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Strong coupling of a spin qubit to a superconducting stripline cavity

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We study electron-spin-photon coupling in a single-spin double quantum dot embedded in a superconducting stripline cavity. With an external magnetic field, we show that either a spin-orbit interaction (for InAs) or an inhomogeneous magnetic field (for Si and GaAs) could produce a strong spin-photon coupling, with a coupling strength of the order of 1 MHz. With an isotopically purified Si double dot, which has a very long spin coherence time for the electron, it is possible to reach the strong-coupling limit between the spin and the cavity photon, as in cavity quantum electrodynamics. The coupling strength and relaxation rates are calculated based on parameters of existing devices, making this proposal experimentally feasible.

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I. INTRODUCTION

Confined electron spins in solid-state nanostructures are promising candidates as building blocks of quantum information processors,[1–3] with the dual advantages of long quantum coherence times[4] and coherent manipulation of individual qubits.[5–7] The coupling of electron spins can be achieved via the exchange interaction,[8] which is short ranged. Scaling up the spin-qubit architecture invariably requires on-chip quantum information transfer, whether via moving the electrons themselves[9–11] or via a "spin bus."[12] However, electron motion could lead to reduced spin coherence,[13] while the spin bus requires strong exchange couplings within a spin chain. An enticing alternative to move spin information is to transfer it coherently to a photon, which is mobile and decoherence resistant. Such high-fidelity information transfer requires the use of cavity quantum electrodynamics[14–20] with a microwave cavity.

The magnetic dipole coupling between a single spin and a photon is very weak, smaller than 1 kHz in a superconducting stripline resonator,[15] which has a strong vacuum field. To achieve a strong spin-photon coupling, the electric dipole coupling and some sort of spin-orbit interaction are required, taking advantage of the so-called electrically driven spin resonance (EDSR).[21–25] During the past decade, many theoretical proposals have been put forward to realize coherent spin-photon coupling, including off-resonant Raman scattering off a single spin,[26,27] coupling two-spin states to cavity photons via electric dipole coupling or gate potential,[28,29] using an InAs nanowire to maximize spin-orbit coupling,[30] and employing ferromagnetic leads to create spin-photon coupling.[31] However, so far there has been no experimental demonstration of strong interaction between a single spin and a single photon, so this remains an important open problem.

Here we propose a spin-photon coupling method with a double quantum dot (DQD) containing a single electron in a superconducting stripline resonator. The DQD has a large electric-dipole moment, and the spin-electric-field coupling is facilitated by either an inhomogeneous magnetic field (from either a current or a nanomagnet) or spin-orbit interaction (in GaAs and InAs). We show that the vacuum Rabi frequency of a single electron spin, under an external magnetic field of 0.1 T, can reach the order of 1 MHz in an inhomogeneous magnetic field, or similar magnitude in InAs mediated by a strong spin-orbit interaction. We also examine the decoherence of a single spin in a DQD, and identify Si as an ideal host material because of the absence of nuclear-spin-induced inhomogeneous broadening. We calculate the spin-relaxation rate of a single electron, and find that it is not a dominant source of decoherence. Combining our results on spin-photon coupling strength and decoherence, we show that a Si DQD in an inhomogeneous magnetic field is the best structure for an electron spin to reach the strong-coupling limit in interacting with the photons in a superconducting stripline cavity.

II. SYSTEM SETUP

The physical system we consider is a semiconductor DQD, with a geometry illustrated in Fig. 1. The Hamiltonian of the system is

$$\begin{aligned} H &= H_{\mathrm{DQD}} + H_{\mathrm{Zeeman}} + H_{\mathrm{SO}} + H_{\mathrm{E}}, \\ H_{\mathrm{Zeeman}} &= \frac{1}{2} g \mu_{\mathrm{B}} \mathbf{B} \cdot \sigma\,, \quad H_{\mathrm{SO}} = H_{\mathrm{BR}} + H_{\mathrm{I}}, \\ H_{\mathrm{BR}} &= \frac{\alpha_{\mathrm{BR}}}{\hbar} (\sigma_x P_y - \sigma_y P_x)\,,\ H_{\mathrm{I}} = \alpha_{\mathrm{I}} \sigma_y x, \quad H_{\mathrm{E}} = eEx. \end{aligned} \tag{1}$$

Here H_{DQD} refers to the orbital part of the single-electron double-dot Hamiltonian, including the confinement potential and the electron kinetic-energy terms.[32] To minimize the effect of the external field on the superconducting cavity, we choose

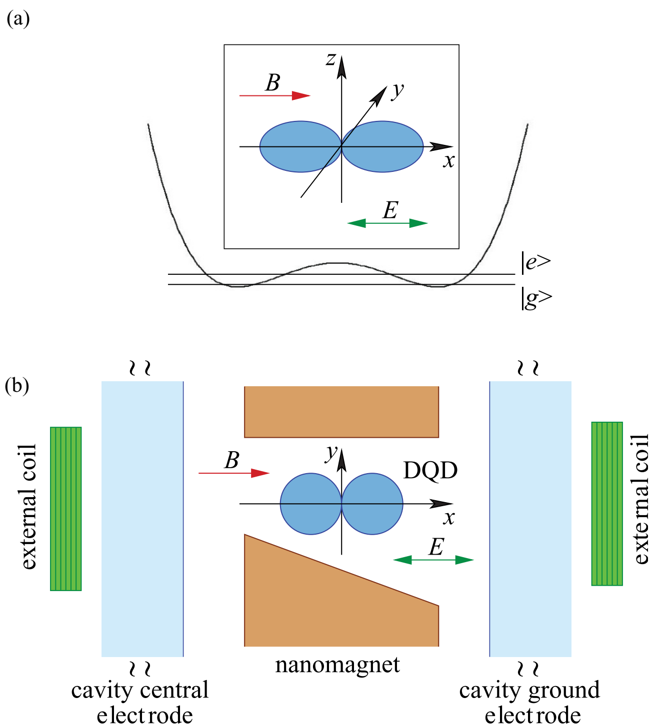

FIG. 1. (Color online) Schematics of the two-dimensional double dot in a cavity. Panel (a) shows that both the cavity electric field and the external magnetic field are along the interdot axis (x axis). Panel (b) shows the top view of the combined DQD-nanomagnet-cavity system for the case of a DQD in an inhomogeneous magnetic field. External coils are used to produce the uniform magnetic field, while close-by nanomagnets provide the transverse inhomogeneous field.

an in-plane uniform magnetic field along the x direction. This choice also minimizes any effect on the electron orbital motion, so that we only need to consider the resulting Zeeman splitting. $H_{\rm SO}$ includes both the normal spin-orbit interaction (such as the Bychkov-Rashba spin-orbit coupling $H_{\rm BR}$ given here[23,33]), and the spin-orbit interaction due to the presence of a spatially inhomogeneous magnetic field, such as the one employed in Refs. 34–36 and given here as $H_{\rm I}$. Last, $H_{\rm E}$ represents the electron interaction with the cavity electric field,[15] which is quantized in terms of the cavity photon operators a and $a^\dagger$:

$$E = \sqrt{\frac{\hbar\omega_{\rm E}}{Cd^2}}\sin\left(\frac{\pi y}{l}\right)(a + a^\dagger), \tag{2}$$

where $\omega_{\rm E}$ is the photon angular frequency, C, l, and d are the capacitance, length, and gap width (between the center and the ground electrodes) of the resonator, respectively. For $\omega_{\rm E} =$ 10 GHz and $d = 10\ \mu$m, the vacuum field is about 0.2 V/m.[15]

III. SPIN-PHOTON COUPLING STRENGTH

Our focuses are to couple the spin of the single electron to the cavity photons, and to investigate whether we can achieve the strong-coupling limit in this system.[37] Here we first project the system Hamiltonian onto the basis states of $|g\rangle|\uparrow\rangle$, $|g\rangle|\downarrow\rangle$, $|e\rangle|\uparrow\rangle$, and $|e\rangle|\downarrow\rangle$, where $|g\rangle = \alpha|L\rangle + \beta|R\rangle$ and $|e\rangle = \beta|L\rangle - \alpha|R\rangle$ are the ground and first excited orbital states of the tunnel-coupled DQD with energies ϵ_g and ϵ_e. $|L\rangle$ ($|R\rangle$) is the single-dot ground orbital state in the left (right) dot, while $|\uparrow\rangle$ and $|\downarrow\rangle$ are the spin eigenstates of $H_{\rm Zeeman}$. The interdot tunnel coupling is $\langle L|H|R\rangle = t$. A straightforward inspection of the matrix elements of the total Hamiltonian shows that the largest spin-photon coupling is achieved when there is no bias between the two dots, when $\epsilon_0 = \epsilon_e - \epsilon_g = 2|t|$, which we choose at 40 μeV, or about 10 GHz. This energy is much smaller than the single-dot excitation energy of $\sim$1 meV, justifying our focus on this sub-Hilbert space for our calculations. The total Hamiltonian now becomes

$$H = \begin{pmatrix} \epsilon_g - \epsilon_Z & 0 & -eEL & -\lambda_x + i\alpha_{\rm I}L \\ 0 & \epsilon_g + \epsilon_Z & \lambda_x - i\alpha_{\rm I}L & -eEL \\ -eEL & \lambda_x + i\alpha_{\rm I}L & \epsilon_e - \epsilon_Z & 0 \\ -\lambda_x - i\alpha_{\rm I}L & -eEL & 0 & \epsilon_e + \epsilon_Z \end{pmatrix}. \tag{3}$$

Here $\epsilon_Z = \frac{1}{2}g\mu_B B$ is the Zeeman energy due to the uniform in-plane magnetic field, E is the electric-field operator, and L is the half interdot distance for the DQD. The matrix element for the inhomogeneous field induced spin-orbit coupling is quite simple ($i\alpha_{\rm I}L$). The physical picture is straightforward as well: the field inhomogeneity leads to different quantization axes for the two dots, so that a spin eigenstate in one dot is a superpositioned state in the other. When driven by an electric field, this spin mixture leads to an ac transverse magnetic field, which in turn produces EDSR.[34] The "intrinsic" (by which we mean that the interaction is a material/structure property, and is not due to an applied field or magnet) spin-orbit coupling parameter is

$$\lambda_x = \frac{\alpha_{\rm BR}LS}{a^2\sqrt{1-S^2}},$$

where

$$S = \langle L|R\rangle = e^{-(L/a)^2}$$

is the interdot wave function overlap, and a is the radius of the Gaussian single-dot ground-state wave function. Here λ_x originates from the $\sigma_y P_x$ term in $H_{\rm BR}$. It is related to the x-direction electric dipole moment of the double dot, and is the main driving force behind the EDSR in the presence of intrinsic spin-orbit interaction.

When $\epsilon_Z \ll \epsilon_0$, we can focus on the spin dynamics by projecting our system Hamiltonian to the sub-Hilbert space spanned by the ground orbital state and the spin eigenstates $|g\rangle \otimes |\uparrow\rangle$ and $|g\rangle \otimes |\downarrow\rangle$. According to Löwdin's perturbation theory,[38] the matrix elements of an effective Hamiltonian (at the lowest order) in the reduced Hilbert space take the form

$$H'_{ij} = H_{ij} + \frac{1}{2}\sum_k \left\{\frac{H_{ik}H_{kj}}{H_{ii} - H_{kk}} + \frac{H_{ik}H_{kj}}{H_{jj} - H_{kk}}\right\}, \tag{4}$$

where the indices i and j refer to states in the targeted sub-Hilbert space, while k refers to states outside this subspace. The off-diagonal term that leads to the cavity-electric-field-driven spin rotation thus takes the form

$$H'_{12} = -2eEL\left(\frac{i\alpha_{\rm I}L}{\epsilon_0} + \frac{\lambda_x\epsilon_Z}{\epsilon_0^2}\right). \tag{5}$$

TABLE I. Spin-photon coupling strength in single and double dots. The estimated magnitudes are obtained by assuming a double dot with half interdot distance $L = 60$ nm in GaAs and 35 nm in Si, $\epsilon_0 = 40$ μeV, and a vacuum electric field of 0.4 V/m. The single dot parameters are $E_0 = 1$ meV for a GaAs dot and 30 meV for an InAs dot. For elongated dots, we choose $E_x = 0.1$ meV for GaAs and 1 meV for InAs.

	H_{12}	Interaction strength
GaAs DQD Inhomogeneous field	$-2ieEL^2\alpha_I\epsilon_0^{-1}$	0.5 MHz ($\alpha_I = 1$ T/μm)
Si DQD Inhomogeneous field	$-2ieEL^2\alpha_I\epsilon_0^{-1}$	0.6 MHz ($\alpha_I = 1$ T/μm)
DQD spin orbit	$-2eEL\lambda_x\epsilon_Z\epsilon_0^{-2}$	50 kHz (GaAs) to 1 MHz (InAs)
SQD Inhomogeneous field	$-ieEa^2\alpha_I E_0^{-1}$	0.2 kHz (InAs) to 5 kHz (Si)
SQD spin-orbit	$-eE\alpha_{\rm BR}\epsilon_Z E_0^{-2}$	10 kHz to 0.1 MHz
Elongated SQD SO	$-eE\alpha_{\rm BR}\epsilon_Z E_x^{-2}$	100 kHz to 1 MHz

This spin-cavity coupling term has several interesting features. First, the denominators for the terms in Eq. (5) are ϵ_0 or ϵ_0^2. For a single circular dot (SQD) these terms would have been $\hbar\omega_0$ or $(\hbar\omega_0)^2$, where $\hbar\omega_0$ is the single-particle excitation energy, on the order of 1 meV in GaAs/Si and 10 meV in InAs. Since $\epsilon_0 \ll \hbar\omega_0$ in a double dot, the spin-photon coupling strength tends to be stronger in a DQD compared to a SQD, even after the reduction due to tunnel coupling is included. Alternatively, one can use an elongated dot to enhance the spin-photon coupling as well, with InAs nanowires as a prime example.[30] Here the longitudinal confinement is weaker, so that the enhancement due to the stronger orbital coupling, like that in a DQD, can be achieved as well. Another feature of Eq. (5) is that the spin-orbit-induced spin rotation term contains an extra factor of ϵ_Z/ϵ_0 as compared to the inhomogeneous-field-induced spin rotation. This extra factor originates from the breaking of the Kramers degeneracy by an external magnetic field, since spin-orbit interaction itself does not break the time-reversal symmetry, while an inhomogeneous magnetic field does.

In Table I we present our results on both DQD and SQD in terms of the effective spin coupling matrix element. For our SQD calculations we included the ground and the first two excited orbitals. Our choices of half interdot distance L for DQDs are determined by the assumption that a single-dot confinement energy in GaAs or Si is about 1 meV, and the interdot tunnel coupling should be about 40 μeV. While the conduction-electron effective mass in GaAs is smaller, leading to larger L, the g factor in GaAs is small ($\sim$0.4), so that under the same nanomagnet the coupling strengths in GaAs and Si are similar. Results for an elongated single dot are also included (from Refs. 30 and 39) for comparison.

IV. STRONG-COUPLING LIMIT: SPIN RELAXATION

To achieve the strong-coupling limit, which would allow coherent information transfer between the spin and the cavity photon, the spin-photon coupling strength needs to satisfy $|H_{12}| > \hbar\gamma$, where γ is the total decoherence rate of the spin-cavity system, including cavity loss and spin decoherence. With a quality factor of $Q > 10^5$ and a photon frequency of 5 GHz,[40] the photon decay rate is <0.05 MHz. Thus photon loss should not pose any significant problem, at least not to demonstration-of-principle-type experiments. Spin decoherence, on the other hand, is a more serious issue. While the true coherence times of spin qubits are generally long [a T_2 of the order of 200 μs has recently been measured in a GaAs DQD (Ref. 4)], the environmental nuclear spins generally cause significant inhomogeneous broadening in III-V quantum dots. For example, the dephasing time due to inhomogeneous broadening from nuclear spins in GaAs is $T_2^* \sim 10$ ns.[5] While this dephasing mechanism does not correspond to true decoherence, it does imply a spin frequency uncertainty on the order of 100 MHz at any particular time. With the cavity frequency fixed over time, such a spin frequency shift would make spin-cavity resonance untenable. Without a viable solution to this dephasing problem,[41–43] it would be impossible to achieve strong spin-photon coupling in a GaAs SQD or DQD. While InAs does have a much stronger spin-orbit coupling, the spin-photon coupling strength is still not of the same order as the dephasing rate (see Table I), making the strong-coupling limit difficult to reach as well.

A more promising candidate to achieve strong spin-photon coupling via a stripline resonator may be an isotopically purified Si DQD. While spin-orbit coupling is small in Si (with a coupling strength about one-tenth that of GaAs), the spin coherence time is very long, especially in isotopically purified ^{28}Si samples, where the nuclear-spin-induced dephasing can be <0.1 MHz,[44] and the true decoherence rate is much smaller.[3] Combining this long decoherence time with an external inhomogeneous magnetic field, achieving the strong-coupling limit becomes a more realistic goal.

With pure dephasing due to nuclear spins not a significant problem in a Si DQD, we still have to clarify the spin-relaxation rates there. While it is known that spin relaxation in an SQD is extremely slow,[22,45] past calculations have also shown that for a single electron in a DQD, there could exist spin hot spots when the electron Zeeman energy and the DQD tunnel splitting is on resonance.[46] We therefore calculate the single-spin relaxation rate in a Si DQD due to the presence of spin-orbit interaction and inhomogeneous magnetic field. The results are presented in Fig. 2. The peak in the relaxation rate occurs when $|g\uparrow\rangle$ and $|e\downarrow\rangle$ are degenerate, i.e., $\epsilon_0 = 2\epsilon_Z$. With $\epsilon_0 = 40$ μeV, the resonant magnetic field is 0.345 T for Si. Even for $B = 0.3$ T, which is quite close to the hot spot, the spin-relaxation rate is only $\sim$1 kHz, posing no problem to the spin-photon coupling scheme.

V. DISCUSSIONS AND CONCLUSIONS

Achieving strong spin-photon coupling would not only allow long-distance quantum communication for spin qubits, but also open a range of new possibilities in spin and photon physics, including spin and photon manipulations, and dispersive spin measurement.[15] The latter could present a viable alternative to the charge sensor and spin-blockade-based

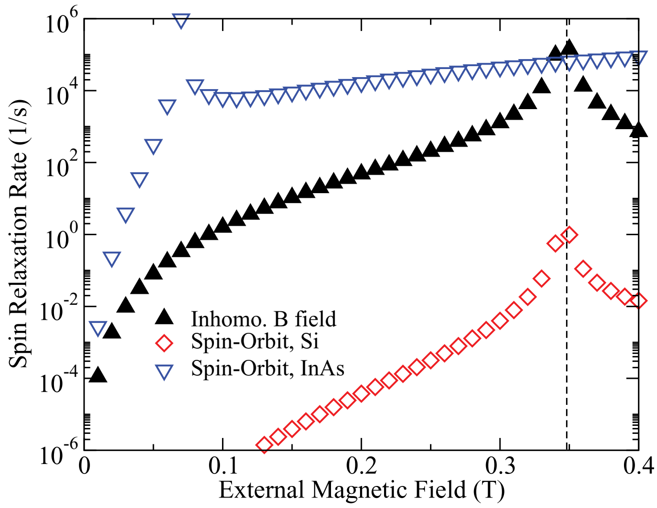

FIG. 2. (Color online) Single-spin relaxation rate in a Si or InAs DQD as a function of the applied field. Here the tunnel splitting of the double dot orbital state is $\epsilon_0 = 40\,\mu$eV (the vertical line represents the resonant condition when $2\epsilon_Z = \epsilon_0$ for Si, with a corresponding magnetic field of 0.345 T), and the interdot distance is 100 nm for Si and 50 nm for InAs. The black filled triangles represent the spin relaxation due to spin mixing from the inhomogeneous magnetic field along the interdot (x) direction, while the red unfilled diamond (for InAs) and blue unfilled triangles (for Si) are due to Rashba spin-orbit coupling. The spin-relaxation rate for InAs peaked at a different magnetic field because of the very different g factor (we used $g = 10$ here).

measurement technique[1,5] that is widely used today for spin qubits. Furthermore, a hybrid design of qubit architecture, with a superconducting stripline cavity as the backbone, and involving various kinds of qubits (such as atomic, ionic, superconducting, and spin qubits) would also become feasible.[18]

As we discussed above, a spin-photon coupling driven by an inhomogeneous magnetic field in Si is probably the best combination to achieve the strong-coupling limit. We note that the strength of this EDSR scheme originates from its "directness," in the sense that the electron spin is directly rotated by the nonuniform magnetic field, as opposed to the case with spin-orbit interaction, which relies on an applied magnetic field to lift the spin degeneracy. For a spin-orbit qubit, such as what is discussed in Refs. 47 and 48, the effects of the spin-orbit interaction is already incorporated in the form of a renormalized anisotropic and confinement-dependent g factor. For such a system, a uniform applied field would produce an apparent inhomogeneous Zeeman splitting across a DQD, so that the EDSR for the qubit can be considered as an EDSR in an inhomogeneous magnetic field. It is also worth noting here that an inhomogeneous magnetic field is not only useful for spin manipulation, but could also lead to an alternative approach for spin measurement, as was discussed in Ref. 49. In other words, a combined DQD-cavity-nanomagnet system has the versatility for studying several different aspects of the coupled spin-photon dynamics.

The main drawback of using the inhomogeneous magnetic field is the added device complexity, with an additional metallic layer in the system and a magnetic material close to the superconducting electrodes of the cavity. To enhance the electric-field-DQD coupling, one could connect the DQD plunger gates directly to the cavity electrodes.[29] Such a configuration should allow a larger gap between the cavity electrodes, so that the nanomagnet can be more easily incorporated.

Compared to the schemes of two-spin coupling to cavity photons,[28] our single-spin approach avoids the Coulomb interaction between the two electrons, which works to reduce the spin-photon coupling strength. Furthermore, maximizing spin-photon (or any other two-level system-to-photon) coupling strength requires a strong vacuum electric field, which can be achieved with a narrower gap for the cavity. However, such reduction in device dimension has to be carefully managed so that it does not conflict with the devices (such as gated quantum dots and nanomagnets) that are incorporated into the cavity.

The spin-orbit-based spin-photon coupling mechanism here is similar to the system discussed in Ref. 30, with both approaches achieving an enhancement of the spin-photon coupling by reducing the orbital state gap, whether through tunnel coupling or by having an elongated quantum dot. The DQD configuration has a clean low-energy spectrum that allows a simpler and more transparent mathematical treatment, and an elongated dot could easily become a DQD due to local disorder, but a DQD has to overcome suppression by tunnel coupling. The spin-photon coupling strengths in these methods are similar after all the factors are considered, as shown in Table I. If a frequency locking mechanism is ever discovered to overcome the inhomogeneous broadening in the III-V quantum dots, an InAs DQD or elongated dot with a single electron[50] would be a good candidate to achieve strong spin-photon coupling as well.

In conclusion, we propose to achieve the strong-coupling limit between an electron-spin qubit and a cavity photon mode by using a single-spin double quantum dot and a superconducting stripline cavity. We show that an inhomogeneous magnetic field could lead to a strong interaction between the spin and the cavity photon. We estimate the coupling strength and relaxation rates based on existing devices and technology, so that the coupling method proposed here should be feasible with a single-electron Si double dot. We also show that in materials such as InAs, the inhomogeneous broadening due to nuclear spins needs to be overcome to achieve strong coupling between electron spins and photons.

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