Quantum Terahertz Electrodynamics and Macroscopic Quantum Tunneling in Layered Superconductors

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We derive a quantum field theory of Josephson plasma waves (JPWs) in layered superconductors, which describes two types of interacting JPW bosonic quanta (one heavy and one lighter). We propose a mechanism of enhancement of macroscopic quantum tunneling (MQT) in stacks of intrinsic Josephson junctions. Because of the long-range interaction between junctions in layered superconductors, the calculated MQT escape rate $\Gamma$ has a nonlinear dependence on the number of junctions in the stack. We show that the crossover between quantum and thermal escape increases when increasing the number of junctions. This allows us to quantitatively describe striking recent experiments in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ stacks.

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The recent surge of interest in stacks of intrinsic Josephson junctions is partly motivated by the desire to develop THz devices, including emitters, filters, detectors, and nonlinear devices. Macroscopic quantum tunneling (MQT) has been, until recently, considered to be negligible in high-$T_c$ superconductors due to the $d$-wave symmetry of the order parameter. Recent unexpected experimental evidence of MQT in layered superconductors could open a new avenue for the applicability of stacks of Josephson junctions in quantum electronics. This requires a quantum theory capable of quantitatively describing these remarkable experiments. In contrast to a single Josephson junction, stacks of intrinsic Josephson junctions are strongly coupled along the direction perpendicular to the layers because the thickness of these layers is of the order of a few nm, which is much smaller than the magnetic penetration length. This results in a profoundly nonlocal electrodynamics that strongly affects quantum fluctuations in layered superconductors.

The two main results of this work are, first, the quantum electrodynamics of Josephson plasma waves (JPWs) and, second, the quantitative description of macroscopic quantum tunneling in stacks of Josephson junctions. Namely, using a general Lagrangian approach, we derive the theory of quantum JPWs, which describes two interacting quantum fields: a heavy JPW and a lighter one. We predict resonances in the amplitudes of quantum processes associated with the creation of pairs of JPW quanta. Using the general approach, we develop a quantitative theory of MQT in stacks of Josephson junctions. Our approach is based on the analysis of coupled sine-Gordon equations that adequately describe the long-range interactions in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ stacks, in contrast to the phenomenological treatment of capacitive coupled Josephson junctions. We derive the MQT escape rate $\Gamma$, which is strongly nonlinear with respect to the number of superconducting layers $N$, and changes to $\Gamma \propto N$ when $N$ exceeds a certain critical value $N_c$. The thermoactivated escape rate $\Gamma_T$ also increases in coupled junctions in comparison with decoupled ones. However, the crossover between quantum and thermal escape occurs at higher temperature for the coupled junctions. More important, our results are in good quantitative agreement with recent experiments.

Quantum theory for layered superconductors.—The electrodynamics of stacks of Josephson junctions can be described by the Lagrangian for the electromagnetic fields $\vec{E}$ and $\vec{H}$ interacting with matter: $\mathcal{L} = \frac{1}{2} \vec{j} \cdot \vec{A} + \frac{1}{\gamma^2} (\vec{E}^2 - \vec{H}^2)$, here without charge degrees of freedom. The current $\vec{j}$ consists of both the Josephson current across the layers (along the $y$ axis, see top inset in Fig. 1(b)) and the London supercurrent along the layers (along the $x$ axis); the vector potential is $\vec{A} = (A_x, A_y, 0)$. This Lagrangian can be rewritten as:

$$\mathcal{L} = \sum_n \int \frac{d^2 \phi_n}{2} \left[ \frac{1}{2} \phi_n^2 + \frac{1}{2} p_n^2 - \frac{1}{2} (\partial_x \phi_n)^2 - \frac{1}{2} (\partial_y \phi_n)^2 - \frac{1}{2} p_n^2 + \cos \phi_n - \frac{1}{2} (\partial_x p_n \phi_n + \partial_y p_n \phi_n + \partial_z p_n \phi_n) \right],$$

where $\phi_n = \chi_{n+1} - \chi_n - 2\pi s A_y^{(n)} / \Phi_0$ is the gauge-invariant interlayer phase difference, and $p_n = (s / \lambda_{ab}) \partial_x \chi_n - 2\pi s A_x^{(n)} / \Phi_0$ is the normalized superconducting momentum in the $n$th layer. Here, we introduce the phase $\chi_n$ of the order parameter, the interlayer distance $s$, the in-plane $\lambda_{ab}$ and out-of-plane $\lambda_c$ penetration depths, the anisotropy parameter $\gamma = \lambda_c / \lambda_{ab}$, and flux quantum $\Phi_0$. The in-plane coordinate $x$ is normalized by $\lambda_c$; the time $t$ is normalized by $1 / \omega_J$, where the plasma frequency is $\omega_J$; also, $\partial_x = \partial / \partial x$, $\partial_y = \lambda_{ab} (f_{n+1} - f_n) / s$, $\partial_z = \partial / \partial t$, and the $z$ axis is pointed along the magnetic field. For simplicity, we now consider 2D fields with $\partial_z = 0$. The
interacting fields $\varphi$ and $p$. This is because the vector potential has two components, $A_x$ and $A_y$, in stacks of Josephson junctions, in contrast to 1D Josephson junctions where one component of the vector potential is enough.

Linearizing Eqs. (2) and substituting $p, \varphi \approx \exp(\alpha x + ik_x y)$ we derive a biquadratic equation, $(\omega^2 - k_y^2 - 1)(\omega^2 - k_x^2 - 1) - k_x^2 k_y^2 = 0$, for the spectrum of the classical JPWs in the continuous limit (i.e., $k_x, k_y \ll 1$) and $\gamma^2 \gg 1$. Here, $k_x$ and $k_y$ are the wave vectors (momenta in the quantum description, here, $h = 1$) of the JPWs. This equation determines two branches, $\omega = \omega_x(\vec{k})$ and $\omega_y(\vec{k})$, of JPWs: $\omega_x(\vec{k}) = [1 + k_x^2/(1 + k_y^2)]^{1/2}$, $\omega_y(\vec{k}) = (k_y^2 + k_x^2)^{1/2}$ up to $1/\gamma^2$. The $a$ branch ($b$ branch) describes Josephson plasmons propagating both along and perpendicular (only perpendicular) to the layers. In order to quantize the JPWs we use the Hamiltonian, $\mathcal{H} = \sum_n \int dx \{ \Pi \varphi_n + p_n \varphi_n \} - L$, with the momenta $\Pi \varphi_n$ and $p_n$ of the $\varphi_n$ and $p_n$ fields, and require the standard commutation relations $[\varphi_n(x), \Pi \varphi_m(y)] = i\delta(x - y)\delta_{n,m}$, $[p_n(x), \Pi \varphi_m(y)] = i\delta(x - y)\delta_{n,m}$ (all others commutators are zero), where $\delta$ is either a delta function or Kronecker symbol. Expanding $\cos \varphi = 1 - \varphi^2/2 + \varphi^4/24 - \cdots$, we can write $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_a + \mathcal{H}_b$, where we include terms up to $\varphi^4$ in $\mathcal{H}_a$, and the anharmonic terms in $\mathcal{H}_b$. Diagonalizing $\mathcal{H}_0$, we obtain the Hamiltonian for the Bosonic free fields $a$ and $b$: $\mathcal{H}_0 = \sum_n \int (dk_n/2\pi) \times \{ \omega_x(\vec{k})a^{+n}a + \omega_y(\vec{k})b^{+n}b \}$. The original fields $\varphi_n, p_n$ in Eq. (1) are related to the free Bosonic fields $a$ and $b$ by $\varphi = (a^+ + a)/(2\omega_a)^{1/2} - Z(b^+ + b)/[\gamma(2\omega_b)^{1/2}]$ and $p = Z(a^+ + a)/(2\omega_a)^{1/2} + \gamma(b^+ + b)/(2\omega_b)^{1/2}$, where $Z = k_x k_y/(k^2 + 1)$. The spectra $\omega_x(\vec{k}), \omega_y(\vec{k})$ show that the “mass” of the $a$ quantum equals one, and for the heavier $b$ quasi-particle is $\gamma$.

The interaction between the $a$ and $b$ fields, including the self-interaction, occurs due to the anharmonic terms in $\mathcal{H}_a = (-1/24)\sum_n \int d\varphi^4$ $\cdots$. In the leading order with respect to the bosonic field interactions, an $a$ particle can create either $a + a$ or $a + b$ pairs. Using the spectra $\omega_{a,b}(\vec{k})$, one can conclude that the amplitudes of these processes have energy thresholds $\omega_{a,b}(\vec{k}) = 3$ or $\gamma + 2$ (similar to the $2mc^2$ rest energy threshold for $e^- + e^+$ pair creation in usual quantum electrodynamics). These result in resonances in the amplitudes of quantum processes (e.g., decay of a quanta).

Enhancement of macroscopic quantum tunneling.—Now we apply our theory to interpret very recent experiments [5] on MQT in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. To observe MQT, an external current $J$, close to the critical value $J_c$, was applied [5]. When tunneling occurs, the phase difference in a junction changes from 0 to $2\pi$, which can be interpreted as the tunneling of a fluxon through the contact. This process can be safely described within a semiclassical approximation and we use the approach developed in Refs. [5,10] to calculate the escape rate.
\( \Gamma = \omega_P \sqrt{30B/\pi} \exp(-B) \) of a fluxon through the potential barrier. Here, \( \omega_P \) is the oscillation frequency of a fluxon near the effective potential minimum, and \( B = \int_{-\infty}^{\infty} \frac{d\tau}{d\tau} L(\tau = it) = 2 \int_{0}^{\pi} ds[\varphi]/2V[\varphi] \) is described by the Lagrangian (1) with the classical fields determined by Eqs. (2). Integration should be performed in the functional space of \( \varphi \) between points \( \sigma_m \) of zeros of the potential \( V \) along a trajectory \( s[\varphi] \), which corresponds to the minimum of the effective action \( B \) (that is, the maximum of the escape rate). This is a complicated numerical problem, which can be replaced by integration over an appropriate set of trial functions. Below we will follow the latter approach.

Following the experimental setup [5], here we consider a stack of intrinsic Josephson junctions [see inset in Fig. 1(b)] having the size \( L \gg s \) along the y direction; i.e., the total number of contacts \( N = L/s \gg 1 \), and the size in the x direction, \( 2d \), is smaller than the Josephson length, \( \lambda_J = \gamma \sqrt{s \lambda_{ab}/2} \). In the limit \( \gamma^2 \gg 1 \), the equations for \( \varphi \) are reduced to standard coupled sine-Gordon equations [9], which in the continuum limit, \( k_s \ll 1 \) and \( \gamma = ns/\lambda_{ab} \), become \( (1 - \partial^2/\partial y^2)[\varphi + \sin x] - \partial^2 \varphi / \partial x^2 = 0 \) with \( \partial \varphi / \partial x = \pm jd/\lambda_c \) at \( x = \pm d/\lambda_c \), where \( j = J/\lambda_c \). We seek a solution \( \varphi \) of the form \( \varphi = \psi(x, y, t) + \arcsin(j) + jx^2/2 \), where the field \( \psi \) obeys

\[
\left( 1 - \frac{\partial^2}{\partial y^2} \right) \frac{\partial^2 \psi}{\partial x^2} + (1 - \cos \psi) + \sqrt{1 - j^2} \sin \psi = 0
\]

with boundary conditions \( \partial \psi / \partial x = 0 \) at \( x = \pm d/\lambda_c \). In Eq. (3) inside square brackets we ignore the terms of the order of \( d^2/\lambda_c^2 \sim 10^{-3} \) compared with \( \sqrt{1 - j^2} \sim 0.1 \).

We can linearize Eq. (3) in all junctions except one, where the fluxon tunnels. The linearized equation can be solved by using the Fourier transformation, \( \psi = \sum_{k_m} \exp(-i\omega t) \exp(i k_m x) \varphi_{km}(y, \omega)/2\pi \), where \( k_m = \lambda_J \pi m/2d \). Here, in order to improve the convergence of the Fourier series, we expand the solution \( \psi(x) \), initially defined for \( -d < x < d \), periodically in \( -\infty < x < \infty \), keeping continuous \( \psi(x) \) and \( \partial \psi(x) / \partial x \) [see inset in Fig. 1(b)]. Since in the experiment [5] the sample connects two bulk superconductors, we can choose the phase difference to be zero at the top \( (y = L) \) and bottom \( (y = L - L) \) layers of the sample, and \( y = 0 \) corresponds to the position of the fluxon tunneling; see inset in Fig. 1(b). Using the continuity of \( \psi(y) \) at \( y = 0 \), we derive the solution of the linearized equation in the form \( \varphi_{km}(y) = \psi_{m}(0) \sinh[q_{m}(L - L)/\sinh[q_{m}(L - L)] / \sinh[q_{m}(L - L)] \) for \( y < 0 \). Here, \( q_{m}^2 = (k_m^2 + \sqrt{1 - j^2 - \omega^2})/\sqrt{1 - j^2 - \omega^2} \).

In the junction at \( y = 0 \), where the fluxon is tunneling, we cannot use the linearized equation. Instead we have

\[ \psi = \psi_{m}^J = J(1 - \cos \psi) + \sqrt{1 - j^2} \sin \psi = \delta J_{ij}/\lambda_c, \]

where \( \delta J_{ij} \) is the current flowing through the junction at \( y = 0 \) due to its coupling with all the other junctions. In order to derive \( \delta J_{ij} \), we use the Maxwell equations \( J_y = -(c/4\pi \lambda_{ab}) \partial H / \partial y, \delta J_x = (c/4\pi \lambda_c) \partial H / \partial x \), and the standard relation [11], \( \partial \psi(y = 0) / \partial x = \sqrt{2} \alpha \lambda_c [J_y(y = 0) - J_y(y = 0)] / \alpha \), between the phase difference and the in-plane current; \( H \) is the magnetic field. Following the approach described in Ref. [2] we obtain \( \delta J_{ij}/\lambda_c = \lambda_c^2 / \sqrt{2} \pi \exp(\gamma \sqrt{s \lambda_{ab} / 2}) \int_{-\infty}^{\infty} \frac{d\omega}{(2\pi)}(\omega - i\omega_\gamma) \exp(-i\omega \tau) \sum_{m=-\infty}^{\infty} k_m^2 \sin[q_{km}(\omega)] \alpha \sin[q_{km}(\omega)]. \]

Neglecting contributions to the tunneling process arising from higher frequencies, \( \omega \approx \omega_J(1 - j^2/4) \), and performing a reverse Fourier transform, \( \psi_{m} = (\lambda_c / 2d) \int \psi(x) \exp(-ik_m x) d\omega \), we derive

\[
\frac{\delta J_{ij}}{\lambda_c} = \frac{\lambda_c^2}{2d \lambda_c} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sqrt{k_m^2} \sin[q_{km}(\omega)] \sin[q_{km}(\omega)] \exp(-i k_m x) \cos k_m x (x - x').
\]

Next, we construct an effective potential \( V[\psi] \) choosing a proper trial function \( \psi \). The tunneling fluxon can be described as a smeared steplike function [see inset in Fig. 1(b)], which can be parameterized by the fluxon position \( x_0 \) and the height \( \psi \) of the step in \( \psi \). We assume that \( \psi \gg x_0 \). This quite natural assumption agrees well with recent numerical simulations [12]. Indeed, the fluxon starts penetrating at the sample edge where the induced current \( \delta J_{ij} \) suppresses the barrier. Staying in this position, the amplitude of \( \psi \) increases, overcoming the barrier. As soon as the barrier is overcome, the fluxon moves classically towards the other sample edge. Integrating \( \psi(x = 0) \) over \( x \), we derive for \( \psi \) the equation: \( d^2 \psi / dt^2 = -\partial V / \partial \psi \). Here the effective potential \( V[\psi] \) can be written as

\[
V[\psi] = \frac{1}{2} \left( \psi^2 - \psi^3 \right) - \frac{1}{2} \left( \psi^2 - \frac{1}{2} \psi^2 \right),
\]

where \( n_{max} = L / \lambda_{ab} / s \) labels the contact through which the fluxon tunnels and \( 2m_{max} + 1 < 2d/2\gamma s \). Harmonics with \( m > m_{max} \) oscillate fast on a scale of the fluxon core, of about \( \gamma s < d \), producing a small correction to the effective potential. Note that the vortex core size of about \( \gamma s \ll \lambda_j \) due to nonlocal effects [2]. For the samples used in the experiment [5] we find \( m_{max} = 1. \) Keeping only the first two harmonics and optimizing the form of the tunneling fluxon with respect to \( x_0 \) we derive

\[
g_n\psi = 20.2 Q \sinh[Qn] \sinh[Q(N - n)],
\]

and \( Q(j) = \pi \gamma s / 2d (1 - j^2)^{1/4} \). Note that this optimization reduces to finding the maximum of \( (1 - \cos k_{1} x_0) / k_{1} x_0 \). The obtained optimal position \( x_0 \) of the tunneling vortex, which is about 0.4d from the sample edge, is close to that found numerically [12] when optimizing the real shape of the fluxon.
Following a semiclassical approach [10] we calculate the effective action \( B = 2 \int_0^\infty \left[ 2V(q) \right]^{1/2} d\phi \), where \( V(q) = 0 \). This can be done either numerically (see green dashed-dotted curve in Fig. 1) or analytically. For an applied current \( J \) close to \( J_c \), we can expand both \( \cos \phi \) and \( \sin \phi \) and obtain \( V(\phi) = -\phi^2 (\phi - \phi_1)/6 \), where \( \phi_1(j) = 3\sqrt{1 - Jc}/2 \). Taking into account that the fluxon can tunnel through any junction of the stack, we derive an analytical expression for \( \Gamma(j) \) (in dimensional units)

\[
\frac{\Gamma(j)}{\Gamma_0(j)} = \sum_{n=0}^N (1 - g_n)^{5/4} \exp \left[ -\frac{36U_0}{5\omega_p} [1 - g_n^{5/2} - 1] \right],
\]

where the summation is taken over all \( N \) contacts. Here, the effective Josephson frequency is \( \omega_p(j) = \omega_j(1 - Jc^2)^{1/4} \), the height of the potential barrier \( U_0(j) = 2E_J(1 - Jc^2)^{3/2}/3 \), the Josephson energy \( E_J = \Phi_0/\pi c \), and the escape rate \( \Gamma_0(j) \) for a single Josephson junction (see, e.g., [5]) is given by \( \Gamma_0(j) = [216\omega_p(j)U_0(j)/\pi^2c]^2 \times \exp[-36U_0(j)/5\omega_p(j)] \). The red and blue solid lines in Fig. 1(a) show \( \Gamma(j) \), which describe well the experimental results in [5]. Some deviation between the experimental data and the theoretical prediction at high currents is due to a significant lowering of the effective potential barrier resulting in a decrease of the accuracy of the semiclassical approximation. The dependence of \( \Gamma \) on the number \( N \) of junctions is nonlinear, more complicated than \( N^2 \) dependence, due to the long-range interaction between different junctions, described by the last term in the expression (5) for the effective potential. This nonlinearity is strong for relatively small \( N \leq N_c = d/\gamma L \), and the escape rate \( \Gamma \) becomes proportional to \( N \) when the thickness \( L \) of the stack exceeds the effective interaction length \( d/\gamma \). However, for the parameters used in the experiment [5] the value \( \Gamma(j) \) obtained here nicely mimics the \( N^2 \) dependence measured in the experiment. The predicted strongly nonlinear dependence of the escape rate on \( N \) [see inset of Fig. 1(a)] could be an experimentally realizable test of our model.

The thermoactivated escape rate, \( \Gamma_T(j) = [\omega_p(j)/2\pi \sum n \exp(-E_j[V_n(\phi)])/kB T] \) with Boltzmann constant \( kB \), also increases due to the mutual interaction between junctions in the stack. In our analytical approach we have \( \max(V_n) = 2\psi_0^2(n)/81 \). The ratio, \( r(T) = \Gamma_T/\Gamma_0 \), between thermal and quantum escape rates is shown in Fig. 1(b) as a function of temperature for the single junction SJ1 and the stack US4 used in Ref. [5]. The thermoactivated escape starts to play a significant role \( (r(T) > 1) \) at \( T = 0.6 \) K for the stack and at \( T = 0.4 \) K for the single junction, in agreement with experiments [5].

Note that a more elaborated theory of MQT in layered superconductors should include the effects of intrinsic dissipation and interaction with the environment (shunting impedance). As it was shown in Ref. [8], the intrinsic dissipation renormalizes \( \Gamma \) by a factor of about 0.9 for the considered case of the c axis junctions. Thus, the main source of dissipation for stacks of intrinsic Josephson junctions is the shunting impedance, which can be ignored if the Josephson inductance, \( 2e^2/\hbar Jc \), is smaller than the inductance of the shunting circuit. It is evident that such a condition was satisfied for experiment [5] since the escape rate for the single-junction sample SJ1 is well described by \( \Gamma_0(j) \), where dissipation is ignored.

Very different types of MQT models in stacks of Josephson junctions, with no quantitative comparison with experimental data, are also studied in [7]. Here we consider the inductive coupling among layers, which is known to be strong, instead of the capacitive coupling among layers used in [7], which is known to be weak. Moreover, theory [7] considers a model for photon-assisted MQT tunneling instead of the current-biased tunneling observed in Ref. [5].

Conclusions.—We consider quantum excitations in stacks of junctions described by two Bosonic fields, one lighter \( a \) and the other heavier \( b \). We also derive the interaction of these quantum fields and predict resonances when either \( a + a \) or \( a + b \) pairs are produced. We suggest a semiclassical theory of the fluxon quantum tunneling in stacks of intrinsic Josephson junctions, which is in good agreement with recent remarkable experimental observations. The obtained results might be potentially useful for future designs of quantum devices.

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