2} |\alpha e^{-i\phi}\rangle \right) \]

\[ + \frac{1}{2} |e_2e_4\rangle \frac{1}{2} |\alpha e^{i\phi}\rangle \right), \]  

(12)

where \( |\Psi_{13}^+\rangle = (|g_1e_3\rangle + |e_1g_3\rangle)/\sqrt{2} \) is the maximally entangled state between atom 1 and atom 3 and \( |\Psi_{24}^+\rangle = (|g_2e_4\rangle + |e_2g_4\rangle)/\sqrt{2} \) is the maximally entangled state between atom 2 and atom 4.

Finally, we impose homodyne detections on the probe fields leaking out of cavities 3 and 4. As shown in Eq. (12), the state of the probe fields leaking out of cavities 3 and 4 can be three possible states \( |\alpha/2\rangle, |\alpha e^{-i\phi}/2\rangle, \) and \( |\alpha e^{i\phi}/2\rangle \). Since the probe fields are bright coherent lights with average photon number \( \bar{n} = |\alpha|^2 \gg 1 \), we have

\[ \langle |\alpha/2| \frac{1}{2} |\alpha e^{-i\phi}\rangle |^2 = \exp[-\bar{n} \sin^2(\phi/2)] \approx 0, \]

\[ \langle |\alpha e^{-i\phi}/2| \frac{1}{2} |\alpha e^{i\phi}\rangle |^2 = \exp[-\bar{n} \sin^2(\phi/2)] \approx 0, \]

\[ \langle |\alpha e^{i\phi}/2| \frac{1}{2} |\alpha e^{-i\phi}\rangle |^2 = \exp(-\bar{n} \sin^2 \phi) \approx 0, \]

which means that the three coherent states \( |\alpha/2\rangle, |\alpha e^{-i\phi}/2\rangle, \) and \( |\alpha e^{i\phi}/2\rangle \) are pairwise orthogonal and thus completely distinguishable. Thus, the homodyne detections on the probe fields are just projective measurements. Corresponding to the three measurement outputs \( |\alpha/2\rangle, |\alpha e^{i\phi}/2\rangle, \) and \( |\alpha e^{-i\phi}/2\rangle \), the states of the system composed of atoms 1 and 3 (atoms 2 and 4) collapse to the maximally entangled state \( |\Psi_{13}^+\rangle (|\Psi_{24}^+\rangle) \) and two separable states \( |g_1g_3\rangle (|g_2g_4\rangle) \) and \( |e_1e_3\rangle (|e_2e_4\rangle) \). The most important case is that the measurement outputs of the probe fields leaking out of cavities 3 and 4 are both \( |\alpha/2\rangle \). In this case, atoms 1 and 3 are in the maximally entangled state \( |\Psi_{13}^+\rangle \) and atoms 2 and 4 are in the maximally entangled state \( |\Psi_{24}^+\rangle \), which means that we generate two maximally entangled states between two pairs of nodes by sharing the same quantum channel.

We now consider the interference effects between the quantum signals from the two pairs of users. These interference effects have been omitted in our previous discussion under the condition that the correction factor \( M = M_1M_2 \) is negligibly small if the chaotic phases introduced have very broad bandwidths. However, these interference effects will affect the information transmission process if the bandwidths of the phase signals are not broad enough. To show this, let us consider how the correction factors \( M_1 \) and \( M_2 \) will change if we tune the corresponding bandwidths of the chaotic signals \( \delta_1 \) and \( \delta_2 \). From Fig. 4(a) we can see that both \( M_1 \) and \( M_2 \) decrease with an increase of the bandwidth of \( \delta_1 \) and \( \delta_2 \) and when we take the bandwidth values of the chaotic signals as 450 MHz, \( M_1 \) and \( M_2 \) are 0.0012 and 0.0033, respectively. With experimentally realizable parameters of the Colpitts chaotic circuits [45,46], it is not too difficult to generate a
FIG. 3. (Color online) (a) Schematic diagram of a transverse electro-optic modulator. The voltage is applied perpendicular to the propagational direction of the input beam and the refractive index of the electro-optic crystal can be changed by varying the voltage $V$, which induces a voltage-dependent phase shift on the input beam. (b) Diagram of the chaotic synchronization circuit between CPS$_1$ and CPS$_3$, where the transmitter drives EOM$_1$ and the receiver drives EOM$_3$.

chaotic phase with bandwidth of 500 MHz and thus $M_1$ and $M_2$ can be very small. Meanwhile, we choose the average photon number $\bar{n} = 10$ and thus we have $M_1M_2 \ll 4/\bar{n}$. This makes it reasonable to omit the $\sqrt{M_1M_2}$ terms in Eqs. (6), (7), and (14). In order to check whether the phase shifts induced by the phase shifters are in the chaotic regime, we show in Fig. 4(b) the Lyapunov exponents of Colpitts circuits with different bandwidths. When the bandwidths of the Colpitts circuits are smaller than 100 MHz, the Lyapunov exponents of the Colpitts circuits are equal to zero, which means that these circuits work in the periodic regime. If we increase the bandwidths of the circuits, the Colpitts circuits will then enter the chaotic regime if the bandwidths are larger than 100 MHz, which corresponds to positive Lyapunov exponents [see Fig. 4(b)]. This is also confirmed by the phase diagrams and the power spectra of the circuits with a bandwidth of 100 MHz shown in Figs. 4(c) and 4(d) and those of the circuits with a bandwidth of 500 MHz in Figs. 4(e) and 4(f).

In order to show the efficiency of the entanglement distribution by the quantum CDMA network, we show in Fig. 5 the fidelities $F_1 = \langle \Psi_{13}^+ | \rho_{13} | \Psi_{13}^+ \rangle$ (see Appendix B) and $F_2 = \langle \Psi_{24}^+ | \rho_{24} | \Psi_{24}^+ \rangle$ versus the bandwidths of the chaotic signals and the average photon number of the probe fields $\bar{n}$, where $|\Psi_{13}^+\rangle$ ($|\Psi_{24}^+\rangle$) is the desired maximally entangled state between atoms 1 and 3 (atoms 2 and 4) and $\rho_{13}$ ($\rho_{24}$) chaotic phase with bandwidth of 500 MHz and thus $M_1$ and $M_2$ can be very small. Meanwhile, we choose the average photon number $\bar{n} = 10$ and thus we have $M_1M_2 \ll 4/\bar{n}$. This makes it reasonable to omit the $\sqrt{M_1M_2}$ terms in Eqs. (6), (7), and (14). In order to check whether the phase shifts induced by the phase shifters are in the chaotic regime, we show in Fig. 4(b) the Lyapunov exponents of Colpitts circuits with different bandwidths. When the bandwidths of the Colpitts circuits are smaller than 100 MHz, the Lyapunov exponents of the Colpitts circuits are equal to zero, which means that these circuits work in the periodic regime. If we increase the bandwidths of the circuits, the Colpitts circuits will then enter the chaotic regime if the bandwidths are larger than 100 MHz, which corresponds to positive Lyapunov exponents [see Fig. 4(b)]. This is also confirmed by the phase diagrams and the power spectra of the circuits with a bandwidth of 100 MHz shown in Figs. 4(c) and 4(d) and those of the circuits with a bandwidth of 500 MHz in Figs. 4(e) and 4(f).

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is the density operator of atoms 1 and 3 (atoms 2 and 4). In our simulations, we set $\phi = \pi/3$, where $\phi$ is the phase shift of the probe fields induced by the qubit-cavity coupling. The trajectories of the fidelity $F_1$ versus the correction factor $M$ and the average photon number $\bar{n}$ are given in Fig. 5(a). We can see clearly that the fidelity $F_1$ can be very high if the factor $M$ is small enough and $\bar{n}$ is not too small, which corresponds to our previous analysis. If we fix $\bar{n} = 10$ and increase the bandwidth of the signals, we can see the increase of the fidelities $F_1$ and $F_2$ as desired [see Fig. 5(b)]. Both $F_1$ and $F_2$ grow very quickly to approach 1 with the increase of the bandwidths of the signals to be larger than 400 MHz, which corresponds to a perfect entanglement distribution.

IV. NONIDEAL CASE: EFFECTS OF THE CHANNEL NOISE

In the previous sections we considered the ideal case in which the channel noises are omitted. To show the efficiency of our method in a more practical case, we consider the effects of the channel noises in this section [58,59]. The channel noises in quantum communication may come from different sources such as the vibration of the optical fiber used for transmitting quantum signals. Most of the channel noises, especially those induced by the fibers, are low-frequency noises with several to several hundreds of kHz, which is far smaller than the characteristic frequency of the information-bearing fields and also smaller than the frequency band of the chaotic phase shifts introduced by the chaotic circuits, which is typically of several hundreds MHz. For these reasons, we can omit the dynamical processes of the channel noises and simply believe that they act as a beam splitter to extract energy from the information-bearing field (see Fig. 6). As shown in Fig. 6, the input-output relationship of the beam splitter BS$_3$ used to represent the effects of the channel noises can be written as

$$a_7 = \sqrt{1 - \eta a_5} + \sqrt{\eta} a_{\text{ns}}, \quad a_8 = \sqrt{1 - \eta a_5} - \sqrt{\eta} a_{\text{ns}},$$  \hspace{1cm} (13)

where $a_{\text{ns}}$ represents the noise mode and $\eta$ denotes the decay rate induced by the noise. From Eqs. (1)–(6) and (13) we can obtain the input-output relationship of the noisy quantum CDMA network as

$$a_3 = \frac{\sqrt{1 - \eta}}{2} a_1 + \frac{\sqrt{1 - \eta} M_1 M_2}{2} a_2 + \frac{\eta M_1}{2} a_{\text{ns}} + \frac{\sqrt{M_1}}{2} a_{\text{BS}},$$

$$a_4 = \frac{\sqrt{1 - \eta}}{2} a_2 + \frac{\sqrt{1 - \eta} M_1 M_2}{2} a_1 + \frac{\eta M_2}{2} a_{\text{ns}} - \frac{\sqrt{M_2}}{2} a_{\text{BS}}.$$  \hspace{1cm} (14)

If we further assume that the field $a_{\text{ns}}$ is in a vacuum state, the output fields of the quantum CDMA network, i.e., $a_3$ and $a_4$, are in the coherent states

$$|\alpha_3\rangle = \left|\frac{\sqrt{1 - \eta}}{2} a_1 + \frac{\sqrt{1 - \eta} M_1 M_2}{2} a_2\rightangle,$$

$$|\alpha_4\rangle = \left|\frac{\sqrt{1 - \eta}}{2} a_2 + \frac{\sqrt{1 - \eta} M_1 M_2}{2} a_1\rightangle.$$  \hspace{1cm} (14)

Recall that the $i$th atom is in the superposition state $|\psi_i\rangle = (|g_i\rangle + |e_i\rangle)/\sqrt{2}$ and the probe field entering the cavity 1 (cavity 2) is a bright coherent light $|\alpha\rangle$ with average photon number $\bar{n} = |\alpha|^2 \gg 1$. By omitting the terms related to $\sqrt{M_1 M_2}$ in Eq. (14), we can easily obtain the state of the total system composed of the four atoms and output fields of the quantum network as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left( |\psi_1\rangle_3 |\sqrt{1 - \eta}/2\alpha\rangle + \frac{1}{2} |g_1 g_3\rangle |\sqrt{1 - \eta}/2\alpha e^{-i\phi}\rangle\right) + \frac{1}{\sqrt{2}} \left( |\psi_4\rangle_2 |\sqrt{1 - \eta}/2\alpha\rangle + \frac{1}{2} |g_2 g_4\rangle |\sqrt{1 - \eta}/2\alpha e^{-i\phi}\rangle\right).$$

If the decay rate $\eta$ induced by the channel noise is not quite high and the probe fields are bright enough with average photon number $\bar{n} = |\alpha|^2 \gg 1/(1 - \eta)$, we have

$$\left|\frac{\sqrt{1 - \eta}}{2}\alpha |\sqrt{1 - \eta}/2\alpha e^{-i\phi}\rangle\right|^2 = e^{-(1-\eta)\bar{n}} \sin^2(\phi/2) \approx 0,$$

$$\left|\frac{\sqrt{1 - \eta}}{2}\alpha |\sqrt{1 - \eta}/2\alpha e^{i\phi}\rangle\right|^2 = e^{-(1-\eta)\bar{n}} \sin^2(\phi/2) \approx 0,$$

$$\left|\frac{\sqrt{1 - \eta}}{2}\alpha e^{-i\phi} |\sqrt{1 - \eta}/2\alpha e^{i\phi}\rangle\right|^2 = e^{-(1-\eta)\bar{n}} \sin^2 \phi \approx 0,$$

which means that the three coherent states $|\sqrt{1 - \eta}\alpha/2\rangle$, $|\sqrt{1 - \eta}\alpha e^{-i\phi}/2\rangle$, and $|\sqrt{1 - \eta}\alpha e^{i\phi}/2\rangle$ are pairwise orthogonal and thus completely distinguishable. Thus we can impose homodyne detections on the fields leaking out of cavities 3 and 4. If the corresponding measurement outputs for the two probe fields are both $\sqrt{1 - \eta}\alpha/2$, the state of atoms 1 and 3

[Diagram of quantum CDMA network with channel noise and beam splitters.]
will collapse to the maximally entangled states $|\Psi_{13}^+\rangle$ and that of atoms 2 and 4 will collapse to the maximally entangled states $|\Psi_{24}^+\rangle$. From the above discussion we can conclude that our method is still valid if only the decay rate induced by the channel noise is not too high such that the decayed probe fields are still bright enough.

However, if the decay rate $\eta$ induced by the channel noises is too high such that the average photon number $\bar{n} = |\alpha|^2$ is comparable to $1/(1-\eta)$, our entanglement distribution strategy will not be so perfect. In this case, we need to analyze the influence of noise on the fidelities $F_1$ and $F_2$. Without loss of generality, let us focus on the fidelity $F_1 = \langle \Psi_{13}^+ | \rho_{13} | \Psi_{13}^+ \rangle$ versus different decay rate $\eta$ and average photon number $\bar{n}$. The discussion for the fidelity $F_2$ is quite similar and thus is omitted. We still choose $\phi = \pi/3$ and assume that the correction factors $M_1 = M_2 \approx 0$. With these system parameters, we show in Fig. 7 how the decay rate $\eta$ affects the entanglement distribution. As can be seen from Fig. 7(a), fidelity $F_1$ can be very high when $\eta$ is small and the average photon number is not large. Figure 7(b) shows the curves of the fidelity $F_1$ versus $\eta$ for several different cases. The red solid curve represents the ideal case, i.e., $F_1 = 1$, which means that atoms 1 and 3 are in the maximally entangled state. The black dashed curve with plus signs denotes the trajectory of the fidelity $F_1$ with increasing $\eta$ ranging from 0 to 1. The blue dash-dotted curve with asterisks shows the fidelity $F_1$ versus $\eta$ without the four EOMs in our quantum CDMA network. By comparing these three curves, we can see that the fidelity $F_1$ will be greatly decreased if we move away the four EOMs in our quantum CDMA network. Meanwhile, with EOMs we can obtain a very high fidelity when the decay rate $\eta$ is not too high.

V. CONCLUSION

We have presented a strategy to distribute quantum entanglement between two pairs of users via a single quantum channel. The interference of the quantum signals from different senders is greatly suppressed by introducing chaotic phase shifts to broaden the quantum signals in the frequency domain. It was shown that the two maximally entangled states can be generated between two pairs of nodes even when we consider the channel noises. It is hoped that our strategy could be applied to other systems such as solid-state quantum circuits and it would provide new perspectives for the field of quantum network control.

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APPENDIX A: CHAOTIC SYNCHRONIZATION OF COLPITTS OSCILLATOR CIRCUITS

In the system we consider, each pair of EOMs is driven by two standard chaotic Colpitts oscillator circuits [44], which are synchronized by the Pecora-Carroll synchronization strategy [45,46], as shown in Fig. 8. The Colpitts chaotic synchronization circuit comprises a transmitter and a receiver. The transmitter is a standard Colpitts oscillator circuit, which will enter the chaotic regime for particular system parameters. The receiver is a part of the standard Colpitts oscillator circuit. In our design, the system parameters of the Colpitts oscillator circuits, such as the resistance $R$, the inductance $L$, the capacitances $C_1$ and $C_2$, and the voltage $V_{CC}$, are chosen as $R = 27.99\Omega$, $L = 17.5$ nH, $C_1 = 13.1$ pF, $C_2 = 12.7$ pF, and $V_{CC} = 15$ V, under which the synchronized voltages $V_{c2}$ and $\tilde{V}_{c2}$ are broadband chaotic signals with bandwidths of 500 MHz [45,46].

![Diagram of synchronized chaotic Colpitts circuits](image)

FIG. 8. (Color online) Schematic diagram of the synchronized chaotic Colpitts circuits, which is composed of a transmitter and a receiver. Here we adopt the Pecora-Carroll synchronization strategy to synchronize the two chaotic Colpitts circuits [47].
In this Appendix we present the calculations of the fidelity $F_1 = \langle \Psi_{13}^+ | \rho_{13} | \Psi_{13}^+ \rangle$. The calculations of the fidelity $F_2 = \langle \Psi_{24}^+ | \rho_{24} | \Psi_{24}^+ \rangle$ are similar to $F_1$, so we omit them. From Eq. (7) we see that if we consider the effect of the correction factor $M$, the state of the system composed of atoms 1 and 2 and those two probe fields that enter cavities 3 and 4 is

$$|\Phi\rangle = \frac{1}{2} |g_1 g_2\rangle \left(\frac{1}{2} a^\dagger e^{-i\phi/2} + \frac{1}{2} M a e^{-i\phi/2}\right) |\psi_1\rangle + \frac{1}{2} |e_1 g_2\rangle \left(\frac{1}{2} a e^{i\phi/2} + \frac{1}{2} M a^\dagger e^{i\phi/2}\right) |\psi_3\rangle + \frac{i}{2} |e_1 e_2\rangle \left(\frac{1}{2} a e^{i\phi/2} + \frac{1}{2} M a^\dagger e^{i\phi/2}\right) |\psi_2\rangle + \frac{i}{2} |e_1 e_2\rangle \left(\frac{1}{2} a^\dagger e^{-i\phi/2} + \frac{1}{2} M a e^{-i\phi/2}\right) |\psi_4\rangle.$$ 

Then we can obtain the state of the total system composed of the four atoms and output fields of the quantum network as

$$|\Psi\rangle = e^{-i \left(\frac{\hbar}{\sqrt{2}} N_{14}^{\text{QC}}\right)} |\Phi\rangle \frac{1}{\sqrt{2}} (|g_3\rangle + |e_3\rangle) \frac{1}{\sqrt{2}} (|g_4\rangle + |e_4\rangle).$$

After the homodyne detections imposed on the probe fields leaking out of cavities 3 and 4, we can obtain the density operator $\rho_{13}$ of atoms 1 and 3 by tracing out the degrees of freedom of atom 2, atom 4, and probe fields 3 and 4 by which the fidelity $F_1$ can be expressed as

$$F_1 = \langle \Psi_{13}^+ | \rho_{13} | \Psi_{13}^+ \rangle$$

$$= \frac{1}{2} \left(\langle g_1 g_3 | g_1 g_3 \rangle \langle g_1 e_3 | g_1 e_3 \rangle \langle e_1 g_3 | e_1 g_3 \rangle \right)$$

$$= \frac{1}{2} + \frac{i}{2} e^{-i |M| (1 - \cos \phi)/2},$$

where $|\Psi_{13}^+\rangle$ is the maximally entangled state between atom 1 and atom 3 and the Hamiltonian $H_{\text{QC}}^{(1)}$ is given in Eq. (9). From Eq. (5) we can calculate the correction factor $M$ with respect to the bandwidths of the Colpitts circuits [see Fig. 4(a)], by which we can obtain the curve of $F_1$ versus the bandwidths of the Colpitts circuits.
