

Optical Selection Rules and Phase-Dependent Adiabatic State Control in a Superconducting Quantum Circuit

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We analyze the optical selection rules of the microwave-assisted transitions in a flux qubit superconducting quantum circuit (SQC). We show that the parities of the states relevant to the superconducting phase in the SQC are well defined when the external magnetic flux $\Phi_e = \Phi_0/2$; then the selection rules are the same as the ones for the electric-dipole transitions in usual atoms. When $\Phi_e \neq \Phi_0/2$, the symmetry of the potential of the artificial “atom” is broken, a so-called Δ -type “cyclic” three-level atom is formed, where one- and two-photon processes can coexist. We study how the population of these three states can be selectively transferred by adiabatically controlling the electromagnetic field pulses. Different from Λ -type atoms, the adiabatic population transfer in our three-level Δ atom can be controlled not only by the amplitudes but also by the phases of the pulses.

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Introduction.—Analogous to natural atoms, superconducting quantum circuits (SQCs) can possess discrete levels. Such artificial atoms provide promising “hardware” for quantum information processing. Microchip electric circuits [1,2] show that quantum optical effects can also appear in artificial atoms, allowing quantum information processing in such circuits by using cavity quantum electrodynamics (e.g., [1–5]).

Quantum optical technology, developed for atomic systems, can be used to manipulate quantum states of artificial atoms. For example, the selective population transfer [6] based on the stimulated Raman adiabatic passage (STIRAP) with Λ -type atoms [7,8] has been applied to superconducting flux qubits [4,9]. How to probe the decoherence of flux qubits by using electromagnetically induced transparency has also been investigated in a SQC [10] formed by a loop, with three Josephson junctions.

We investigate a generalized STIRAP approach for the novel type of artificial atoms presented here. It is well known that the parities of eigenstates are well defined for usual atomic systems. Because of their atomic symmetry, described by $SO(3)$ or $SO(4)$, one-photon transitions between two energy levels require that the two corresponding eigenstates have opposite parities, but a two-photon process needs these states to have the same parities. However, this situation can be significantly changed for artificial atoms due to its easily controllable (by the external magnetic flux Φ_e) effective potential.

Here, we focus on the flux qubit circuit [11,12], analyzing its parity. When $\Phi_e = \Phi_0/2$, the qubit potential energy of the superconducting phases is symmetric, and the interaction Hamiltonian between the time-dependent microwave field and the qubit also has a well-defined parity. In this case, the optical selection rules of the microwave-

assisted transitions between different qubit states are the same as for the electric-dipole ones in usual atoms: one- and two-photon transitions cannot coexist. However, if $\Phi_e \neq \Phi_0/2$, the symmetries of both the potential and the interaction Hamiltonian are broken. Then the selection rules do not hold, and an unusual phenomenon appears: one- and two-photon processes can coexist [13,14]. In this case, all transitions between any two states are possible. Then the population can be cyclically transferred with the assistance of (the amplitudes and/or phases of) microwave pulses. Thus, we achieve a pulse-phase-sensitive adiabatic manipulation of quantum states in this three-level artificial atom. Usually, only the amplitude was considered for adiabatic control.

Broken symmetry of the superconducting phase and selection rules.—We consider a qubit circuit composed of a superconducting loop with three Josephson junctions (e.g., [11,15]). The two larger ones have equal Josephson energies $E_{J1} = E_{J2} = E_J$ and capacitances $C_{J1} = C_{J2} = C_J$, while for the third junction $E_{J3} = \alpha E_J$ and $C_{J3} = \alpha C_J$, with $\alpha < 1$. The Hamiltonian is

$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m),$$

with $M_p = 2C_J(\Phi_0/2\pi)^2$ and $M_m = M_p(1 + 2\alpha)$. The effective potential $U(\varphi_p, \varphi_m)$ is $U(\varphi_p, \varphi_m) = 2E_J(1 - \cos\varphi_p \cos\varphi_m) + \alpha E_J[1 - \cos(2\pi f + 2\varphi_m)]$, where $\varphi_p = (\varphi_1 + \varphi_2)/2$ and $\varphi_m = (\varphi_1 - \varphi_2)/2$ are defined by the phase drops φ_1 and φ_2 across the two larger junctions; $f = \Phi_e/\Phi_0$ is the reduced magnetic flux.

Figure 1(a) summarizes numerical results of the f -dependent spectrum for H_0 up to the sixth eigenvalue, with $\alpha = 0.8$ and $E_J = 40E_c$ (as in Ref. [11]). Figure 1(a)

shows that near the point $f = 0.5$, e.g., $f = 0.496$, the lowest three energy levels are well separated from other higher energy levels. Then the lowest two energy levels form a two-level artificial atom, called a qubit, with an auxiliary third energy level. Figure 1(b) plots f -dependent ratios $D_{ij} = (\varepsilon_3 - \varepsilon_2)/(\varepsilon_i - \varepsilon_j)$ of the transition frequency between the fourth and third energy levels with the other three among the lowest three energy levels ($j < i < 3$). It is found that $D_{ij} \neq 1$, so the transition frequencies of different eigenstates are not equal when f is near 0.5. Then when we manipulate the lowest three states, the fourth state will be well separated and not be populated.

When a time-dependent microwave electromagnetic field $\Phi_a(t)$ is applied through the loop, photon-assisted transitions occur. For small $\Phi_a(t)$, the φ_m -dependent perturbation Hamiltonian reads $H_1(\varphi_m, t) = I\Phi_a(t) = I\Phi_a^{(0)} \cos(\omega_{ij}t)$, where $I = -(2\pi\alpha E_J/\Phi_0) \sin(2\pi f + 2\varphi_m)$ is the circulating supercurrent when $\Phi_a(t) = 0$, and the amplitude $\Phi_a^{(0)}$ is now assumed to be time independent. The transitions are determined by the matrix elements $t_{ij} = \langle i|I\Phi_a(t)|j\rangle$ for the i th and j th eigenstates $|i\rangle$ and $|j\rangle$, with eigenvalues ε_i and ε_j , respectively.

Analytically, the potential $U(\varphi_p, \varphi_m)$ is an even function of φ_p and φ_m when $2f$ is an integer; however, $H_1(\varphi_m, t)$ is

an odd function of φ_m . Thus, at these specific points, the *parities* of $H_1(\varphi_m, t)$ and the eigenstates of H_0 are well defined. Then, the selection rules for the transition matrix elements t_{ij} at these points have the same behavior as the ones for the *electric-dipole transitions* in atoms due to the *odd parity* of $H_1(\varphi_m, t)$. However, if f deviates from these points, the symmetries are broken and thus the dipole selection rules do not hold. Figure 1(c) shows the f -dependent transition elements $|t_{ij}|$ for resonant microwave frequencies between any two of the lowest three states: $|0\rangle$, $|1\rangle$, and $|2\rangle$. It shows that the transition $|0\rangle \leftrightarrow |2\rangle$ is forbidden at $f = 0.5$, but the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|0\rangle \leftrightarrow |1\rangle$ reach their maxima at this point. Then these three states have ladder-type or Ξ -type transitions [6,7], as shown in Fig. 1(d). The states $|0\rangle$ and $|2\rangle$ have the same parities when $f = 0.5$. Figure 1(c) also demonstrates that *all* photon-assisted transitions are possible when $f \neq 0.5$, as in Fig. 1(e), showing what we call Δ -type (or triangle-shaped) transitions. So a Λ -type “atom,” allowing transitions $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ but prohibiting $|0\rangle \leftrightarrow |1\rangle$, cannot be realized in this circuit.

Adiabatic energy levels of Δ -artificial atoms.—In ladder-type transitions [6,7], the population of the lowest state $|0\rangle$ can be adiabatically transferred to the highest state $|2\rangle$ by applying two appropriate classical pulses. Λ -type transitions are usually required [6–8] to adiabatically manipulate the populations of two states $|0\rangle$ and $|1\rangle$ by a third (auxiliary) state $|2\rangle$. In contrast with the usual Λ -type model [10], in our case, when $f \neq 0.5$, the transitions among the lowest three states *can be cyclic*.

Now we consider three electromagnetic pulses applied through the loop. We assume $f \neq 0.5$, but near it. The time-dependent flux is $\Phi_a(t) = \sum_{m>n=0}^2 [\Phi_{mn}(t)e^{-i\omega_{mn}t} + \Phi_{mn}^*(t)e^{i\omega_{mn}t}]$, and the $\Phi_{mn}(t)$ vary slowly on the time-scale of the pulses, where ω_{mn} are the pulse carrier frequencies. If ω_{mn} is resonant or near resonant with the transitions among the nonadiabatic (i.e., diabatic) states $|m\rangle$ ($m = 0, 1, 2$), the total Hamiltonian in the interaction picture can be written under the rotating wave approximation as $H_{\text{int}} = \sum_{m>n=0}^2 [\Omega_{mn}(t)e^{i\Delta_{mn}t}|m\rangle\langle n| + \text{H.c.}]$, where the complex Rabi frequencies $\Omega_{mn}(t) = \langle m|I\Phi_{mn}(t)|n\rangle$ and the detuning $\Delta_{mn} = \omega_m - \omega_n - \omega_{mn}$, with $\omega_m = \varepsilon_m/\hbar$.

The instantaneous adiabatic eigenvalues of H_{int} are given by $E_k = 2|\Omega(t)|\sqrt{1/3}\cos\{[\theta + 2(k-1)\pi]/3\}$ ($k = 1, 2, 3$). Here, $\cos\theta = 3\sqrt{3}\text{Re}[\Omega_{01}(t)\Omega_{12}(t)\Omega_{20}(t)e^{i\omega't}]/|\Omega(t)|^3$, $|\Omega(t)|^2 = |\Omega_{12}(t)|^2 + |\Omega_{20}(t)|^2 + |\Omega_{01}(t)|^2$, $\omega' = \Delta_{01} + \Delta_{12} - \Delta_{02}$. The eigenvalues are sensitive to the total phase ϕ of the product $\Omega_{01}(t)\Omega_{12}(t)\Omega_{20}(t) = \Omega'(t)$ and detuning ω' . It can be found that $\theta = 0$ or π when $|\Omega_{01}(t)| = |\Omega_{12}(t)| = |\Omega_{20}(t)|$ and $\beta = \omega't + \phi = n\pi$. In such case, there are energy level crossings and, thus, the adiabatic description of the time evolution is no longer correct. Comparing with the typical Λ -type atom [6], the time-evolved zero eigenvalue $E_3 = 0$ [corresponding to $E_1 = |\Omega(t)|$, $E_2 = -|\Omega(t)|$] can also be found for H_{int}

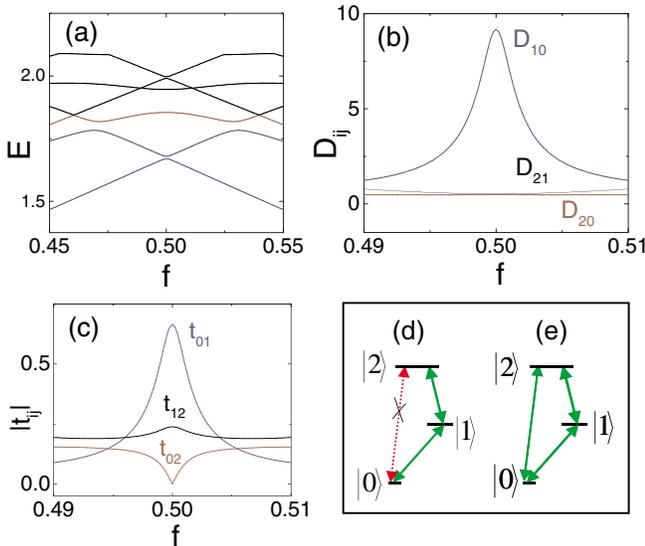


FIG. 1 (color online). (a) Energy levels, in units of E_J , of the flux qubit vs reduced flux f for states $|0\rangle$ to $|5\rangle$. (b) f -dependent ratios of transition frequencies: $D_{ij} = (\varepsilon_3 - \varepsilon_2)/(\varepsilon_i - \varepsilon_j)$ with $j < i < 3$. (c) Moduli $|t_{ij}|$ of the transition matrix elements between states $|i\rangle$ and $|j\rangle$ vs f , for the lowest three levels. Transition diagram of three energy levels vs f : (d) corresponds to $f = 0.5$ and (e) to $f \neq 0.5$, where the green solid (red dashed) line means allowed (forbidden) transitions. (e) A triangle-shaped or Δ -type energy diagram for $f \neq 0.5$, where all photon-assisted transitions are possible and the electric-dipole selection rules do not hold.

when $\beta = (2p + 1)\pi/2$, for integer p . In such case, the evolution is adiabatic.

Let us now consider Rabi frequencies $\Omega_{mn}(t)$ as Gaussian envelopes, e.g., $\Omega_{21}(t) = \Omega_0 \exp[i\phi_2 - (t - \tau_1)^2/\tau^2]$, $\Omega_{10}(t) = 0.9\Omega_0 \exp[i\phi_1 - (t - \tau_2)^2/\tau^2]$, and $\Omega_{20}(t) = 0.85\Omega_0 \exp[i\phi_3 - (t - \tau_3)^2/\tau^2]$, where τ is the pulse width, ϕ_i is the pulse phase, and $\Omega_0 > 0$. To avoid energy crossings, the pulse central times τ_i ($i = 1, 2, 3$) are chosen such that $|\Omega_{12}| \neq |\Omega_{01}| \neq |\Omega_{02}|$ during the time evolution. For the case $\phi = 0$ and $\omega' = 0$, Fig. 2(a) shows the dependence of the eigenvalues E_k on the pulse central times τ_i . If τ_i 's are equal or nearly equal for two pulses [e.g., $|\Omega_{01}(t)|$ and $|\Omega_{02}(t)|$], then two eigenvalues (e.g., E_2 and E_3) are closer to each other when the overlap region between two pulses is large. Generally, the conditions $\phi = 0$ and $\omega' = 0$ are not always satisfied. Combining our expressions for ϕ and ω' , Fig. 2(b) plots a few snapshots of the eigenvalues $E_k(t)$ vs the phase β of $\Omega'(t)$, for the given pulse central times $\tau_1 = \tau_2/2 = \tau_3/2 = 2\tau$. Figure 2(b) shows that two of the eigenvalues E_k are closer to each other for three points ($\phi = -\pi, 0, \pi$) in the range $|\phi| \leq \pi$, when $t = 3\tau$ and $\omega' = 0$. If $\omega' \neq 0$ and $\phi \neq 0$, the phase $\beta = \omega't + \phi$ always changes in time. Two eigenvalues are close to each other when $\beta \cong n\pi$. If the pulses related to Ω_{mn} do not have significant overlap (e.g., when $t = 3.5\tau$), then the three energy levels are well separated. If the maximum amplitudes of two pulses among $|\Omega_{01}(t)|$, $|\Omega_{12}(t)|$, and $|\Omega_{02}(t)|$ are the same, then the central times for these two pulses should be different to avoid the energy level crossings in the significant overlap area of these three pulses. However, if two central times, among the three pulses, are the same, then their maximum

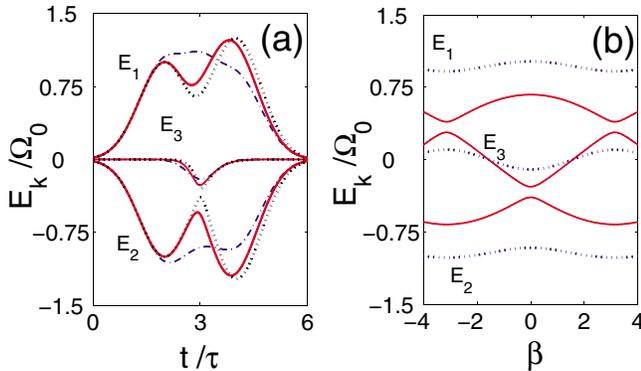


FIG. 2 (color online). (a) The rescaled time t/τ dependence of the eigenvalues E_k of H_{int} with the detuning $\omega' = 0$ for different pulse central times: $\tau_1 = 2\tau_2/3 = \tau_3/2 = 2\tau$ (dash-dotted blue curves); $\tau_1 = \tau_3/2 = 2\tau$ and $\tau_2 = 3.7\tau$ (solid red curves), and $\tau_1 = \tau_2/2 = \tau_3/2 = 2\tau$ (dotted black curves). (b) The phase-dependent eigenvalues E_k at $t = 3\tau$ (solid red curves) and $t = 3.5\tau$ (dotted blue curves), with time delays $\tau_1 = \tau_2/2 = \tau_3/2 = 2\tau$. Here eigenvalues are given in order E_1, E_3 , and E_2 from the top curve to the bottom curve for the same color.

amplitudes should be different in order to guarantee an adiabatic evolution.

Adiabatic control of quantum states.—The population transfer from one state to another is implemented via an adiabatic evolution, where the system remains in the k th eigenstate $|E_k\rangle = N_k^{-1}(t)[a_{k,2}(t)|2\rangle + a_{k,1}(t)|1\rangle + |0\rangle]$ of the instantaneous Hamiltonian H_{int} , with $N_k^2(t) = |a_{k,2}(t)|^2 + |a_{k,1}(t)|^2 + 1$. The relative population amplitudes $a_{k,2}(t) = [e^{i(\Delta_{02}t - i\phi_3)}/b_k(t)][|\Omega_{10}(t)\Omega_{21}(t)|e^{i\beta} + E_k|\Omega_{20}(t)|]$ and $a_{k,1}(t) = [e^{i(\Delta_{12}t - i\phi_2)}/b_k(t)] \times [|\Omega_{10}(t)\Omega_{20}(t)|e^{-i\beta} + E_k|\Omega_{21}(t)|]$ are determined by the applied pulses. Here $b_k(t) = E_k^2(t) - |\Omega_{10}(t)|^2$.

The control of the quantum state can be realized by choosing the classical pulses such that the diabatic components in the adiabatic basis states $|E_k\rangle$ can be changed during the adiabatic evolution. For the pulses with the same central times as in Fig. 2, but arbitrary ω' and ϕ , we find that the probabilities $P_{k,m}$ of the diabatic components $|m\rangle$ (in order $m = 0, 1, 2$) for the eigenstates $|E_1\rangle$ and $|E_2\rangle$ are $(|0\rangle + |1\rangle)/\sqrt{2}$ before the overlap central time area, but they evolve to $(|1\rangle + |2\rangle)/\sqrt{2}$ after the overlap central time area. However, it evolves from $|0\rangle$ to $|1\rangle$ for $|E_3\rangle$, which is more desirable since it coincides with the diabatic (bare) ground state in the past. As an illustration, Fig. 3(a) shows the pulse-phase and time-dependent probabilities $P_{3,m}(t, \phi)$ for the eigenstate $|E_3\rangle$ with $\omega' = 0$ and the pulses given in Fig. 2 with central times $\tau_1 = 2\tau_2/3 = \tau_3/2 = 2\tau$. It clearly shows that the bare ground state $|0\rangle$ coincides with the adiabatic basis state $|E_3\rangle$ when $t \rightarrow 0$. Thus, if the system evolves adiabatically, the ground state $|0\rangle$ can evolve to a superposition of three states $|m\rangle$ at $t = 3\tau$, then gradually to a superposition $\alpha|1\rangle + \beta|2\rangle$ of the two upper states, and finally to the first excited state $|1\rangle$. Generally, the weights of the superpositions discussed above depend on ϕ , ω' , and the pulse magnitudes $\Omega_{01}(t)$, $\Omega_{12}(t)$, $\Omega_{20}(t)$. Now let us discuss how the total phase ϕ affects the adiabatic evolution. The adiabatic evolution requires that the nonadiabatic coupling $\langle E_k | \frac{d}{dt} E_l \rangle$ and adiabatic energy differences $|E_k - E_l|$ satisfy [16] the condi-

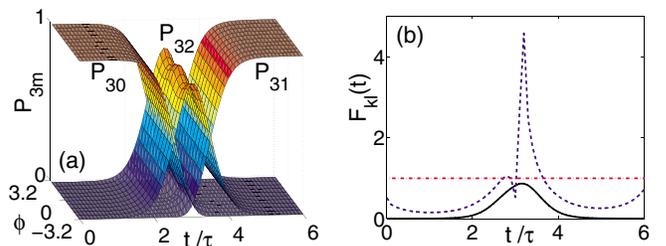


FIG. 3 (color online). (a) The rescaled time t/τ and (phase) ϕ -dependent probabilities $P_{3,m}$ of the diabatic components in the third adiabatic eigenstate $|E_3\rangle$. (b) The representative top curves of $F_{kl}(t)$ are plotted for two phases $\phi = \pi$ (dashed blue curve) and $\phi = \pi/2$ (solid black curve) with the pulses given in Fig. 2 for central times $\tau_1 = 2\tau_2/3 = \tau_3/2 = 2\tau$; here, the detunings are taken as $\Delta_{mn} = 0$. The dash-dotted red line denotes $F_{kl}(t) = 1$.

tion $F_{kl} = |\langle E_k | \frac{d}{dt} E_l \rangle / E_k - E_l| \ll 1$. For given pulses, it implies that the adiabatic condition depends not only on the total detuning ω' and individual detunings Δ_{mn} , but also on the total phase ϕ . In Fig. 3(b), the time evolutions of F_{kl} are illustrated by showing only one representative top curve for the central times ($\tau_1 = 2\tau_2/3 = \tau_3/2 = 2\tau$) in the resonant case (e.g., $\Delta_{mn} = 0$), for two special phases $\phi = \pi$ and $\phi = \pi/2$. Figure 3(b) shows that the adiabatic condition $F_{kl} \ll 1$ is valid when $\phi = \pi/2$ but it is invalid when $\phi = \pi$, for fixed envelopes of the Rabi frequencies. By comparing with Fig. 2, it is also found that the adiabatic evolution is invalid even when there are no energy levels crossing in the overlap area of the pulses, since the non-adiabatic coupling is very large in this area when $\phi = \pi$.

Discussions and conclusions.—We analyzed selection rules of the artificial atom formed by a SQC, and our analytical results were numerically confirmed. We find that when the reduced external magnetic flux $2f$ is an integer, the potential of the artificial atom has a well-defined symmetry of the superconducting phases φ_p and φ_m , while the interaction Hamiltonian has *odd parity*. Therefore the microwave-assisted transitions in this artificial atom are the same as electric-dipole ones. However, when $2f$ is not an integer, the symmetry of the potential is broken and the parity of the interaction Hamiltonian is also not well defined. In this case, transitions between any two levels are possible.

Based on the analysis of the selection rules, we discuss the microwave-assisted adiabatic population transfer among the lowest three energy levels when $2f$ is not an integer. In this case, the population of the three levels can be transferred cyclically, and a triangular Δ configuration is formed. Different from Λ atoms [6,7], the energies of the adiabatic states in this Δ atom are sensitive not only to the amplitude but also the total phase ϕ of the pulses and detuning ω' between different microwave fields and atomic transition frequencies. The adiabatic condition is strongly affected by the pulse phase ϕ when fixing other pulse parameters. This pulse-phase-sensitive transition is due to a broken symmetry, in which one and two-photon processes can coexist. By adjusting the pulse phases, central times and intensities, as well as the detunings, the populations of the artificial atom can be adiabatically controlled. Therefore, desired or target quantum states can be prepared using this controllable pulse manipulation.

Finally, we emphasize the following: (i) the adiabatic manipulation can be completed in about $0.36 \mu\text{s}$, if the pulse width is taken [4] as, e.g., $\tau = 60 \text{ ns}$; (ii) this artificial atom can be used to demonstrate three-level masers [7]; (iii) it can be regarded as a natural candidate to realize the quantum heat engine proposed in Ref. [17]; (iv) a superposition of two upper energy levels can be adiabatically prepared from the ground state, then a quantum beat experiment should be accessible in this microchip electric

circuit; (v) when one of the Rabi frequencies, e.g., $\Omega_{01}(t)$, $\Omega_{12}(t)$, or $\Omega_{02}(t)$, and the environmental effects are negligible, the Δ atom can be reduced to either a Λ -, V -, or Ξ -type atom [7]; (vi) the interaction between the quantized microwave field and this Δ atom can generate quasi- and nonclassical photon states [18]; (vii) since the total phase of the pulses plays an important role in the state control of Δ -type atoms, this controllable pulse phase might be used to suppress decoherence. In summary, the artificial Δ atoms introduced here provide many exciting future opportunities for quantum state control.

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- [1] J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003); Physica (Amsterdam) **18E**, 33 (2003); J. Q. You, J. S. Tsai, and F. Nori, Phys. Rev. B **68**, 024510 (2003).
 - [2] A. Wallraff *et al.*, Nature (London) **431**, 162 (2004); I. Chiorescu *et al.*, *ibid.* **431**, 159 (2004).
 - [3] A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004).
 - [4] C. P. Yang, S. I. Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003); Phys. Rev. Lett. **92**, 117902 (2004).
 - [5] A. M. Zagoskin *et al.*, Phys. Rev. A **70**, 060301(R) (2004).
 - [6] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).
 - [7] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
 - [8] L. M. Duan *et al.*, Nature (London) **414**, 413 (2001); J. I. Cirac *et al.*, Phys. Rev. Lett. **78**, 3221 (1997); C. P. Sun, Y. Li, and X. F. Liu, *ibid.* **91**, 147903 (2003).
 - [9] Z. Zhou, S. I. Chu, and S. Han, Phys. Rev. B **66**, 054527 (2002); **70**, 094513 (2004).
 - [10] K. V. R. M. Murali *et al.*, Phys. Rev. Lett. **93**, 087003 (2004).
 - [11] C. H. van der Wal *et al.*, Science **290**, 773 (2000); I. Chiorescu *et al.*, *ibid.* **299**, 1869 (2003); Y. Yu *et al.*, Phys. Rev. Lett. **92**, 117904 (2004); S. Saito *et al.*, *ibid.* **93**, 037001 (2004).
 - [12] J. Q. You, Y. Nakamura, and F. Nori, Phys. Rev. B **71**, 024532 (2005).
 - [13] This phenomenon has also been found in enantiomers: chiral molecules and their mirror image; see Ref. [14].
 - [14] P. Křál and M. Shapiro, Phys. Rev. Lett. **87**, 183002 (2001); P. Křál *et al.*, *ibid.* **90**, 033001 (2003); I. Thanopoulos *et al.*, *ibid.* **92**, 113003 (2004).
 - [15] T. P. Orlando *et al.*, Phys. Rev. B **60**, 15398 (1999).
 - [16] C. P. Sun, Phys. Rev. D **41**, 1318 (1990).
 - [17] M. O. Scully *et al.*, Science **299**, 862 (2003).
 - [18] C. P. Sun *et al.*, quant-ph/0506011.