Temperature dependence of the Casimir force for bulk lossy media

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We discuss the limitations for the applicability of the Lifshitz theory to describe the temperature dependence of the Casimir force between bulk lossy metal slabs of *finite* sizes. We pay attention to the important fact that Lifshitz's theory is not applicable when the characteristic wavelength of the fluctuating field, responsible for the temperature-dependent terms in the Casimir force, are longer than the size of the sample. As a result, the widely discussed linearly decreasing temperature dependence of the Casimir force can be observed only for dirty and large metal samples at high enough temperatures. Moreover, for the correct description of the Casimir effect at low enough temperatures, a careful consideration of the concrete geometry of the interacting samples is essential.

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I. INTRODUCTION

The Casimir effect is one of the most interesting macroscopic manifestations of the zero-point vacuum oscillations of the quantum electromagnetic field. This effect manifests itself as an attractive force arising between two uncharged bodies due to the difference of the zero-point oscillation spectrum in the absence and in the presence of these bodies (see, e.g., Refs. [1–4]).

The Casimir effect has attracted considerable attention because of its numerous applications in quantum field theory, atomic physics, condensed matter physics, gravitation, and cosmology [1–5]. The noticeable progress in the measurements of the Casimir force [6] has opened the way for various potential applications in nanoscience [7], particularly in the development of nanomechanical systems [2,4,7].

In spite of intensive studies on the Casimir effect, it is surprising that such an important problem as the temperature dependence of this effect is still an issue of lively discussion (see, e.g., Refs. [8–15]). The zero-temperature contribution to the force, originating from quantum fluctuations of the electromagnetic field, is well understood. However, the contribution $F_{\rm rad}(T)$ to the Casimir force originating from thermal fluctuations is a source of numerous problems.

First, within the Lifshitz theory [16], there is no continuous transition for the forces between ideal metals and real metals [9]. The Lifshitz formula predicts an increase of $F_{\rm rad}(T)$ when increasing T only for ideal metals without relaxation. At the same time, for lossy media with relaxation frequency $\nu \neq 0$, this formula gives a decrease of $F_{\rm rad}(T)$ in a wide region of temperatures. This decreasing term is related to the transparency of real metals for s-polarized (transverse electric) low-frequency fields [17]. In other words, the behavior of $F_{\rm rad}(T)$ changes abruptly, in a jump-like manner, for *infinitesimal* ν, in comparison to the case when $\nu = 0$. This discontinuous jump is not physical.

Second, within the Drude model, for a perfect crystal lattice of infinite size, the Casimir-Lifshitz entropy does not go to zero when $T \to 0$. This would be unphysical, because it would violate Nernst's theorem.

Similar problems exist for dielectrics and semiconductors [18–20]. In the Casimir force, the term that decreases linearly with temperature arises from the p-polarized low-frequency modes, when a finite conductivity is taken into account [21]. These problems are still the focus of discussions (see, e.g., Ref. [11]). It was shown in Ref. [22] that the consistency with the Bohr-van-Leeuwen theorem should also be taken into account when choosing between different models.

As shown in Refs. [21,23,24], all the problems with the violation of the Nernst theorem in the parallel-plane geometry are due to the noncommutativity of the limits $T \to 0$ and $\omega \to 0$. In the Casimir force, the main contribution to the term that decreases linearly with temperature comes from the low-frequency fluctuations with $\omega \sim \nu$. Thus, this term can only be present in the high-temperature regime, for $kT \gtrsim \hbar\nu$. The low-frequency s-polarized modes correspond to the eddy currents for interacting metal plates [25], and thus the problem is similar to the Johnson noise in wires [26].

Refs. [17,27,28] indicated that the problems mentioned previously can be solved if one takes into account the spatial dispersion of the low-frequency metal conductivity. *Nonlocal* effects play an important role in metals with small enough relaxation frequencies, $\nu \ll \omega_{\rm ir} = \omega_p v_F/c$, where v_F is the Fermi speed of the electrons and ω_p is the plasma frequency. All pure metals satisfy this requirement at low enough temperatures. Thus, considering nonlocal effects is required for checking consistency with the Nernst theorem. For semiconductors, taking into account nonlocal effects also leads to consistency with the third law of thermodynamics [14]. Note, however, that some problems with the temperature dependence of the Casimir force remain unsolved. An open problem is the violation of the Nernst theorem for metals, if the relaxation frequency tends to zero sufficiently fast as $T \to 0$ (see, e.g., Ref. [25]). Another question: why has the decreasing temperature dependence of the Casimir force never been observed?

In this paper, we demonstrate that there exist simple limitations for the applicability of Lifshitz's theory that are related to the *finite sizes* of the interacting bodies. The effects of finite size of the interacting bodies in the Casimir effect have been taken into account in Ref. [29] where the role of boundaries was studied for the case of ideal metals. The interplay between the effects of temperature, relaxation, and sphere-plane geometry was considered in Ref. [30]. Special attention was focused on the case of large distances between a metallic plane and a sphere, compared to the radius of the latter, when the so-called Proximity Force Approximation is not valid. A negative entropy was predicted not only for the Drude model, but also for the plasma and perfect reflector models. It was shown that the appearance of negative entropies is not related to the presence of dissipation. Here, we show that the term in the Casimir force which is linearly decreasing with temperature can only appear for large and/or dirty metals if the following inequality is satisfied:

$$\nu \gg 2\pi c/L. \tag{1}$$

Here, L is the width of the sample. Usually, condition (1) is not satisfied in the measurements of the Casimir force. For instance, the experiments in Ref. [31] dealt with a tiny metal sphere of radius $R = 151.3\ \mu$m. In this case, Eq. (1) is not valid if the relaxation frequency is less than $10^{14}\ \mathrm{s}^{-1}$ for distances l of the order of 1 μm [when $L \sim (Rl)^{1/2} \approx 12\ \mu$m]. In addition, we show that Lifshitz's theory cannot be applied to describe the temperature dependence of the Casimir force in the temperature interval $T < T_L = 2\pi c\hbar/kL$. Here, k is the Boltzmann constant. In this interval, the main contribution to the temperature-dependent term in the Casimir force comes from fluctuating fields with wavelengths longer than the sample size. In other words, the *finite* size of the sample must be taken into account in this temperature range.

II. ANALYSIS OF THE TEMPERATURE DEPENDENCE OF THE LIFSHITZ FORMULA

Let us now consider the term which decreases linearly with temperature in the Casimir force between *infinite* plates of lossy metals. The purpose of this section is to express this term, which comes from fluctuating fields with small frequencies, when $\omega \lesssim \nu$, in a convenient form.

Following Refs. [17,23], we analyze the Lifshitz expression for the Casimir force taken from Ref. [16] in the form of an integral over *real* frequencies ω. We use the Drude model for the permittivity ε,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}. \tag{2}$$

In this case, the thermal term F_{rad} in the Casimir force per unit area can be written in the following form:

$$F_{\mathrm{rad}} = \frac{\hbar}{\pi^2 c^3}\mathrm{Re}\int_0^\infty d\omega \int dp\, p^2\omega^3 \frac{1}{\exp(2\hbar\omega/kT) - 1} \times \left\{\left[\left(\frac{s+p}{s-p}\right)^2 \exp(-2ip\omega l/c) - 1\right]^{-1} + \left[\left(\frac{s+\varepsilon p}{s-\varepsilon p}\right)^2 \exp(-2ip\omega l/c) - 1\right]^{-1}\right\}, \tag{3}$$

where $s = \sqrt{\varepsilon(\omega) - 1 + p^2}$, l is the separation between the interacting bodies, and Re denotes the real part. The integration trajectory over p consists of two parts: from 1 to 0 over the real axis, and from $i0$ to $+i\infty$ over the imaginary axis.

We examine the difference ΔF_{rad} between the contributions to the Casimir force from thermal fluctuations for a dissipationless metal ($\nu = 0$) and for a metal with weak dissipation ($\nu \to 0$),

$$\Delta F_{\mathrm{rad}} = F_{\mathrm{rad}}|_{\nu\to 0} - F_{\mathrm{rad}}|_{\nu=0}. \tag{4}$$

Namely, ΔF_{rad} describes the "linearly decreasing with T" part of the Casimir force $F_{\mathrm{rad}}(T)$ that appears in a jumplike manner at $\nu \neq 0$. It is important to note that only the first term in the curly brackets in Eq. (3) [integrated over p from $i0$ to $+i\infty$, and over ω from 0 to $+\infty$] produces this discontinuity. So, the difference ΔF_{rad} can be written as

$$\Delta F_{\mathrm{rad}} = \frac{\hbar}{\pi^2 c^3}\mathrm{Re}\int_0^\infty d\omega \int_{0i}^{+i\infty} dp\, p^2\omega^3 \frac{1}{\exp(2\hbar\omega/kT) - 1} \times \left\{\left[\left(\frac{s+p}{s-p}\right)^2 \exp(-2ip\omega l/c) - 1\right]^{-1} - \left[\left(\frac{s|_{\nu=0}+p}{s|_{\nu=0}-p}\right)^2 \exp(-2ip\omega l/c) - 1\right]^{-1}\right\}. \tag{5}$$

Introducing the notation,

$$t = \frac{\omega}{\nu}, \quad x = -\frac{2ip\omega l}{c}, \quad \alpha = \frac{c}{2l\omega_p}, \tag{6}$$

and assuming that $\hbar\nu \ll kT$, we obtain

$$\Delta F_{\mathrm{rad}} = -\frac{kT}{8\pi^2 l^3}\mathrm{Im}\int_0^\infty dt\ I(\alpha,t),$$
$$I(\alpha,t) = \frac{1}{t}\int_0^{+\infty} \frac{dx x^2}{\left(\alpha x + \sqrt{\alpha^2 x^2 + \frac{t}{t+i}}\right)^4 \left(\frac{t+i}{t}\right)^2 e^x - 1}, \tag{7}$$

where Im denotes the imaginary part.

It is seen from this equation that the difference ΔF_{rad} does *not* depend on ν, and that the main contribution to this integral comes from $x \sim 1$ and $t \lesssim 1$. According to Eq. (6), the characteristic values of ω are either of the order or less than ν.

III. TEMPERATURE DEPENDENCE OF THE CASIMIR FORCE FOR SAMPLES OF FINITE SIZE

In the Lifshitz theory, the main contribution to the term $F_{\mathrm{rad}}(T)$, which decreases linearly with T, comes from small frequencies satisfying these two inequalities:

$$\omega \lesssim kT/\hbar, \quad \omega \lesssim \nu. \tag{8}$$

Obviously, the wavelengths of the fluctuating fields with such frequencies should be much smaller than the size of the sample. Otherwise, the sample cannot be considered as semi-infinite. In other words, the Lifshitz theory gives a (*physically* correct) $F_{\mathrm{rad}}(T)$ decreasing with temperature *if*

$$\nu \gg 2\pi c/L, \tag{9}$$

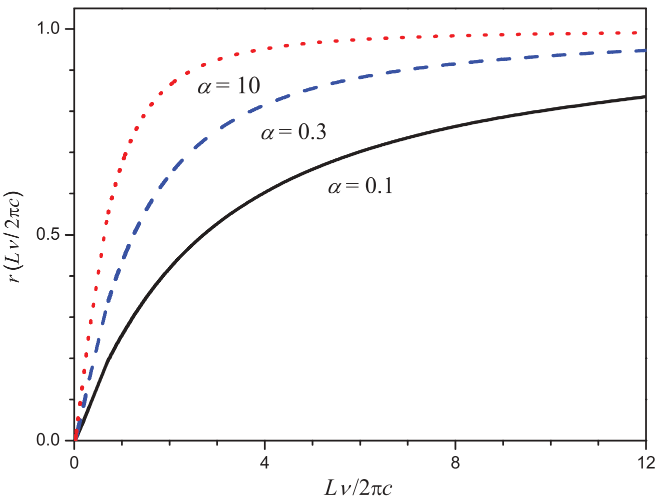

FIG. 1. (Color online) The reduction coefficient r in Eq. (12) versus the parameter $L\nu/2\pi c$, for different values of the parameter $\alpha = c/2l\omega_p$ indicated near the curves.

and

$$T \gg T_L = 2\pi\hbar c/kL. \tag{10}$$

Naturally, we cannot calculate the Casimir force for bodies of arbitrary shape and finite size. However, it is possible to estimate this force by introducing a cutting-off procedure. We integrate in Eq. (7) not over the whole range of frequencies, but allow only the fluctuations with wavelength smaller than the sample size L,

$$\Delta F_{\rm rad}(T,\nu L/2\pi c) = r(\nu L/2\pi c)\Delta F_{\rm rad}(T,\infty). \tag{11}$$

Here we introduce the reduction coefficient,

$$r(\nu L/2\pi c) = \frac{\mathrm{Im}\int_{2\pi c/L\nu}^{\infty} dt\ I(\alpha,t)}{\mathrm{Im}\int_0^{\infty} dt\ I(\alpha,t)}. \tag{12}$$

Figure 1 shows the decrease of the reduction coefficient r, when decreasing the sample sizes for different values of the parameter α. One can see that linearly decreasing temperature dependence can be observed for large enough samples of dirty metals only when $L\nu/2\pi c \gg 1$.

We have also analyzed the radiation term in a wide temperature range for slabs of finite sizes. Figure 2 shows the contribution to the Casimir force from fluctuations with wavelengths smaller than the sample size. Only these inputs are calculated correctly using the Lifshitz formula. However, they become exponentially small for temperatures $T < T_L = 2\pi c\hbar/kL$. The corresponding portions of the curves are shown with (black, blue, red) dashes. Obviously, the low-frequency fluctuations with wavelengths larger than the sample sizes can strongly affect the radiation term in this temperature range. It is very important to note that the temperature interval $T < T_L$ overlaps with a significant part of the room-temperature range for small samples with $L < 0.1$ mm, and T_L is about 1300 K for slabs with $L = 10\ \mu$m. This means that the finiteness of samples should be undoubtedly taken into account when calculating the temperature dependence of the Casimir force for small slabs.

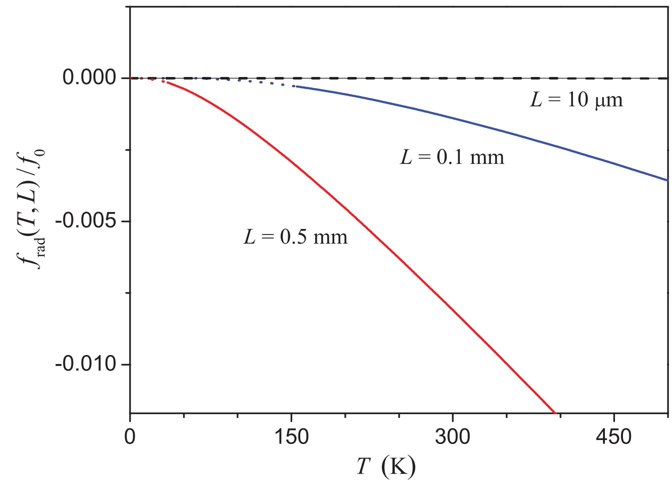

FIG. 2. (Color online) Temperature dependence of the dimensionless radiation term of the Casimir force calculated with cutting off the contribution of the low-frequency fluctuating fields. Dashed portions of the curves correspond to temperatures smaller than $T_L = 2\pi c\hbar/kL$. Here, fluctuating fields with $\omega < 2\pi c/L$ can play a crucial role, compared to the exponential input of the high-frequency fluctuations. The values of parameters used here are $\nu = 5.4 \times 10^{13}$ s^{-1}, $\omega_p = 1.4 \times 10^{16}$ s^{-1}, and $a = 500$ nm. The sample sizes L are indicated near the curves. The radiation term is normalized to the value of the Casimir force between the ideal metal slabs: $f_0 = \pi^2\hbar c/240a^4$.

Note that the estimate presented here for the Casimir force concerns with the interaction between two parallel plates of finite sizes. However, we believe that the results in this section will be qualitatively valid for bodies of other shapes (e.g., for the plane-sphere geometry considered in Ref. [30]).

IV. CONCLUSIONS

Here we discussed the applicability of the Lifshitz theory to describe the temperature dependence of the Casimir force between bulk lossy metal slabs of finite sizes. We pay attention to the important fact that Lifshitz's theory is not applicable when the characteristic wavelengths of the fluctuating fields, responsible for the temperature-dependent terms in the Casimir force, are longer than the sizes of the samples. We have shown that the widely discussed linearly decreasing temperature dependence of the Casimir force can be observed only for dirty and large metal samples at high enough temperatures. Moreover, for the correct description of the Casimir effect at low enough temperatures, it is necessary to take into account the specific geometry of the interacting samples.

Note that the condition in (10) did not hold in the experiment in Ref. [31] (with a tiny metal sphere of radius $R = 151.3\ \mu$m). For all temperatures used in that experiment, the wavelengths of the fluctuating fields responsible for the temperature decrease of the Casimir force (expected within the Lifshitz theory with the Drude model for the permittivity) were of the order of R and longer than the radius $r \sim (Rl)^{1/2}$ of the effective interacting region of the tiny sphere. Therefore, it is not surprising that the temperature decrease of the Casimir force was not observed in Ref. [31].

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