Superconducting qubits

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Funding from July 2006: NSA, LPS, ARO

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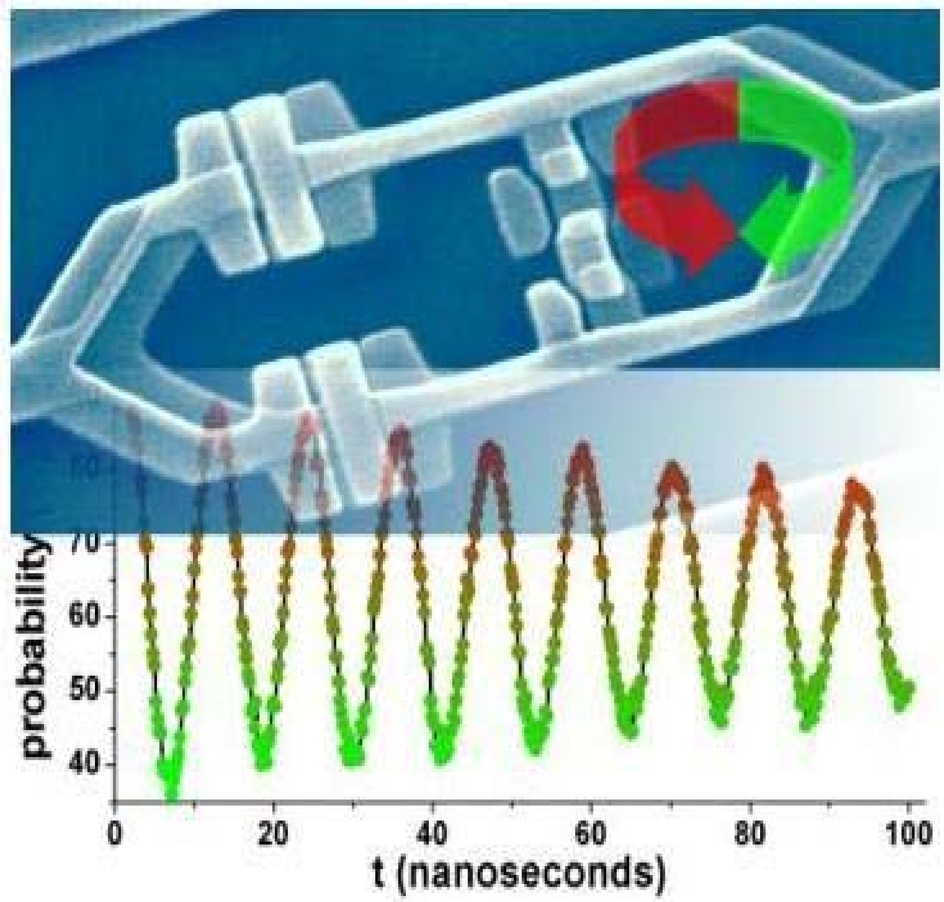
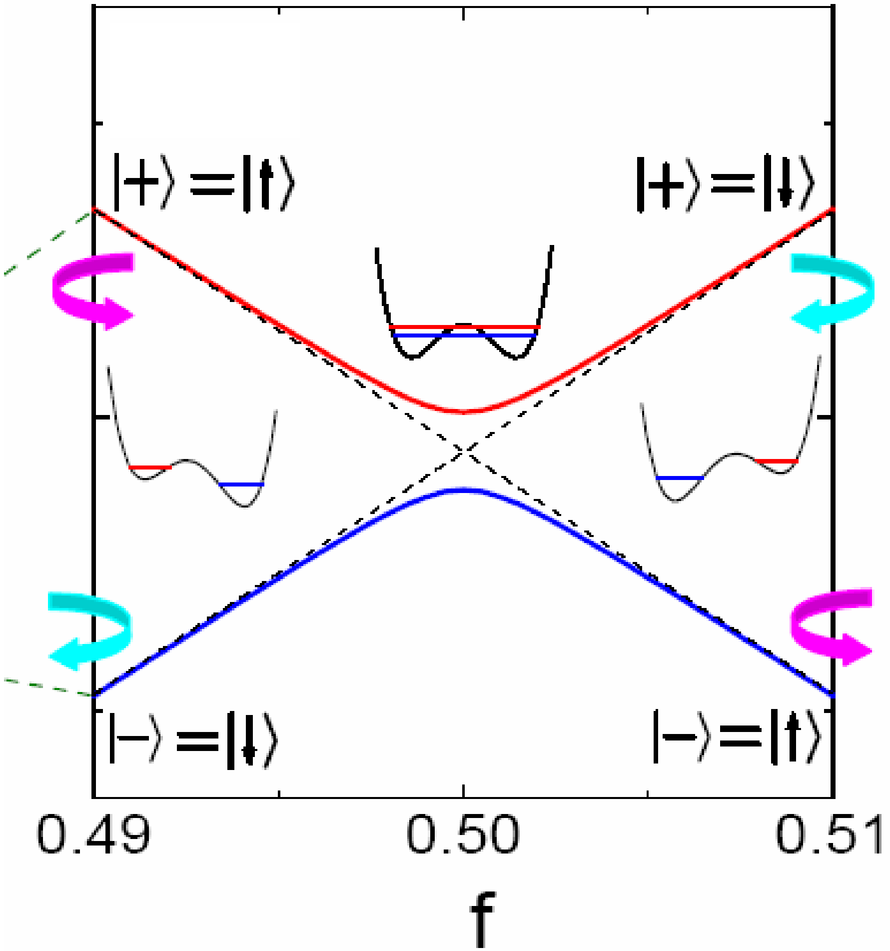
Qubit = Two-level quantum system



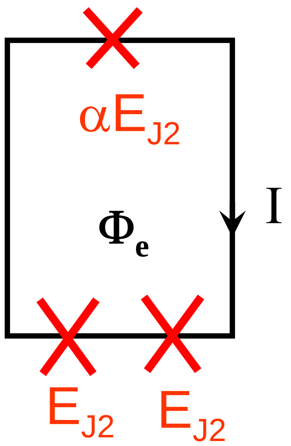
Chiorescu et al, Science 299, 1869 (2003)

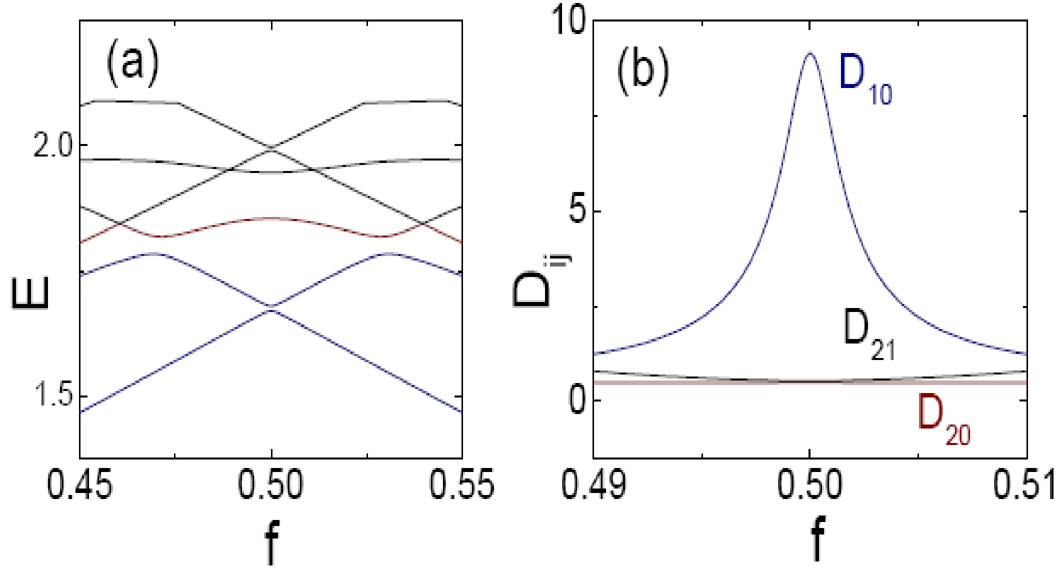


You and Nori, Phys. Today 58 (11), 42 (2005)

Reduced magnetic flux: $f = \Phi_e / \Phi_0$. Here: Φ_e = external DC bias flux

Flux qubit (here we consider the three lowest energy levels)





$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m, f)$$

$$U = 2E_J\left(1 - \cos\varphi_p \cos\varphi_m\right) + \alpha E_J\left[1 - \cos\left(2\varphi_m + 2\pi f\right)\right] \qquad f = \frac{\Phi_e}{\Phi_0}$$

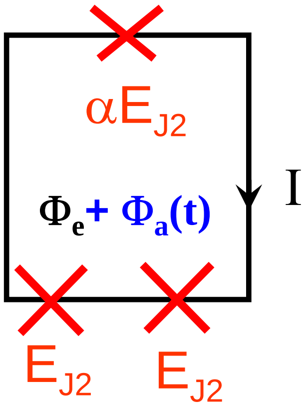
Phases and momenta (conjugate variables) are

$$\varphi_p = (\phi_1 + \phi_2)/2; \quad \varphi_m = (\phi_1 - \phi_2)/2; \quad P_k = -i\hbar\, \partial/\partial\varphi_k \quad (k = p, m)$$

Effective masses

$$M_p = \left(\Phi_0 / 2\pi\right)^2 2C; \quad M_m = 2M_p(1 + 2\alpha) \ \text{ with capacitance } C \text{ of the junction}$$

I. Flux qubit: Symmetry and parity



$$H = H_0 + V(t)$$

$$H_0 |m\rangle = E_m |m\rangle$$

Time-dependent magnetic flux

$$\Phi_a(t) = \Phi_a^{(0)} \cos(\omega_{ij} t)$$

$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m, f)$$

$$V(t) = -\frac{2\alpha\pi\Phi_a^{(0)} E_J}{\Phi_0} \sin(2\pi f + 2\varphi_m) \cos(\omega_{ij} t)$$

Transition elements are

$$t_{ij} = -\frac{2\alpha\pi\Phi_a^{(0)} E_J}{\Phi_0} \langle i | \sin(2\pi f + 2\varphi_m) | j \rangle$$

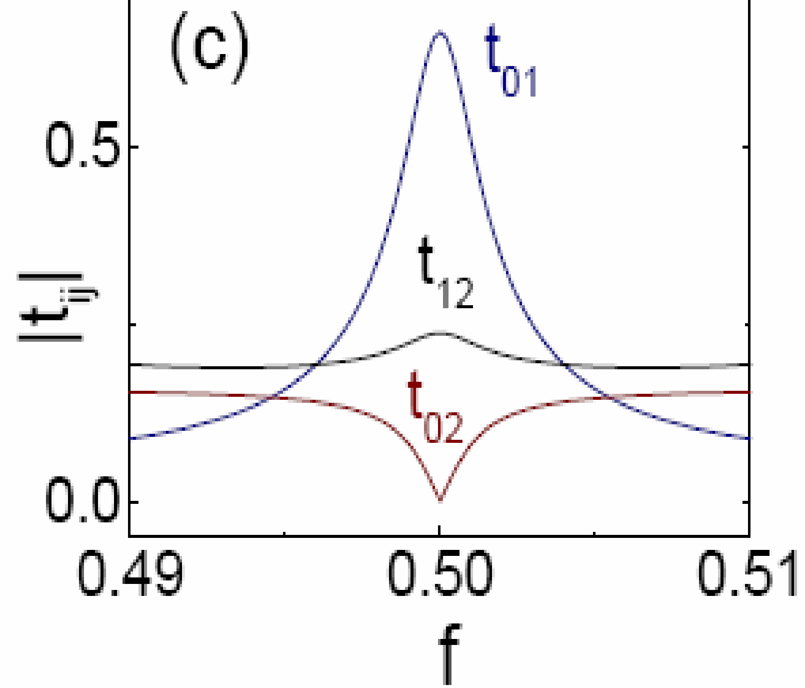
Liu, You, Wei, Sun, Nori, *PRL* 95, 087001 (2005)

Parity of $U(\varphi_m, \varphi_p) \equiv U$

$$U = 2E_J(1 - \cos\varphi_p \cos\varphi_m) + \alpha E_J[1 - \cos(2\varphi_m + 2\pi f)]$$

$$f = 1/2 \Rightarrow U(\varphi_m, \varphi_p) \text{ even function of } \varphi_m \text{ and } \varphi_p$$

$$U = 2E_J(1 - \cos\varphi_p \cos\varphi_m) + \alpha E_J[1 + \cos(2\varphi_m)]$$



I. Flux qubit: Symmetry and parity

In standard atoms, electric-dipole-induced selection rules for transitions satisfy the relations for the angular momentum quantum numbers:

$$\Delta l = \pm 1 \quad \text{and} \quad \Delta m = 0, \pm 1$$

In superconducting qubits, there is no obvious analog for such selection rules.

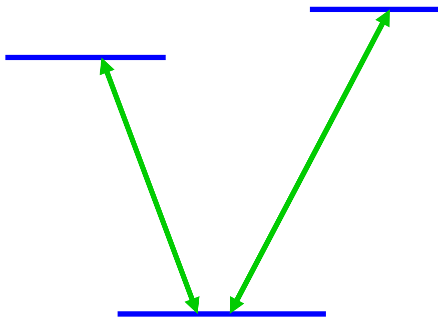
Here, we consider an analog based on the symmetry of the potential $U(\varphi_m, \varphi_p)$ and the interaction between:

-) superconducting qubits (usual atoms) and the

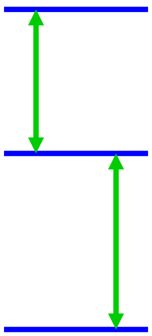
-) magnetic flux (electric field).

Liu, You, Wei, Sun, Nori, *PRL* (2005)

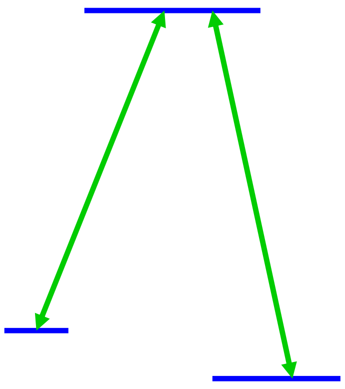
Different transitions in three-level atoms



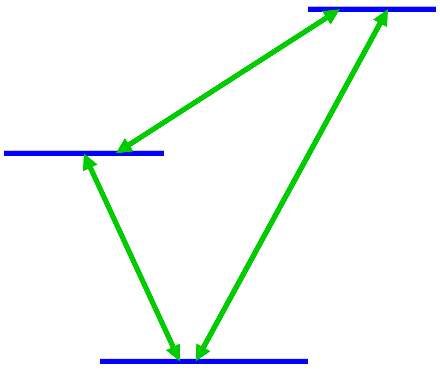
V - type



***Ξ* - type or ladder**



***Λ* - type**



No *Δ* – type because of the electric-dipole selection rule.

Some differences between artificial and natural atoms:

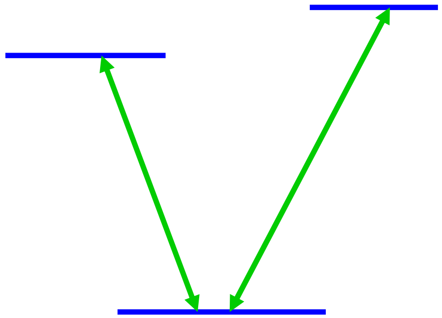
In natural atoms, it is *not* possible to obtain cyclic transitions by only using the electric-dipole interaction, due to its well-defined symmetry.

However, these transitions can be naturally obtained in the flux qubit circuit, due to the broken symmetry of the potential of the flux qubit, when the bias flux deviates from the optimal point.

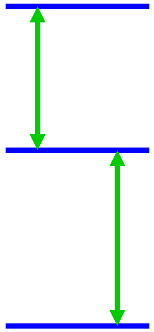
The magnetic-field-induced transitions in the flux qubit are similar to atomic electric-dipole-induced transitions.

Liu, You, Wei, Sun, Nori, *PRL* (2005)

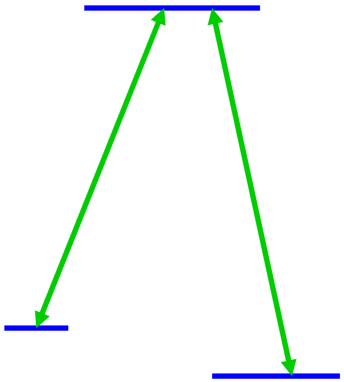
Different transitions in three-level systems



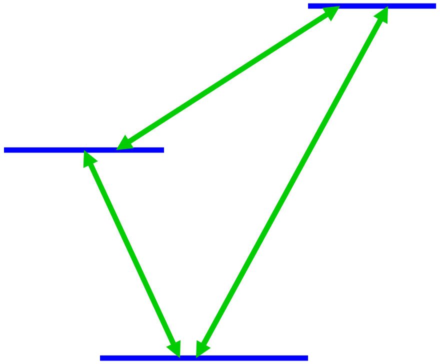
V - type



Ξ- type or ladder (f $= 1/2$)



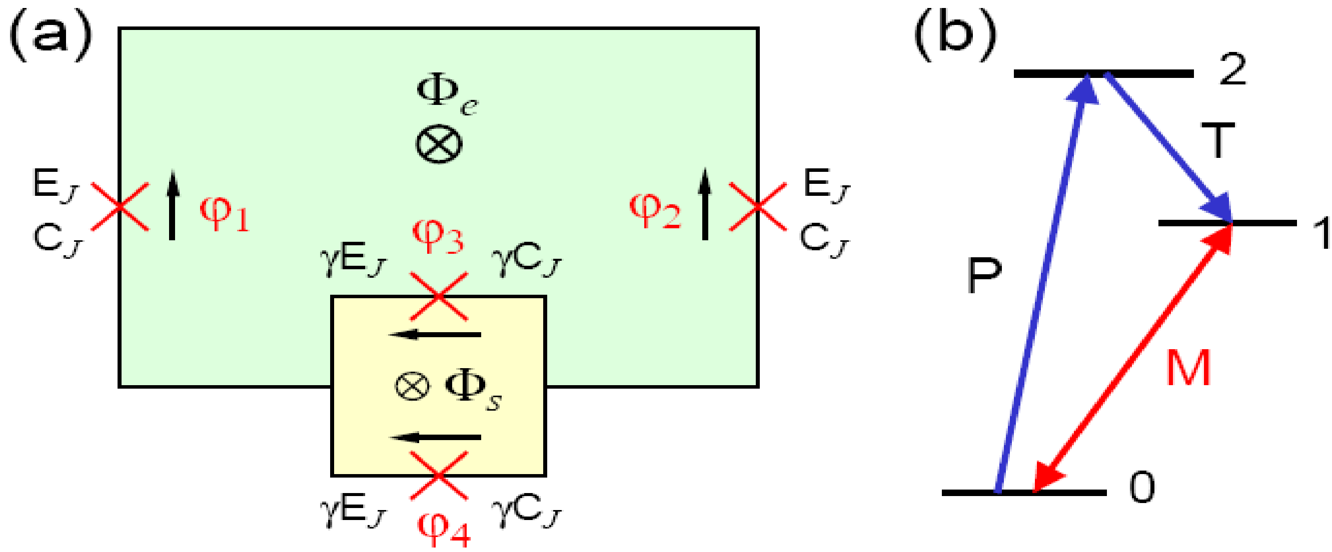
Λ - type



Δ – type
Can be obtained using flux qubits (f away from 1/2)

Liu, You, Wei, Sun, Nori, *PRL* (2005)

Flux qubit: micromaser

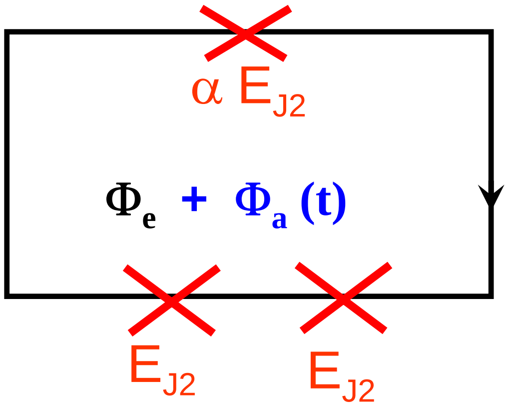


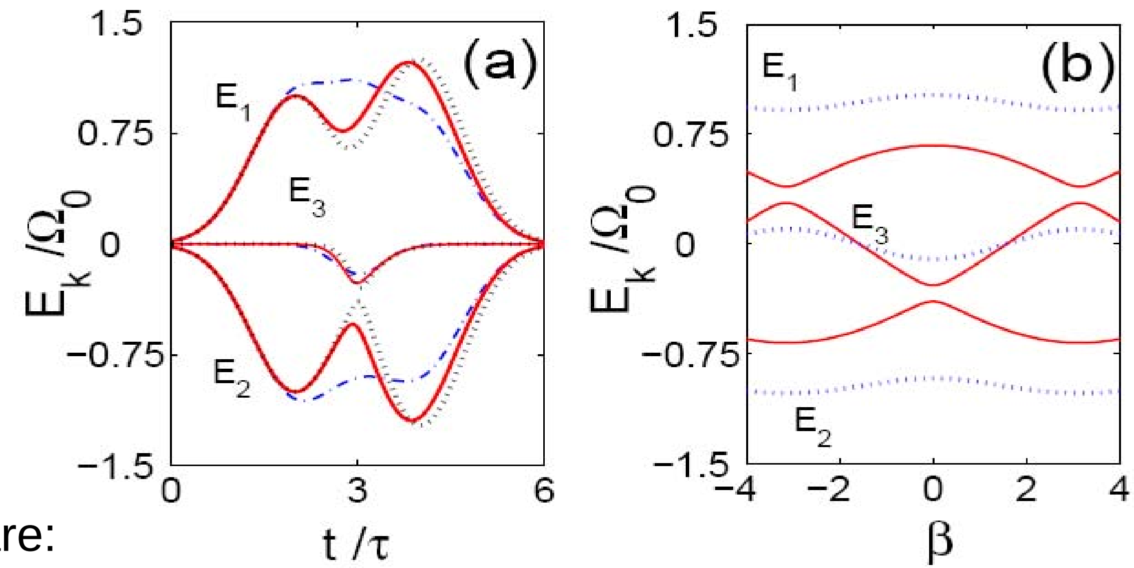
You, Liu, Sun, Nori, quant-ph / 0512145

We propose a *tunable* on-chip *micromaser* using a superconducting quantum circuit (SQC).

By taking advantage of externally controllable state transitions, a <u>state population inversion</u> can be achieved and preserved for the two working levels of the SQC and, when needed, the SQC can *generate a single photon.*

Flux qubit: Adiabatic control and population transfer

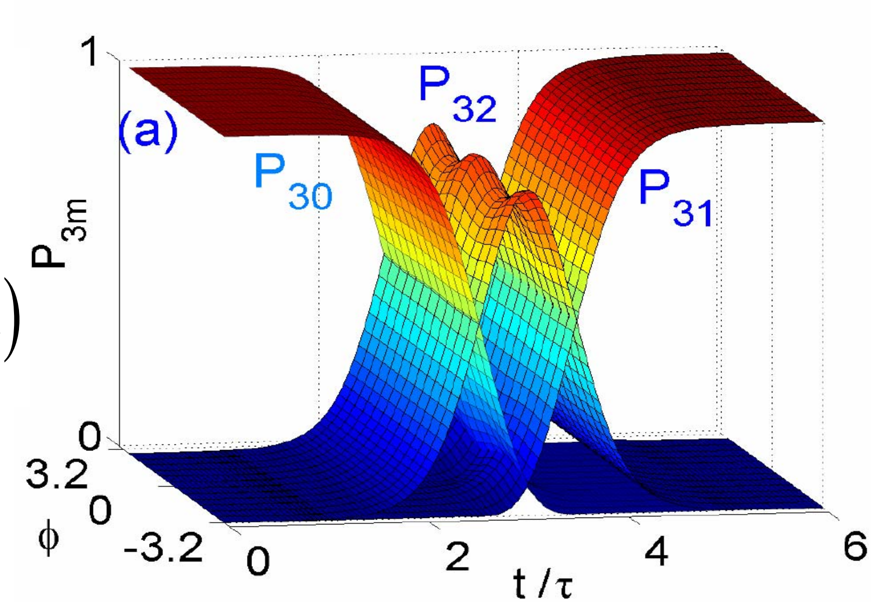




The applied magnetic fluxes and interaction Hamiltonian are:

$$\Phi_a(t) = \sum_{m>n=0}^{2} \left[\Phi_{mn}(t) \exp(-i\omega_{mn} t) + \text{H.c.} \right]$$

$$H_{\text{int}} = \sum_{m>n=0}^{2} \left(\Omega_{mn}(t) \exp(i\Delta_{mn} t) |m\rangle\langle n| + \text{H.c.} \right)$$



Liu, You, Wei, Sun, Nori, PRL 95, 087001 (2005)

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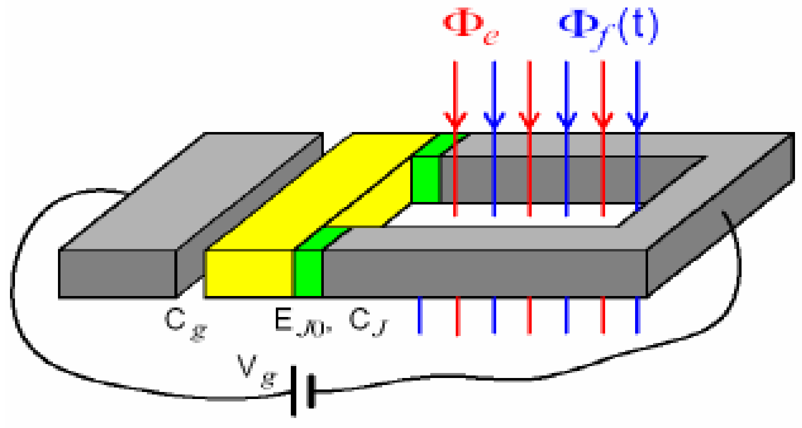
Cavity QED: Charge-qubit inside cavity

$H = E_c(n - C_g V_g / 2e)^2 - E_J(\Phi_e) \cos\phi$,

ϕ = average phase drop across the JJ

$E_c = 2e^2/(C_g + 2C_{J0})$ = island charging energy;

$E_J(\Phi_e) = 2 E_{J0} \cos(\pi \Phi_e/\Phi_0)$.



You and Nori, PRB 68, 064509 (2003)

Here, we assume that the qubit structure is embedded in a microwave cavity with only a single photon mode λ providing a quantized flux

$$\Phi_f = \Phi_\lambda a + \Phi^*_\lambda a^\dagger = |\Phi_\lambda| (e^{-i\theta} a + e^{i\theta} a^\dagger),$$

with Φ_λ given by the contour integration of $\mathbf{u}_\lambda d\boldsymbol{l}$ over the SQUID loop.

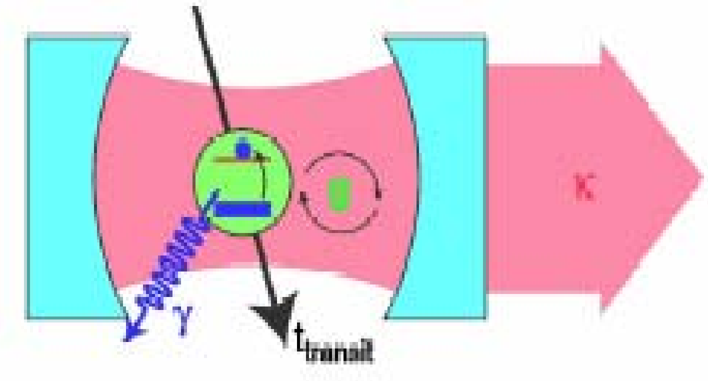
Hamiltonian: $H = ½ E \rho_z + \hbar\omega_\lambda (a^\dagger a + ½) + H_{Ik}$,

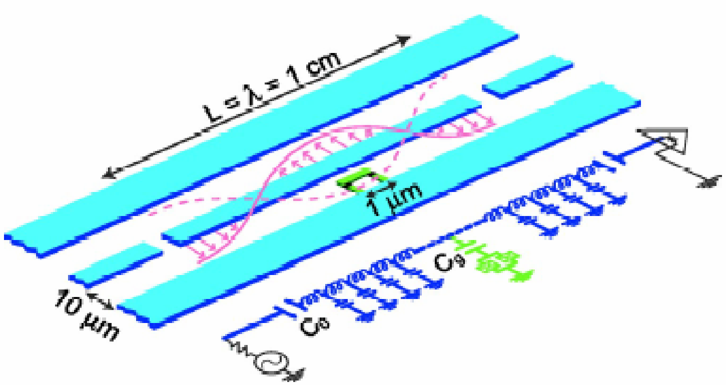
$$H_{Ik} = \rho_z f(a^\dagger a) + [e^{-ik\theta} |e\rangle\langle g| a^k g^{(k)}(a^\dagger a) + H.c.]$$

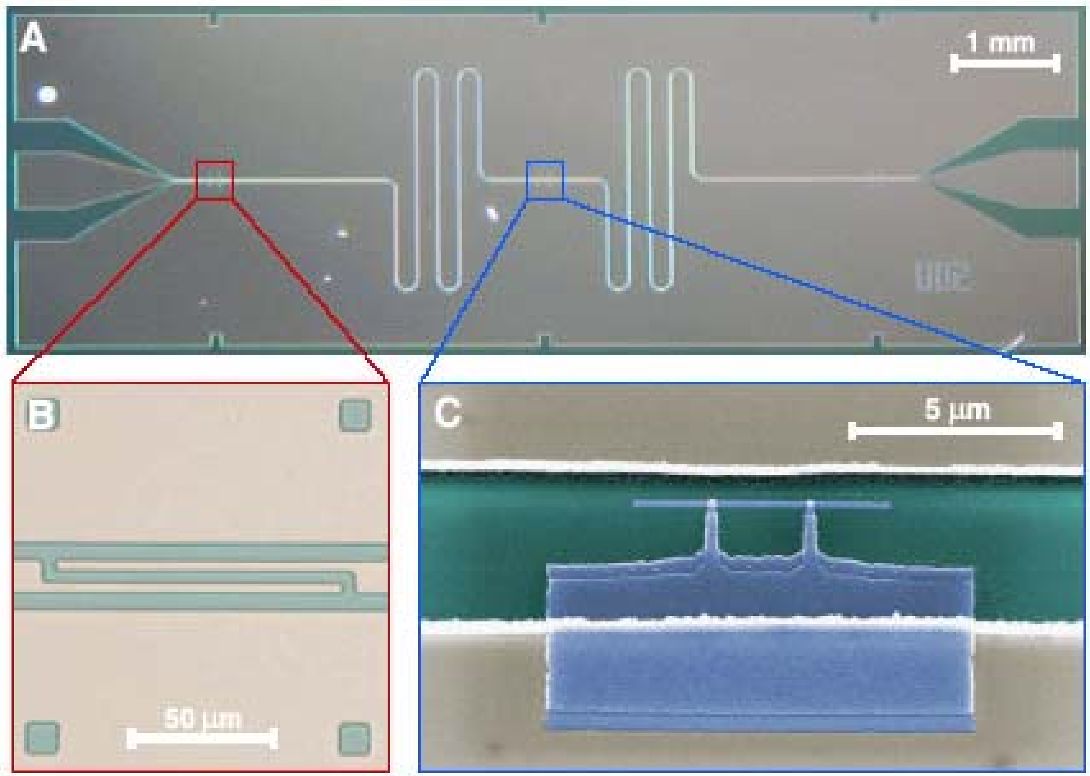
This is *flux*-driven. The *E*-driven version is in: You, Tsai, Nori, PRB (2003)

II. Circuit QED

Charge-qubit coupled to a transmission line







Yale group

$$H = \frac{\hbar}{2}\omega\left(\Phi_e, n_g\right)\sigma_z + \hbar\omega\, a^\dagger a + \hbar\left[g\sigma_+ a + H.c.\right]$$

$\omega(\Phi_e,n_g)$ can be changed by the gate voltage n_g and the magnetic flux Φ_e .

II. Cavity QED on a chip

Based on the interaction between the radiation field and a superconductor, we propose a way to engineer quantum states using a SQUID charge qubit inside a microcavity.

This device can act as a deterministic single photon source as well as generate any Fock states and an arbitrary superposition of Fock states for the cavity field.

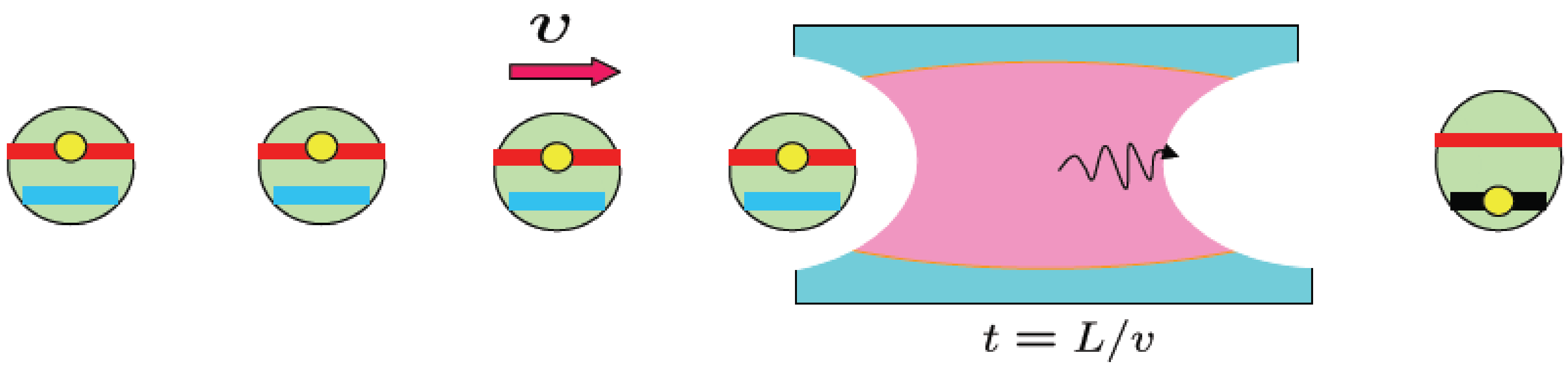
The controllable interaction between the cavity field and the qubit can be realized by the tunable gate voltage and classical magnetic field applied to the SQUID.

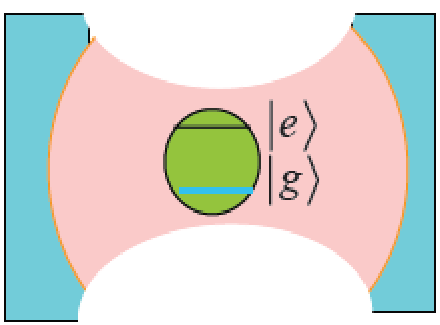
Liu, Wei, Nori, *EPL* 67, 941 (2004); *PRA* 71, 063820 (2005); *PRA* 72, 033818 (2005)

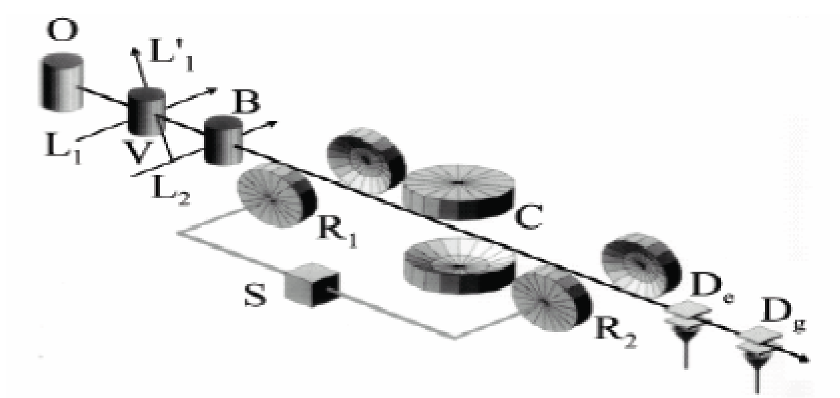
Comparison of our proposal with a micromaser

Carrier process: thermal excitation for micromaser

First red sideband excitation: the excited atoms enter the cavity, decay, and emit photons







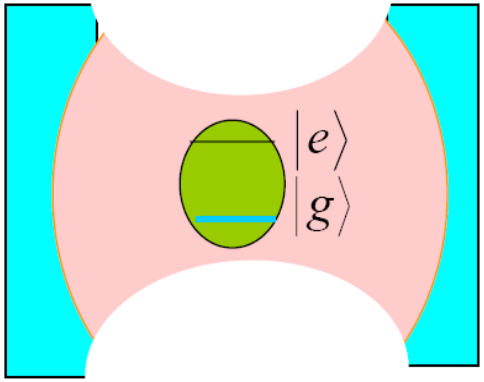
X. Maitre, et al., PRL 79, 769 (1997)

Comparison of our proposal with a micromaser

	JJ qubit photon generator	Micromaser
Before	JJ qubit in its ground state then excited via $n_g = 1/2, \quad \Phi_C = \Phi_0$	Atom is thermally excited in oven
Interaction with microcavity	JJ qubit interacts with field via $n_g = 1, \quad \Phi_C = \Phi_0/2$	Flying atoms interact with the cavity field
After	Excited JJ qubit decays and emits photons	Excited atom leaves the cavity, decays to its ground state providing photons in the cavity.

Liu, Wei, Nori, *EPL* (2004); *PRA* (2005); *PRA* (2005)

Interaction between the JJ qubit and the cavity field



Liu, Wei, Nori,
EPL 67, 941 (2004);
PRA 71, 063820 (2005);
PRA 72, 033818 (2005)

$$H = \underbrace{\hbar\omega a^\dagger a}_{\text{cavity field}} - \underbrace{2E_C(1-2n_g)\sigma_z}_{\text{charging energy}}$$

$$\underbrace{- E_J \cos\left[\frac{\pi}{\Phi_0}(\Phi_c + ga + g^* a^\dagger)\right]\sigma_x}_{\text{interaction term}}$$

with $g = i\int_S u(r)\cdot ds$ and $\hbar\omega = 2E_C$

(1) The interaction between the cavity field and the SQUID is controlled by the gate charge n_g and the dc applied flux Φ_C.

(2) S is the area of the SQUID.

(3) $u(r)$ is a mode function of a single-mode cavity field.

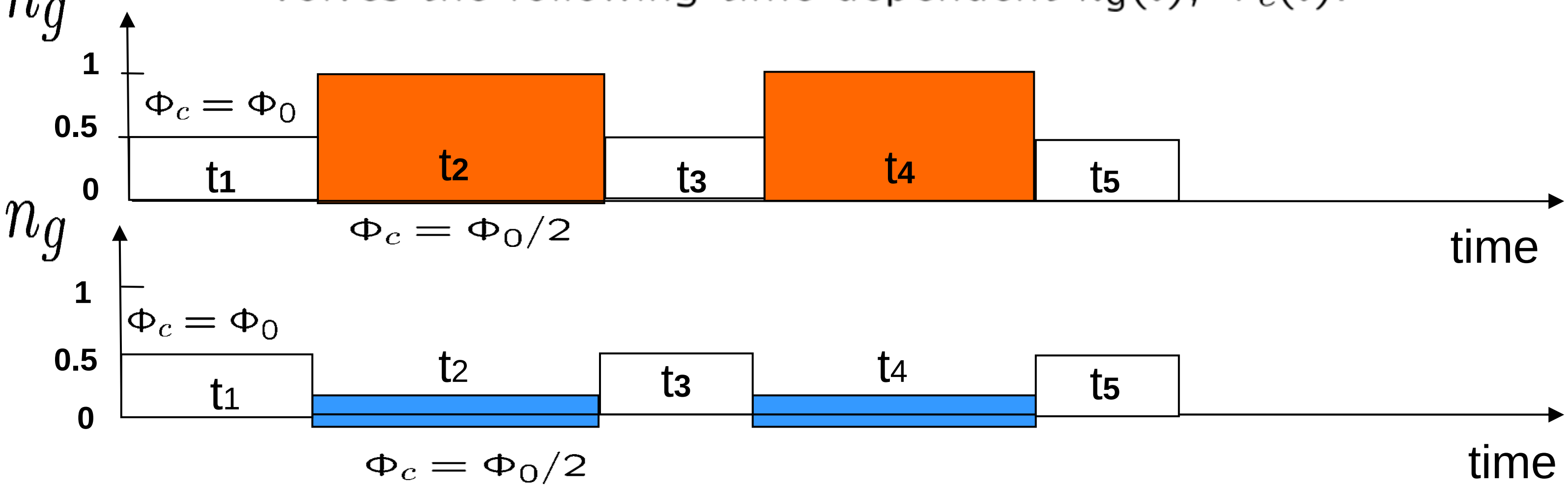
II. Cavity QED: Controllable quantum operations

Controllable operation can be realized by the Hamiltonian

$$H = \underbrace{\hbar\omega a^\dagger a}_{\textit{cavity field}} \underbrace{- 2E_C(1-2n_g)\sigma_z}_{\textit{charging energy}} \underbrace{- E_J \cos\left(\frac{\pi\Phi_c}{\Phi_0}\right)(\sigma_+ + \sigma_-)}_{\textit{carrier}}$$

$$+ \underbrace{\frac{\pi E_J}{\Phi_0}\sin\left(\frac{\pi\Phi_c}{\Phi_0}\right)(ga\sigma_+ + g^* a^\dagger \sigma_-)}_{\textit{Red sideband excitation}} + \underbrace{\frac{\pi E_J}{\Phi_0}\sin\left(\frac{\pi\Phi_c}{\Phi_0}\right)(ga\sigma_- + g^* a^\dagger \sigma_+)}_{\textit{Blue sideband excitation}}$$

Operation with red (blue) sideband excitation and carrier involves the following time-dependent $n_g(t)$, $\Phi_c(t)$.

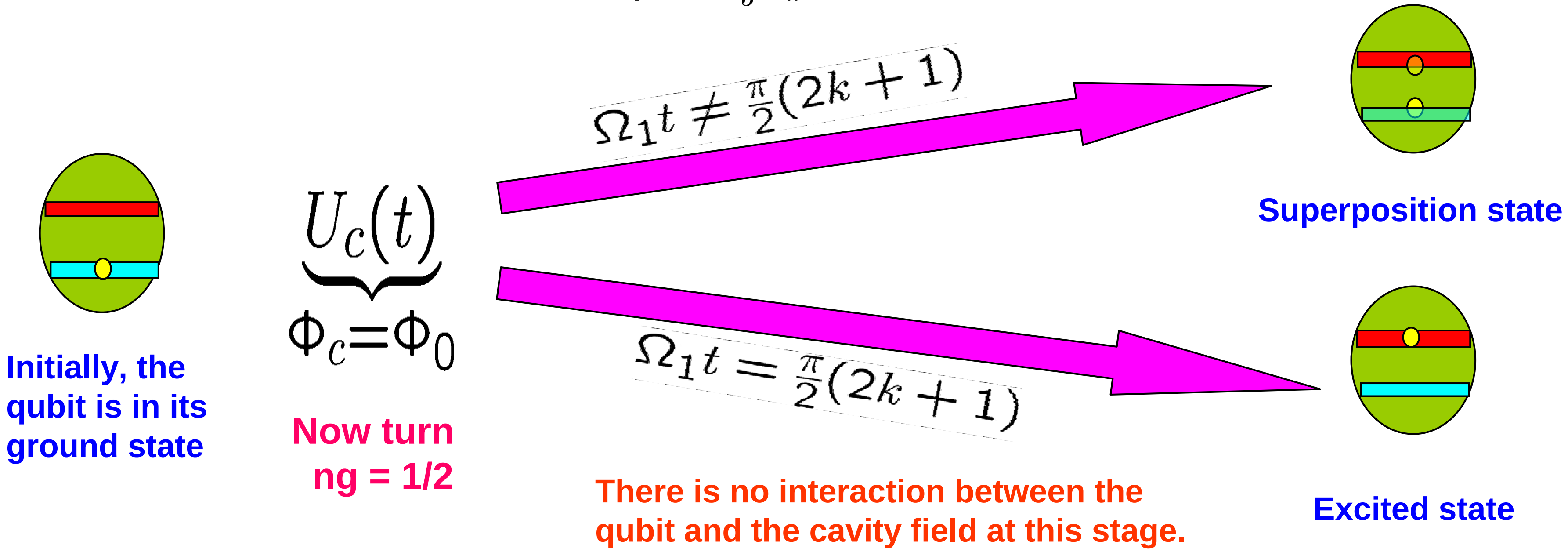


II. Cavity QED on a chip

Carrier brings the qubit to superpositions or excited states

When the JJ charge qubit works at the degeneracy point $n_g = 1/2$, the qubit can be prepared in the state $\beta_1|\downarrow\rangle + \beta_2|\uparrow\rangle$ or $|\downarrow\rangle$ by the quantum operation (here $\Omega_1 = E_J/\hbar$)

$$\underbrace{U_c(t) = \cos(\Omega_1 t)I + \sin(\Omega_1 t)(|e\rangle\langle g| + |g\rangle\langle e|)}_{H_c = E_J\sigma_x}$$



II. Cavity QED on a chip

Red sideband process: JJ qubit emits a photon

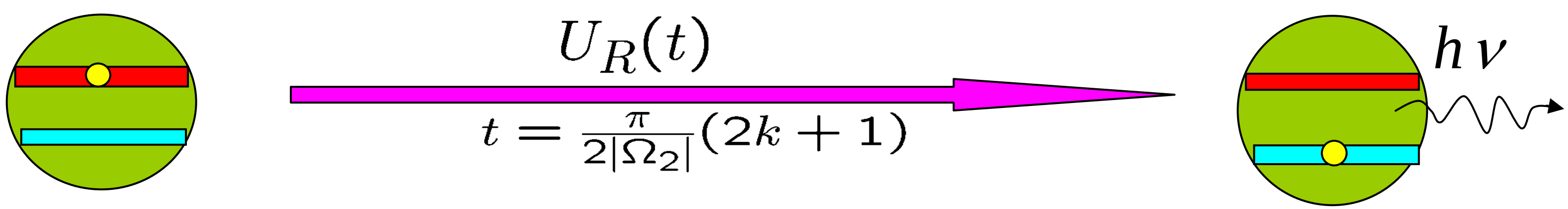
The gate voltage and magnetic flux are set to $n_g = 1$ and $\Phi_c = \Phi_0/2$. Then the qubit resonantly interacts with the cavity field.

$$\underbrace{U_R(t) = R_{ee}|e\rangle\langle e| + R_{gg}|g\rangle\langle g| - iR_{ge}|g\rangle\langle e| - iR_{eg}|e\rangle\langle g|}_{H_R = \frac{\pi E_J}{\Phi_0}(ga\sigma_+ + ga^\dagger\sigma_-)}$$

where

$$R_{eg} = \left[e^{i\theta}\frac{\sin|\Omega_2|t\sqrt{a^\dagger a}}{\sqrt{a^\dagger a}}\right]a = R_{ge}^\dagger, \quad \Omega_2 = \frac{\pi|g|E_J}{\hbar\Phi_0}e^{i\theta}$$

$$R_{ee} = \cos(|\Omega_2|t\sqrt{aa^\dagger}), \quad R_{gg} = \cos(|\Omega_2|t\sqrt{a^\dagger a})$$



Initially, the qubit is in its excited state $n_g = 1$

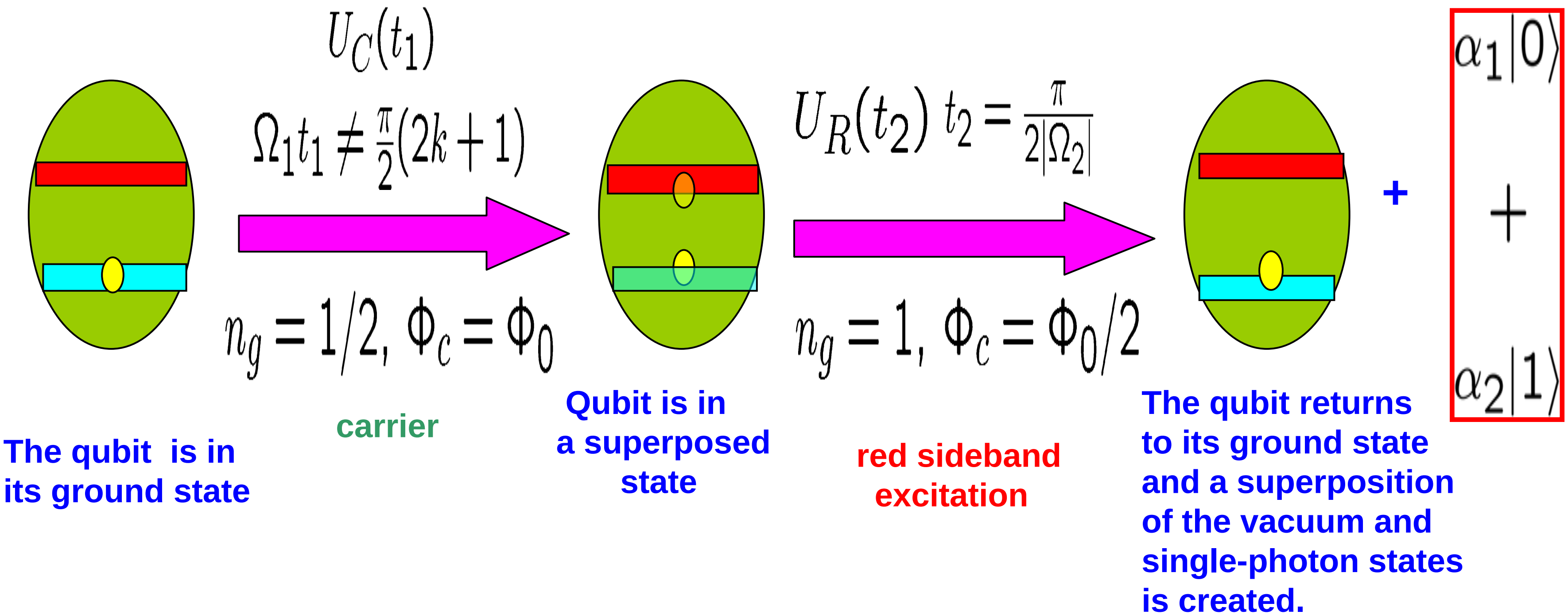
Red sideband excitation is provided by turning on the magnetic field such that $\Phi_c = \Phi_0/2$.

Finally, the qubit is in its ground state and one photon is emitted.

II. Cavity QED on a chip

How to create superpositions of photon states

$\alpha_1|0\rangle + \alpha_2|1\rangle$ with $\alpha_1 = \cos(\Omega_1 t_1)$ and $\alpha_2 = e^{-i\theta}\sin(\Omega_1 t_1)$



When the red sideband excitation satisfies the condition $t_2 = \pi/2|\Omega_2|$, it creates a superposition of the vacuum and single photon states.

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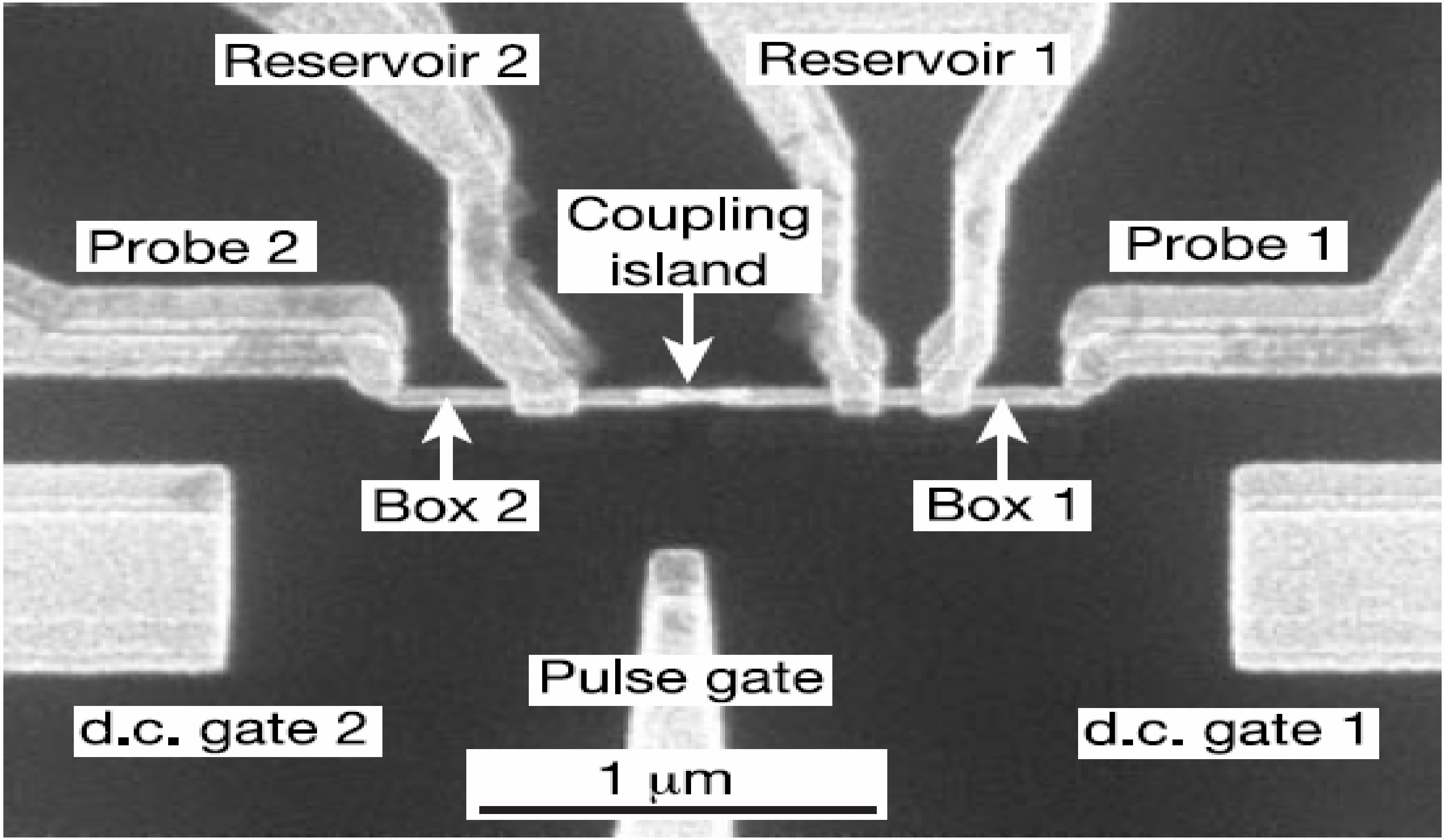
VI. Quantum tomography

VII. Conclusions

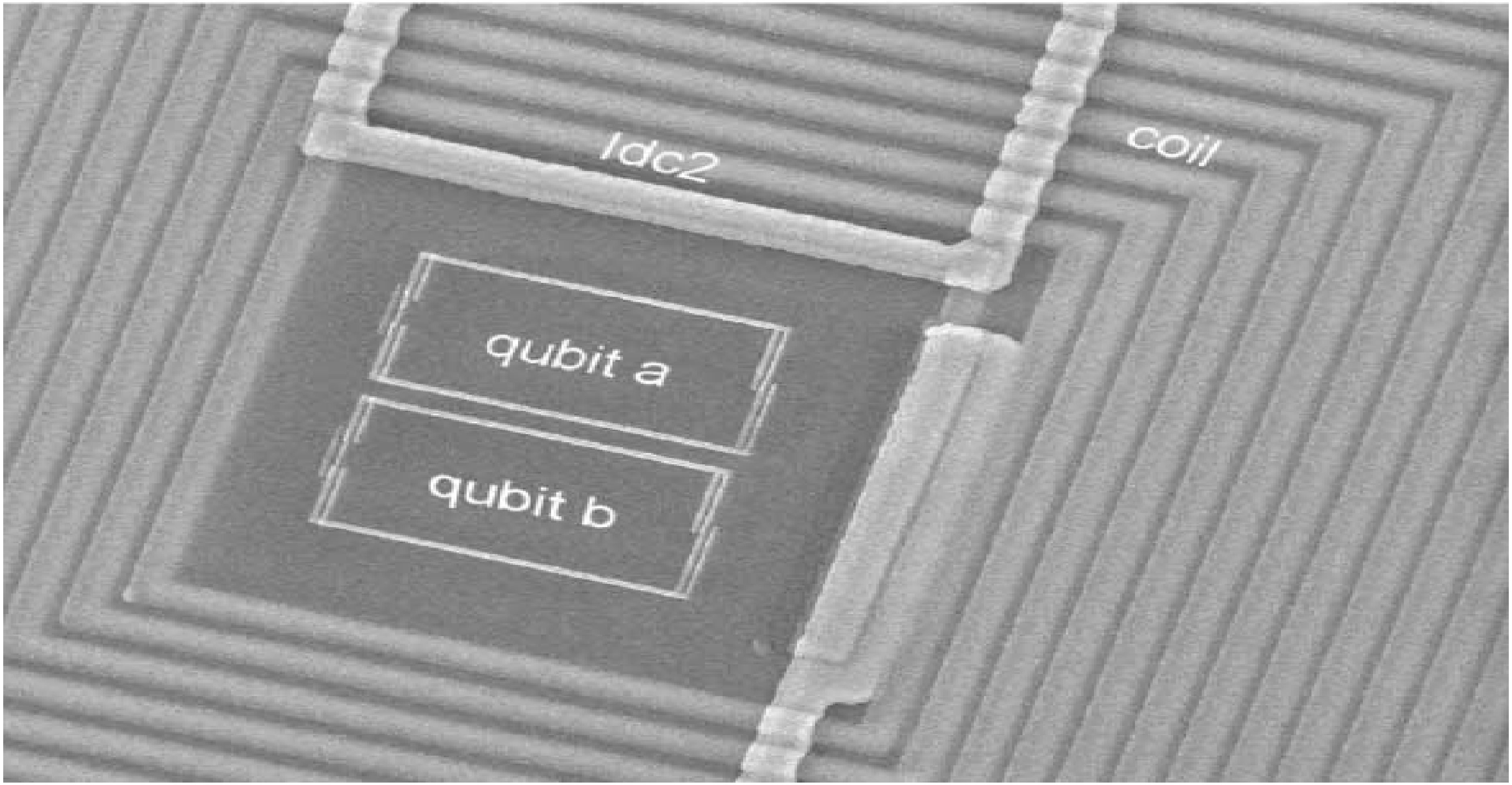
Capacitively coupled charge qubits



NEC-RIKEN

Entanglement; conditional logic gates

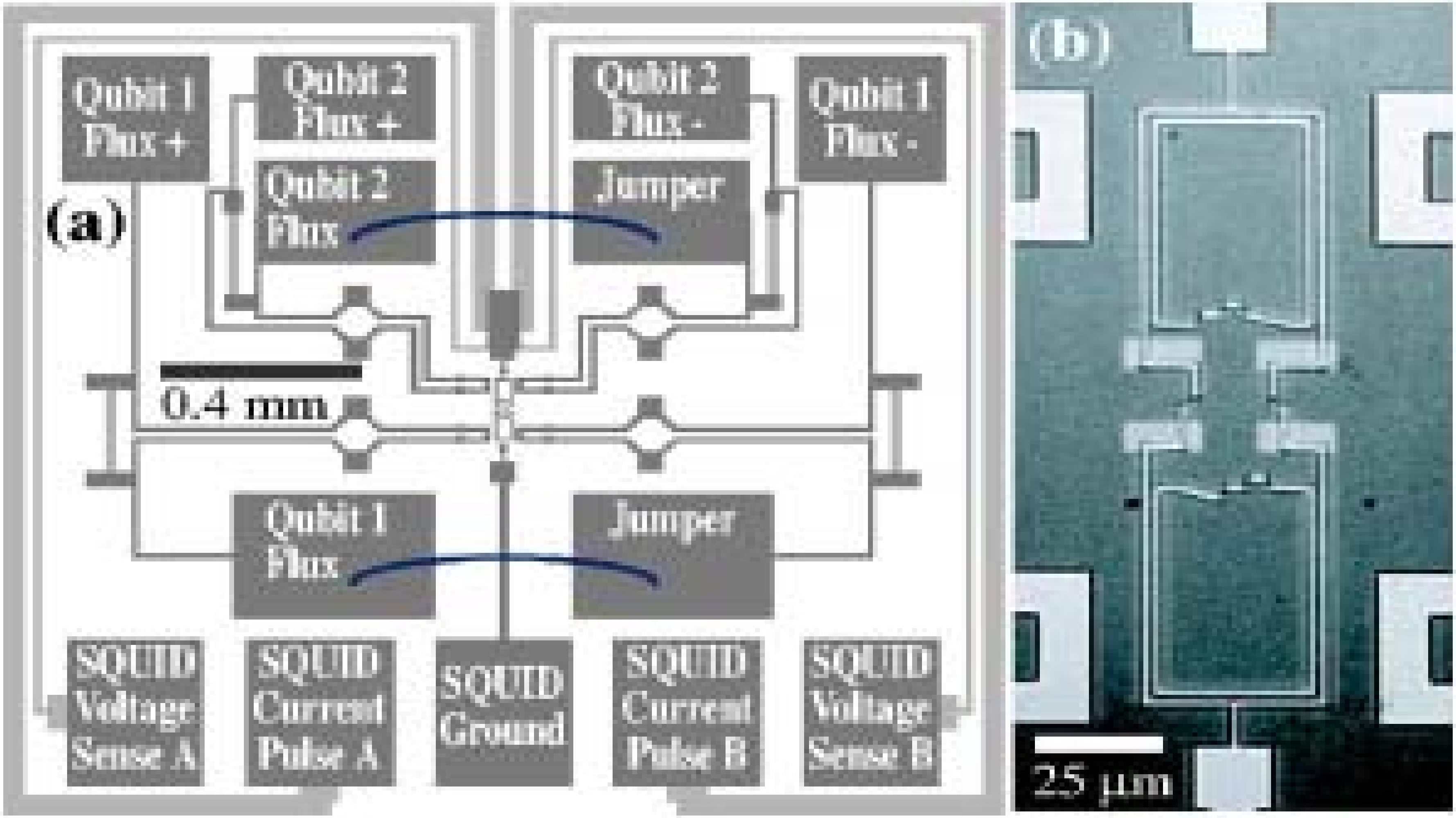
Inductively coupled flux qubits



A. Izmalkov et al., PRL 93, 037003 (2004)

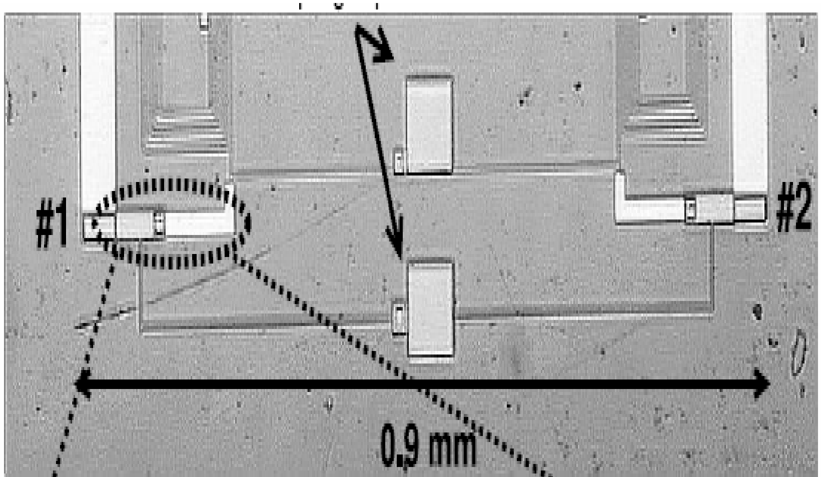
Entangled flux qubit states

Inductively coupled flux qubits

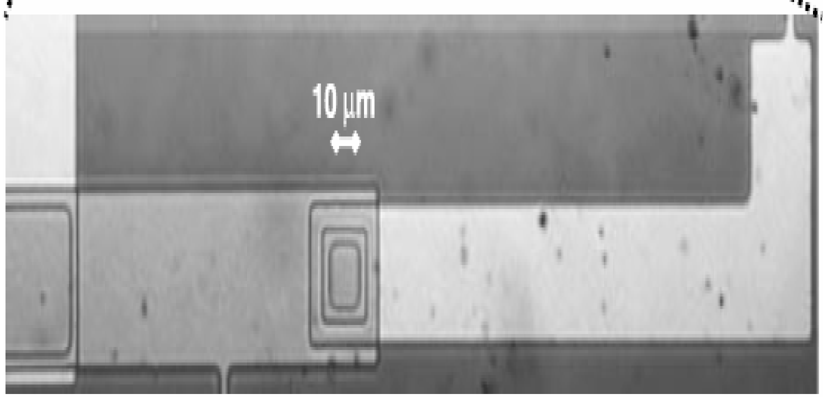


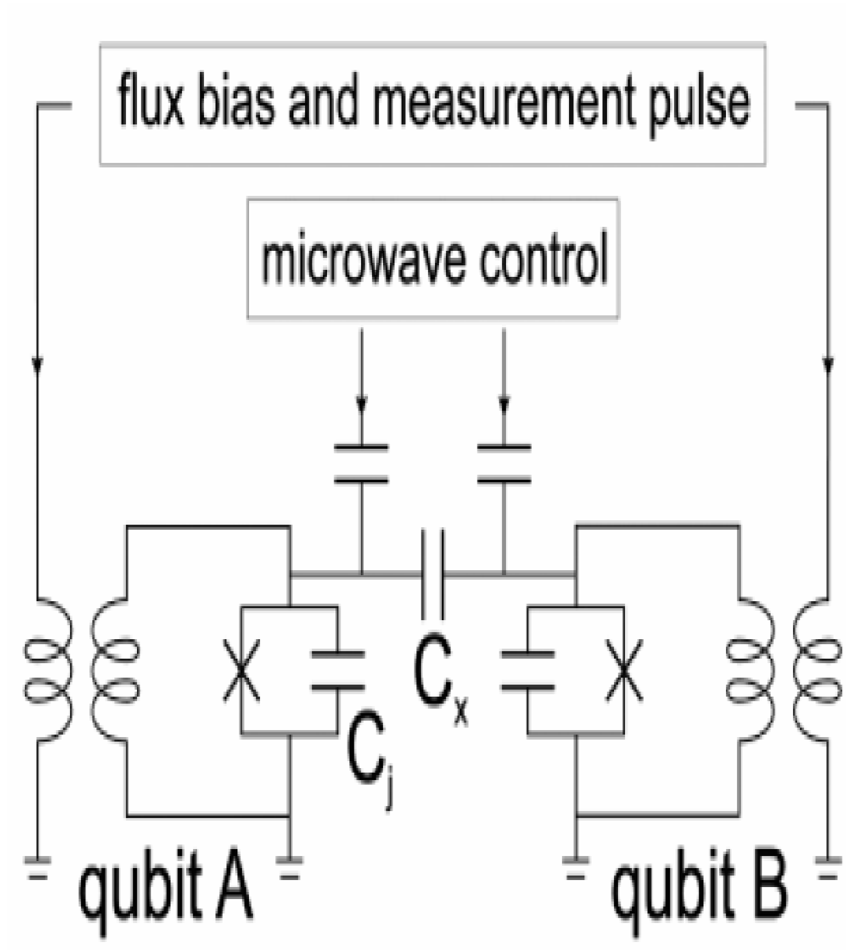
J. Clarke's group, Phys. Rev. B 72, 060506 (2005)

Capacitively coupled phase qubits



Berkley et al., Science (2003)



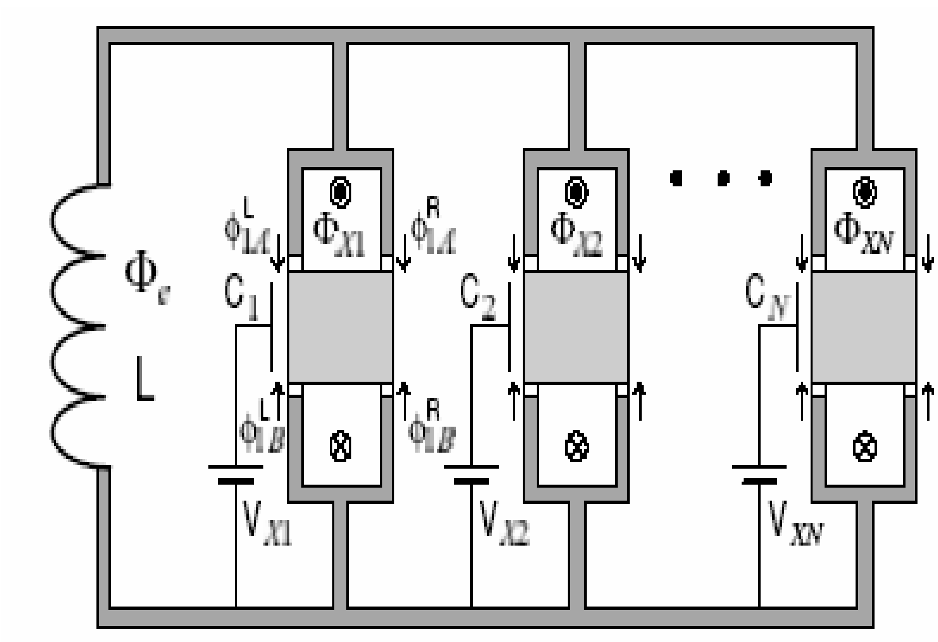


McDermott et al., Science (2005)

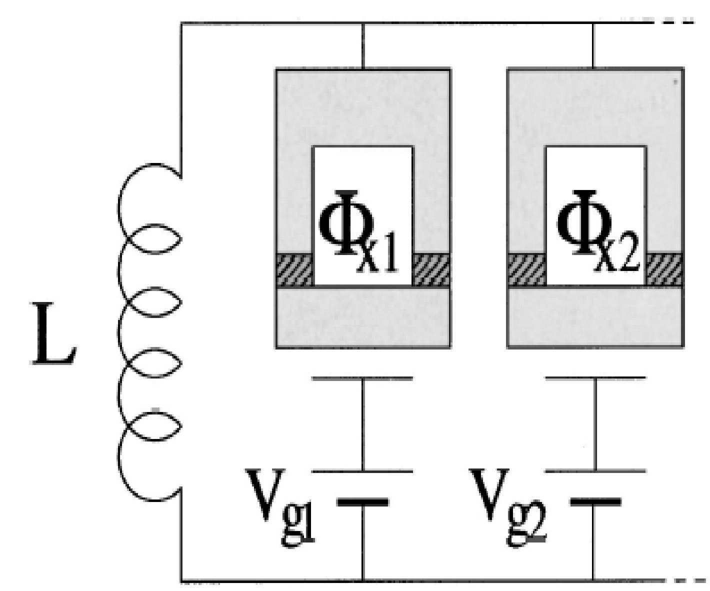
Entangled phase qubit states

Switchable qubit coupling proposals

E.g., by changing the magnetic fluxes through the qubit loops.



You, Tsai, Nori, *PRL* (2002)

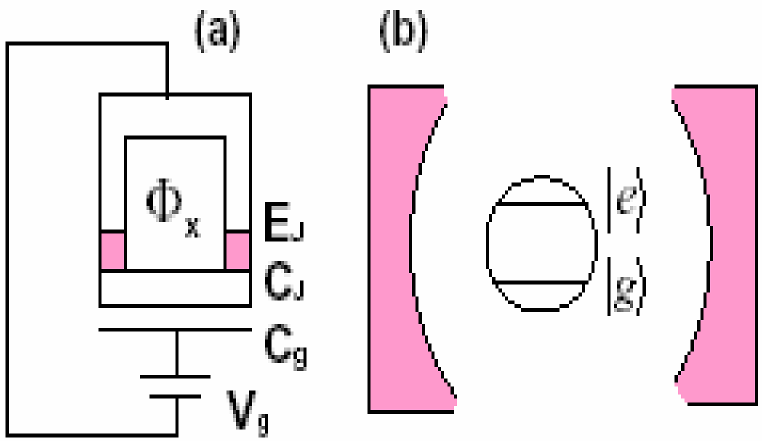


Y. Makhlin et al., *RMP* (2001)

Coupling: $$\chi\left(\Phi_e^{(1)},\Phi_e^{(2)}\right) \propto \cos\left(\pi\frac{\Phi_e^{(1)}}{\Phi_0}\right)\cos\left(\pi\frac{\Phi_e^{(2)}}{\Phi_0}\right)$$

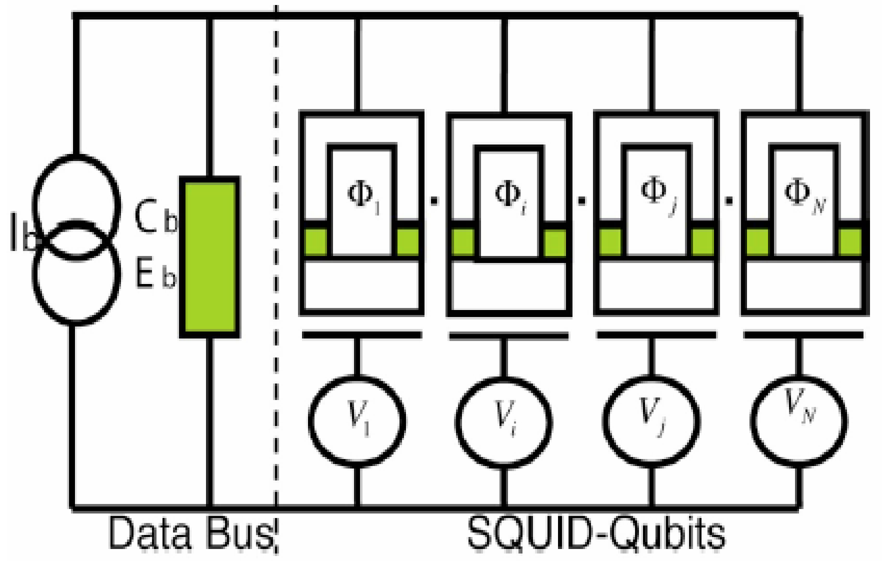
Switchable coupling: data bus

A switchable coupling between the qubit and a data bus could also be realized by changing the magnetic fluxes through the qubit loops.



Liu, Wei, Nori, EPL 67, 941 (2004)

Single-mode cavity field



Wei, Liu, Nori, PRB 71, 134506 (2005)

Current biased junction

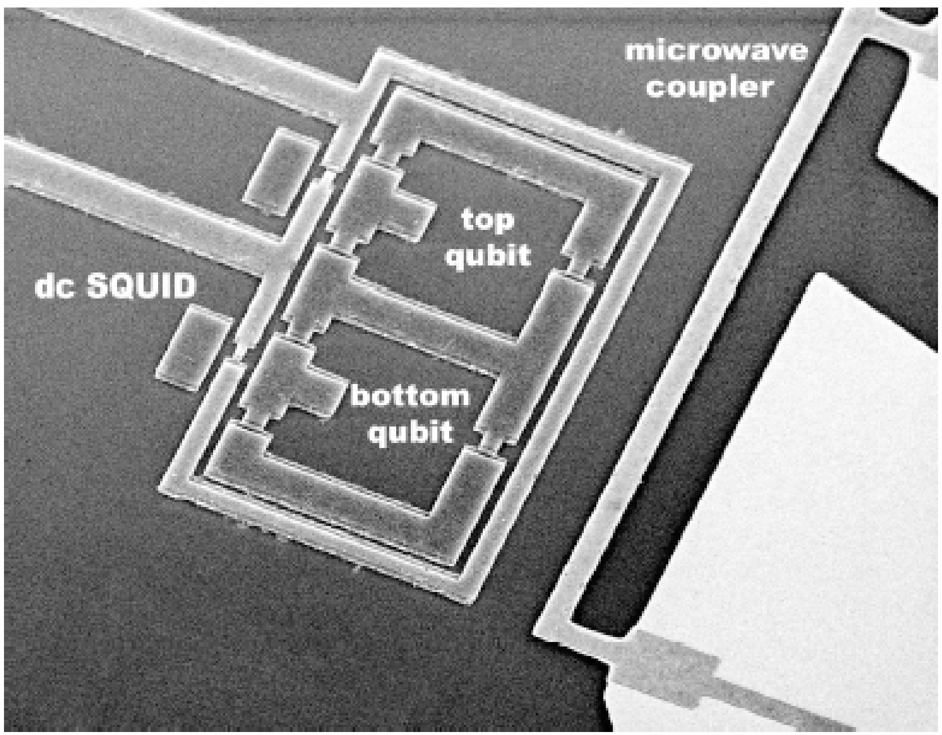
The bus-qubit coupling constant is proportional to $\cos\left(\pi \frac{\Phi_X}{\Phi_0}\right)$

How to couple flux qubits

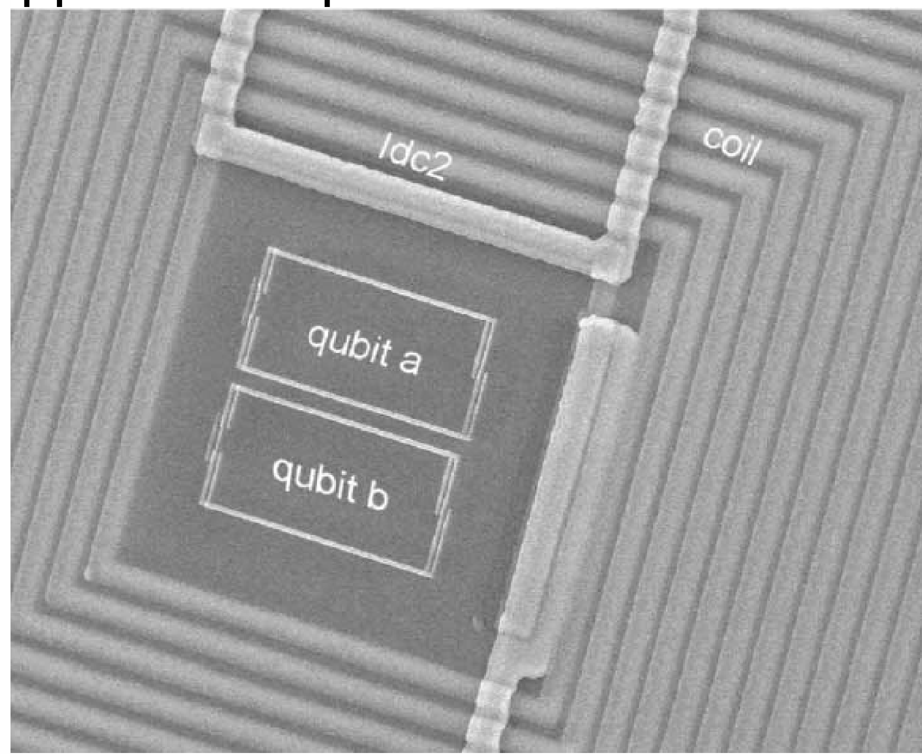
We made several proposals on how to couple qubits.

No auxiliary circuit is used in several of these proposals to mediate the qubit coupling.

This type of proposal could be applied to experiments such as:



J.B. Majer et al., PRL94, 090501(2005)

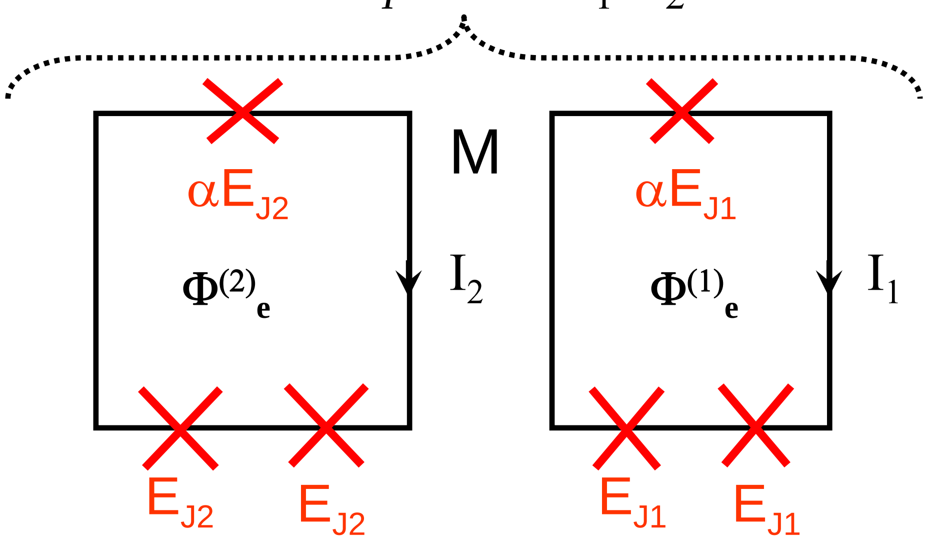


A. Izmalkov et al., PRL 93, 037003 (2004)

Hamiltonian without VFMF (Variable Frequency Magnetic Flux)

$$H_0 = H_{q1} + H_{q2} + H_I \quad = \quad \text{Total Hamiltonian}$$

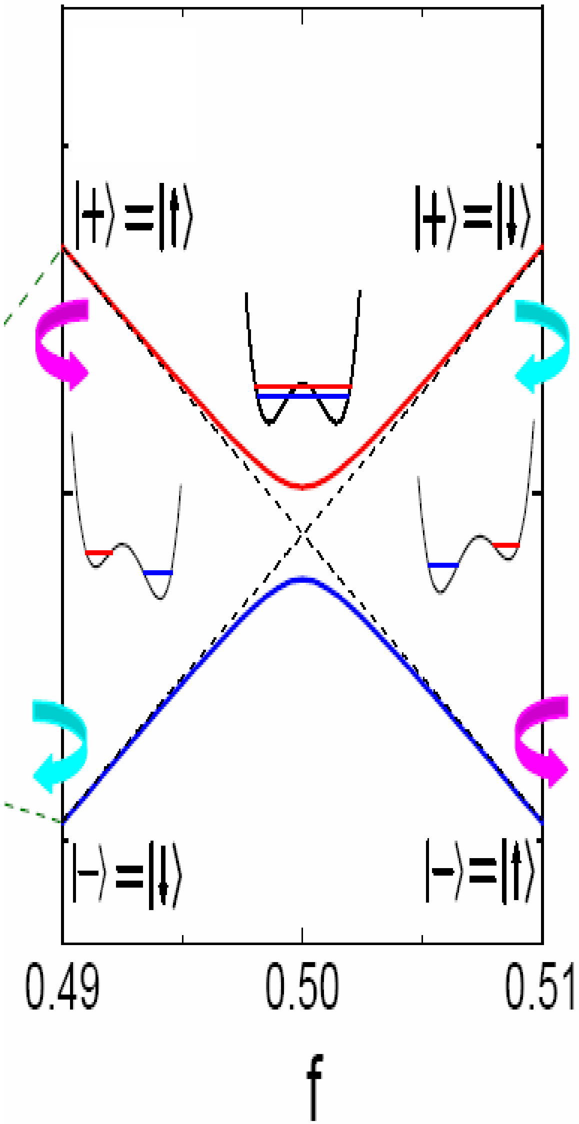
$$H_I = M\ I_1\ I_2$$



$$H_{ql} = \frac{P_{ml}^2}{2M_{ml}} + \frac{P_{pl}^2}{2M_{pl}} + 2E_{Jl} + \alpha E_{Jl} - 2E_{Jl}\cos\varphi_m^{(l)}\cos\varphi_P^{(l)} - \alpha E_{Jl}\cos\left(2\pi \mathrm{f}_1 + 2\varphi_m^{(l)}\right)$$

l=1,2

Hamiltonian in qubit basis



You and Nori, Phys. Today 58 (11), 42 (2005)

$$H_0 = \frac{\hbar}{2}\left[\omega_1\sigma_z^{(1)} + \omega_2\sigma_z^{(2)}\right] + \left[g\sigma_+^{(1)}\sigma_-^{(2)} + \text{H.c.}\right]$$

Qubit frequency ω_l is determined by the loop current $I^{(l)}$ and the tunneling coefficient t_l

$$\omega_l = \sqrt{2I^{(l)}\left[\Phi_e^{(l)} - \Phi_0/2\right]^2 + t_l^2}$$

Decoupled Hamiltonian

$$\Delta = \omega_1 - \omega_2 >> |g|$$

$$H_0 \approx \frac{\hbar}{2}\left[\omega_1 + 2\frac{|g|^2}{\Delta}\right]\sigma_z^{(1)} + \frac{\hbar}{2}\left[\omega_2 - 2\frac{|g|^2}{\Delta}\right]\sigma_z^{(2)}$$

$$|g|/(\omega_1 - \omega_2) \approx 0$$

$$H_0 \approx \frac{\hbar}{2}\omega_1\sigma_z^{(1)} + \frac{\hbar}{2}\omega_2\sigma_z^{(2)}$$

III. Controllable couplings via VFMFs

We propose an experimentally realizable method to control the coupling between two flux qubits (PRL 96, 067003 (2006)).

The dc bias fluxes are always fixed for the two inductively-coupled qubits. The detuning $\Delta = |\omega_2 - \omega_1|$ of these two qubits can be initially chosen to be sufficiently large, so that their initial interbit coupling is almost negligible.

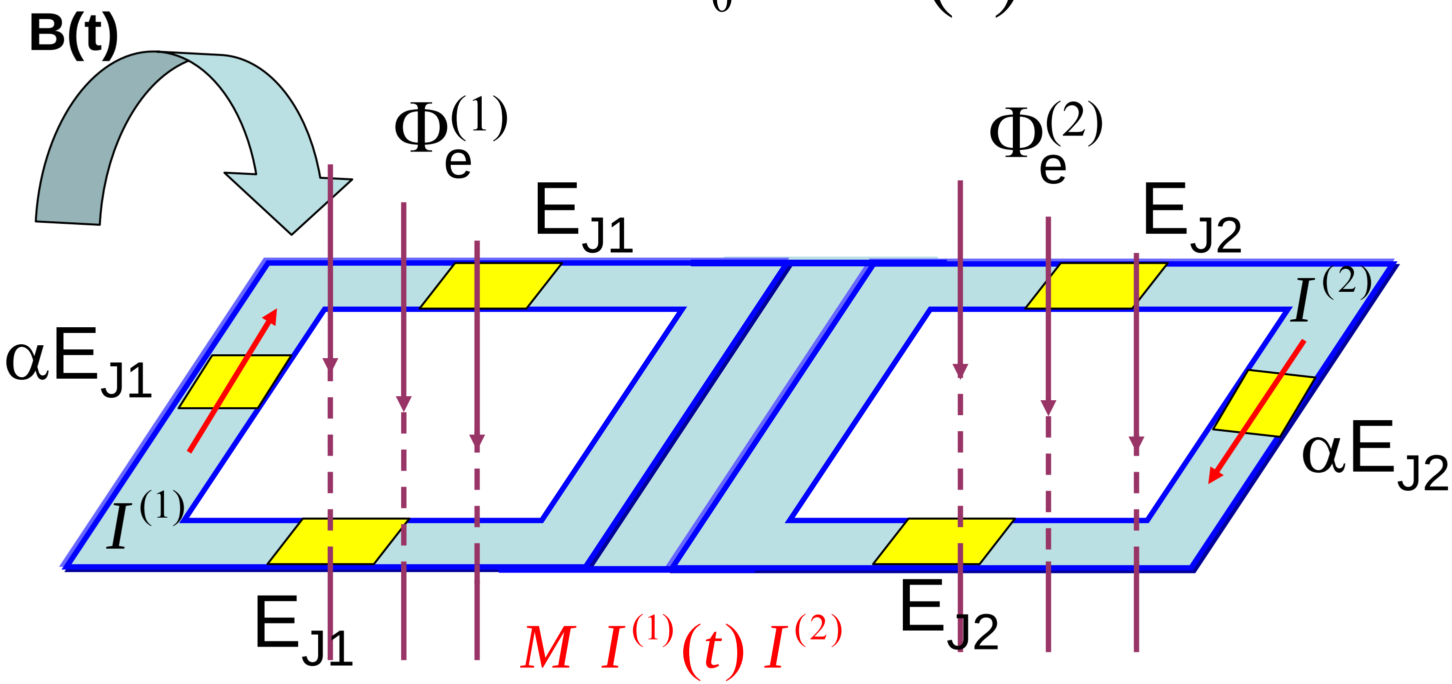
When a time-dependent, or variable-frequency, magnetic flux (VFMF) is applied, a frequency of the VFMF can be chosen to compensate the initial detuning and to couple two qubits.

This proposed method avoids fast changes of either qubit frequencies or the amplitudes of the bias magnetic fluxes through the qubit loops

III. Controllable couplings via VFMFs

Applying a Variable-Frequency Magnetic Flux (VFMF)

$$H = H_0 + H(t)$$



$$I^{(1)}(t) \approx I^{(1)} + \tilde{I}(t)$$

Liu, Wei, Tsai, and Nori, *Phys. Rev. Lett.* 96, 067003 (2006)

III. Controllable couplings via VFMFs

Coupling constants with VFMF

When $|g|/(\omega_1-\omega_2) \ll 1$, the Hamiltonian becomes

$$H=\frac{\hbar}{2}\omega_1\sigma_z^{(1)}+\frac{\hbar}{2}\omega_2\sigma_z^{(2)}+\left[\Omega_1\sigma_+^{(1)}\sigma_+^{(2)}\exp(-i\omega t)+\text{H.c.}\right]+\left[\Omega_2\sigma_+^{(1)}\sigma_-^{(2)}\exp(-i\omega t)+\text{H.c.}\right]$$

$$f_l=\frac{1}{2}\text{ and parity}$$

$$\Omega_1\propto\left\langle e_2\,|\,I^{(2)}\,|\,g_2\right\rangle\times\left\langle e_1\,|\cos(2\varphi_P^{(1)}+2\pi f_1)|\,g_1\right\rangle$$

$$\Omega_2\propto\left\langle g_2\,|\,I^{(2)}\,|\,e_2\right\rangle\times\left\langle e_1\,|\cos(2\varphi_P^{(1)}+2\pi f_1)|\,g_1\right\rangle$$

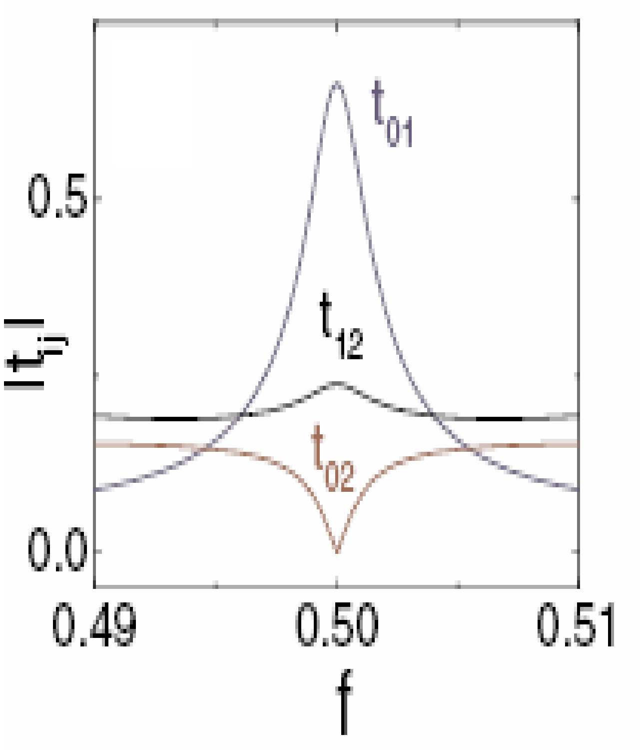
f_1=1/2 even parity

$|g_l\rangle$ and $|e_l\rangle$ have different parities when f_l=1/2

$$I^{(2)}=C_2\sum_{i=1}^{3}\frac{I_{ic}^{(2)}}{C_i}\sin\varphi_i^{(2)}$$

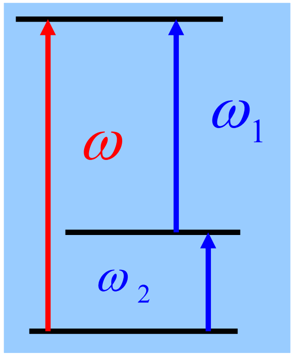
with $$\frac{1}{C_2}=\sum_{i=1}^{3}\frac{1}{C_{Ji}^{(2)}}$$

f_2=1/2 odd parity



Liu et al., *PRL* 95, 087001 (2005)

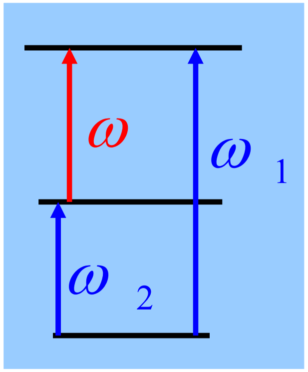
III. Controllable couplings via VFMFs



Frequency or mode matching conditions

$$H_{\text{int}} = \left\{ \Omega_1 \sigma_+^{(1)} \sigma_+^{(2)} \exp\left[-i \left(\omega - \omega_1 - \omega_2 \right) t \right] + \text{H.c.} \right\}$$

$$+ \left\{ \Omega_2 \sigma_+^{(1)} \sigma_-^{(2)} \exp\left[-i \left(\omega + \omega_2 - \omega_1 \right) t \right] + \text{H.c} \right\}$$



$$\omega_1 + \omega_2 = \omega$$

$$\omega_1 - \omega_2 = \omega$$

If $\omega_1 - \omega_2 = \omega$, then the exp[…] of the second term equals one, while the first term oscillates fast (canceling out). Thus, the second term dominates and the qubits are coupled with coupling constant Ω_2

If $\omega_1 + \omega_2 = \omega$, then the exp[…] of the first term equals one, while the second term oscillates fast (canceling out). Thus, the first term dominates and the qubits are coupled with coupling constant Ω_1

Thus, the coupling between qubits can be controlled by the frequency of the variable-frequency magnetic flux (VFMF) matching either the detuning (or sum) of the frequencies of the two qubits.

III. Controllable couplings via VFMFs

Mode matching conditions

$$\omega_1 - \omega_2 = \omega$$

$$H_1 = \Omega_2\, \sigma_+^{(1)}\, \sigma_-^{(2)} + \text{H.c.}$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|e_1, g_2\rangle + |g_1, e_2\rangle\right)$$

$$t = \frac{\hbar \pi}{2|\Omega_2|}$$

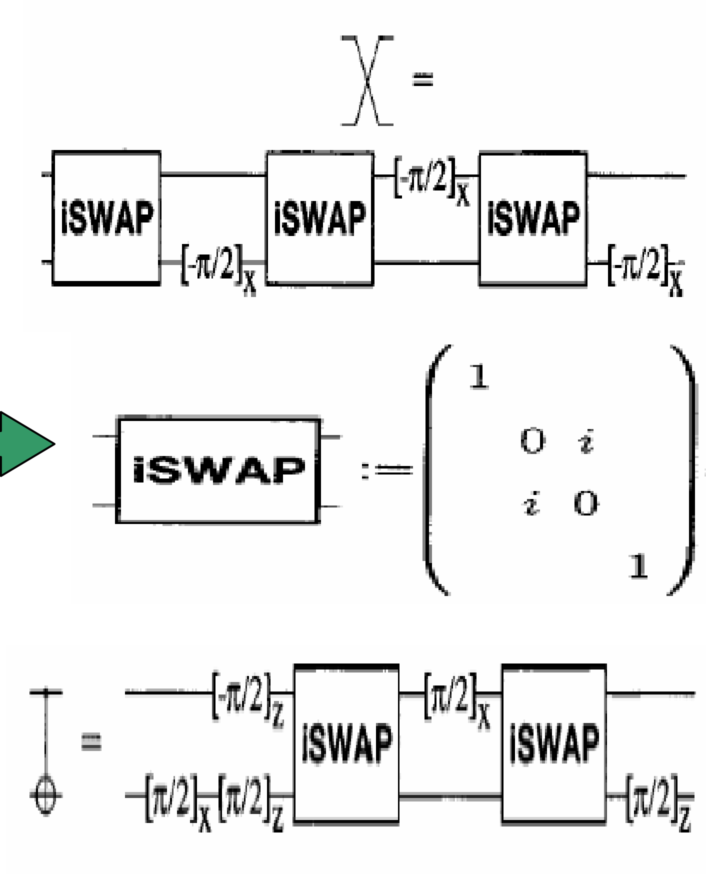
$$\omega_1 + \omega_2 = \omega$$

$$H_2 = \Omega_1\, \sigma_+^{(1)}\, \sigma_+^{(2)} + \text{H.c.}$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|g_1, g_2\rangle + |e_1, e_2\rangle\right)$$

$$t = \frac{\hbar \pi}{2|\Omega_1|}$$

Logic gates

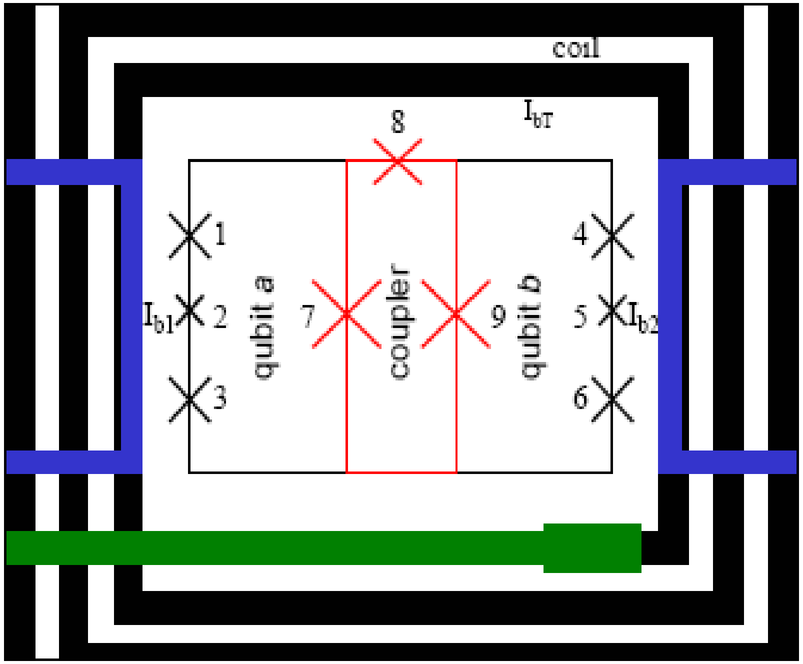


Quantum tomography can be implemented via an ISWAP gate, even if only one qubit measurement can be performed at a time.

Experimentally realizable circuits for VFMF controlled couplings

We propose a coupling scheme, where two or more flux qubits with different eigenfrequencies share Josephson junctions with a coupler loop devoid of its own quantum dynamics.

Switchable two-qubit coupling can be realized by tuning the frequency of the AC magnetic flux through the coupler to a combination frequency of two of the qubits.



Grajcar, Liu, Nori, Zagoskin, cond-mat/0605484.
DC version used in Jena experiments cond-mat/0605588

The coupling allows any or all of the qubits to be simultaneously at the degeneracy point and their mutual interactions can change sign.

Switchable coupling proposals

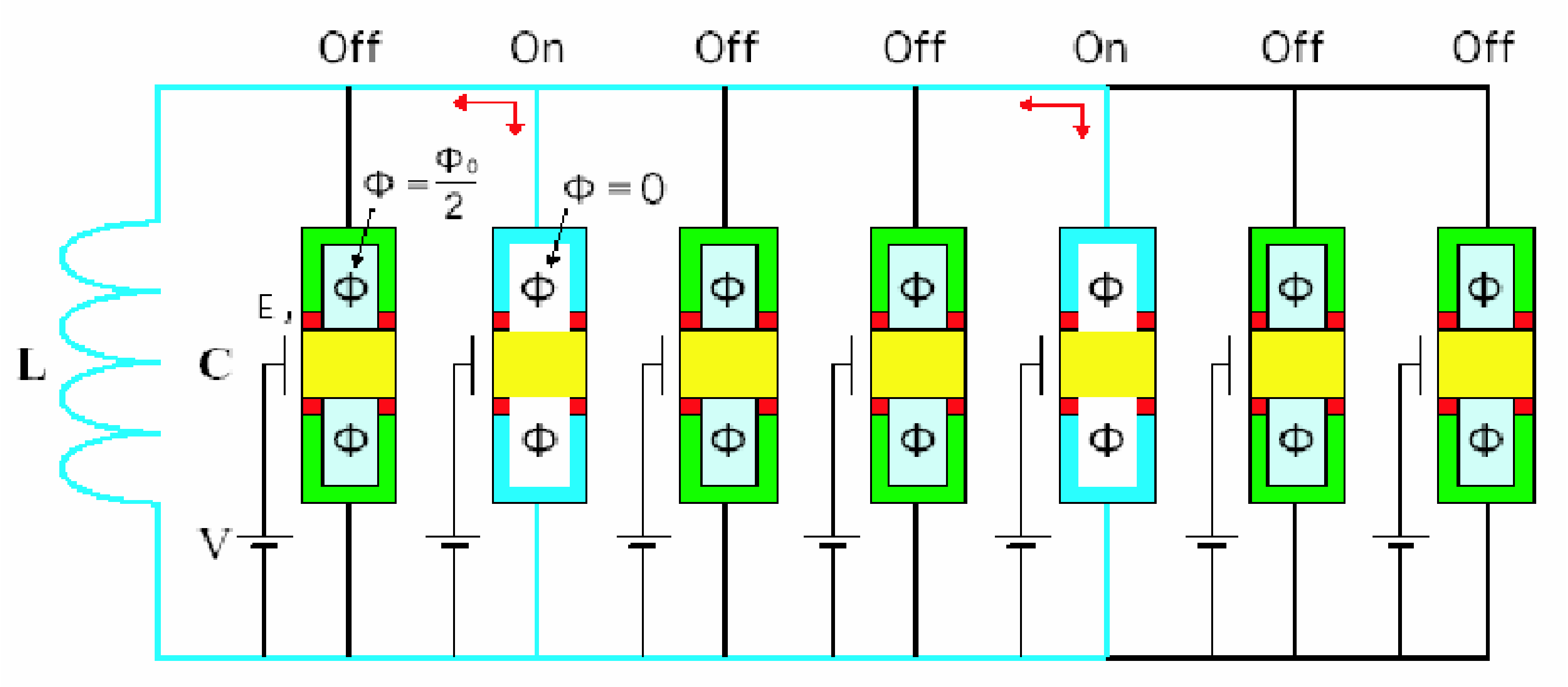
Feature / Proposal	Weak fields	Optimal point	No additional circuitry
Rigetti et al.	No	Yes	Yes
Liu et al.	OK	No	Yes
Bertet et al. Niskanen et al.	OK	Yes	No
Ashhab et al.	OK	Yes	Yes

Depending on the experimental parameters, our proposals might be useful options in certain situations.

Contents

IV. Scalable circuits

Couple qubits via a common inductance



You, Tsai, and Nori, *Phys. Rev. Lett.* 89, 197902 (2002)

Switching on/off the SQUIDs connected to the Cooper-pair boxes, can couple any selected charge qubits by the common inductance (not using LC oscillating modes).

IV. Scalable circuits

We propose a scalable circuit with superconducting qubits (SCQs) which is essentially the same as the successful one now being used for trapped ions.

The SCQs act as "trapped ions" and are coupled to a "vibrating" mode provided by a superconducting LC circuit, acting as a data bus (DB).

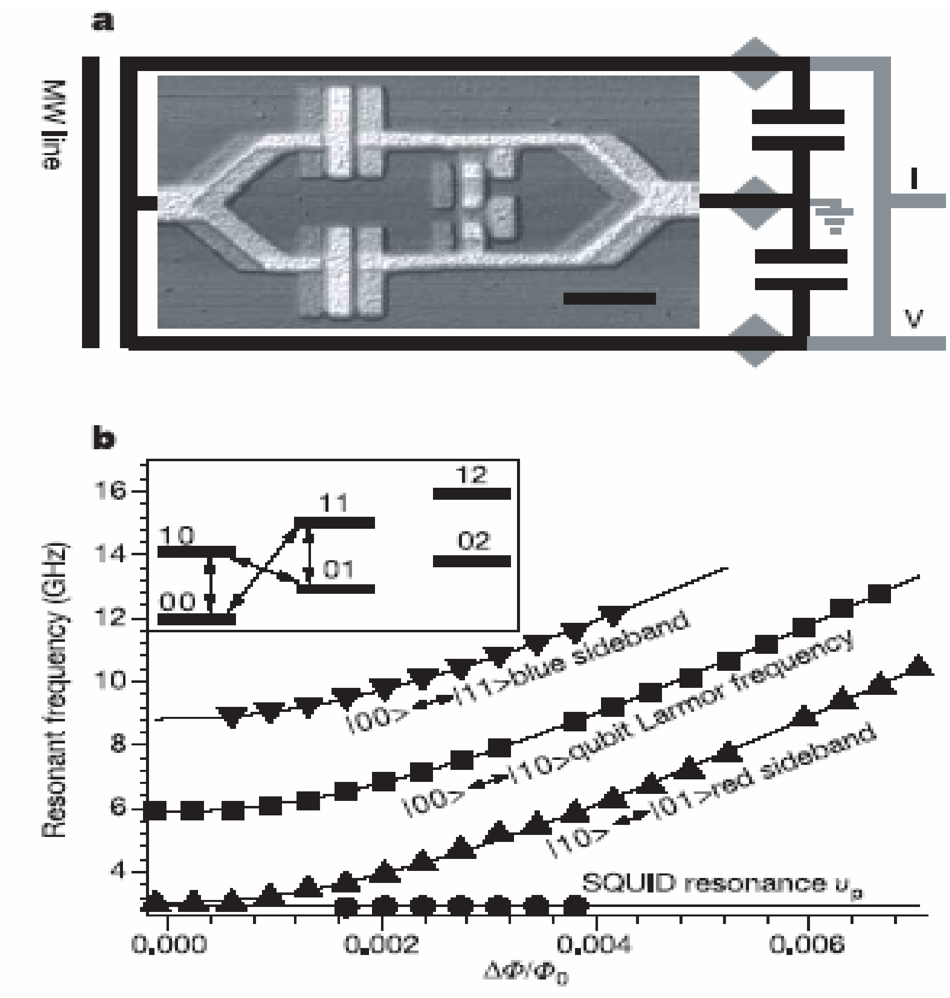
Each SCQ can be separately addressed by applying a time-dependent magnetic flux (TDMF).

Single-qubit rotations and qubit-bus couplings and decouplings are controlled by the frequencies of the TDMFs. Thus, qubit-qubit interactions, mediated by the bus, can be selectively performed.

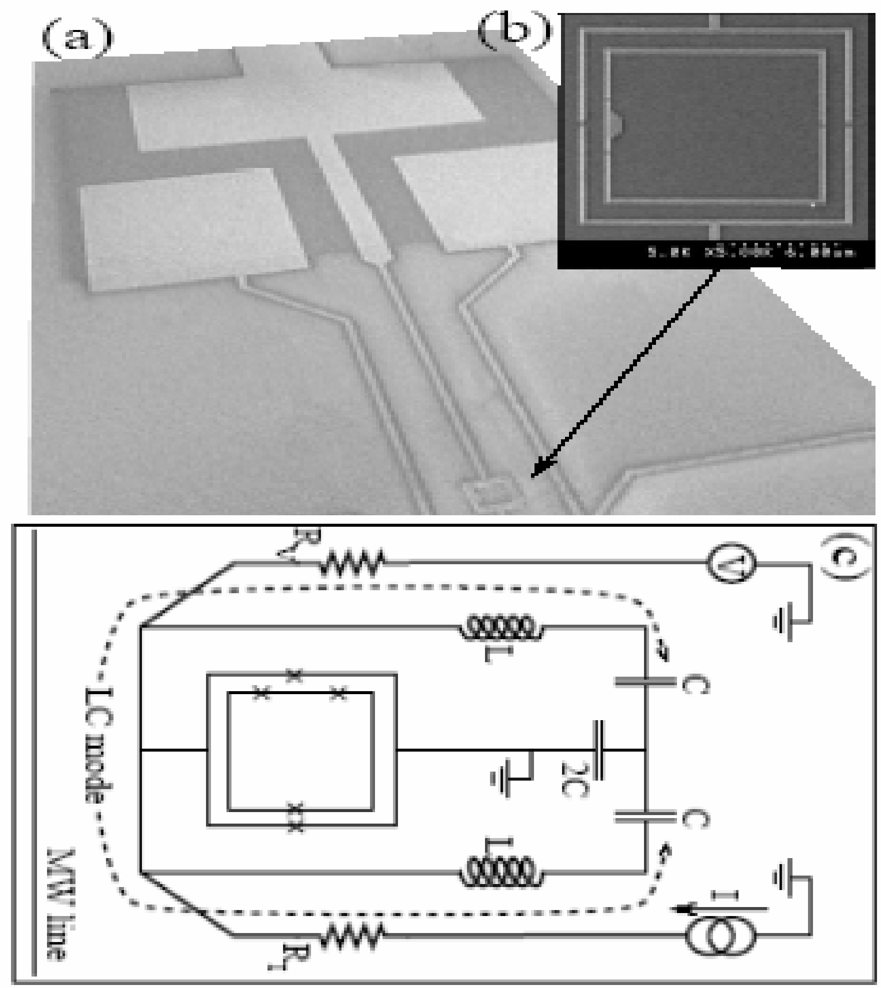
Liu, Wei, Tsai, and Nori, cond-mat/0509236

LC-circuit-mediated interaction between qubits

Level quantization of a superconducting LC circuit has been observed.



Delft, Nature, 2004



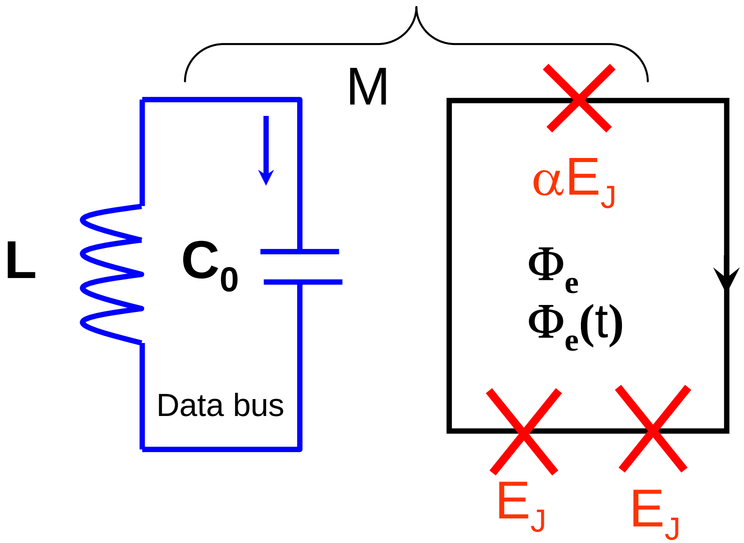
NTT, PRL 96, 127006 (2006)

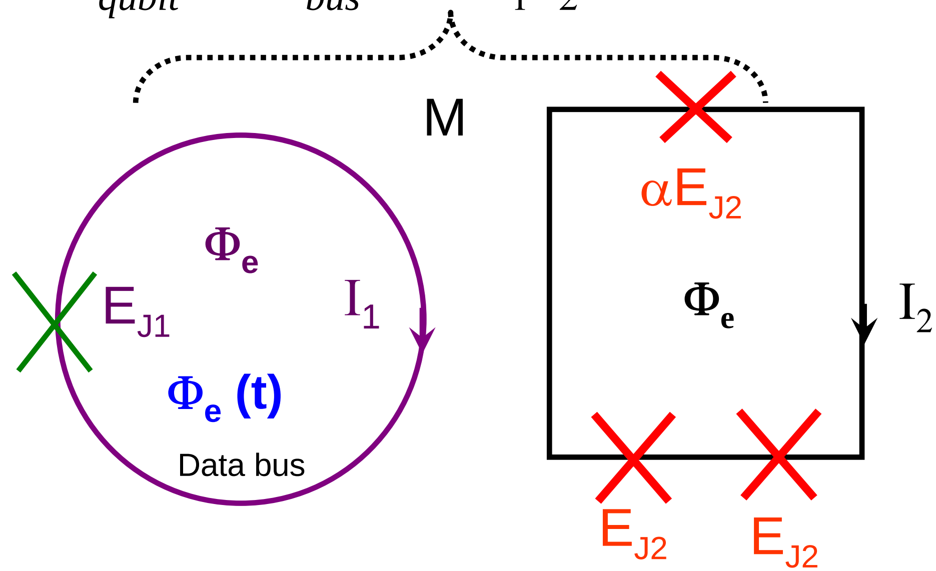
IV. Scalable circuits

Controllable interaction between the data bus and a flux qubit

Inductive coupling via M

$$H = H_{qubit} + H_{bus} + MI_1 I_2$$





The circuit with an LC data bus models the Delft circuit in Nature (2004), which does not work at the optimal point for a TDMF to control the coupling between the qubit and the data bus.

This TDMF introduces a non-linear coupling between the qubit, the LC circuit, and the TDMF.

Replacing the LC circuit by the JJ loop as a data-bus, with a TDMF, then the qubit can work at the optimal point

Liu, Wei, Tsai, Nori, cond-mat/0509236

Controllable interaction between data bus and a flux qubit

$$H=\frac{1}{2}\omega_q\sigma_z-\left(\Omega_1\sigma_++\Omega_1^*\sigma_-\right)\left[\exp\left(-i\omega_C t\right)+\exp\left(i\omega_C t\right)\right]$$
$$+\left(\hbar\omega+\frac{1}{2}\right)a^\dagger a-\left(a^\dagger+a\right)\left(\Omega_2\sigma_-+\Omega_2^*\sigma_+\right)$$
$$-\left(a^\dagger+a\right)\left(\Omega\sigma_++\Omega^*\sigma_-\right)\left[\exp\left(-i\omega_C t\right)+\exp\left(i\omega_C t\right)\right]$$

Large detuning: $|\omega_q - \omega| >> |\Omega_2|$

$$H=\frac{1}{2}\omega_q\sigma_z-\left(\Omega_1\sigma_++\Omega_1^*\sigma_-\right)\left[\exp\left(-i\omega_C t\right)+\exp\left(i\omega_C t\right)\right]$$
$$+\left(\hbar\omega+\frac{1}{2}\right)a^\dagger a-\left(a^\dagger+a\right)\left(\Omega\sigma_++\Omega^*\sigma_-\right)\left[\exp\left(-i\omega_C t\right)+\exp\left(i\omega_C t\right)\right]$$

Mode match and rotating wave approximation

$\omega_q=\omega_c$, Carrier

$$H=\frac{1}{2}\omega_q\sigma_z-\left(\Omega_1\sigma_+\exp\left(-i\omega_C t\right)+\Omega_1^*\exp\left(i\omega_C t\right)\sigma_-\right)$$

$\omega_c=\omega_q-\omega$, Red

$$H=\frac{1}{2}\omega_q\sigma_z+\left(\hbar\omega+\frac{1}{2}\right)a^\dagger a$$
$$+\Omega\sigma_+a\exp\left(-i\omega_C t\right)+\Omega^*\sigma_-a^\dagger\exp\left(i\omega_C t\right)$$

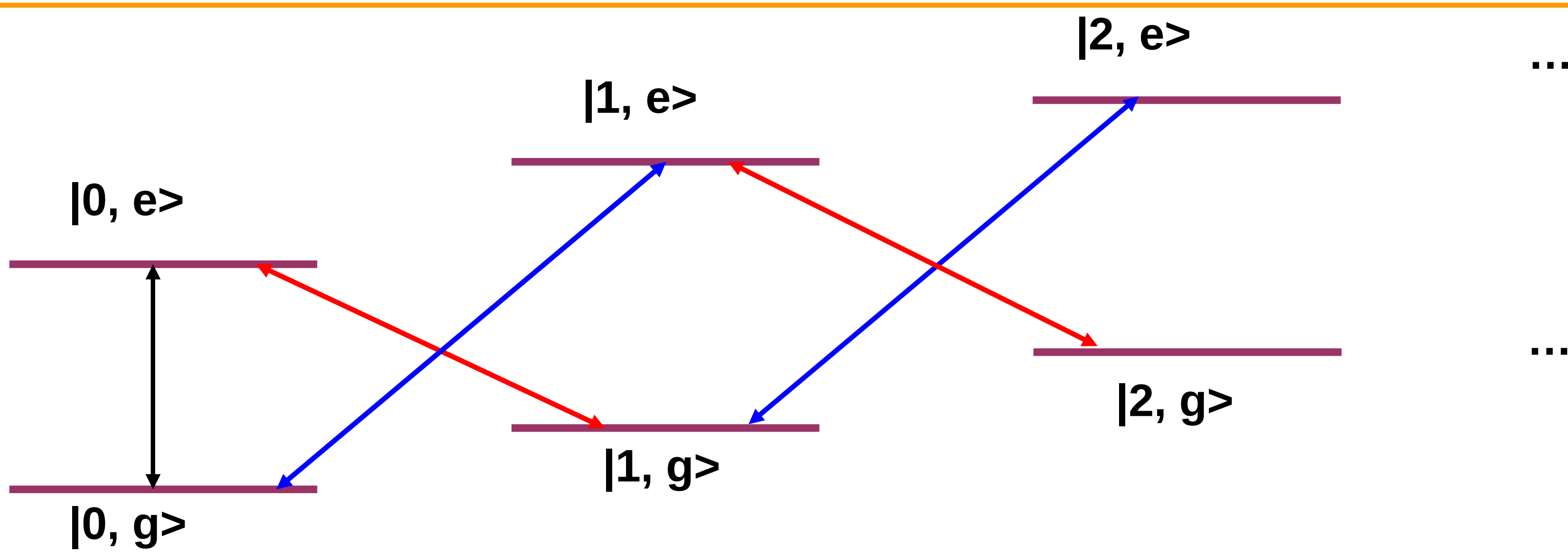
$$H=\Omega\sigma_+a\exp\left(i\Delta t-i\omega_C t\right)+\text{H.c.}$$

$$\Delta=\omega_q-\omega$$

$\omega_c=\omega_q+\omega$, Blue

$$H=\frac{1}{2}\omega_q\sigma_z+\left(\hbar\omega+\frac{1}{2}\right)a^\dagger a$$
$$+\Omega\sigma_+a^\dagger\exp\left(-i\omega_C t\right)+\Omega^*\sigma_-a\exp\left(i\omega_C t\right)$$

Three-types of excitations



|n,g> ⟷ |n,e>

|n+1,g> ⟷ |n,e>

|n,g> ⟷ |n+1,e>

Carrier process:

$\omega_q = \omega_c$

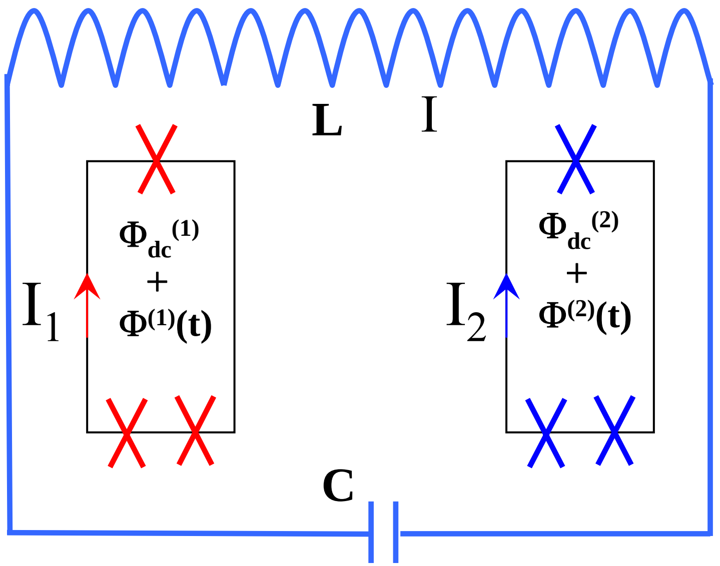
Red sideband excitation:

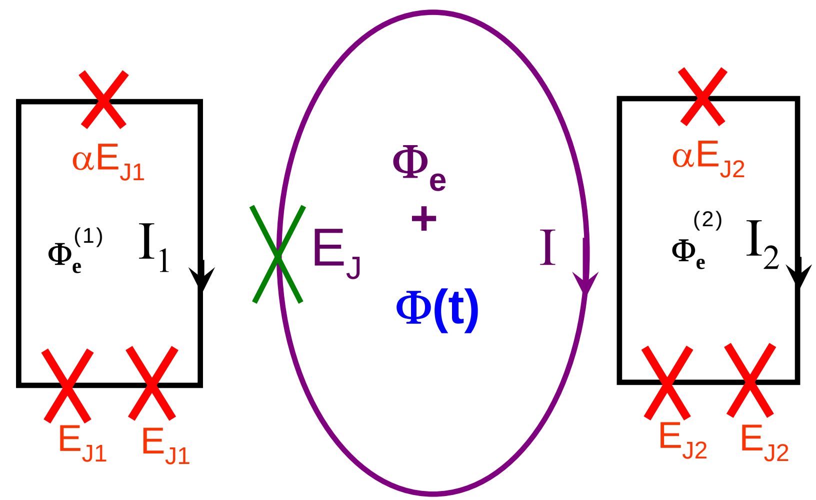
$\omega_c = \omega_q - \omega$

Blue sideband excitation:

$\omega_c = \omega_q + \omega$

A data bus using TDMF to couple several qubits





A data bus could couple several tens of qubits.

The TDMF introduces a nonlinear coupling between the qubit, the LC circuit, and the TDMF.

Liu, Wei, Tsai, Nori, cond-mat/0509236

Comparison between SC qubits and trapped ions

Qubits	Trapped ions	Superconducting circuits
Quantized mode bosonic mode	Vibration mode	LC circuit
Classical fields	Lasers	Magnetic fluxes

Contents

I. Flux qubits

II. Cavity QED on a chip

III. Controllable couplings via variable frequency magnetic fields

IV. Scalable circuits

V. **Dynamical decoupling**

VI. Quantum tomography

VII. Conclusions

V. Dynamical decoupling

Main idea:

Let us assume that the coupling between qubits is not very strong (coupling energy $<$ qubit energy)

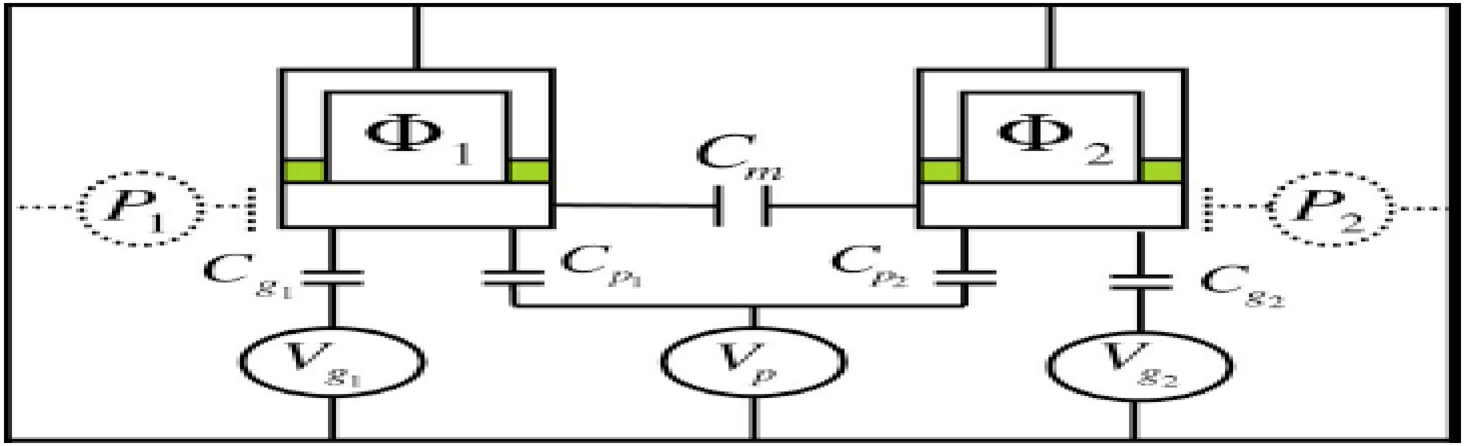
Then the interaction between qubits can be effectively incorporated into the single qubit term (as a perturbation term)

Then single-qubit rotations can be approximately obtained, even though the qubit-qubit interaction is fixed.

Wei, Liu, Nori, *Phys. Rev. B* 72, 104516 (2005)

V. Dynamical decoupling

Test Bell's inequality



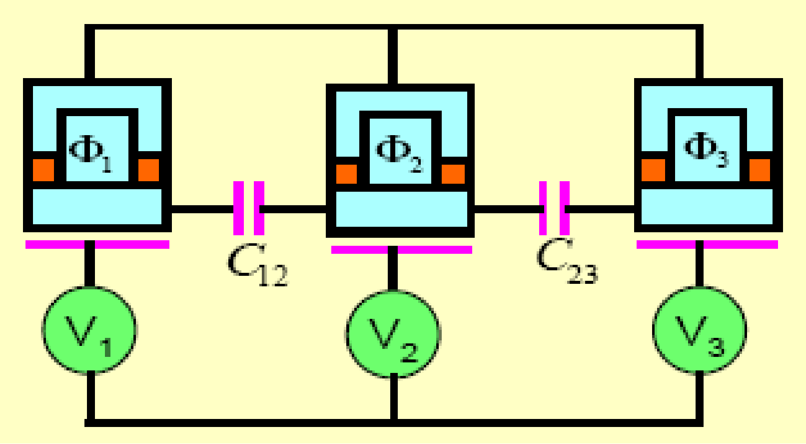
Wei, Liu, Nori, Phys. Rev. B 72, 104516 (2005)

1) Propose an effective dynamical decoupling approach to overcome the "fixed-interaction" difficulty for effectively implementing elemental logical gates for quantum computation.

2) The proposed single-qubit operations and local measurements should allow testing Bell's inequality with a pair of capacitively coupled Josephson qubits.

V. Dynamical decoupling

Generating GHZ states



Wei, Liu, Nori, *Phys. Rev. Lett.* 97, in press (2006); quant-ph/0510169

1) We propose an efficient approach to produce and control the quantum entanglement of three macroscopic coupled superconducting qubits.

2) We show that their Greenberger-Horne-Zeilinger (GHZ) entangled states can be deterministically generated by appropriate conditional operations.

3) The possibility of using the prepared GHZ correlations to test the macroscopic conflict between the noncommutativity of quantum mechanics and the commutativity of classical physics is also discussed.

Contents

VI. Quantum tomography

We propose a method for the *tomographic reconstruction of qubit states* for a general class of solid state systems in which the Hamiltonians are represented by spin operators, e.g., with Heisenberg-, XXZ-, or XY- type exchange interactions.

We analyze the implementation of the projective operator measurements, or spin measurements, on qubit states. All the qubit states for the spin Hamiltonians can be reconstructed by using experimental data.

This general method has been applied to study how to reconstruct any superconducting charge qubit state.

Liu, Wei, Nori, Europhysics Letters 67, 874 (2004); Phys. Rev. B 72, 014547 (2005)

VI. Quantum tomography

Quantum states

A single qubit state can be expressed in the basis $\{|0\rangle, \ |1\rangle\}$ as a density matrix

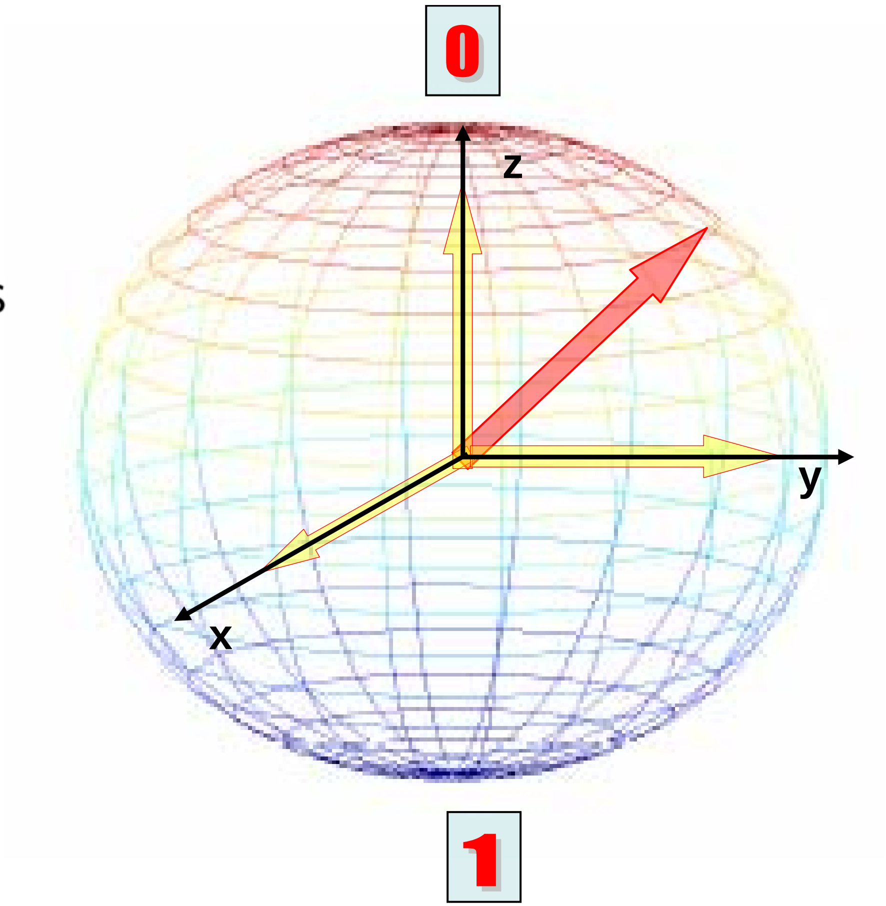
$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix},$$

which can be rewritten as

$$\rho = \frac{1}{2}(1 + \sum_k r_k \sigma_k)$$

with three Pauli matrices σ_k (k=x, y, z), and

$$r_z = \rho_{00} - \rho_{11},$$
$$r_x = \rho_{01} + \rho_{10},$$
$$r_y = i(\rho_{01} - \rho_{10}).$$



Liu, Wei, and Nori, Europhys. Lett. 67, 874 (2004)

VI. Quantum tomography

r_k can be determined via measurements of σ_k: $r_k = Tr(\rho\,\sigma_k)$

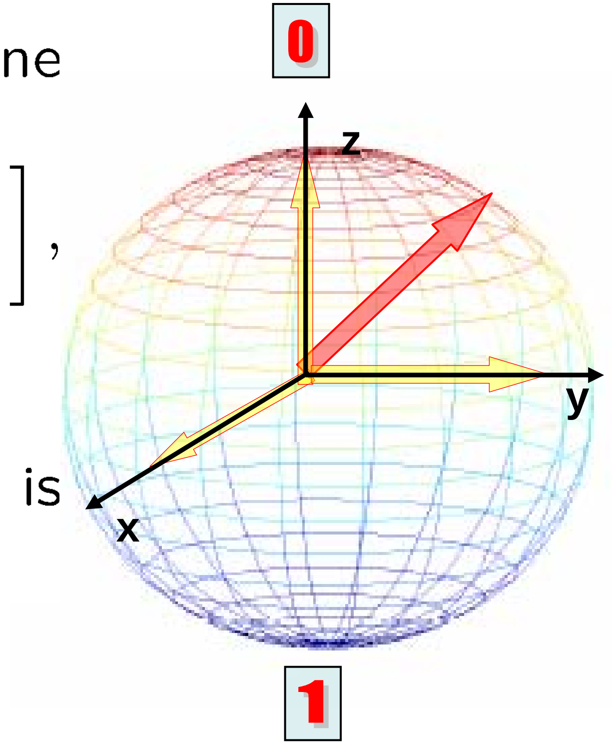
r_z determines the probabilities of $|0\rangle$ and $|1\rangle$.

r_x and r_y determine the relative phase of the state.

The experimental measurement $|1\rangle\langle 1|$ is done along the z axis, that is,

$$|1\rangle\langle 1| = \frac{1}{2}(I - \sigma_z) = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right],$$

which is used to obtain r_z.



The resulting probability of measuring $|1\rangle\langle 1|$ is $p = Tr(\rho|1\rangle\langle 1|) = \frac{1}{2}(1 - r_z) = \rho_{11}$.

VI. Quantum tomography

1. r_x and r_y cannot be directly obtained via the experimentally realizable measurement $|1\rangle\langle 1|$.

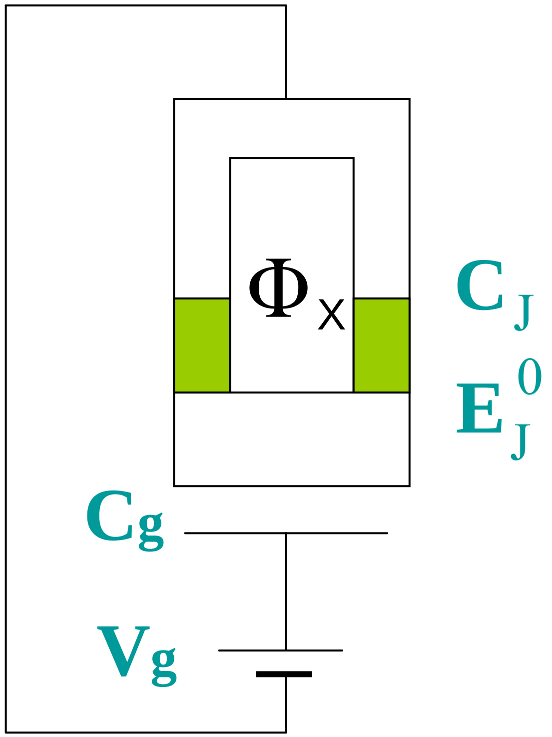
2. A quantum operation (rotation) W needs to be performed so that the r_x and r_y are transformed to a measurable position.

3. After the operation W is made on the qubit state, the measured probability is

$$p = Tr(W\rho W^{\dagger}|1\rangle\langle 1|).$$

4. r_y (r_x) can be obtained by a rotation $\pi/2$ around the x (y) axis.

Superconducting charge qubit



Hamiltonian

$$H = -E_{\text{ch}}\,\sigma_z - E_{\text{J}}\,\sigma_x$$

with

$$E_{\text{ch}} = \frac{e^2}{C_{\text{g}}+2C_{\text{J}}}(1-2n_g)$$

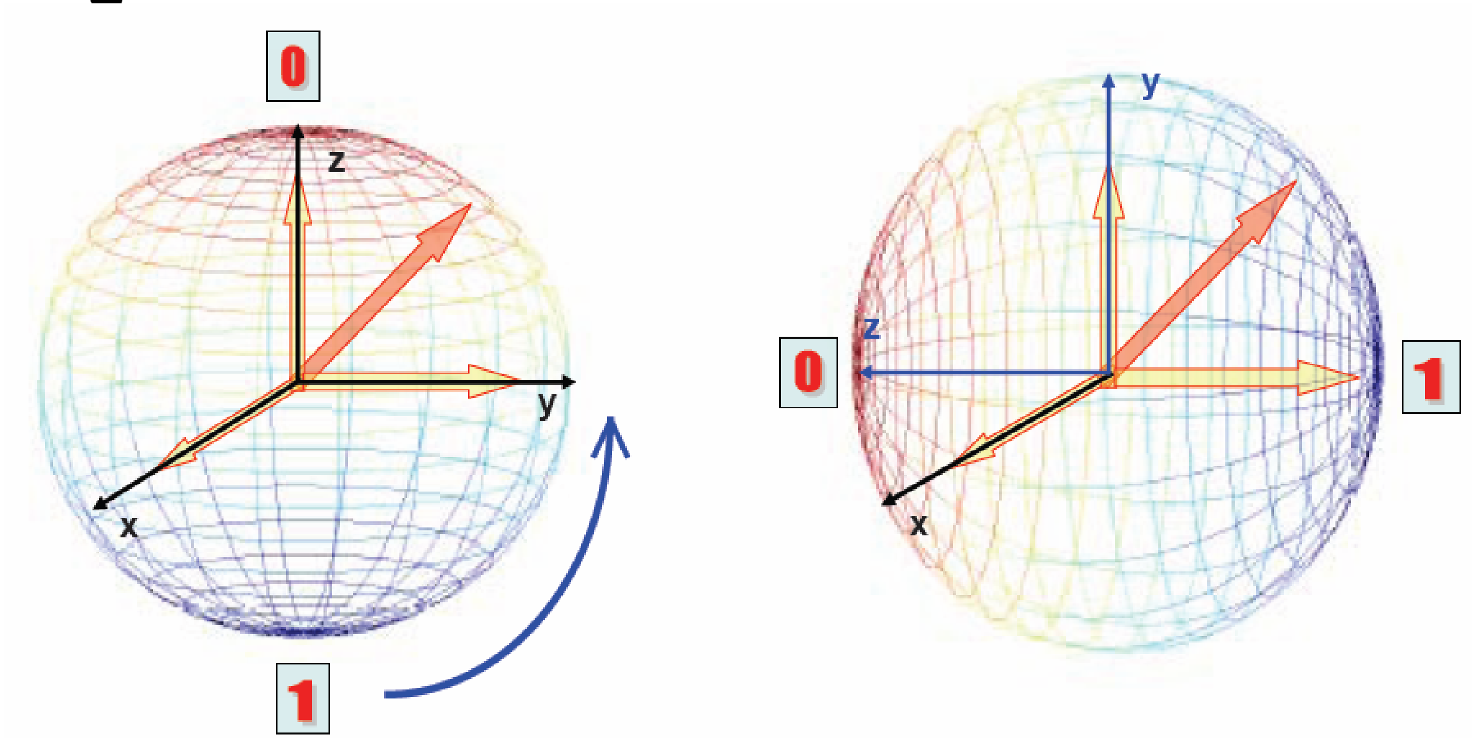
$$E_{\text{J}} = E_{\text{J}}^0 \cos\left(\pi\frac{\Phi_X}{\Phi_0}\right)$$

Quantum tomography for superconducting charge qubits

Liu, Wei, Nori, Phys. Rev. B 72, 014547 (2005)

VI. Quantum tomography

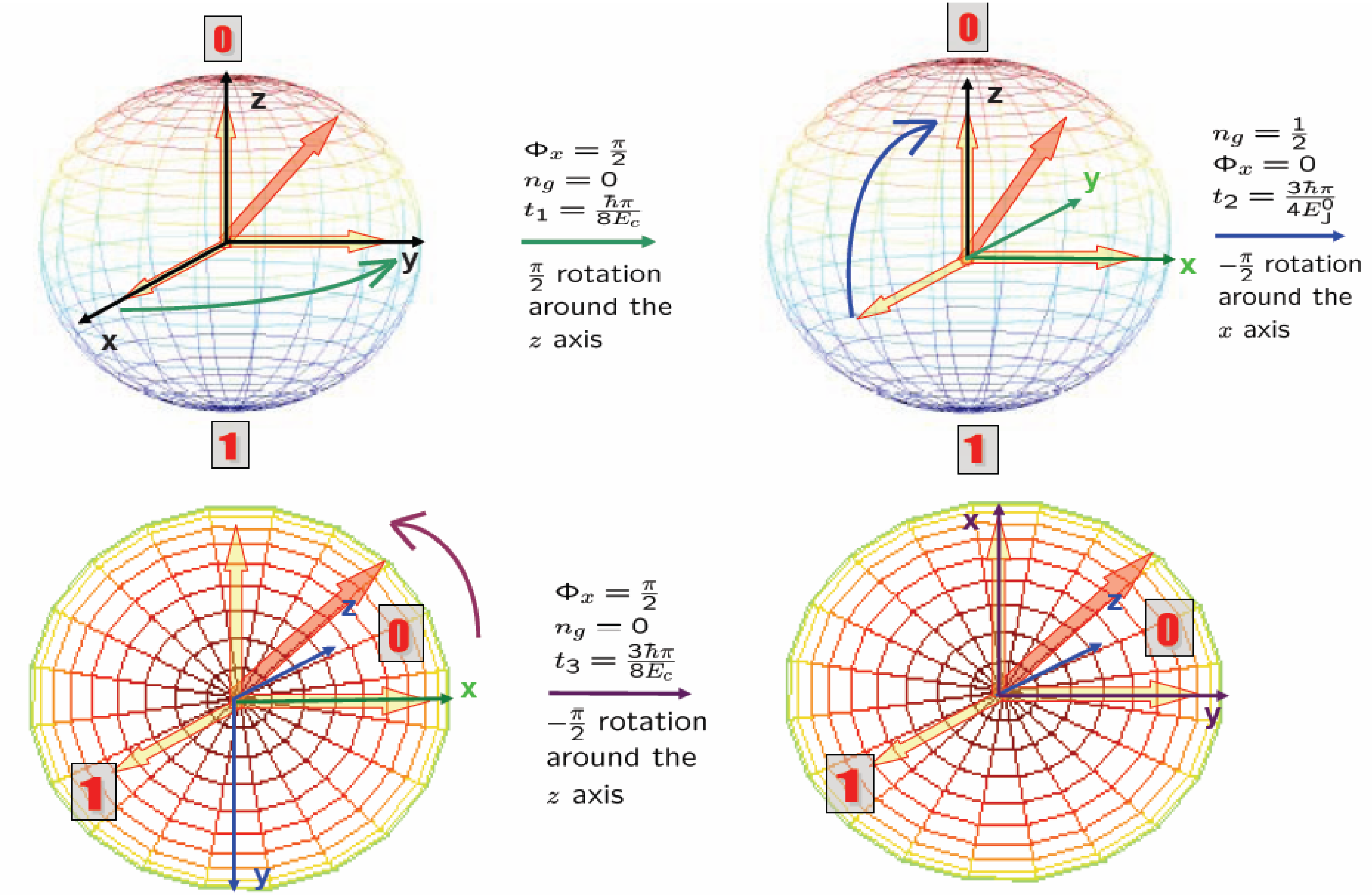
$\frac{\pi}{2}$ rotation around the x axis



This rotation can be realized by setting $\Phi_x = 0$ and $n_\mathrm{C} = \frac{1}{2}$ with an evolution time $t_x = \frac{\hbar\pi}{4E_\mathrm{J}^0}$.

VI. Quantum tomography

$\frac{\pi}{2}$ rotation around the y axis



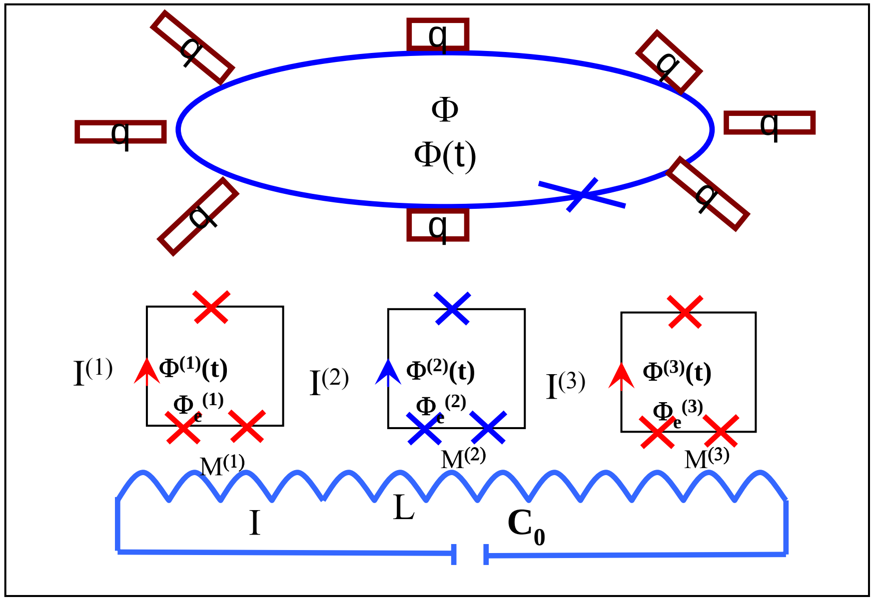
Liu , Wei, Nori, Phys. Rev. B72, 014547 (2005)

Contents

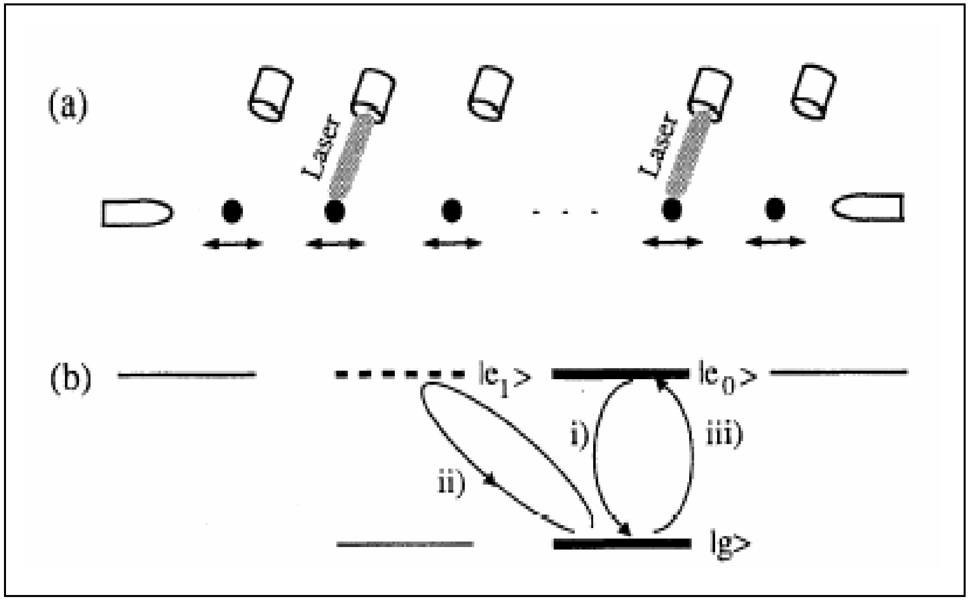
VII. Conclusions

1. Studied superconducting charge, flux, and phase qubits.
2. We proposed and studied circuit QED
3. Proposed how to control couplings between different qubits. These methods are experimentally realizable.
4. Studied how to dynamically decouple qubits with always-on interactions
5. Introduced and studied quantum tomography on solid state qubits.

Comparison between SC qubits and trapped ions



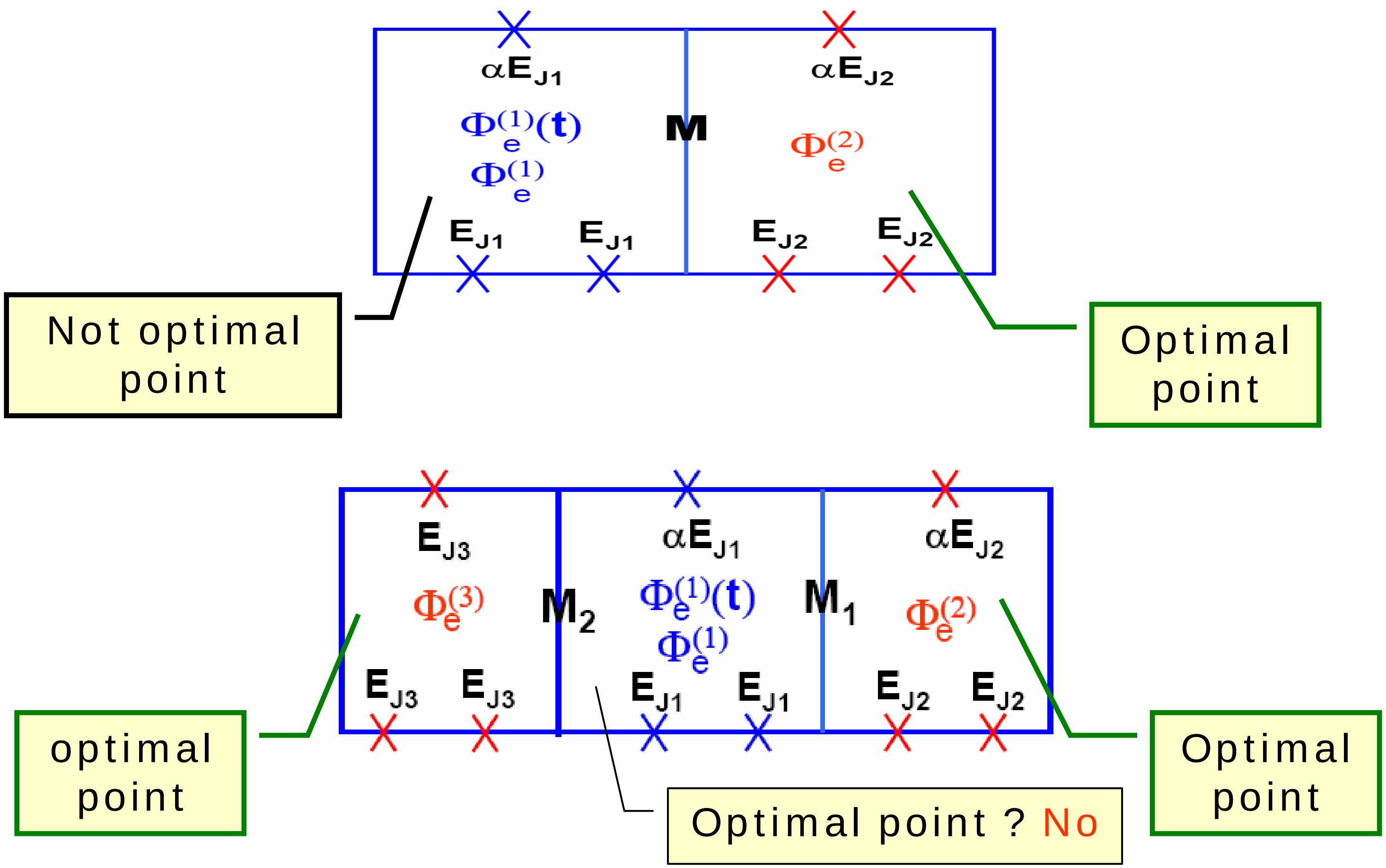
Liu, Wei, Tsai, Nori, cond-mat/0509236



Cirac and Zoller, PRL74, 4091 (1995)

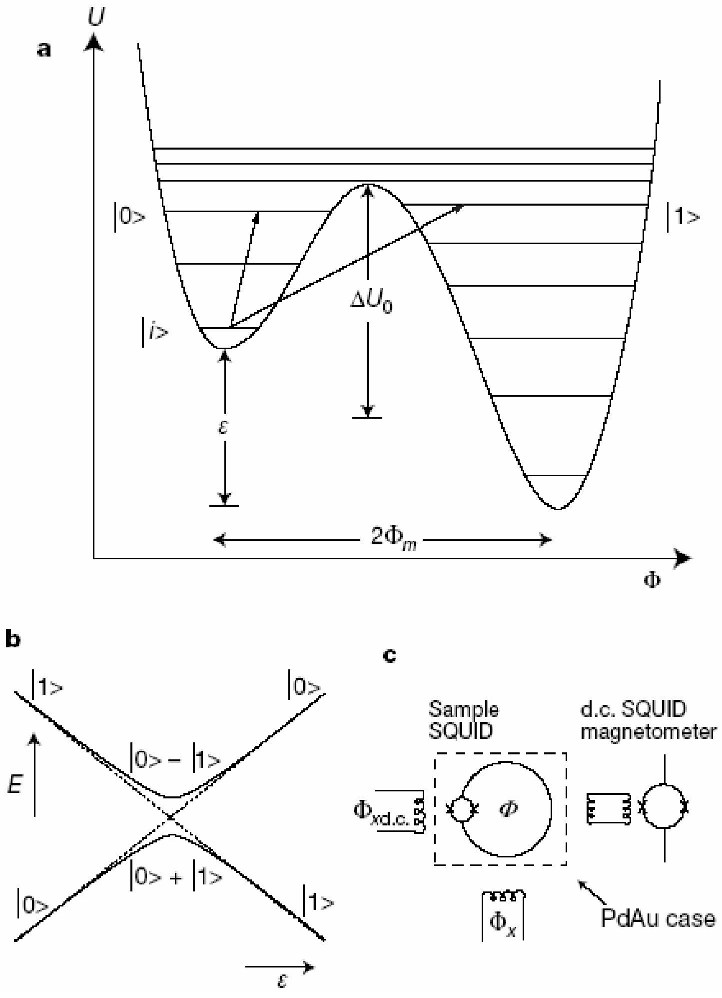
III. Controllable couplings via VFMFs

The couplings in these two circuits work similarly

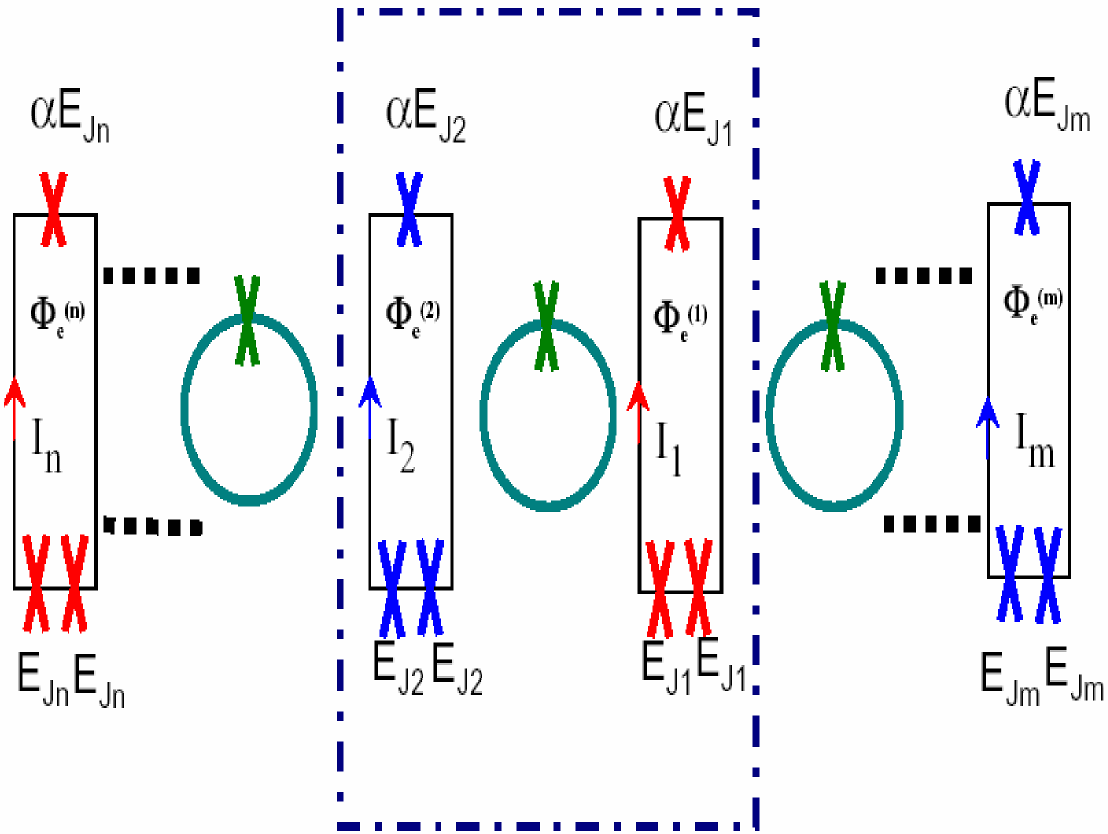


IV. Scalable circuits

rf SQUID mediated qubit interaction



Friedman et al., Nature (2000)



Liu et al, unpublished

Radius of rf SQUID ~100 μm; Radius of the qubit with three junctions~1—10 μm. Nearest neighbor interaction.