Enhancement of mechanical effects of single photons in modulated two-mode optomechanics

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We propose an approach to enhance the mechanical effects of single photons in a two-mode optomechanical system. This is achieved by introducing a resonance-frequency modulation to the cavity fields. When the modulation frequency and amplitude satisfy certain conditions, the mechanical displacement induced by single photons could be larger than the quantum zero-point fluctuation of the oscillating resonator. This method can be used to create distinct mechanical superposition states.

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I. INTRODUCTION

One of the most interesting regimes in cavity quantum optomechanics [1,2] occurs when the radiation pressure of a single photon is strong enough to push a mechanical oscillator with a displacement greater than the oscillator's zero-point fluctuations before escaping the cavity. In such a regime, one can generate distinct superposition states of a macroscopic object (the oscillator) [3,4], and the quantum nonlinearity in the ultrastrong optomechanical coupling regime can lead to various quantum effects, including photon blockade [5–8], non-Gaussian states of the mechanical oscillator [9–13], modified spectra of photon scattering [6,14], and dressed-state-representation dissipations [15]. Specifically, this regime is defined by the condition

$$g_0 > \omega_M \gg \kappa_c, \tag{1}$$

where g_0 is the single-photon optomechanical coupling strength, ω_M is the frequency of the mechanical oscillator, and κ_c is the cavity field damping rate.

The requirement of $g_0 > \omega_M$ in (1) is understood from a generic single-mode time-independent optomechanical system with the Hamiltonian $\mathcal{H} = \omega_c a^\dagger a + \omega_M b^\dagger b - g_0 a^\dagger a(b + b^\dagger)$, where a (b) and $a^\dagger$ ($b^\dagger$) are the annihilation and creation operators of the cavity field (mechanical oscillator). By solving the Schrödinger equation, one finds that the oscillator, when driven by a single photon, can evolve to a coherent state $|2g_0/\omega_M\rangle$ from the ground state after a time $t = \pi/\omega_M$. Therefore, $g_0 > \omega_M$ provides a significant displacement away from the ground state.

However, the realization of condition (1) has been challenging for current experiments because g_0 is small in most optomechanical systems, and ω_M has to be sufficiently high to overcome thermal noise effects. In particular, the quantum limit of phonon number n_b obtained by resolved sideband cooling is $n_b = \kappa_c^2/(16\omega_M^2)$ [16–20]. It should be noted that condition (1) is based on the single-mode time-independent Hamiltonian $\mathcal{H}$, and thus an interesting question is whether it is possible to achieve ultrastrong coupling effects in multimode and time-dependent systems. Indeed, a similar question has been explored in electromechanical systems recently [21,22], and it is found that a suitable modulation can be used to effectively enhance photonic nonlinearities in electromechanical systems. In addition, an interferometric scheme has been proposed to detect optomechanical coherent interaction at the single-photon level [23].

The methods of enhancing radiation pressure effects of a single photon in optomechanical systems have been recently discussed by several authors [24–28]. In this paper, we approach this problem by modulating the photon-tunneling process in the "membrane-in-the-middle" (MIM) configuration. The MIM optomechanical system has been studied experimentally [29–37] and theoretically [38–44]. Here we show that by changing the frequency of the cavities in a time-dependent manner, the photon tunneling oscillation can be modulated. We discover a set of conditions such that a single photon can significantly displace the membrane, even though g_0/ω_M is much less than 1. Below we derive an effective Hamiltonian of the system and present numerical evidence of the ultrastrong coupling effect.

II. THE MODEL

We consider an MIM optomechanical system (Fig. 1) in which a vibrational mode of the membrane is coupled to the left and right cavity field modes via the radiation pressure difference between the two cavities, and tunneling of photons through the membrane is allowed. The Hamiltonian of the system [38–44] is modeled by ($\hbar = 1$)

$$\begin{aligned} H = {} & \omega_L a_L^\dagger a_L + \omega_R a_R^\dagger a_R + J(a_L^\dagger a_R + a_R^\dagger a_L) \\ & + \omega_M b^\dagger b + g_0(a_L^\dagger a_L - a_R^\dagger a_R)(b + b^\dagger), \end{aligned} \tag{2}$$

where $a_{L(R)}$ and b are the annihilation operators of the left (right) cavity and the mechanical membrane, with the respective resonance frequencies $\omega_{L(R)}$ and ω_M. The parameters J and g_0 are the coupling strengths of the photon hopping and the optomechanical interaction, respectively.

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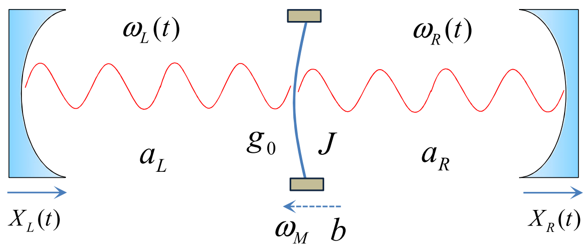

FIG. 1. (Color online) Schematic of the optomechanical system with the "membrane-in-the-middle" configuration. The cavity fields couple with the mechanical oscillation via the radiation pressure coupling g_0, and the optical fields in the left and right cavities couple to each other via the photon-hopping interaction J. To realize the cavity-frequency modulation in Eq. (3), the two outside mirrors could be forced to oscillate with the same displacements $X_L(t) = X_R(t) = (\Delta_0/\omega_c)L\cos(2Jt)$ by using an external device such as a piezoelectric transducer.

A key feature in our model is the introduction of a weak frequency modulation to the left and the right cavities such that ω_L and ω_R in Eq. (2) are given by

$$\omega_L(t) = \omega_c + \Delta_0\cos(2Jt), \tag{3a}$$

$$\omega_R(t) = \omega_c - \Delta_0\cos(2Jt), \tag{3b}$$

where the modulation amplitude Δ_0 is assumed to be much smaller than J. For simplicity, Δ_0 and J are taken to be positive numbers. The sinusoidal time dependence of the cavity-field frequencies could be realized by modulating the lengths of the cavities. When the lengths of the left and right cavities change as $L - (\Delta_0/\omega_c)L\cos(2Jt)$ and $L + (\Delta_0/\omega_c)L\cos(2Jt)$, respectively, then the frequencies of the two cavities are modulated as in Eq. (3). Here L is the cavity length when the membrane and the outside mirrors are at rest. We assume that the motion of two outside mirrors is prescribed classically, and the back-action on such mirrors is neglected.

To realize the modulation of the cavity lengths, one could use piezoelectric transducers, which convert the electrical signal to mechanical vibration [45]. We could also mount the end mirrors on mechanical resonators, such as cantilevers, which oscillate periodically. In addition, one could shake the outside mirrors by shining each of them with two lasers of frequency difference $2J$, since the beating between the two lasers yields a radiation pressure force modulated at a frequency $2J$ [46].

The evolution of the system state $|\psi(t)\rangle$ is governed by the Schrödinger equation $H(t)|\psi(t)\rangle = id|\psi(t)\rangle/dt$. To study how the frequency modulation affects the quantum dynamics, it is useful to introduce the transformation $|\psi(t)\rangle = T(t)|\tilde{\psi}(t)\rangle$, where $T(t)$ is a unitary operator defined by $T(t) \equiv V_1(t)V_2(t)V_3(t)$ with

$$V_1(t) = \exp[-i\omega_c t(a_L^\dagger a_L + a_R^\dagger a_R)], \tag{4a}$$

$$V_2(t) = \exp[-iJt(a_L^\dagger a_R + a_R^\dagger a_L)], \tag{4b}$$

$$V_3(t) = \exp\left\{-i\left[\frac{\Delta_0}{2}(a_L^\dagger a_L - a_R^\dagger a_R) + \omega_M b^\dagger b\right]t\right\}. \tag{4c}$$

The evolution of $|\tilde{\psi}(t)\rangle$ is governed by the transformed Hamiltonian $\tilde{H}(t) = T^\dagger(t)HT(t) - iT^\dagger(t)dT(t)/dt$, which has the form

$$\begin{aligned}\tilde{H}(t) = {} & \frac{\Delta_0}{4}(a_L^\dagger a_L - a_R^\dagger a_R)(e^{4iJt} + e^{-4iJt}) \\ & + \frac{\Delta_0}{4}[a_R^\dagger a_L(e^{i(4J-\Delta_0)t} - e^{-i(4J+\Delta_0)t}) + \text{H.c.}] \\ & + \frac{g_0}{2}(a_L^\dagger a_L - a_R^\dagger a_R)(be^{-i(\omega_M-2J)t} + b^\dagger e^{i(\omega_M-2J)t}) \\ & + \frac{g_0}{2}(a_L^\dagger a_L - a_R^\dagger a_R)(be^{-i(\omega_M+2J)t} + b^\dagger e^{i(\omega_M+2J)t}) \\ & + \frac{g_0}{2}[a_L^\dagger a_R b^\dagger(e^{i[\Delta_0+(\omega_M-2J)]t} - e^{i[\Delta_0+(\omega_M+2J)]t}) \\ & + a_L^\dagger a_R b(e^{i[\Delta_0-(\omega_M+2J)]t} - e^{i[\Delta_0-(\omega_M-2J)]t}) + \text{H.c.}].\end{aligned} \tag{5}$$

The motivation for performing the above transformation is that the parameters ω_M and J, which are typically much greater than g_0 and Δ_0, all appear in oscillating phase factors. The form of $\tilde{H}(t)$ allows us to identify fast oscillating terms and perform the rotating-wave approximation. Specifically, under the conditions

$$|\omega_M - 2J| \leqslant \frac{g_0}{2}, \quad J \gg \frac{5}{16}\Delta_0, \quad \Delta_0 \gg g_0, \tag{6}$$

we may retain only the slowly oscillating terms $\frac{g_0}{2}(a_L^\dagger a_L - a_R^\dagger a_R)[be^{-i(\omega_M-2J)t} + b^\dagger e^{i(\omega_M-2J)t}]$ in Eq. (5), and discard all other terms. This is justified within the rotating-wave approximation, because in order to discard those terms, their oscillation frequencies must be much larger than their corresponding coupling coefficients under conditions (6). Hence Eq. (5) is approximated by

$$\tilde{H}(t) \approx \frac{g_0}{2}(a_L^\dagger a_L - a_R^\dagger a_R)(be^{-i(\omega_M-2J)t} + \text{H.c.}). \tag{7}$$

Such a Hamiltonian can be cast into a familiar form in a rotating frame defined by $|\tilde{\psi}(t)\rangle = S(t)|\tilde{\psi}'(t)\rangle$, with the unitary operator $S(t) = \exp\{i[\omega_c(a_L^\dagger a_L + a_R^\dagger a_R) + (\omega_M - 2J)b^\dagger b]t\}$. The transformed Hamiltonian $\tilde{H}' = S^\dagger(t)\tilde{H}(t)S(t) - iS^\dagger(t)dS(t)/dt$ then reads

$$\begin{aligned}\tilde{H}' \approx {} & \omega_c(a_L^\dagger a_L + a_R^\dagger a_R) + (\omega_M - 2J)b^\dagger b \\ & + \frac{g_0}{2}(a_L^\dagger a_L - a_R^\dagger a_R)(b + b^\dagger).\end{aligned} \tag{8}$$

The right side is precisely the same form of the Hamiltonian of the MIM optomechanical system without the tunneling term, but with an effective mechanical frequency $\omega_M - 2J$, which is significantly reduced. Consequently, the mechanical displacement induced by a single photon, which is now proportional to $g_0/[2(\omega_M - 2J)]$, can be significant.

As a remark, our model can be analyzed by working in the Schwinger representation. Define the angular momentum operators $S_x = (a_L^\dagger a_R + a_R^\dagger a_L)/2$, $S_y = i(a_R^\dagger a_L - a_L^\dagger a_R)/2$, and $S_z = (a_L^\dagger a_L - a_R^\dagger a_R)/2$, then the Hamiltonian (2), up to a constant-of-motion term $\omega_c(a_L^\dagger a_L + a_R^\dagger a_R)$, can be expressed as

$$H = 2\Delta_0 \cos(2Jt)S_z + 2JS_x + \omega_M b^\dagger b + 2g_0 S_z(b + b^\dagger), \tag{9}$$

which is the quantum Rabi model for $S = 1/2$ (single-photon case) and $\Delta_0 = 0$. The Δ_0 term corresponds to a driving term. It is worth noting that the quantum Rabi model in the ultrastrong-coupling regime, which is difficult to be realized in typical systems, can be simulated by reducing the effective qubit frequency [47].

III. SINGLE-PHOTON MECHANICAL DISPLACEMENT

Let us consider an initial state of the system in which the membrane is in the ground state and a photon is in the left cavity mode,

$$|\psi(0)\rangle = |\tilde{\psi}'(0)\rangle = |1\rangle_L|0\rangle_R|0\rangle_M. \tag{10}$$

Then, by using the approximate Hamiltonian (8), we have

$$|\psi(t)\rangle \approx T(t)S(t)e^{-i\tilde{H}'t}|\tilde{\psi}'(0)\rangle \equiv |\psi_{\text{approx}}(t)\rangle \tag{11}$$

after transforming back to the original frame. Explicitly, the approximate state $|\psi_{\text{approx}}(t)\rangle$ is given by

$$\begin{aligned}|\psi_{\text{approx}}(t)\rangle = e^{-i\theta(t)}e^{-i\omega_c t}[&\cos(Jt)|1\rangle_L|0\rangle_R \\ &- i\sin(Jt)|0\rangle_L|1\rangle_R]|\beta(t)\rangle_M,\end{aligned} \tag{12}$$

where $|\beta(t)\rangle_M$ is a coherent state of the membrane, and $\theta(t)$ and $\beta(t)$ are given by

$$\theta(t) = \left(\frac{\Delta_0}{2} - \frac{g_0^2}{4(\omega_M - 2J)}\right)t + \frac{g_0^2 \sin[(\omega_M - 2J)t]}{4(\omega_M - 2J)^2}, \tag{13a}$$

$$\beta(t) = \frac{g_0}{2(\omega_M - 2J)}(e^{-i\omega_M t} - e^{-2iJt}). \tag{13b}$$

Hence the membrane can reach a maximum displacement $g_0/(\omega_M - 2J)$ at the time $t_s = \pi/(\omega_M - 2J)$, assuming $\omega_M > 2J$. A large displacement amplitude of the membrane can be obtained if $2J$ is close to ω_M. In particular, at $2J = \omega_M$, the displacement amplitude in Eq. (13b) takes the form $\beta(t) = -ig_0 t \exp(-i\omega_M t)/2$, which grows linearly in time until it becomes too large that Hamiltonian (2) breaks down. This is in contrast to unmodulated systems (i.e., $\Delta_0 = 0$) in which the membrane can gain less than one phonon for the same initial condition [48].

To check the validity of our approximation, we performed a numerical calculation of the state $|\psi(t)\rangle$ with the Hamiltonian H in Eq. (2) subjected to the frequency modulation defined by Eq. (3) and the same initial condition as above. The numerically exact $|\psi(t)\rangle$ is then compared with the $|\psi_{\text{approx}}(t)\rangle$ of Eq. (12). Specifically, we examine the fidelity $F(t) = |\langle\psi(t)|\psi_{\text{approx}}(t)\rangle|^2$.

We remark that in solving numerically the Schrödinger equation using the Hamiltonian (2) and the initial state $|\psi(0)\rangle = |1\rangle_L|0\rangle_R|0\rangle_M$, we express the system state at later time as

$$|\psi(t)\rangle = \sum_{m=0}^{\infty}[A_m(t)|1\rangle_L|0\rangle_R + B_m(t)|0\rangle_L|1\rangle_R]|m\rangle_M. \tag{14}$$

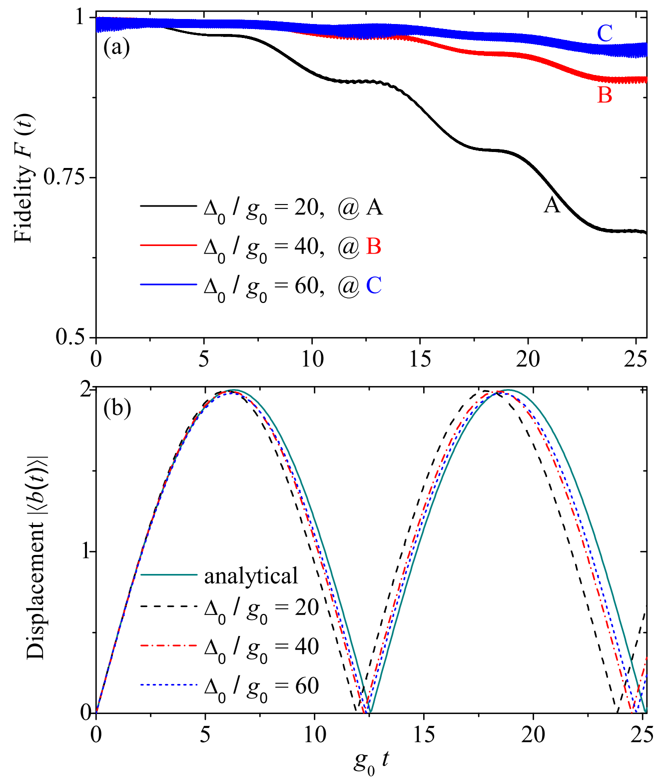

FIG. 2. (Color online) (a) The fidelity $F(t)$ versus the scaled time $g_0 t$ when the parameter Δ_0/g_0 takes various values. (b) Comparison of the mechanical displacements $|\langle b(t)\rangle|$ in the numerical case (for $\Delta_0/g_0 = 20$, 40, and 60) with that in the analytical case. Other parameters are given by $\omega_M/g_0 = 201$ and $J/g_0 = 100.25$.

Then, by the Schrödinger equation, the probability amplitudes A_m and B_m are governed by the following set of coupled differential equations:

$$\begin{aligned}\dot{A}_m(t) = &-i[\omega_c + m\omega_M + \Delta_0\cos(2Jt)]A_m(t) - iJB_m(t) \\ &- ig_0[\sqrt{m+1}A_{m+1}(t) + \sqrt{m}A_{m-1}(t)],\end{aligned} \tag{15a}$$

$$\begin{aligned}\dot{B}_m(t) = &-i[\omega_c + m\omega_M - \Delta_0\cos(2Jt)]B_m(t) - iJA_m(t) \\ &+ ig_0[\sqrt{m+1}B_{m+1}(t) + \sqrt{m}B_{m-1}(t)],\end{aligned} \tag{15b}$$

which are solved numerically. The fidelity between the two states $|\psi(t)\rangle$ and $|\psi_{\text{approx}}(t)\rangle$ can be obtained as

$$\begin{aligned}F(t) &= |\langle\psi(t)|\psi_{\text{approx}}(t)\rangle|^2 \\ &= \left|e^{-\frac{|\beta(t)|^2}{2}}\sum_{m=0}^{\infty}\frac{\beta^m(t)}{\sqrt{m!}}[\cos(Jt)A_m^*(t) - i\sin(Jt)B_m^*(t)]\right|^2.\end{aligned} \tag{16}$$

In Fig. 2(a), we plot the fidelity as a function of the evolution time for various values of Δ_0/g_0. We see that a good fidelity [i.e., $F(t) \approx 1$] can be obtained for larger values of Δ_0/g_0, which is in accordance with the condition in Eq. (6). The fidelity curves exhibit some fast oscillations, which are caused by the time factors $\exp(\pm i2Jt)$ and $\exp(\pm i\omega_M t)$ ($\omega_M, 2J \gg g_0$). We also plot the time dependence of the

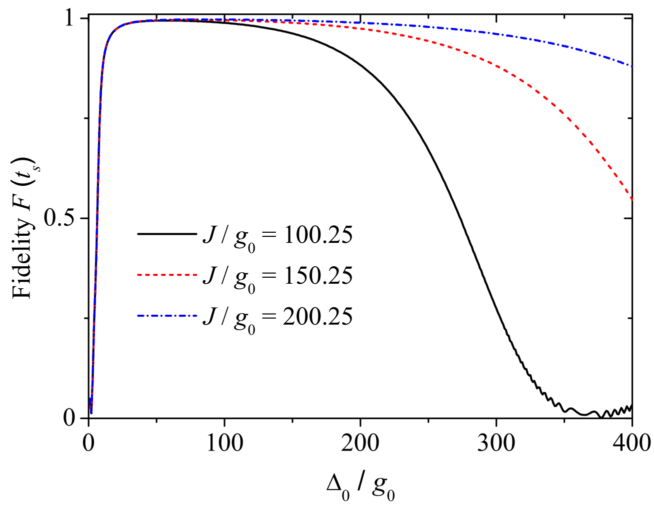

FIG. 3. (Color online) The fidelity $F(t_s)$ at time t_s versus the ratio Δ_0/g_0 for various values of J/g_0, where $t_s = \pi/(\omega_M - 2J)$ is the time for the first peak value of $|\beta(t)|$. For different values of J, the values of ω_M are taken to keep $\omega_M - 2J = g_0/2$.

mechanical displacement $|\langle b(t)\rangle|$ in Fig. 2(b). Here the solid curve corresponds to the analytical case $|\langle b(t)\rangle| = |\beta(t)|$, which is independent of Δ_0, as given in Eq. (13b). The other three curves in panel (b) are determined by the numerical solutions with the corresponding values of Δ_0/g_0 given in panel (a). It can be seen that the exact numerical results match well the approximate analytical result, except a slight shift of the period.

Although a high ratio Δ_0/g_0 would increase the fidelity as shown in Fig. 2, there is an upper limit of Δ_0 according to Eq. (6). Specifically, for the approximate Hamiltonian (7) to be valid, we also need $\Delta_0 \ll 16J/5$. In other words, there is a finite range of Δ_0 for our model to operate with. This is illustrated in Fig. 3 where the fidelity $F(t_s)$ at $t_s = \pi/(\omega_M - 2J)$ is plotted as a function of Δ_0/g_0. The choice of time t_s is useful because it is the time for the first maximum displacement. In plotting all the curves in Fig. 3, we use the parameters with $\omega_M - 2J = g_0/2$, so that the maximum mechanical displacement is 2. From Fig. 3, we see that fidelity is almost 1 in a range of Δ_0/g_0 when the conditions in Eq. (6) are satisfied. The range of Δ_0/g_0 giving $F(t_s) \approx 1$ becomes wider as the ratio J/g_0 increases, as illustrated by the blue dash-dotted curve in Fig. 3.

IV. DISCUSSIONS

Our analysis so far has focused on the coherent evolution of the modulated system. To address the effect of cavity-field damping, we let κ_c be the cavity-field damping rate for both the left and right cavities, then with $\omega_M - 2J \approx g_0/2$, $t \approx 1/g_0$ is the time scale for the membrane displacement being greater than the width of the mechanical ground state. Therefore the condition $g_0 \gg \kappa_c$ would ensure that the photon has a sufficient time to yield a significant mechanical displacement (with $|\beta| > 1$) before it leaks out of the cavity.

More specifically, based on the approximate Hamiltonian (8) and the initial state (10), it is not difficult to estimate that the phonon number $\langle b^\dagger b\rangle$ is of the order $g_0^2/[4(\omega_M - 2J)^2 + 4\kappa_c^2]$ at a time long compared with the cavity-field lifetime, but short compared with the lifetime of the mechanical oscillator, i.e., $1/\gamma_M \gg t \gg 1/\kappa_c$, where γ_M is the damping rate of the membrane. Therefore, if $g_0 \gg \kappa_c$ and $\omega_M - 2J < g_0$, then the average phonon number of our modulated system can be larger than 1, after the photon leaks out of the cavity.

We point out that our approach can be used to create distinct mechanical superposition states [49–56]. The procedure consists of three steps: (i) Prepare the system in initial states $(1/\sqrt{2})(|1\rangle_L|0\rangle_R \pm |0\rangle_L|1\rangle_R)|0\rangle_M$. (ii) Then let the system evolve for a suitable time, for example, $t = 2\pi/(\omega_M - 2J)$. (iii) Measure the state of the photon [44]. As long as the photon is measured in either the left cavity or the right cavity, then the mechanical resonator will be projected to a mechanical superposition state.

We note that mechanical superposed states can be measured by using quantum state tomography [57,58]. By strongly driving another cavity mode (avoiding the cross talk with the single-photon state) at red sideband, the optomechanical coupling can be linearized into a beam-splitter interaction. Assume a large decay of the assistant cavity field such that it can be eliminated adiabatically, then the state of the mechanical mode can be mapped to the output light [59]. As a result the mechanical state can be known by reconstructing the state of the output light beam [60]. In addition, the technique of spectrometric reconstruction can also be used to obtain the state information of the mechanical mode by detecting the single-photon spectrum [57].

Moreover, we would like to mention that the approach in this paper can also be used to enhance the optical nonlinearities [21]. In optomechanics, the magnitude of the optical nonlinearity is g_0^2/ω_M. It can be increased by effectively decreasing the frequency, from ω_M to $\omega_M - 2J$, under the same coupling strength g_0.

Finally, we discuss an example that illustrates the conditions required for the experimental demonstration of the mechanism presented in this paper. The example consists of a vibrational mode of a silicon nitride membrane with an eigenfrequency $\omega_M/2\pi = 1$ MHz [36,37], and the mechanical damping rate is less than a few hertz and so it may be ignored. The membrane is placed in the middle of a Fabry-Perot cavity of length L. A pair of nearly degenerate normal modes of the cavity is chosen such that the frequency splitting $2J \approx \omega_M$ at the avoided crossing. To obtain mode splitting in the megahertz range, the transverse modes of the cavity may be exploited [30,33,36]. Consider $L = 1$ mm, then g_0 is about 1 kHz, which is much smaller than ω_M. However, by applying a weak modulation with $\Delta_0 = 40g_0 \approx 40$ kHz, the effect of radiation pressure from a single photon can be significantly enhanced according to our theory, provided that the cavity-field damping rate κ_c is smaller than g_0. Since κ_c is of the order of few megahertz in experiments [37], this is still a main challenge to achieve $g_0 > \kappa_c$. Note that $g_0 > \kappa_c$ is also the condition for the MIM optomechanical system to exhibit enhanced photon-photon interactions [42]. Therefore the progress of designs and fabrications of ultrahigh Q cavities will be crucial to access ultrastrong-coupling quantum effects in optomechanics.

V. CONCLUSION

In conclusion, we have proposed a method to enhance the mechanical effects of single photons in a two-mode optomechanical system by introducing a frequency modulation to the cavity fields. The quantum dynamics, which is driven by photon hopping between the two modulated cavities and optomechanical interactions, can be captured approximately by the Hamiltonian (8) under the condition (6). Such a Hamiltonian indicates that a single photon can displace the membrane with $\beta > 1$ even though $g_0/\omega_M \ll 1$. This method can be used to create distinct superposition states of the mechanical resonator and enhance the optical nonlinearities. Finally we remark that these types of systems can be studied via quantum emulations or quantum simulations [61,62], as described in [63,64]. These describe circuit analogs of optomechanical systems, allowing an additional way to study the effects predicted here.

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