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Information about the state of a charge qubit gained by a weakly coupled quantum point contact

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Abstract
We analyze the information that one can learn about the state of a quantum two-level system, i.e. a qubit, when probed weakly by a nearby detector. We consider the general case where the qubit Hamiltonian and the qubit's operator probed by the detector do not commute. Because the qubit's state keeps evolving while being probed and the measurement data is mixed with a detector-related background noise, one might expect the detector to fail in this case. We show, however, that under suitable conditions and by proper analysis of the measurement data, useful information about the initial state of the qubit can be extracted. Our approach complements the usual master-equation and quantum-trajectory approaches, which describe the evolution of the qubit's quantum state during the measurement process but do not keep track of the acquired measurement information.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Solid-state systems are among the most promising candidates for future quantum information processing devices. One type of such systems are superconductor- and semiconductor-based charge qubits [1]. These qubits are commonly measured by capacitively coupling them to quantum point contacts (QPC) or single-electron transistors (SET), such that the current in the detector is sensitive to the charge state of the qubit [2–6][4,5]. By measuring the current passing through the detector, one can infer the state of the qubit. One limitation that arises in practical situations is that, in order to minimize the effects of the detector on the qubit before the measurement, the qubit–detector coupling strength is set to a value that is small compared with the qubit's energy scale[6]. As a result one must deal with some form of weak-measurement regime. This type of weak, charge-sensitive readout works well when the qubit is biased such that the charge states are eigenstates of the Hamiltonian and therefore do not mix during the measurement. In this case, one can allow the detector to probe the qubit for as long as is needed to obtain a high signal-to-noise ratio, without having to worry about any coherent qubit dynamics (note that, since we are mainly interested in the measurement process, we ignore any additional qubit decoherence mechanisms in the system, which could impose constraints on the allowed measurement time).

In contrast to the simple situation described above, when the detector weakly probes the charge state of the qubit

[4] We shall not consider measurement methods based on probing the capacitance of the qubit [7].
[5] For a review on different readout methods in solid-state qubits, see e.g. [8].
[6] Interestingly, weak measurement can have the advantage of being insensitive to undesirable mixing or 'contamination' between the states of the measurement basis [9].

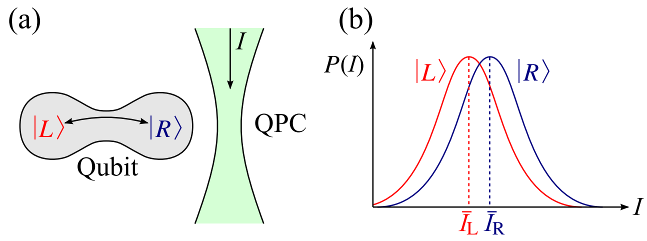

Figure 1. Schematic diagrams of (a) a charge qubit measured by a QPC and (b) the probability distributions of the possible QPC current values for the two charge states of the qubit. The finite widths of the probability distributions are a result of the finite measurement time. When the distance between the two center points in figure 1(b) $|\bar{I}_{\rm R} - \bar{I}_{\rm L}|$ is much smaller than the widths of the distributions, the QPC performs a weak measurement on the qubit in the short interval under consideration. In panel (b), we have assumed that $\bar{I}_{\rm R} > \bar{I}_{\rm L}$, which would be the case if the qubit is defined by an extra positive charge (e.g. a hole) tunneling between the two wells. Taking the opposite case, i.e. $\bar{I}_{\rm L} > \bar{I}_{\rm R}$, would not have any effect on the analysis of this paper.

while the Hamiltonian induces mixing dynamics between the different charge states, it becomes unclear how to interpret a given measurement signal. Since the signal typically contains a large amount of detector-related noise and the measurement unavoidably destroys the coherence present in the qubit's initial state, it might seem that this type of measurement cannot be used to determine the initial state of the qubit, i.e. at the time that the experimenter decides to perform the measurement. Indeed, there have been a number of studies analyzing the measurement-induced decoherence and the evolution of the qubit's state in this situation [2–6], but not the question of how to take the measurement data and extract from it information about the initial state of the qubit. This question is a key issue for qubit-state readout and is the main subject of this paper. We shall show below that high-fidelity information can be extracted from the measurement data, provided that additional decoherence mechanisms are weak and the readout signal can be monitored at a sufficiently short timescale. It turns out that not only the measurement result, but also the measurement basis is determined stochastically in this case. In spite of the uncontrollability of the measurement basis, the measurement results correspond properly to the initial state of the qubit. As an example, we show how they can be used to perform quantum state tomography (QST) on the qubit. These results show that under suitable conditions and by proper analysis of the measurement data, useful information about the state of the qubit can be obtained.

2. Model

We consider a system composed of a charge qubit capacitively coupled to a QPC, as illustrated in figure 1. The qubit can be viewed as a system where a charged particle is trapped in a double-well potential and can occupy, and tunnel between, the localized ground states of the two wells. We shall denote these states by $|L\rangle$ and $|R\rangle$.

During the measurement a voltage is applied to the QPC, and a current flows through it. We assume that the QPC measures the qubit in the $\{|L\rangle, |R\rangle\}$ basis; the current through the QPC depends on whether the qubit is in the state $|L\rangle$ or $|R\rangle$. We further assume that the QPC does not induce any decoherence in the qubit's state except that associated with the measurement-induced projection. For the purpose of fully characterizing the operation of the QPC as a detector for the state of the qubit, it is convenient to make a few statements about the QPC's operation when the qubit Hamiltonian is diagonal in the charge basis and the qubit is initialized in one of the states of the charge basis. In this case, there is no mechanism by which the states $|L\rangle$ and $|R\rangle$ mix during the system dynamics. If the qubit is initially in the state $|L\rangle$, the long-time-averaged QPC current is given by $\bar{I}_{\rm L}$, and the qubit remains in the state $|L\rangle$. A similar statement applies to the state $|R\rangle$ of the qubit, with corresponding QPC current $\bar{I}_{\rm R}$. The QPC current therefore serves as an indicator of the qubit's state in the charge basis $\{|L\rangle, |R\rangle\}$, as long as the qubit Hamiltonian does not mix the states of this basis.

On any finite timescale, there will be fluctuations in the QPC current, and the observed value might deviate from $\bar{I}_{\rm L}$ or $\bar{I}_{\rm R}$. The longer the period over which the averaging is made, the smaller the fluctuations. One can therefore define a measurement timescale that determines how long one needs to wait in order to distinguish between the states $|L\rangle$ and $|R\rangle$. The relation between this timescale and the qubit's Hamiltonian-induced precession period separates two measurement regimes: fast versus slow measurement, or alternatively strong versus weak qubit–detector coupling. Note that this distinction is irrelevant when the qubit Hamiltonian is diagonal in the charge basis, since there is no mixing between the states $|L\rangle$ and $|R\rangle$ in this case.

For the remainder of this paper, we analyze the general case where the qubit Hamiltonian is not necessarily diagonal in the charge basis. We shall use the basis in which the qubit Hamiltonian is diagonal, thus

$$\hat{H}_{\rm q} = -E\hat{\sigma}_z/2, \tag{1}$$

where E is the energy splitting between the qubit's two energy levels, and $\hat{\sigma}_z$ is the z-axis Pauli matrix. We shall denote the ground and excited states of the Hamiltonian by $|0\rangle$ and $|1\rangle$, respectively. The states of the charge basis can be expressed as

$$\begin{aligned} |R\rangle &= \cos\frac{\beta}{2}|0\rangle + \sin\frac{\beta}{2}|1\rangle \\ |L\rangle &= \sin\frac{\beta}{2}|0\rangle - \cos\frac{\beta}{2}|1\rangle, \end{aligned} \tag{2}$$

where β represents the angle between the charge basis and the energy eigenbasis.

3. Measurement- and Hamiltonian-induced dynamics

We start our analysis by considering a short-time interval between times t and $t+\delta t$. We assume that during this time interval a large number of electrons tunnel through the QPC, such that it is natural to define a QPC current $I(t)$ during this short interval. We also assume that a weak-measurement regime exists for a properly chosen value of δt, which means that the QPC-current probability densities (for the states $|L\rangle$ and $|R\rangle$) are broad and almost completely overlap, as shown in figure 1(b). For definiteness we shall take these probability densities to be time-independent, Gaussian functions. The interval δt is taken to be much longer than the coherence time of the QPC, such that the QPC's operation during this interval is independent of the QPC's state at earlier times. Finally, we take δt to be much shorter than the precession period of the qubit.

We now construct matrices (or propagators) that describe the qubit-state evolution depending on the observed QPC current $I(t)$: when a given value of $I(t)$ is observed in the QPC, the quantum state of the qubit is projected (possibly partially) according to the observed value. Neglecting decoherence that is not associated with the measurement, the projection of the qubit's state is described by a 2×2 matrix that we shall call $\hat{U}_{\rm M}[I, \delta I, \delta t]$, where δI defines the size of a finite interval of QPC currents that we identify with a single value:

$$\rho_{\rm q}(t+\delta t) \propto \hat{U}_{\rm M}[I, \delta I, \delta t]\, \rho_{\rm q}(t)\, \hat{U}_{\rm M}^{\dagger}[I, \delta I, \delta t], \qquad (3)$$

where $\rho_{\rm q}$ denotes the qubit's density matrix, and $\dagger$ represents the complex conjugate transpose. One could say that with the introduction of δI we are turning the probability distributions in figure 1(b) into histograms with discrete possible outcomes (this discretization will also be used in our numerical calculations below). In order to identify the appropriate form for $\hat{U}_{\rm M}[I, \delta I, \delta t]$, we note that the probability of obtaining the corresponding outcome is

$$P[I, \delta I, \delta t] = {\rm Tr}\{\hat{U}_{\rm M}^{\dagger}[I, \delta I, \delta t]\hat{U}_{\rm M}[I, \delta I, \delta t]\rho_{\rm q}(t)\}. \qquad (4)$$

Let us denote by $P_j[I, \delta I, \delta t]$ the probability that the value I (up to the dicretization parameters δI and δt) of the QPC current is observed given that the qubit is in state j. We now find that the simplest, and in some sense ideal, matrix $\hat{U}_{\rm M}[I, \delta I, \delta t]$ that satisfies equation (4) is given by

$$\hat{U}_{\rm M}[I, \delta I, \delta t] = \sqrt{P_{\rm L}[I, \delta I, \delta t]}|L\rangle\langle L| + \sqrt{P_{\rm R}[I, \delta I, \delta t]}|R\rangle\langle R|. \qquad (5)$$

This matrix could be followed by a unitary transformation that does not affect equation (4). Any such transformation can be incorporated into the analysis straightforwardly.

In addition to the measurement-induced evolution described by equation (5), the qubit Hamiltonian induces a unitary evolution in the qubit's state during the time interval t to $t+\delta t$: taking $\hbar = 1$

$$\hat{U}_{\rm H}[\delta t] = \exp\{-{\rm i}\hat{H}_{\rm q}\delta t\} \approx 1 + {\rm i}\frac{E}{2}\delta t\hat{\sigma}_z. \qquad (6)$$

The matrices $\hat{U}_{\rm M}[I, \delta I, \delta t]$ and $\hat{U}_{\rm H}[\delta t]$ can now be combined to give the total evolution matrix

$$\hat{U}[I(t), \delta I, \delta t] = \hat{U}_{\rm M}[I(t), \delta I, \delta t] \times \hat{U}_{\rm H}[\delta t]. \qquad (7)$$

Note that both $\hat{U}_{\rm M}[I, \delta I, \delta t]$ and $\hat{U}_{\rm H}[\delta t]$ are approximately proportional to the unit matrix in the limit $\delta t \to 0$, with lowest-order corrections of order δt. The operators $\hat{U}_{\rm M}[I, \delta I, \delta t]$ and $\hat{U}_{\rm H}[\delta t]$ therefore commute to first order in δt.

When a given QPC output signal $I(t)$ (from the initial time $t = 0$ until $t = t_{\rm f}$) is observed, one can take the corresponding short-time evolution matrices explained above and use them to construct the total evolution matrix $\hat{U}_{\rm Total}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]$. Using the unit matrix as the total evolution matrix for $t = 0$, we find that

$$\hat{U}_{\rm Total}[I(t: 0 \to t_{\rm f}), \delta I, \delta t] = \hat{U}[I(t_{\rm f} - \delta t), \delta I, \delta t] \times \cdots \times \hat{U}[I(0), \delta I, \delta t]. \qquad (8)$$

Once the 2×2 matrix $\hat{U}_{\rm Total}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]$ is calculated, one can divide it into two parts, a measurement matrix $\hat{U}_{\rm Meas}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]$ followed by a unitary transformation $\hat{U}_{\rm Rot}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]$:

$$\hat{U}_{\rm Total}[I(t: 0 \to t_{\rm f}), \delta I, \delta t] = \hat{U}_{\rm Rot}[I(t: 0 \to t_{\rm f}), \delta I, \delta t] \times \hat{U}_{\rm Meas}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]. \qquad (9)$$

The matrix $\hat{U}_{\rm Meas}$ has the form

$$\hat{U}_{\rm Meas} = \sqrt{P_1}|\psi_1\rangle\langle\psi_1| + \sqrt{P_2}|\psi_2\rangle\langle\psi_2|, \qquad (10)$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are two orthogonal states. The states $|\psi_1\rangle$ and $|\psi_2\rangle$ represent the measurement basis that corresponds to the output signal $I(t)$, and the parameters P_i are the probabilities that the outcome defined by $I(t)$, δI and δt is obtained given that the qubit was initially in the state $|\psi_1\rangle$. With a simple calculation, one can see that the measurement fidelity is given by (see e.g. [10])

$$\text{fidelity} = \left|\frac{P_1 - P_2}{P_1 + P_2}\right|. \qquad (11)$$

To summarize, the QPC output-current signal can be used to derive the matrix $\hat{U}_{\rm Total}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]$. This matrix can then be used to determine the measurement basis, the measurement result (i.e. ± 1 along the measurement axis), the fidelity (or in other words the degree of certainty about the obtained measurement result) and the final state of the qubit (given by the measurement result transformed by the unitary, i.e. rotation, matrix $\hat{U}_{\rm Rot}[I(t: 0 \to t_{\rm f}), \delta I, \delta t]$). Note that when the measurement fidelity approaches one, the final state is a pure state that can be determined even without any *a priori* knowledge about the initial state.

4. Results and discussion

We now present the results of our numerical calculations. The calculations were performed by analyzing a sequence of discrete events, with each event representing a time step of size δt. We also discretize the possible values of QPC current. We have verified that the results presented below are insensitive to the exact discretization parameters, as long as we take $E\delta t \ll 1$ and there are a large number of possible QPC current values. The qubit is initialized in a given state that depends on the specific calculation. In each time step, the value of the QPC current is determined stochastically using equation (4). Based on the obtained value, the qubit's state evolves as described in equation (3). Following this weak-measurement step, a unitary transformation of the form of equation (6) is applied to the qubit's state. After a sufficiently long QPC signal is obtained, this signal (in all its detail) is taken and used to extract the measurement matrix

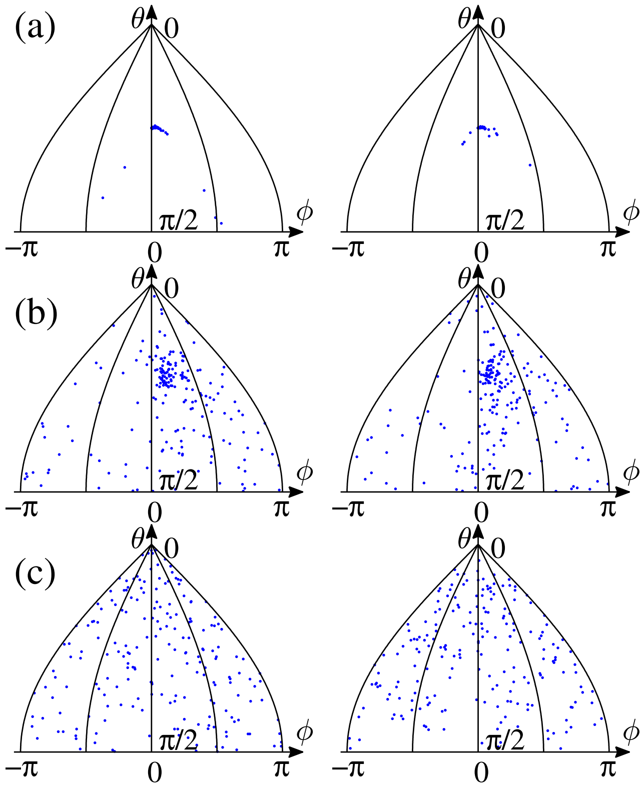

Figure 2. The spherical coordinates θ and ϕ that define the stochastically determined measurement bases obtained in simulations of the experiment under consideration (note that $(\theta, \phi) = (0, 0)$ for the energy eigenbasis and $(\theta, \phi) = (\beta, 0)$ for the charge basis). Each figure contains 200 points. In figure 2(a) $E\tau_m/(2\pi) = 0.01$, i.e. deep in the strong-coupling regime. In figure 2(b), $E\tau_m/(2\pi) = 0.2$ (intermediate-coupling regime), and in figure 2(c) $E\tau_m/(2\pi) = 5$ (weak-coupling regime). In all the figures, $\beta = \pi/4$. In each set, the figure on the left is generated using the initial state $|L\rangle$, and the one on the right is generated using the initial state $|0\rangle$. Each set is an identical pair, up to statistical fluctuations, demonstrating that the initial state plays no role in determining the measurement basis.

$\hat{U}_{\text{Meas}}$ explained above. This matrix is then used to extract the measurement basis and fidelity.

The strength of the qubit–QPC coupling is determined by the relation between two parameters in the numerical calculations: (1) the width, or standard deviation σ, of the QPC-current distribution functions and (2) the distance between the average values of these distribution functions $(\bar{I}_R - \bar{I}_L)$. It is more convenient, however, to present the results in terms of a different parameter that characterizes the qubit–QPC coupling strength, namely $E\tau_m/(2\pi)$, where τ_m is the timescale needed to obtain sufficient QPC signal to read out the state of the qubit (for the time being one can think of this definition as applying to the case when $\beta = 0$; but see below). If one considers N of the small steps considered above, the standard deviation of the QPC's averaged signal scales as $\sigma/\sqrt{N}$ (note that σ is the standard deviation for one step). The measurement time τ_m can now be naturally defined as the product of the time step δt and the value of N at which $2\sigma/\sqrt{N} = |\bar{I}_R - \bar{I}_L|$. The measurement time is therefore given by

$$\tau_m = \frac{4\sigma^2\delta t}{|\bar{I}_R - \bar{I}_L|^2}. \tag{12}$$

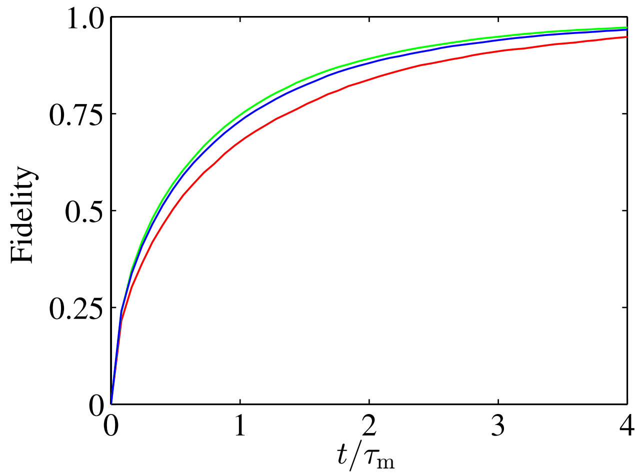

Figure 3. The measurement fidelity as a function of measurement duration for three different values of the angle β between the charge basis and the energy eigenbasis: $\beta = 0$ (red; lowest line), $\pi/4$ (green) and $\pi/2$ (blue). Here $E\tau_m/(2\pi) = 5$, i.e. deep in the weak-coupling regime. The fidelity increases as the measurement duration increases, but the fidelity is essentially independent of the angle β.

First, in figure 2 we show the spherical coordinates of the (stochastically determined) measurement bases for different levels of qubit–detector coupling. In the strong-coupling, fast-measurement regime (figure 2(a)), the measurement basis is always the charge basis, which is the natural measurement basis for the detector under consideration. As the qubit–detector coupling strength is reduced (figure 2(b)), the measurement bases start to deviate from the charge basis, and they develop some statistical spread. This region could be called the intermediate-coupling regime. In the weak-coupling, slow-measurement regime (figure 2(c)), the measurement bases are spread over all the possible directions.

In figure 3, we plot the measurement fidelity as a function of measurement duration for three different values of β, keeping all other parameters fixed. We can see that the fidelity approaches one for long enough measurement duration, regardless of the angle β. In fact, the fidelity is almost independent of β. This result shows that even though more complicated analysis is needed to extract useful measurement information when $\beta \neq 0$, the information acquisition rate is only slightly affected by the coherent, Hamiltonian-induced precession.

The fact that the measurement basis is generally unpredictable, and therefore uncontrollable, is a rather strange phenomenon from a fundamental point of view. From a practical point of view, one can wonder whether anything useful can be done with such measurements that are performed in a stochastically determined basis. If one absolutely requires a measurement in a given basis, measurement results in different bases would be less useful. One could then treat the deviation of the observed measurement basis from the desired one as an experimental error and deal with it accordingly.

In the above discussion, we have ignored any decoherence other than that associated with the measurement-induced projection. If the measurement time τ_m is much smaller than the timescale of decoherence caused by other mechanisms, the measurement is completed with minimal effects of any additional decoherence. Our results are valid in this case. A number of different types of detectors, including QPCs, are approaching this limit where the decoherence is limited by the measurement [11], indicating that our results could be observable in such systems.

4.1. QST

One example of a procedure where the uncontrollability of the measurement basis can be harmless is QST. In QST, one is given a large number of copies of the same quantum state, and the goal is to deduce this state. Typically, one measures the operator $\hat{\sigma}_x$ for one third of the copies, and similarly for $\hat{\sigma}_y$ and $\hat{\sigma}_z$. Once the average values $\langle\hat{\sigma}_x\rangle$, $\langle\hat{\sigma}_y\rangle$ and $\langle\hat{\sigma}_z\rangle$ are known, the density matrix is reconstructed straightforwardly:

$$\rho = \frac{1}{2}\left(1+\langle\hat{\sigma}_x\rangle\hat{\sigma}_x+\langle\hat{\sigma}_y\rangle\hat{\sigma}_y+\langle\hat{\sigma}_z\rangle\hat{\sigma}_z\right). \qquad (13)$$

In the present case, the measurement bases are chosen stochastically, and in principle no two of them are the same. One must therefore reconstruct the unknown quantum state using a procedure that allows for data taken from measurements made in any combination of bases. One such procedure is the minimization of the function

$$\mathcal{T}(r,\theta,\phi) = \sum_j \left[1 - r\cos\Omega(\theta,\phi,\theta_j,\phi_j)\right]^2, \qquad (14)$$

where r, θ and ϕ are the spherical coordinates of a point in the Bloch sphere; j is an index labeling the different runs of the experiment; the direction (θ_j,ϕ_j) defines the qubit state obtained in a given measurement; and $\Omega(\theta,\phi,\theta_j,\phi_j)$ is the angle between the directions (θ,ϕ) and (θ_j,ϕ_j). Assuming that the measurement bases cover all possible directions (see e.g. figure 2(c)), the convergence of this procedure to the correct density matrix should be similar to the convergence of the standard QST procedure explained in the previous paragraph.

We have simulated QST by repeating the measurement procedure a large number of times, obtaining a set of measurement results (in the form of premeasurement qubit states), and then minimizing the function $\mathcal{T}(r,\theta,\phi)$ with respect to r, θ and ϕ. We have chosen several initial states covering the Bloch sphere, and the tomography procedure consistently produced the initial state of the qubit for the parameters of figures 2(b) and (c). For strong qubit–detector coupling (see figure 2(a)), the procedure becomes unreliable, because the vast majority of the measurements are performed in one basis.

5. Conclusion

In conclusion, we have analyzed the question of what information can be extracted from the output signal of a QPC that weakly probes the charge state of a charge qubit when the qubit Hamiltonian induces oscillations between the different charge states. We have shown that the measurement basis is determined stochastically every time the measurement is repeated. In the case of weak qubit–detector coupling, the possible measurement bases cover all the possible directions. The measurement basis and result can both be extracted from the QPC output signal. We have also shown that the information acquisition rate is independent of the angle β between the direction defining the charge basis and that defining the qubit Hamiltonian. In other words, given enough time, the detector will produce a high-fidelity measurement result, regardless of the value of β. These results show that, under suitable conditions and by proper analysis, the detector's ability to obtain high-fidelity information about the state of the qubit is not affected by the competition between the measurement and coherent-precession dynamics. More detailed analysis of the results discussed in this paper is presented elsewhere [12].

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