

Quantum dynamics of spatial decoherence of two atoms in a ring cavity

Li Zheng,^{1,2} Chui-Ping Yang,^{2,3,4} and Franco Nori^{2,3}

¹*Information Science and Engineering College, Dalian Polytechnic University, Dalian 116034, China*

²*Advanced Science Institute, RIKEN, Wako-shi, Saitama 351-0198, Japan*

³*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*

⁴*Department of Physics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China*

(Received 20 July 2010; published 7 December 2010)

We study the spatial decoherence dynamics for the relative position of two atoms in a single-mode ring cavity. We find that the spatial decoherence of the two atoms depends strongly on their relative position. Taking into account the spatial degrees of freedom, we investigate the entanglement dynamics of the internal states of the two atoms. It is shown that the entanglement decays to almost zero in a finite time, and the disentanglement time depends on the width of the wave packets describing the atomic spatial distribution.

DOI: [10.1103/PhysRevA.82.062106](https://doi.org/10.1103/PhysRevA.82.062106)

PACS number(s): 03.65.Yz, 03.65.Ud, 42.50.Pq

I. INTRODUCTION

Superposition and its many-particle version—entanglement—are two basic features of quantum physics, distinguishing the quantum world from the classical world. Because of its intriguing properties, quantum entanglement has attracted considerable attention as an important resource for quantum information processing [1,2]. However, quantum coherence can be destroyed due to the physical system interacting with the environment, which has been recognized as a main obstacle to realizing quantum information processing. Hence a better understanding of the mechanisms of quantum decoherence is not only crucial for the understanding of the quantum-classical transition (see, e.g., Refs. [3–5]), but also essential for the implementation of quantum information processing.

In the last few years, theoretical studies in this context have involved a variety of systems (see, e.g., Refs. [6–12]). Moreover, experiments have also been done to demonstrate the dynamic process of decoherence as well as the collapse and revival of the quantum coherence (see, e.g., Refs. [13–15]). In recent years, several physical systems have been studied to learn more about environment-induced decoherence (see, e.g., Refs. [16,17]).

The influence of atomic spatial motion on quantum dynamics has been considered in different contexts (see, e.g., Refs. [18,19]). For a system of two cold atoms placed in a noisy vacuum field, the back-action of emitted photons on the wave-packet evolution about the relative position of the two cold atoms was discussed in Ref. [20]. It was shown that the photon recoil resulting from the atomic spontaneous emission can induce the localization of the relative position of the two atoms, through the entanglement between the spatial motion of individual atoms and their emitted photons.

In contrast with previous works, here we consider the case where a pair of atoms simultaneously interact with a single-mode ring cavity. In fact, this kind of system is a typical system in which the cavity field acts as a data bus inducing two qubits to be entangled [21,22]. The system is efficiently used in many schemes for proposing how to realize quantum logic gates and teleportation in cavity QED [23]. In this paper, we will study the spatial decoherence of the atomic relative position and the disentanglement of the internal states of these

two atoms induced by the back-action of the photons emitted from these atoms. We find that the spatial decoherence of the two atoms depends strongly on their relative position. Our results show that the entanglement of the internal states decays to almost zero in a finite time, and the disentanglement time is determined by the width of the atomic wave packets.

This paper is organized as follows. In Sec. II we present an effective Hamiltonian of the total system and then give the solution for the eigenequation of the effective Hamiltonian. In Sec. III we discuss the time evolution of the density operator of the total system and obtain the reduced density matrix in the relative-coordinate picture. Furthermore, a decoherence factor for arbitrary cavity-field and general atomic spatial states is introduced. In Sec. IV, under the assumption that the two atoms' relative position is initially in a superposition state of two Gaussian wave packets, we demonstrate the spatial decoherence when the cavity field is initially in a coherent state. In Sec. V we study the disentanglement dynamics of the two atoms' internal states. Finally, a concluding summary is given in Sec. VI.

II. MODEL

Our system consists of two identical two-level atoms that interact with a single-mode cavity field (Fig. 1). The two atoms are denoted here as atoms 1 and 2. The mass of each atom is denoted by m_0 , and their atomic transition frequency is ω_0 . Assume that the two atoms are spatially separated and located at the positions \hat{x}_1 and \hat{x}_2 , respectively, and the corresponding momenta for atoms 1 and 2 are \hat{p}_1 and \hat{p}_2 . Then the momentum for the center of mass (c.m.) of the two atoms is $\hat{P} = \hat{p}_1 + \hat{p}_2$, and the relative momentum of the two atoms is $\hat{p} = (\hat{p}_1 - \hat{p}_2)/2$. The c.m. position and the relative position are $\hat{X} = (\hat{x}_1 + \hat{x}_2)/2$ and $\hat{x} = \hat{x}_1 - \hat{x}_2$, respectively.

Under the rotating-wave approximation, the Hamiltonian of this system reads as

$$\hat{H} = \frac{\hat{p}_1^2}{2m_0} + \frac{\hat{p}_2^2}{2m_0} + \frac{1}{2}\hbar\omega_0(\hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)}) + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g[\hat{a}(\hat{\sigma}_+^{(1)}e^{ikx_1} + \hat{\sigma}_+^{(2)}e^{ikx_2}) + \text{H.c.}], \quad (1)$$

where $\hat{\sigma}_z^{(i)} = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|$, $\hat{\sigma}_+^{(i)} = |e_i\rangle\langle g_i|$ and $\hat{\sigma}_-^{(i)} = |g_i\rangle\langle e_i|$ ($i = 1, 2$) are the atomic operators for the i th atom with

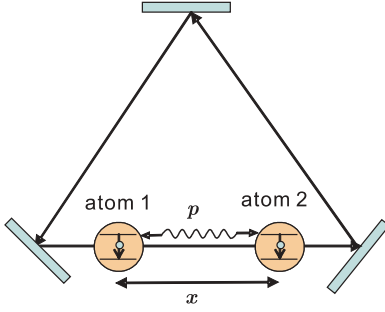


FIG. 1. (Color online) A schematic diagram of the system considered in this paper. A single-mode ring cavity contains two atoms 1 and 2. Here p is the relative momentum of the two atoms, and x is the relative position of the two atoms.

respect to the excited state $|e\rangle$ and the ground state $|g\rangle$ of the atoms, \hat{a}^\dagger and \hat{a} are the creation and annihilation operators for the cavity field, ω and k are the frequency and wave number of the cavity field, and g is the atom-field coupling constant. There may be a counterpropagating running-wave mode with a wave vector $-k$ in such a ring cavity, which may affect the atom-field dynamics and contribute to the interaction part. However, as discussed in Refs. [18,24], we can consider that an atom traverses only an arm of the optical ring cavity, and only one cavity mode is excited by an external laser. Thus, it is reasonable to consider a single propagating (e.g., a clockwise-running) wave mode in a ring cavity.

We first factorize the evolution operator $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$ into a product as

$$\hat{U}(t) = \hat{W}(x_1)\hat{W}(x_2)\hat{U}_e(t)\hat{W}(x_2)^\dagger\hat{W}(x_1)^\dagger, \quad (2)$$

where $\hat{W}(x_i)$ ($i = 1, 2$) is a unitary transformation defined by

$$\hat{W}(x_i) = \exp\left(\frac{ikx_i}{2}\right)|e_i\rangle\langle e_i| + \exp\left(\frac{-ikx_i}{2}\right)|g_i\rangle\langle g_i|, \quad (3)$$

which concerns the coupling of the internal levels with the spatial degrees of the atom i . The operator $\hat{U}_e(t) = \exp(-i\hat{H}_e t/\hbar)$ is easily proved to be determined by the effective Hamiltonian $\hat{H}_e = \hat{H}_0 + \hat{H}_1$ of the system with

$$\hat{H}_0 = \frac{\hat{p}_1^2}{2m_0} + \frac{\hat{p}_2^2}{2m_0} + \frac{\hbar^2 k^2}{4m_0}, \quad (4)$$

$$\hat{H}_1 = \hbar \sum_{i=1,2} \left[\frac{\Omega_i}{2} (|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|) + g(\hat{a}^\dagger |g_i\rangle\langle e_i| + \text{H.c.}) \right] + \hbar\omega\hat{a}^\dagger\hat{a}. \quad (5)$$

Here $\Omega_1 = \omega_0 + p_1 k/m_0$ and $\Omega_2 = \omega_0 + p_2 k/m_0$. In the following discussion, we consider that the atomic transition is resonant with the cavity mode, i.e., $\omega_0 = \omega$. In this case, we have $\omega_0 = ck$ (c is the light velocity in vacuum). Note that $p_i k/m_0 = V_i k$ ($i = 1, 2$), where V_i is the velocity of the atom i . Thus, when the velocity of the atom i is far smaller than the light velocity in vacuum (i.e., $V_i \ll c$), we have $p_i k/m_0 \ll \omega_0$. On the other hand, the condition $p_i k/m_0 \ll g$ needs to be met, such that the influence of the momentum-dependent energy shifts of the atomic internal levels on the system dynamics is negligibly small. To see the availability of this condition, let us consider a ^{87}Rb atom with two circular Rydberg levels

$|g\rangle$ and $|e\rangle$ (corresponding to principal quantum numbers 50 and 51). The transition frequency between $|g\rangle$ and $|e\rangle$ is ~ 51.1 GHz [25]. Thus, we have $k \sim 10^3 \text{ m}^{-1}$ for the case when the transition between $|g\rangle$ and $|e\rangle$ is resonant with the cavity mode. The coupling constant g is on the order of 10^5 Hz [25–27]. For the laser-cooled and optically trapped ^{87}Rb atom, we can assume that the atom has a measured temperature of $T \sim 180 \mu\text{K}$ [28]. A simple calculation shows that the atomic velocity is $V \sim 23 \text{ cm/s}$. Hence, we have $Vk \sim 230 \text{ Hz} \ll 10^5 \text{ Hz}$, which demonstrates that the approximation of the condition $|p_i k/m_0| \ll g$ could be satisfied in practice. The analysis given here shows that the term $p_i k/m_0$ in Ω_i can be neglected, leading to $\Omega_i = \omega_0 + p_i k/m_0 \approx \omega_0$. We note that the same approximation was made in Ref. [20].

The total excitation number $\hat{a}^\dagger\hat{a} + |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2|$ is conserved during the interaction. By resolving the eigenequation of \hat{H}_1 , in the subspace spanned by states with the total excitation number $(n+2)$, the following eigenstates of \hat{H}_1 are obtained:

$$|\Psi\rangle_1^{(n)} = \sqrt{2}f_{2n}|e_1, e_2, n\rangle - \sqrt{2}f_{1n}|g_1, g_2, n+2\rangle, \quad (6)$$

$$|\Psi\rangle_2^{(n)} = \frac{\sqrt{2}}{2}|g_1, e_2, n+1\rangle - \frac{\sqrt{2}}{2}|e_1, g_2, n+1\rangle, \quad (7)$$

$$|\Psi\rangle_3^{(n)} = f_{1n}|e_1, e_2, n\rangle + \frac{1}{2}|g_1, e_2, n+1\rangle + \frac{1}{2}|e_1, g_2, n+1\rangle + f_{2n}|g_1, g_2, n+2\rangle, \quad (8)$$

$$|\Psi\rangle_4^{(n)} = -f_{1n}|e_1, e_2, n\rangle + \frac{1}{2}|g_1, e_2, n+1\rangle + \frac{1}{2}|e_1, g_2, n+1\rangle - f_{2n}|g_1, g_2, n+2\rangle, \quad (9)$$

with the eigenvalues

$$E_{1,2}^{(n)} = (n+1)\hbar\omega = E_0, \quad E_3^{(n)} = E_0 + \hbar A_n, \quad (10)$$

$$E_4^{(n)} = E_0 - \hbar A_n,$$

where $A_n = \sqrt{2(2n+3)}g$, $f_{1n} = \sqrt{(n+1)/[2(2n+3)]}$, $f_{2n} = \sqrt{(n+2)/[2(2n+3)]}$, and n is an arbitrary nonnegative integer.

III. DECOHERENCE FACTOR

In this section we discuss the time evolution of the system and investigate the spatial decoherence factor for the atom-atom relative position.

Assume that the initial density operator of the whole system is given by

$$\hat{\rho}(0) = \hat{\rho}_s(0) \hat{\rho}_i(0) \hat{\rho}_f(0), \quad (11)$$

where $\hat{\rho}_s(0) = |\psi(0)\rangle\langle\psi(0)|$ is the initial density operator for the spatial motion of the two atoms, $\hat{\rho}_i(0) = |\phi(0)\rangle\langle\phi(0)|$ is the initial density operator for the internal state of the two atoms, and $\hat{\rho}_f(0)$ is the initial density operator for the cavity field.

Let us now assume that the state $|\psi(0)\rangle$ is expanded (in the momentum representation) as

$$|\psi(0)\rangle = \int \int_{-\infty}^{\infty} dp_1 dp_2 C_{p_1 p_2} |p_1, p_2\rangle, \quad (12)$$

where $|p_1\rangle$ is the momentum eigenstate of atom 1, $|p_2\rangle$ is the momentum eigenstate of atom 2, and $C_{p_1 p_2}$ is the distribution function satisfying the normalization condition

$\int_{-\infty}^{\infty} dp_1 dp_2 |C_{p_1 p_2}|^2 = 1$. The state $|\phi(0)\rangle$ is assumed to be a pure separable state $|e_1, e_2\rangle$. In terms of Fock states $|n\rangle$ and $|n'\rangle$, the density operator $\hat{\rho}_f(0)$ is, in general, written as

$$\hat{\rho}_f(0) = \sum_{n, n'} c_{n, n'} |n\rangle \langle n'|. \quad (13)$$

The time evolution of $\hat{\rho}(0)$ can be obtained by $\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t)$. Following the eigenstates (6)–(9), eigenvalues (10) of \hat{H}_1 , and based on Eqs. (2), (3) and (11)–(13), we find that the density operator $\hat{\rho}(t)$ can be written as

$$\hat{\rho}(t) = \sum_{n, n'} c_{n, n'} |\Psi_n(t)\rangle \langle \Psi_{n'}(t)|, \quad (14)$$

where

$$\begin{aligned} |\Psi_n(t)\rangle = & \int \int_{-\infty}^{\infty} dp_1 dp_2 C_{p_1 p_2} \exp \left\{ -\frac{it}{2m_0\hbar} \left[\left(p_1 - \frac{\hbar k}{2} \right)^2 \right. \right. \\ & \left. \left. + \left(p_2 - \frac{\hbar k}{2} \right)^2 \right] \right\} [D_1(n, t) |p_1, p_2\rangle \otimes |e_1, e_2, n\rangle \\ & + D_2(n, t) (|p_1 - \hbar k, p_2\rangle \otimes |g_1, e_2, n+1\rangle \\ & + |p_1, p_2 - \hbar k\rangle \otimes |e_1, g_2, n+1\rangle) \\ & + D_3(n, t) |p_1 - \hbar k, p_2 - \hbar k\rangle \otimes |g_1, g_2, n+2\rangle] \end{aligned} \quad (15)$$

and

$$D_1(n, t) = 2f_{1n}^2 \cos(A_n t) + 2f_{2n}^2, \quad (16)$$

$$D_2(n, t) = -if_{1n} \sin(A_n t), \quad (17)$$

$$D_3(n, t) = 2f_{1n} f_{2n} [\cos(A_n t) - 1]. \quad (18)$$

We assume that the distribution function $C_{p_1 p_2}$ in Eq. (12) satisfies $C_{p_1 p_2} = C_P C_p$; i.e., the initial state $|\psi(0)\rangle$ for the spatial motion of the two atoms can be written as $|\psi(0)\rangle = |\mu(0)\rangle \otimes |\varphi(0)\rangle$, with $|\mu(0)\rangle = \int_{-\infty}^{\infty} dP C_P |P\rangle$ describing the initial c.m. state of the atoms with a momentum distribution function C_P corresponding to the c.m. momentum eigenstate $|P\rangle$, and $|\varphi(0)\rangle = \int_{-\infty}^{\infty} dp C_p |p\rangle$ describing the initial relative position state of the atoms with a distribution function C_p corresponding to the relative momentum eigenstate $|p\rangle$. Both C_P and C_p satisfy the normalization condition, i.e., $\int_{-\infty}^{\infty} |C_P|^2 dP = 1$ and $\int_{-\infty}^{\infty} |C_p|^2 dp = 1$.

By tracing over the cavity field, the c.m. motion, and the internal states of the atoms, we obtain from Eq. (14) the following elements of the reduced density matrix (in a relative coordinate picture):

$$\rho(x, x', t) = \varphi(x, t) \varphi^*(x', t) F(x, x', t), \quad (19)$$

where $\varphi(x, t) = \int_{-\infty}^{\infty} dp C_p \exp[-ip^2 t / (m_0 \hbar) + ipx / \hbar]$ is the free-evolution state of $|\varphi(0)\rangle$ expressed in a relative coordinate picture, and $F(x, x', t)$ is the decoherence factor, which is given by

$$\begin{aligned} F(x, x', t) = & \sum_{n=0}^{\infty} c_{n, n} \{ D_1^2(n, t) + D_3^2(n, t) \\ & + 2|D_2(n, t)|^2 \cos[k(x - x')/2] \}. \end{aligned} \quad (20)$$

The decoherence factor $F(x, x', t)$ given here will be used in our analysis of the spatial decoherence of two atomic wave packets.

IV. SPATIAL DECOHERENCE OF TWO ATOMIC WAVE PACKETS

We consider that the initial state $|\varphi(0)\rangle$ of the two atoms' relative position is a superposition of two Gaussian wave packets centered at a and $-a$, respectively; i.e., we have (in the x representation)

$$\varphi(x, 0) = \frac{1}{\sqrt{2\delta}} [G_+(x) + G_-(x)], \quad (21)$$

where $G_{\pm}(x) = (\sqrt{2\pi}d)^{-1/2} \exp[-(x \pm a)^2 / (4d^2)]$ and $\delta = 1 + \exp[-a^2 / (2d^2)]$. Here δ is a normalization constant, and, for simplicity, we have assumed that the two Gaussian distributions have the same spread d , which is limited in $d_{\min} \leq d \leq a$. Here d_{\min} is much smaller than the wavelength λ of the cavity field, but a zero spread is not permitted to avoid the atoms indistinguishable by quantum dispersion [29].

Assume that the time for the atoms staying in the cavity is so short that we can assume the spatial distribution remains unchanged with respect to the initial spatial state for the atomic free evolution. Then Eq. (19) can be written as a product of the initial state and the decoherence factor

$$\rho(x, x', t) \simeq \varphi(x, 0) \varphi^*(x', 0) F(x, x', t) = \rho(x, x', 0) F(x, x', t), \quad (22)$$

where $\rho(x, x', 0) = \varphi(x, 0) \varphi^*(x', 0)$ describes the initial state of the atomic relative position. The initial state is illustrated in Fig. 2. Here the two peaks, along the $x' = x$ direction, correspond to the diagonal terms of $\rho(x, x', 0)$, while the other two peaks along the $x' = -x$ direction correspond to the off-diagonal terms of $\rho(x, x', 0)$, which represent the coherence between the two wave packets.

Following Eq. (20), it can be proved that for the case of $x' = x$, $F(x, x, t) = 1$, thus we have $\rho(x, x, t) = \rho(x, x, 0)$; i.e., the diagonal terms of the density matrix $\rho(x, x', t)$ remain unchanged during the time evolution. However, for the off-diagonal terms with $x' = -x \neq m\lambda$ (m is an arbitrary nonzero integer), it can be seen that $|F(x' = -x, t)| \leq 1$. Thus we will mainly analyze the evolution of the off-diagonal terms of a reduced density matrix.

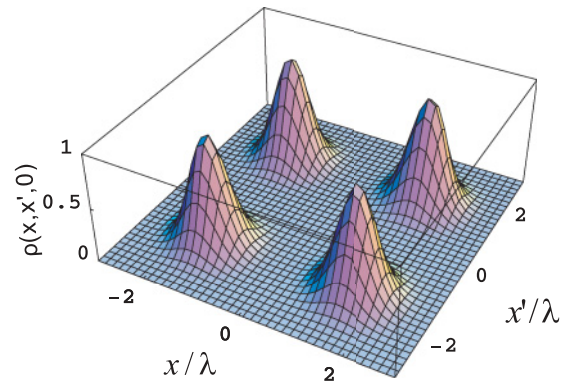


FIG. 2. (Color online) Plot of the density matrix $\rho(x, x', 0)$ representing the initial state for the relative position of the two atoms. It is a superposition of two Gaussian wave packets. The two peaks along the $x' = -x$ diagonal direction represent the coherence between the two wave packets.

For the cavity field initially in a coherent state $|\alpha\rangle$, we have $c_{n,n} = \exp(-|\alpha|^2)\alpha^{2n}/n!$ where α is a complex number. Thus, Eq. (20) becomes

$$F(x, x', t) = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{n!} \left\{ D_1^2(n, t) + 2|D_2(n, t)|^2 \cos\left[\frac{k(x-x')}{2}\right] + D_3^2(n, t) \right\}. \quad (23)$$

Along the $x' = -x$ direction, the decoherence factor takes the following form:

$$F(x' = -x, t) = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{n!} [D_1^2(n, t) + 2|D_2(n, t)|^2 \cos(kx) + D_3^2(n, t)], \quad (24)$$

which is shown in Fig. 3. It can be seen that

(i) For $x = m\lambda$, $F(x' = -x, t)$ remains equal to 1 during the time evolution [Fig. 3(a)].

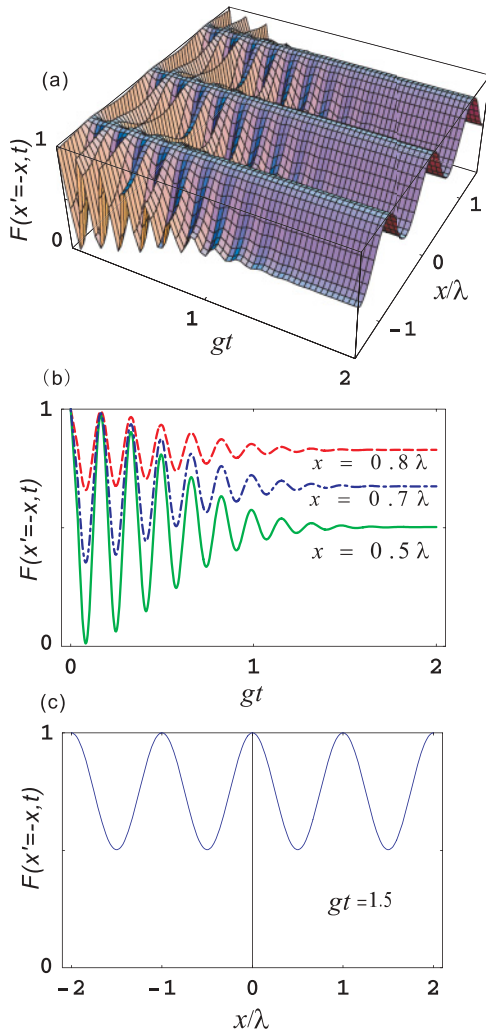


FIG. 3. (Color online) (a) Evolution of the decoherence factor $F(x' = -x, t)$ with time t and relative position x . Decoherence factor $F(x' = -x, t)$ for (b) some selected relative positions and (c) a selected time. Panels (a), (b), and (c) are plotted for the cavity field initially in a coherent state with $\alpha = 10$.

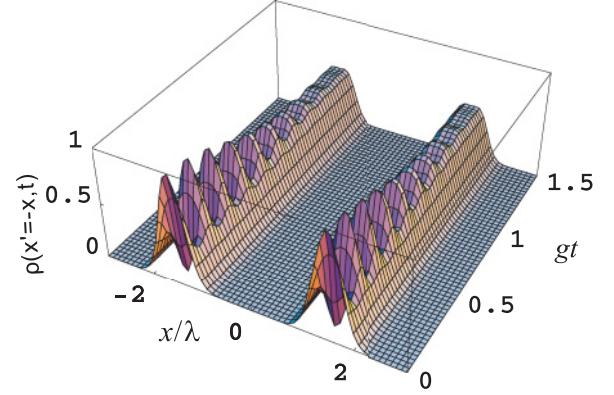


FIG. 4. (Color online) Density matrix $\rho(x' = -x, t)$, which evolves from the initial state (21) with $a \neq m\lambda$ for the cavity field initially in a coherent state with $\alpha = 10$.

(ii) For $x \neq m\lambda$, $F(x' = -x, t)$ decays to a constant after a finite time, and the value of the constant is determined by the value of x [Fig. 3(b)].

(iii) For a given time t , $F(x' = -x, t)$ oscillates with x in a cosine law [Fig. 3(c)], and for the case of $x = (m + 1/2)\lambda$, the decay of $F(x' = -x, t)$ reaches the maximum with $F(x' = -x, gt \geq 1.5) \approx 0.5$ [Fig. 3(c)].

We presented an analysis on the decoherence factor for the case of the initial state $\varphi(x, 0)$ described by Eq. (21). According to that analysis, the following can be concluded:

(i) For the case $a = m\lambda$, the two peaks along the $x' = -x$ direction remain unchanged during the time evolution. So the relative motion of the two atoms decouples with the cavity field, and thus there is no decoherence induced by the photon recoil.

(ii) For the case $a \neq m\lambda$, the two peaks along the $x' = -x$ direction partially decay in a finite time and there is no revival, and the decay of the two peaks depends on a in a cosine law, with a maximum decay at $a = (m + 1/2)\lambda$.

The evolution of $\rho(x' = -x, t)$ for the initial state $\varphi(x, 0)$ with $a \neq m\lambda$ is shown in Fig. 4, which demonstrates that the initial state $\varphi(x, 0)$ undergoes a partial decoherence after a finite time and a revival does not occur. Thus, the coherence of the atomic relative position is strongly destroyed. The Wigner functions for the initial density matrix $\rho(x, x', 0)$ shown in Fig. 2 and the density matrix $\rho(x, x', t)$ given in Eq. (22) are shown in Fig. 5. Strong oscillations together with negative values [Fig. 5(a)] indicate quantum coherence between the two wave packets, and it turns out that the oscillations are partially damped by decoherence [Fig. 5(b)]. This may be induced by the entanglement between the two-atom spatial motion and the cavity field resulting from the photonic back-action. This entangling process inevitably destroys the coherence of the spatial motion of the two atoms. Since the single-mode cavity field is just an environment with a few degrees of freedom, the spatial coherence is not completely destroyed.

V. DISENTANGLEMENT DYNAMICS OF TWO ATOMS

To begin, let us assume that

(i) The initial spatial state $|\psi(0)\rangle$ of the two atoms is a separable state:

$$|\psi(0)\rangle = |\mu\rangle_1 \otimes |\mu\rangle_2. \quad (25)$$

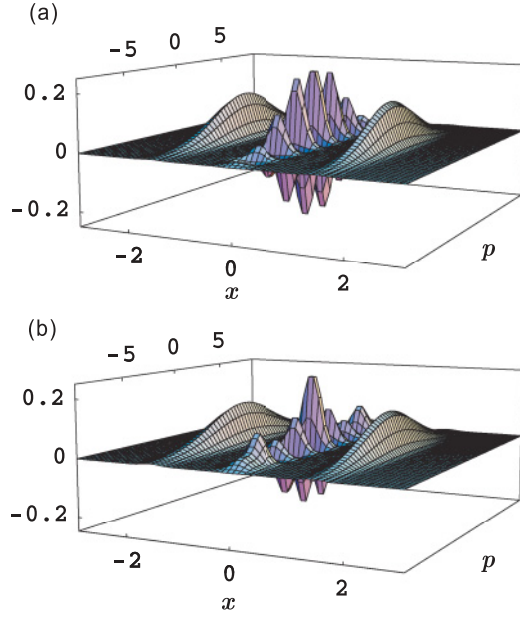


FIG. 5. (Color online) (a) The Wigner function for the initial density matrix $\rho(x, x', 0)$ shown in Fig. 2. (b) The Wigner function for the density matrix in Eq. (22) whose off-diagonal terms are shown in Fig. 4. Panels (a) and (b) are plotted for $gt = 2$. Here p is the relative momentum of the two atoms, and x is the relative position of the two atoms.

Here $|\mu\rangle_1$ and $|\mu\rangle_2$ represent two Gaussian wave packets describing the spatial distribution of the two atoms, respectively. In the coordinate representation, $|\mu\rangle_i$ is expressed as ($i = 1, 2$)

$$\mu(x_i, 0) = \int_{-\infty}^{\infty} dp_i C_{p_i} \exp\left(\frac{i}{\hbar} p_i x_i\right) = \left(\frac{1}{2\pi d^2}\right)^{1/4} \times \exp\left[-\frac{(x_i + a_i)^2}{4d^2}\right], \quad (26)$$

where a_i is the center of the Gaussian function $\mu(x_i, 0)$ and d is the width of the Gaussian function $\mu(x_i, 0)$. The coefficient C_{p_i} is given by

$$C_{p_i} = \left(\frac{2d^2}{\pi\hbar^2}\right)^{1/4} \exp\left(-\frac{d^2 p_i^2}{\hbar^2} + \frac{i a_i p_i}{\hbar}\right). \quad (27)$$

For convenience, we assume that $a_1 = -a_2 = a/2$. The initial density operator for the spatial motion of the two atoms is $\hat{\rho}_s(0) = |\psi(0)\rangle\langle\psi(0)|$.

(ii) The initial internal state $|\phi(0)\rangle$ of the two atoms is an entangled state, which is given by

$$|\phi(0)\rangle = \cos\gamma |g_1, g_2\rangle + \sin\gamma |e_1, e_2\rangle. \quad (28)$$

The initial density operator for the internal state of the two atoms is $\hat{\rho}_i(0) = |\phi(0)\rangle\langle\phi(0)|$.

(iii) The cavity field is initially in a vacuum state, i.e., $\hat{\rho}_f(0) = |0\rangle\langle 0|$.

The initial density operator of the whole system is $\hat{\rho}(0) = \hat{\rho}_s(0) \hat{\rho}_i(0) \hat{\rho}_f(0)$. At time t , the density operator of the system is $\hat{\rho}(t) = \hat{U}(t) \hat{\rho}(0) \hat{U}^\dagger(t)$. Here $\hat{U}(t)$ is the unitary operator in Eq. (2). After tracing $\hat{\rho}(t)$ over the electromagnetic field and the spatial degrees of freedom of the atoms, we obtain the

following reduced density matrix for the internal state of the two atoms:

$$\hat{\rho}_i(t) = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \quad (29)$$

which is written in a basis formed by $|e_1, e_2\rangle$, $|e_1, g_2\rangle$, $|g_1, e_2\rangle$, and $|g_1, g_2\rangle$. Here

$$\begin{aligned} a &= D_1^2(0, 0) \sin^2 \gamma, \\ b &= c = |D_2(0, 0)|^2 \sin^2 \gamma, \\ d &= \cos^2 \gamma + D_3^2(0, 0) \sin^2 \gamma, \\ w &= D_1(0, 0) \cos \gamma \sin \gamma e^{-i2\omega t} e^{-s(t)}, \\ z &= b e^{-\xi(t)}, \end{aligned}$$

with $\xi(t) = s(t) + d^2 k^2 - ika$ and $s(t) = \hbar^2 k^2 t^2 / (4d^2 m_0^2)$. The parameter $s(t)$ is obtained after tracing $\hat{\rho}(t)$ over the atomic spatial degrees of freedom, which causes the strong time dependence of the concurrence as will be shown.

A popular measure of entanglement for $\hat{\rho}_i(t)$ is given by the concurrence [30], which is defined as

$$C(\hat{\rho}_i) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (30)$$

where λ_i are the eigenvalues of the non-Hermitian matrix $\hat{\rho}_i \tilde{\hat{\rho}}_i$ in descending order, and $\tilde{\hat{\rho}}_i = (\hat{\sigma}_y^1 \otimes \hat{\sigma}_y^2) \hat{\rho}_i^* (\hat{\sigma}_y^1 \otimes \hat{\sigma}_y^2)$. Here $\hat{\rho}_i^*$ is the complex conjugate of $\hat{\rho}_i$, and $\hat{\sigma}_y^i$ is the usual Pauli operator for atom i ($i = 1, 2$). The concurrence C varies from $C = 0$ for a nonentangled state to $C = 1$ for a maximally entangled state. The concurrence of the density matrix $\hat{\rho}_i(t)$ in Eq. (29) is [31,32]

$$C(\hat{\rho}_i(t)) = 2 \max\{0, |z| - \sqrt{ad}, |w| - \sqrt{bc}\}, \quad (31)$$

which is shown in Fig. 6. Equation (31) demonstrates that the concurrence evolves with a nonsmoothly exponential decay due to the factor e^{-s} , as shown in Fig. 6.

Alternatively, the initial internal state for the two atoms could be another type of entangled state given by

$$|\phi(0)\rangle = \cos\gamma |e_1, g_2\rangle + \sin\gamma |g_1, e_2\rangle. \quad (32)$$

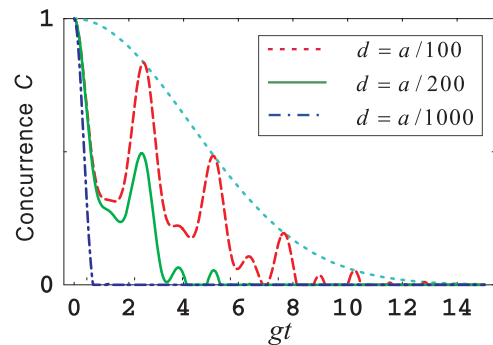


FIG. 6. (Color online) Time evolution of the concurrence C for the case when the initial internal state of the two atoms is an entangled state $|\phi(0)\rangle = \cos\gamma |g_1, g_2\rangle + \sin\gamma |e_1, e_2\rangle$, with $\gamma = \pi/4$. The dotted line is for e^{-s} with $d = a/100$.

By doing a calculation similar to the above, it can be seen that for the entanglement state $|\phi(0)\rangle$ considered here, the reduced density matrix $\hat{\rho}_i(t)$ for the internal state of the two atoms is

$$\hat{\rho}_i(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ 0 & 0 & 0 & w \end{pmatrix}, \quad (33)$$

where

$$\begin{aligned} b &= A_+^2(t) \cos^2 \gamma + A_-^2(t) \sin^2 \gamma \\ &\quad + 2A_+(t)A_-(t) \cos \gamma \sin \gamma e^{-\delta} \cos(ak), \\ c &= A_-^2(t) \cos^2 \gamma + A_+^2(t) \sin^2 \gamma \\ &\quad + 2A_+(t)A_-(t) \cos \gamma \sin \gamma e^{-\delta} \cos(ak), \\ w &= |B(t)|^2 + 2 \cos \gamma \sin \gamma e^{-\delta} \cos(ak), \\ z &= e^{-s(t)} [A_+(t)A_-(t) [\cos^2 \gamma e^{-\alpha} + \sin^2 \gamma e^{-\beta}] \\ &\quad + [A_+^2(t) + A_-^2(t) e^{-\eta}] \cos \gamma \sin \gamma], \end{aligned}$$

with

$$\begin{aligned} A_{\pm}(t) &= \frac{\cos(\sqrt{2}gt) \pm 1}{2}, \\ B(t) &= -i \sin(\sqrt{2}gt) / \sqrt{2}. \end{aligned}$$

Here $\delta = d^2k^2$, $\alpha = d^2k^2 + iak$, $\beta = d^2k^2 - iak$ and $\eta = 4d^2k^2 + i2ak$.

The concurrence for the reduced density matrix $\hat{\rho}_i(t)$ in Eq. (33) is given by $C(\hat{\rho}_i(t)) = 2|z|$, which is shown in Fig. 7. One can see that there is a common factor $e^{-s(t)}$ with $s(t) = \hbar^2k^2t^2/(4d^2m_0^2)$, appearing in the expression of z . Because of this factor $e^{-s(t)}$, the value of $|z|$ decays to zero with time asymptotically. Here Figs. 6 and 7 show that the oscillations of the entanglement are damped exponentially, but the decay of the entanglement occurs faster when the width d of the initial Gaussian function becomes smaller, and the entanglement decays to almost zero in a finite time. So the back-action of the emitted photons may induce an entanglement between the two-atom internal states and the two-atom spatial motion states, and this correlation destroys the coherence of the internal states of the two atoms. The smaller the width of the wave packets, the larger the uncertainty

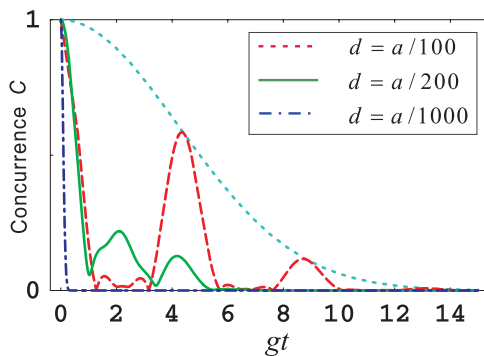


FIG. 7. (Color online) Time evolution of the concurrence C for the case when the initial internal state of the two atoms is an entangled state $|\phi(0)\rangle = \cos \gamma |e_1, g_2\rangle + \sin \gamma |g_1, e_2\rangle$, with $\gamma = \pi/4$. The dotted line is for e^{-s} with $d = a/100$.

of the atomic spatial momentum, which results in a faster destruction of the atomic internal-state entanglement.

VI. CONCLUSION

We have considered a system of two atoms interacting with a single-mode cavity field. Since the spatial degrees of freedom of the atoms are considered, the Hamiltonian of the whole system becomes complicated. To solve the Schrödinger equation, we have introduced two unitary transformations (involving the coupling of the internal levels with the spacial degrees of the atoms), presented an effective Hamiltonian of the system, and given the analytic solutions to the eigenequation of the effective Hamiltonian. Based on these, we have investigated the decoherence dynamics for the relative position of the two atoms and presented a decoherence factor for a general cavity-field state and an arbitrary atomic spatial state, which might be useful for future related work.

Under the assumption that the atomic relative position is in a superposition of two Gaussian wave packets, we have demonstrated the spatial decoherence of the two atoms' relative position for a cavity field initially in a coherent state. Our results show that the spatial decoherence of the two atoms depends strongly on the relative position of the two atoms. Interestingly, we found that when the relative position of the two atoms is an integral multiple of the wavelength of the cavity field, the spatial coherence of the relative position of the two atoms is not destroyed by the photon recoil. However, when the relative position of the two atoms is not an integral multiple of the wavelength of the cavity field, spatial decoherence of the relative position of the two atoms happens.

Furthermore, we have studied the entanglement dynamics of the internal states of the two atoms interacting with a single-mode cavity field. Our results show that the entanglement, measured by the concurrence, decays to almost zero in a finite time. Thus, the back-action of the photons emitted from the two atoms may be a fundamental process destroying the entanglement of atoms.

From this work, it can be concluded that there exists a phenomenon that the moving qubits (e.g., atoms) placed in a cavity may suffer from the spatial decoherence. Therefore, it is important to overcome the influence of the spatial motion of qubits on their entanglement.

ACKNOWLEDGMENTS

We are grateful to C. P. Sun, A. Miranowicz, S. B. Zheng, X. G. Wang, and P. Zhang for very useful comments and help. F.N. acknowledges partial support from the Laboratory of Physical Sciences, National Security Agency, Army Research Office, Defense Advanced Research Projects Agency, Air Force Office of Scientific Research, National Science Foundation Grant No. 0726909, JSPS-RFBR contract No. 09-02-92114, Grant-in-Aid for Scientific Research (S), MEXT Kakenhi on Quantum Cybernetics, and Funding Program for Innovative R&D on S&T (FIRST). C.P.Y. acknowledges support from the National Natural Science Foundation of China under Grant No. 11074062, the Natural Science Foundation of Zhejiang Province under Grant No. Y6100098, and funds from Hangzhou Normal University. This work was also supported by the China Scholarship Council and the Foundation of the Education Department of Liaoning Province Grant No. 20060160.

- [1] J. Stolze and D. Suter, *Quantum Computing* (Wiley, Weinheim, Germany, 2008).
- [2] W. P. Schleich and H. Walther, eds., *Elements of Quantum Information* (Wiley, Weinheim, Germany, 2008).
- [3] L. F. Wei, Y. X. Liu, C. P. Sun, and F. Nori, *Phys. Rev. Lett.* **97**, 237201 (2006).
- [4] A. N. Omelyanchouk, S. N. Shevchenko, A. M. Zagoskin, E. Il'ichev, and F. Nori, *Phys. Rev. B* **78**, 054512 (2008).
- [5] S. N. Shevchenko, A. N. Omelyanchouk, A. M. Zagoskin, S. Savel'ev, and F. Nori, *New J. Phys.* **10**, 073026 (2008).
- [6] X. G. Wang, A. Miranowicz, Y. X. Liu, C. P. Sun, and F. Nori, *Phys. Rev. A* **81**, 022106 (2010).
- [7] P. Zhang, X. F. Liu, and C. P. Sun, *Phys. Rev. A* **66**, 042104 (2002).
- [8] C. P. Sun, X. F. Liu, D. L. Zhou, and S. X. Yu, *Eur. Phys. J. D* **17**, 85 (2001).
- [9] J. Q. You, J. S. Tsai, and F. Nori, *Phys. Rev. B* **68**, 024510 (2003).
- [10] J. Q. You and F. Nori, *Phys. Today* **58**, 42 (2005).
- [11] X.-B. Wang, J. Q. You, and F. Nori, *Phys. Rev. A* **77**, 062339 (2008).
- [12] J. Zhang, Y. X. Liu, C.-W. Li, T.-J. Tarn, and F. Nori, *Phys. Rev. A* **79**, 052308 (2009).
- [13] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **77**, 4887 (1996).
- [14] C. J. Myatt *et al.*, *Nature (London)* **403**, 269 (2000).
- [15] J. R. Friedman *et al.*, *Nature (London)* **406**, 43 (2000).
- [16] A. V. Rau, J. A. Dunningham, and K. Burnett, *Science* **301**, 1081 (2003).
- [17] K. W. Chan, C. K. Law, and J. H. Eberly, *Phys. Rev. Lett.* **88**, 100402 (2002).
- [18] X. G. Wang and C. P. Sun, *J. Mod. Opt.* **42**, 515 (1995).
- [19] L. You, *Phys. Rev. A* **64**, 012302 (2001).
- [20] L. Zheng, C. Li, Y. Li, and C. P. Sun, *Phys. Rev. A* **71**, 062101 (2005).
- [21] S. B. Zheng and G. C. Guo, *Phys. Rev. Lett.* **85**, 2392 (2000).
- [22] A. Miranowicz, S. K. Özdemir, Y. X. Liu, M. Koashi, N. Imoto, and Y. Hirayama, *Phys. Rev. A* **65**, 062321 (2002).
- [23] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Rev. Mod. Phys.* **81**, 865 (2009).
- [24] T. Sleator and M. Wilkens, *Phys. Rev. A* **48**, 3286 (1993).
- [25] J. M. Raimond, M. Brune, and S. Haroche, *Rev. Mod. Phys.* **73**, 565 (2001).
- [26] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **77**, 4887 (1996); S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, *ibid.* **87**, 037902 (2001).
- [27] S. B. Zheng, *Phys. Rev. A* **70**, 034301 (2004).
- [28] E. Urban, T. A. Johnson, T. Henage, L. Isenhower, D. D. Yavuz, T. G. Walker, and M. Saffman, *Nature Phys.* **5**, 110 (2009).
- [29] F. Lastra, S. Wallentowitz, M. Orszag, and M. Hernández, *J. Phys. B* **42**, 065504 (2009).
- [30] W. K. Wootters, *Phys. Rev. Lett.* **80**, 2245 (1998).
- [31] T. Yu and J. H. Eberly, *Quantum Inf. Comput.* **7**, 459 (2007).
- [32] T. Yu and J. H. Eberly, *Science* **323**, 598 (2009).