# Generation and Control of Greenberger-Horne-Zeilinger Entanglement in Superconducting Circuits

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Going beyond the entanglement of microscopic objects (such as photons, spins, and ions), here we propose an efficient approach to produce and control the quantum entanglement of three *macroscopic* coupled superconducting qubits. By conditionally rotating, one by one, selected Josephson-charge qubits, we show that their Greenberger-Horne-Zeilinger (GHZ) entangled states can be deterministically generated. The existence of GHZ correlations between these qubits could be experimentally demonstrated by effective single-qubit operations followed by high-fidelity single-shot readouts. The possibility of using the prepared GHZ correlations to test the macroscopic conflict between the noncommutativity of quantum mechanics and the commutativity of classical physics is also discussed.

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*Introduction.*—Entanglement is one of the most essential features of quantum mechanics and has no analogue in classical physics. Mathematically, it means that the wave function of a system composed of many particles cannot be separated into independent wave functions, one for each particle. Physically, entangled particles can display remarkable and counterintuitive quantum effects. For example, a measurement made on one particle collapses the total wave function and thus instantaneously determines the states of the other particles, even if they are far apart.

The existence of entanglement has been experimentally demonstrated [1] with, e.g., two photons separated far apart (e.g., up to 500 m) and two *closely spaced* trapped ions (e.g., *separated a few micrometers apart*). The obvious violation of Bell's inequality in these two-qubit experiments *statistically* verifies the conflict between the locality of classical physics and the nonlocality of quantum mechanics. Only recently, the experimental study of entanglement has been successfully extended to a system composed of more than two qubits. For example, three-photon Greenberger-Horne-Zeilinger (GHZ) entangled states [2] have been demonstrated, and then used to test the conflict between classical local-realism and quantum nonlocality using *definite* predictions [3], rather than the *statistical* ones based on Bell's inequalities. Yet, besides the problem of detector efficiency, the expected GHZ state in optical experiments could not be deterministically prepared [2] because: (i) each entangled photon pair was generated in a small subset of all pairs created in certain spontaneous processes, and (ii) the nondeterministic detection of a trigger photon among two pairs of entangled photons was required.

Instead of fast-escaping photons, massive or macroscopic quantum systems [4] have also been extensively studied to realize controllable multipartite quantum entanglement. The three-qubit entanglement of *microscopic* Rydberg atoms [5] and trapped ions [6] was prepared experimentally. Moreover, the GHZ state of massive *macroscopic* "particles" has also been demonstrated in liquid NMR [7]. However, the existence of nonlocal correlations in these particles cannot be settled, as the correlated information between them will be completely mangled in their readouts of *ensemble averages*.

Superconducting qubits [8] provide an attractive platform to control the genuine (rather than ensemble-pseudo-pure) macroscopic quantum state. The sizes of the "particles" (e.g., Cooper-pairs boxes) and the distance between them, are typically on the order of microns. If the interbit couplings are *switchable*, then methods [2,5,6], working well in photon- and trapped-ion systems, could be applied [9] to generate and verify the GHZ entanglement between the Josephson qubits. However, in all published (so far) experiments the interactions between Josephson qubits [8] are *fixed* (either capacitively or inductively), and thus the usually required single-qubit gates cannot, in principle, be *strictly* implemented.

For the currently existing experimental circuits with *always-on coupling*, here we propose an effective approach

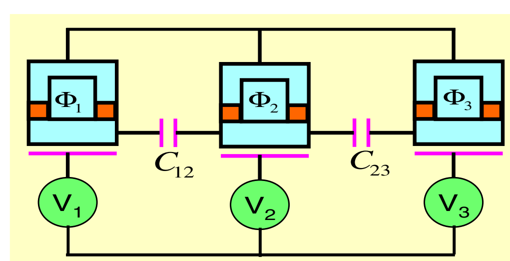


FIG. 1 (color online). Three capacitively coupled SQUID-based charge qubits. The quantum states of three Cooper-pair boxes (i.e., qubits) are manipulated by controlling the applied gate voltages $V_j (j = 1, 2, 3)$, and external magnetic fluxes $\Phi_j$ (penetrating the SQUID loops). The circuit can be generalized to include more qubits.

 

to deterministically generate three-qubit GHZ states by conditionally rotating the selected qubits one by one. The existence of the desirable GHZ entanglement is then reliably verified by using *effective* single-qubit operations. The prepared GHZ entanglement should allow to test quantum nonlocality by definite predictions at a macroscopic level.

*Preparation of GHZ states.*—We consider the three-qubit circuit sketched in Fig. 1, that is, only adding one qubit to the experimentally existing one [10]. Three superconducting-quantum-interference-device (SQUID) loops with controllable Josephson energies produce three Josephson qubits, fabricated a small distance apart [e.g., up to *a few micrometers* [10], as the case of entangled trapped ions in Ref. [6] ] and coupled via the capacitances $C_{12}$ and $C_{23}$. The dynamics of the system can be effectively restricted to the subspace spanned by the computational basis, and be thus described by the following simplified Hamiltonian

$$\hat{H} = \frac{1}{2}\sum_{j=1}^{3}[E_C^{(j)}\sigma_z^{(j)} - E_J^{(j)}\sigma_x^{(j)}] + \sum_{j=1}^{2} K_{j,j+1}\sigma_z^{(j)}\sigma_z^{(j+1)}. \quad (1)$$

Here, $E_C^{(j)} = 2e^2[\tilde{C}_{\Sigma_j}^{-1}(2n_{g_j} - 1) + \sum_{k\neq j}\tilde{C}_{j,k}^{-1}(2n_{g_k} - 1)]$ with $n_{g_j} = C_{g_j}V_j/(2e) \sim 0.5$, is the effective charging energy of the $j$th qubit, whose effective Josephson energy is $E_J^{(j)} = 2\varepsilon_J^{(j)}\cos(\pi\Phi_j/\Phi_0)$ with $\varepsilon_J^{(j)}$ the Josephson energy of the single-junction and $\Phi_0$ the flux quantum. The effective coupling energy between the $j$th qubit and the $(j+1)$th one is $K_{j,j+1} = e^2\tilde{C}_{j,j+1}^{-1}$. Above, $C_{\Sigma_j}$ is the sum of all capacitances connected to the $j$th box, and other effective capacitances are defined by $\tilde{C}_{\Sigma_1} = C_{\Sigma_1}/(1 + C_{12}^2 C_{\Sigma_3}/\tilde{C})$, $\tilde{C}_{\Sigma_2} = \tilde{C}/(C_{\Sigma_1}C_{\Sigma_3})$, $\tilde{C}_{\Sigma_3} = C_{\Sigma_3}/(1 + C_{23}^2 C_{\Sigma_1}/\tilde{C})$, $\tilde{C}_{12} = \tilde{C}/(C_{\Sigma_3}C_{12})$, $\tilde{C}_{23} = \tilde{C}/(C_{\Sigma_1}C_{23})$, $\tilde{C}_{13} = \tilde{C}/(C_{12}C_{23})$, with $\tilde{C} = \prod_{j=1}^{3} C_{\Sigma_j} - C_{12}^2 C_{\Sigma_3} - C_{23}^2 C_{\Sigma_1}$. The pesudospin operators are defined as $\sigma_z^{(j)} = |0_j\rangle\langle 0_j| - |1_j\rangle\langle 1_j|$ and $\sigma_x^{(j)} = |0_j\rangle\langle 1_j| + |1_j\rangle\langle 0_j|$. As the interbit-couplings are always on, the charge energy $E_C^{(j)}$ of the $j$th qubit depends not only on the gate voltage applied to the $j$th qubit, but also on those applied to the other two Cooper-pair boxes. Compared to the coupling $K_{j,j+1}$ between nearest-neighboring qubits, the interaction of two non-nearest-neighbor qubits (i.e., $K_{13} = e^2/\tilde{C}_{13}$ between the first and the third qubits), is very weak and thus has been safely neglected [11]. Indeed, for the typical experimental parameters: $C_J \sim 600$ aF, $C_m \sim 30$ aF, and $C_g = 0.6$ aF in Ref. [10], we have $K_{13}/K_{12} = K_{13}/K_{23} < C_m/C_J = 0.05$ and $K_{12}/2\varepsilon_J \sim 1/4$.

In principle, the coupled qubits cannot be individually manipulated, as the nearest-neighbor capacitive couplings $K_{j,j+1}$ are sufficiently strong. However, once the state of the circuit is known, it is still possible to design certain operations for only evolving the selected qubits and keeping the remaining ones unchanged. Our preparation begins with the ground state of the circuit $|\psi(0)\rangle = |000\rangle$, which can be easily initialized. The expected GHZ state could be produced by the following three-step pulse process [11]

$$\begin{aligned}|\psi(0)\rangle = |000\rangle &\xrightarrow{\hat{U}_2(t_2)} \frac{1}{\sqrt{2}}(|000\rangle \pm i|010\rangle) \xrightarrow{\hat{U}_1(t_1)} \frac{1}{\sqrt{2}}(|000\rangle \\ &\mp |110\rangle) \xrightarrow{\hat{U}_3(t_3)} \frac{1}{\sqrt{2}}(|000\rangle \pm i|111\rangle) \\ &= |\psi_{\rm GHZ}^{\pm}\rangle. \end{aligned} \quad (2)$$

The first evolution $\hat{U}_2(t_2)$, with $\sin[E_J^{(2)}t_2/(2\hbar)] = \pm 1/\sqrt{2}$, is used to superpose two logic states of the second qubit. This is achieved by simply using a pulse that switches on the Josephson energy $E_J^{(2)}$ and sets the charging energy $E_C^{(2)} = -2(K_{12} + K_{23})$. The second [or third] evolution $\hat{U}_1(t_1)$ [or $\hat{U}_3(t_3)$] is achieved by switching on the Josephson energy of the first [third] qubit and setting its charging energy as $E_C^{(1)} = 2K_{12}$ (or $E_C^{(3)} = 2K_{23}$). The corresponding duration is set to satisfy the conditions $\sin[E_J^{(j)}t_j/(2\hbar)] = 1$ and $\cos(\gamma_j t_j/\hbar) = 1$, with $\gamma_j = \sqrt{(2K_{j2})^2 + (E_J^{(j)}/2)^2}$, with $j = 1, 3$, in order to conditionally flip the $j$th qubit, that is, flip it if the second qubit is in the $|1\rangle$ state, and keep it unchanged if the second qubit is in the $|0\rangle$ state.

The fidelity of the GHZ state prepared above can be experimentally measured by quantum-state tomography [6,7,12]. However, it would be desirable to confirm the existence of a GHZ state without using tomographic measurements on a sufficient number of identically prepared copies. Optical experiments [2] have achieved this via single-shot readout and we propose a superconducting-qubit analog of this approach. The single-shot readout of a Josephson-charge qubit has been experimentally demonstrated [13] by using a single-electron transistor (SET) [14]. Before and after the readout, the SET is physically decoupled from the qubit. The GHZ state generated above implies that the three SETs, if they are individually coupled to each one of the three Cooper-pair boxes at the same time, will simultaneously either receive charge signals or receive no signal. The former case indicates that the circuit is in the state $|111\rangle$, while the latter one corresponds to the state $|000\rangle$. However, the existence of these two terms, $|111\rangle$ and $|000\rangle$, in these single-shot readouts, is just a necessary but not yet sufficient condition for demonstrating the GHZ entanglement. Indeed, a statistical mixture of those two states may also give the same measurement results. In order to confirm that the state (2), e.g., $|\psi_{\rm GHZ}^{+}\rangle$, is indeed in a coherent superposition of the states $|000\rangle$ and $|111\rangle$, we consider the following operational sequence

$$|\psi_{\rm GHZ}^{+}\rangle \xrightarrow{\tilde{U}_2} \frac{1}{2}(|000\rangle - |101\rangle + i|010\rangle + i|111\rangle) \xrightarrow{\hat{P}_2} \frac{1}{\sqrt{2}}(|0_1 0_3\rangle + |1_1 1_3\rangle) \xrightarrow{\tilde{U}_1 \otimes \tilde{U}_3} \frac{1}{\sqrt{2}}(|0_1 1_3\rangle + |1_1 0_3\rangle), \quad (3)$$

which is similar to the verification of the optical GHZ correlations [2]. Above, $\hat{P}_2 = |1_2\rangle\langle 1_2|$ is a projective measurement of the second qubit. The suffixes are introduced in the second and third steps to denote the order of the qubits. When we finally readout the first and third qubits at the same time, the simultaneous absence of the terms $|0_1 0_3\rangle$ and $|1_1 1_3\rangle$ due to destructive interference indicates the desired coherent superposition of the terms in the prepared GHZ state (2). The question now is how to realize the required single-qubit operations $\tilde{U}_j = \exp[i\pi\sigma_x^{(j)}/4]$, $j = 1, 2, 3$, keeping the remaining qubits unchanged, in this circuit with untunable interbit interactions (like the currently available experimental ones).

In order to effectively implement the single-qubit rotation $\tilde{U}_2$ performed only on the second qubit, while keeping the first and third qubits unchanged, we let the circuit evolve under the Hamiltonian $\hat{H}_2 = -\varepsilon_J^{(2)}\sigma_x^{(2)} + K_{12}\sigma_z^{(1)}\sigma_z^{(2)} + K_{23}\sigma_z^{(2)}\sigma_z^{(3)}$, by only switching on the Josephson energy of the second qubit, e.g., $E_J^{(2)} = 2\varepsilon_J^{(2)}$. Since $\zeta_{12} = K_{12}/(2\varepsilon_J^{(2)}) < 1$, $\zeta_{23} = K_{23}/(2\varepsilon_J^{(2)}) < 1$ [e.g., $\lesssim 1/4$ for the typical experimental parameters [10] ], we can treat the second and third terms in $\hat{H}_2$ as perturbations of the first one there. Indeed, neglecting quantities smaller than the second-order perturbations [15], the Hamiltonian $\hat{H}_2$ can be effectively approximated to [16]

$$\hat{H}_{\text{eff}}^{(2)} = -\varepsilon_J^{(2)}[1 + 2\zeta_{12}^2 + 2\zeta_{23}^2 + 4\zeta_{12}\zeta_{23}\sigma_z^{(1)}\sigma_z^{(3)}]\sigma_x^{(2)}. \quad (4)$$

In the state (2) the logic states of the first and third qubits are always identical. Thus, by setting the corresponding duration $\tau_2$ as $\tau_2 = \hbar\pi/4\varepsilon_J^{(2)}[1 + 2\zeta_{12}^2 + 2\zeta_{23}^2 + 4\zeta_{12}\zeta_{23}]$, the required single-qubit operation $\tilde{U}_2 = \exp(-i\hat{H}_{\text{eff}}^{(2)}\tau_2/\hbar) = \exp(i\pi\sigma_x^{(2)}/4)$ could be effectively performed on the second qubit in state (2). Similarly, the Hamiltonian $\hat{H}_{13} = \sum_{j=1,3}\{-\varepsilon_J^{(j)}\sigma_x^{(j)} + K_{j2}\sigma_z^{(j)}\sigma_z^{(2)}\}$, induced by simultaneously switching on the Josephson energies of the first and third qubits, can be effectively approximated to

$$\hat{H}_{\text{eff}}^{(13)} = -\sum_{j=1,3}\varepsilon_J^{(j)}[1 + 2\zeta_{j2}^2\sigma_z^{(2)}]\sigma_x^{(j)}, \quad (5)$$

by neglecting the higher-order terms of $\zeta_{j2} = K_{j2}/(2\varepsilon_J^{(j)}) < 1$, with $j = 1, 3$. The shifts of Josephson energies $\Delta\tilde{E}_J^{(j)} = 4\varepsilon_J^{(j)}\zeta_{j2}^2\sigma_z^{(2)}$ depend on the state of the second Cooper-pair box, which collapsed into the state $|0\rangle$ after the projective measurement $\hat{P}_2 = |1_2\rangle\langle 1_2|$ (because such a measurement tunnels the existing excess Cooper pairs into the connected SET). Thus, the effective Hamiltonian $\hat{H}_{\text{eff}}^{(13)}$ yields the evolution $\hat{U}_{13}(\tau_{13}) = \exp(-i\hat{H}_{\text{eff}}^{(13)}\tau_{13}/\hbar) = \prod_{j=1,3}\exp\{i\tau_{13}[\varepsilon_J^{(j)}(1 + 2\zeta_{j2}^2)]\sigma_x^{(j)}/\hbar\}$. Obviously, if the duration $\tau_{13}$ satisfies the condition $\tau_{13}[\varepsilon_J^{(j)}(1 + 2\zeta_{j2}^2)]/\hbar = \pi/4$, then the required single-qubit operations $\tilde{U}_j = \exp[i\pi\sigma_x^{(j)}/4]$, $j = 1, 3$, could be simultaneously implemented.

*Possible application.*—The prepared GHZ state, e.g., $|\psi_{\text{GHZ}}^+\rangle$, should allow, at least in principle, to test the *macroscopic* conflict between the noncommutativity of quantum mechanics and the commutativity of classical physics by definite predictions [3]. Using the EPR's reality criterion, each observable corresponds to an "element of reality" (even if it is not measured). That is, the quantum operators $\sigma_\alpha^{(j)}$, ($\alpha = x, y, z; j = 1, 2, 3$) are linked to the classical numbers $m_\alpha^{(j)}$, which have the value $+1$ or $-1$. The so-called $\sigma_\alpha^{(j)}$ measurement is the projection of the quantum state into one of the eigenstates of $\sigma_\alpha^{(j)}$. The prepared GHZ state is the eigenstate of the three operators: $A_{yxx} = \sigma_y^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}$, $A_{xyx} = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}$, and $A_{xxy} = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}$, with a common eigenvalue $+1$. Thus, *classical* reality implies that $1 = (m_y^{(1)}m_x^{(2)}m_x^{(3)})(m_x^{(1)}m_y^{(2)}m_x^{(3)}) \times (m_x^{(1)}m_x^{(2)}m_y^{(3)}) = m_y^{(1)}m_y^{(2)}m_y^{(3)}$. The second formula indicates that, if we perform the $\prod_{j=1}^3\sigma_y^{(j)}$ measurement (i.e., *yyy* experiment) on the state $|\psi_{\text{GHZ}}^+\rangle$, the eigenstate $|\tilde{-}\rangle$ only shows in pairs. Here, $|\tilde{+}\rangle$ (or $|\tilde{-}\rangle$) denotes the eigenstate of the operator $\sigma_y$ with eigenvalue $+1$ (or $-1$) and corresponds to the classical number $m_y = +1$ (or $-1$). While, for this *yyy* experiment *quantum* mechanics predicts that the state $|\tilde{-}\rangle$ never shows simultaneously in pairs, because the prepared GHZ state can be rewritten as $|\psi_{\text{GHZ}}^+\rangle = (|\tilde{+}\,\tilde{+}\,\tilde{-}\rangle + |\tilde{+}\,\tilde{-}\,\tilde{+}\rangle + |\tilde{-}\,\tilde{+}\,\tilde{+}\rangle + |\tilde{-}\,\tilde{-}\,\tilde{-}\rangle)/2$. Obviously, this contradiction comes from the fact that the observable $\sigma_x^{(j)}$ anticommutes with the observable $\sigma_y^{(j)}$ and the operator identity $(\sigma_y^{(1)}\sigma_x^{(2)}\sigma_x^{(3)})(\sigma_x^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}) \times (\sigma_x^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}) = -\sigma_y^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}$, which is "opposite" to its classical counterpart.

The protocol described above could be directly [e.g., for the optical system [2] ] performed by reading out the eigenstates of the operators $\sigma_x$ and $\sigma_y$, respectively. However, in the present solid-state qubit, the eigenstates of $\sigma_z$ are usually read out. Thus, additional operations, e.g., the Hadamard transformation $\hat{S}_x = (\sigma_z + \sigma_x)/\sqrt{2}$, and the unitary transformation $\hat{S}_y = [(1+i)\hat{I} + (1-i)\sum_\alpha\sigma_\alpha]/(2\sqrt{2})$, are required to transform the eigenstates of $\sigma_x$ and $\sigma_y$ to those of $\sigma_z$, respectively. These additional - single-qubit operations could be implemented by combining the rotations of the selected qubit along the $x$ axis (by using the effective Hamiltonian proposed above) and those along the $z$ axis [by effectively refocusing the fixed-interactions [15] ].

*Conclusion and discussions.*—The experimental realization of our proposal for producing and testing GHZ correlations is possible, although it may also face various technological challenges, like other theoretical designs [17] for quantum engineering. Of course, the fabrication of the proposed circuit is not difficult, as it only adds one qubit to experimentally existing superconducting nanocircuits [10]. Moreover, rapidly switching on/off the

Josephson energy, to realize the fast quantum manipulations, is experimentally possible. In fact, assuming a SQUID loop size of 10 $(\mu\text{m})^2$, changing the flux by about a half of a flux quantum in $10^{-10}$ s, requires sweeping the magnetic field at a rate of about $10^5$ T/s, almost reachable by current techniques [18].

Also, the prepared GHZ states are the eigenstates of the idle circuit (i.e., no operations on it) without any charge and Josephson energies (by setting the controllable parameters as $\Phi_j = \Phi_0/2$ and $n_{g_j} = 0.5$) and thus are relatively long lived, at least theoretically. Indeed, the couplings $\hat{V} = \sum_{j=1}^{3} \sigma_z^{(j)} (\sum_{k=1}^{3} \lambda_{jk} X_k)$ between the relevant baths and the circuit commute with the nonfluctuating Hamiltonian of the idle circuit $\hat{H}_0 = \sum_{j=1,2} K_{j,j+1} \sigma_z^{(j)} \sigma_z^{(j+1)}$. Here, $\lambda_{jk}$ equals to either 1 for $j = k$ or $\tilde{C}_{\Sigma_k} \tilde{C}_{jk}$ for $j \neq k$, and $X_k = (eC_{g_k}/\tilde{C}_{\Sigma_k}) \sum_{\omega_k} (g^*_{\omega_k} \hat{a}^\dagger_{\omega_k} + g_{\omega_k} \hat{a}_{\omega_k})$ with $\hat{a}_{\omega_k}$, $\hat{a}^\dagger_{w_k}$ being the Boson operators of the $k$th bath, and $g_{\omega_k}$ the coupling strength between the oscillator of frequency $\omega_k$ and the nondissipative system. Thus, only pure dephasing, i.e., the zero frequency value of the noise spectrum contributes to overall decoherence rates [19]. However, the working frequency of the present circuit is always nonzero. This implies that the lifetimes of the prepared GHZ correlations are still sufficiently long, and thus various required quantum manipulations could still be coherently implemented.

Perhaps the biggest challenge comes from the fast single-shot readouts [13,20] of multiqubits at the same time. This is a common required task of almost all quantum algorithms and important for physical realizations of quantum computing. In order to avoid the cross talk between qubits during the readouts, the readout time $t_m$ should be ''much'' shorter than the characteristic time $t_c \sim \hbar/K_{j,j+1}$ of communications. This requirement has been achieved by the existing phase-qubit circuits [21]: $t_m \sim 1$ ns, and $t_c \sim 4$ ns for the demonstrated coupling energy $K \sim$ 80 MHz. For the existing charge-qubit circuits [10], where the interbit coupling-energy $K \sim 3$ GHz yields $t_c \sim$ 100 ps, the duration of the single-shot readout pulse should be not longer than several tens of picosecond. Thus, the weaker interbit coupling, e.g., lowered to hundreds of KHz, is required for the current SET technique, whose response time is usually hundreds of nanoseconds [13,14].

In summary, based on conditionally manipulating the selected qubits, we have shown how to engineer the macroscopic quantum entanglement of Josephson qubits with fixed couplings. Our proposal allows us to deterministically prepare three-qubit GHZ entangled states and allows a macroscopic test of the contradiction between the noncommutativity of quantum mechanics and the commutativity of classical physics.

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