Collective Interaction-Driven Ratchet for Transporting Flux Quanta

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We propose and study a novel way to produce a dc transport of vortices when applying an ac electrical current to a sample. Specifically, we study superconductors with a graduated random pinning density, which transports interacting vortices as a ratchet system. We show that a ratchet effect appears as a consequence of the long range interactions between the vortices. The pinned vortices create an asymmetric periodic flux density profile, which results in an asymmetric effective potential for the unpinned interstitial vortices. The latter exhibit a net longitudinal rectification under an applied transverse ac electric current.

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Stochastic transport on asymmetric potentials, the ratchet effect, has been recently studied in the context of biology, physics, and applied mathematics. The central question in these systems is how the random (Brownian) motion of a particle is rectified in a spatially asymmetric system. This can result in a net transport or current of the particle even in the absence of an external dc drive [1–4]. The ratchet effect is important for certain motor proteins and molecular motors. In addition, technological applications such as new particle separation techniques [5] and smoothing of atomic surfaces during electromigration [6] also utilize the ratchet effect. Most studies of ratchets have been conducted for a *single* particle; however, often systems contain many particles, and the *collective* (or cumulative) interactions between these particles may significantly change the transport properties from the single particle case. Moreover, most ratchet studies focus on a single particle moving on a *one*-dimensional (1D) asymmetric potential, as opposed to motion in 2D or 3D.

In this Letter, we propose and study a new type of 2D ratchet system which utilizes gradients of pointlike disorder, rather than a uniformly varying underlying potential. In particular, we study fluxons in superconductors containing a periodic arrangement of a graduated density of point defects, a geometry motivated by recent experiments [7]. Such defects can be created by either controlled irradiation techniques or direct-write electron-beam lithography. For a sufficiently large externally applied magnetic field, the fluxons fill most of the pinning sites and create a periodic asymmetric, or sawtooth, 2D flux profile. A certain field-dependent fraction of the vortices does not become pinned at individual pinning sites but can move in the interstitial regions between pinning sites in the presence of an applied ac drive. Although the moving interstitial vortices do not directly interact with the short-ranged pinning sites, they feel the long-range interaction of the vortices trapped at the pinning sites and therefore move in an effective asymmetric potential. The coherence length ξ provides the length scale controlling pinning, and this is much smaller than the length scales of interactions (given by the penetration depth λ). For finite temperature T and for an applied transverse ac electric current, we observe a net dc longitudinal transport of interstitial fluxons. Devices built using these ideas could be useful for removing unwanted flux in SQUIDs, and for making devices where flux can be focused via lensing. This type of ratchet may also be useful for the transport of colloids and charges, in which point defect gradients can be constructed.

The ratchet system described here differs significantly from other recent proposals for creating ratchets in superconductors. These range from the use of Josephson junctions in SQUIDs and arrays [8], to the use of a standard 1D-type potential-energy ratchet (e.g., [1–3]) to drive fluxons out of superconducting samples [9]. The concept of 2D asymmetric channel walls was proposed in Ref. [10]. All of these ratchet proposals rely on *single particles* interacting with an external potential to produce the dc response, whereas in our system *collective interactions* between particles are required to produce dc transport.

In order to create ratchet potentials in actual superconducting samples, an easily controllable method of introducing pinning into the material is required. The ratchet geometry described here can be created with existing experimental techniques. For example, irradiating the sample with heavy ions produces columnar pinning, which is very effective at trapping the vortices and much stronger than naturally occurring pinning, except at low temperatures [11]. In a recent experiment [7], controlled irradiation was used to create columnar pinning with a sawtooth-shaped, spatially varying density, producing a ratchet potential that can induce collective transport [7] proposed in this research.

We find that the ratchet geometry studied here is effective at transporting flux when operated at fields above the first matching field, where the density of vortices equals the density of pinning sites. For a given driving frequency, the ratcheting effect is optimized for narrow ranges in temperature and ac drive amplitude.

Simulation.—We consider a 2D slice of a system of superconducting vortices interacting with a pinning background [12]. The applied magnetic field is $\mathbf{H} = H\hat{\mathbf{z}}$, and we use periodic boundary conditions in x and y coordinates. The overdamped equation of motion for a vortex in a bulk superconductor is

$$\mathbf{f}^{(i)} = \mathbf{f}_{vv}^{(i)} + \mathbf{f}_L(t) + \mathbf{f}_p^{(i)} + \mathbf{f}_T = \mathbf{v}^{(i)}, \qquad (1)$$

where the total force on vortex i due to the repulsion from other vortices is given by $\mathbf{f}_{vv}^{(i)} = \sum_{j=1}^{N_v} f_0 K_1(|\mathbf{r}_i - \mathbf{r}_j|/\lambda)\hat{\mathbf{r}}_{ij}$. Here, K_1 is a modified Bessel function, $\mathbf{r}_i(\mathbf{v}_i)$ is the location (velocity) of the ith vortex, N_v is the number of vortices, $f_0 = \Phi_0^2/8\pi^2\lambda^3$, and $\hat{\mathbf{r}}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j|$. We measure all lengths in units of the penetration depth λ. Most of the results presented here are for systems of size $24\lambda \times 18\lambda$ containing $N_v = 336$ vortices and $N_p = 122$ pins; we also considered samples up to $192\lambda \times 36\lambda$ containing $N_v = 5624$ vortices. The Lorentz driving force from an applied ac current $\mathbf{J} = J\hat{\mathbf{y}}\sin\omega t$ is modeled as a uniform driving force $\mathbf{f}_L = \sin 2\pi\nu t$ on the vortices in the x direction. The pinning sites in the material are assumed to be randomly placed columnar defects (such as are created by irradiation in the experiment of Ref. [7]) and are modeled by parabolic traps of radius $r_p = 0.2\lambda$. Each vortex experiences a pinning force of $\mathbf{f}_p^{(i)} = \sum_k^{N_p}(f_p/r_p) \times (|\mathbf{r}_i - \mathbf{r}_k^{(p)}|)\Theta(r_p - |\mathbf{r}_i - \mathbf{r}_k^{(p)}|)\hat{\mathbf{r}}_{ik}^{(p)}$, where $\mathbf{r}_k^{(p)}$ is the location of pin k, Θ is the Heaviside step function, and $\hat{\mathbf{r}}_{ik}^{(p)} = (\mathbf{r}_i - \mathbf{r}_k^{(p)})/|\mathbf{r}_i - \mathbf{r}_k^{(p)}|$. We take $f_p = 2.0 f_0$; however, since pinned vortices remain pinned during the ratcheting motion, the results here are not sensitive to pinning strength. The samples have a sawtooth spatial distribution of pinning site density, repeated every 12λ (see Fig. 1), which serves to produce a ratchet potential in a manner described below. The forces due to thermal fluctuations, $\mathbf{f}_T$, are implemented via Langevin white noise. Our dimensionless temperature is $T = k_B T_{\mathrm{actual}}/\lambda f_0$.

Rectification.—When we apply an ac transverse electrical current to a sample at finite temperature, we observe a slow net dc longitudinal motion of fluxons in the positive x direction, indicating that we have succeeded in creating a vortex rectifier or diode. The individual pins, which interact with the vortices only over a *short* range, cannot by themselves produce the type of potential required to generate a ratchet. Instead, it is pinned vortices, which interact with unpinned vortices over a much *longer* range, that provide the properly shaped potential. For the strong pinning strengths considered here, a vortex that is trapped by a pinning site never depins afterwards. Since we focus in the regime above the first matching field, there are more vortices than pinning sites. Thus, not all of the vortices can be trapped. The mobile interstitial vortices are the ones that participate in the rectified 2D motion. They interact with the gradient in vortex density established by the gradient in pinning density. The local vortex density $n_v(x)$ integrated along the y direction is shown in Fig. 1(a). Figure 1(b) shows the top view, along z, of the x-y cross section of the sample, and the location of the pinning sites (open circles). A flux profile similar to that in Fig. 1(a) has been observed experimentally through magneto-optical imaging of an irradiated sample containing pins arranged as in Fig. 1(b) [13]. This structure, made of columnar pins, provides a ratchet that works based on the collective interactions of the movable objects. This type of structure would not function for a single vortex, but requires many interacting vortices to be present in order for it to work.

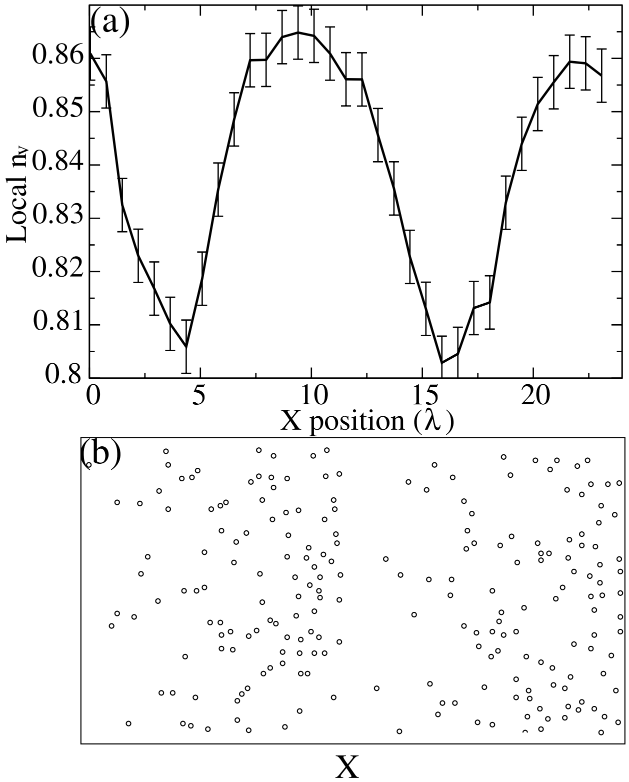

FIG. 1. (a) Local flux density n_v as a function of position X in a $24\lambda \times 36\lambda$ sample. (b) Locations of pinning sites in the sample showing a sawtooth gradient in pinning density. The variation in pinning density leads to a variation in the vortex density.

Field.—To further illustrate the collective nature of the rectification process, in Fig. 2 we show the rectified net fluxon velocity $\langle v\rangle = \sum_i^{N_v} \hat{\mathbf{x}} \cdot \mathbf{v}_i/N_v$ obtained as the vortex density in the sample is varied, for a fixed temperature of $T = 0.045$. When there are fewer vortices than pins, $N_v/N_p < 1$, each vortex is trapped by a pin during the initial annealing period. When a driving force of amplitude $f_L = 0.1 f_0$ and frequency $\nu = 0.003\,125$ is applied, the vortices are unable to escape from the pinning sites and remain stationary. Thus, we see no ratchet signal when there are no unpinned vortices: $\langle v\rangle = 0$ for $N_v/N_p < 1$. Once all of the pins have been filled, additional vortices remain in the interstitial region between pins. The motion of

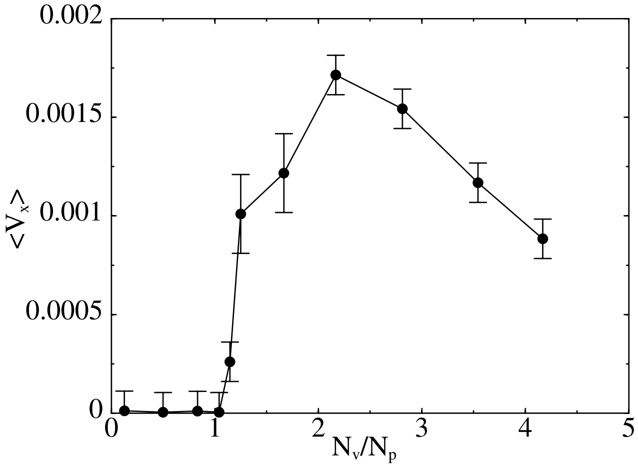

FIG. 2. The dependence of dc voltage $\langle v \rangle$ on the ratio of vortices to pins N_v/N_p in the sample for temperature $T = 0.045$, driving force $f_L = 0.1 f_0$, and frequency $\nu = 0.003\,125$. An optimal filling fraction appears near $N_v/N_p = 2$. Below $N_v/N_p = 1$, no dc signal appears because all of the vortices are trapped in pinning sites.

these interstitial vortices is influenced by the pinned vortices, but not directly by the short-ranged pins. As soon as interstitial vortices appear, a ratchet signal is obtained, as clearly indicated by the abrupt increase in $\langle v \rangle$ above $N_v/N_p = 1$ in Fig. 2 [14]. Initially, the magnitude of the ratchet signal continues to increase as the number of interstitial vortices increases, but at higher applied magnetic fields the strong vortex-vortex interactions begin to inhibit the ratcheting motion. In Fig. 2 $\langle v \rangle$ begins to decrease slowly above $N_v/N_p = 2$.

Temperature.—We explore the properties of the vortex diode under varying conditions by measuring V_x. Initially, we apply a driving force of amplitude $f_L = 0.1 f_0$ and frequency $\nu = 0.003\,125$, and vary the temperature from $T = 0$ to $T = 0.125$. The resulting $\langle v \rangle$ is shown in Fig. 3. These simulations clearly indicate an optimal or "resonant" temperature regime in which the dc fluxon velocity is maximized by the fluxon pump or diode. This optimal temperature regime can be explained as a trade-off between allowing the fluxons to fully explore the ratchet geometry, and washing out the driving force or pinning at high temperatures.

Frequency.—We show the fluxon velocity $\langle v \rangle$ at varying frequencies for a sample at $T = 0.045$ at two different driving forces of $f_L = 0.1 f_0$ and $f_L = 0.2 f_0$ in Fig. 4. The ratcheting effect dies away at high frequencies because the vortices do not have enough time to explore the ratchet potential. We also observe a saturation of the voltage response at low frequencies as we reach the dc limit. This is because, in the low-frequency limit, the interstitial vortices can repeatedly sample the asymmetry of the underlying ratchet potential, and the difference in the transport current in the two directions is fully exploited.

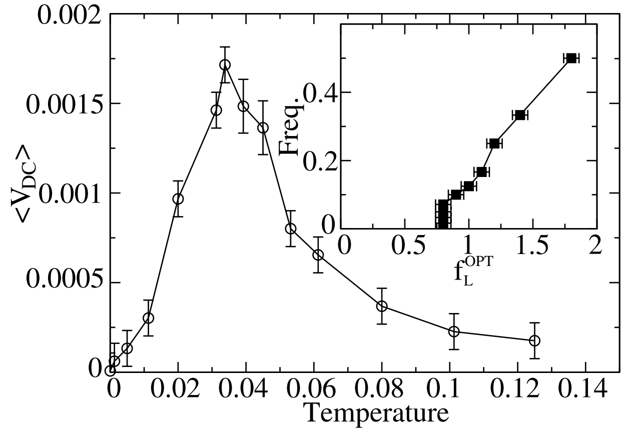

FIG. 3. Main panel: The dependence of dc voltage $\langle v \rangle$ on temperature T for a sample with driving force $f_L = 0.1 f_0$ and frequency $\nu = 0.003\,125$. An optimal temperature appears near $T = 0.035$. Inset: Relation between the applied frequency ν and the optimal ac driving force amplitude, f_L^{OPT}, for $T = 0.045$. At low frequencies f_L^{OPT} saturates to $f_L^{\mathrm{OPT}} = 0.8 f_0$.

Amplitude.—We find that there is an optimal driving amplitude f_L^{OPT}, as shown in Fig. 5 for samples at $T = 0.045$ driven with frequencies ranging from $\nu = 0.000\,521$ to $\nu = 1$. The optimal amplitude shifts to higher driving forces as the driving frequency is increased, as illustrated in the inset of Fig. 3, since the vortex can explore the same portion of the pinning in a shorter time interval if the driving force is increased. At low frequencies a saturation value of $f_L^{\mathrm{OPT}} = 0.8 f_0$ is reached.

Conclusion.—Using a new type of ratchet system that fundamentally depends on the collective interactions of the movable objects, we observe a dc vortex rectification starting with an input ac electrical current. Our proposal employs superconductors with periodic, graduated random pinning density of columnar defects. This ratchet differs

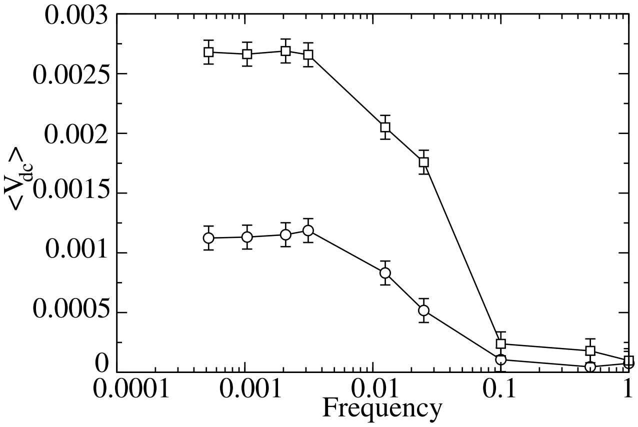

FIG. 4. The dependence of dc voltage $\langle v \rangle$ on frequency ν for a sample with $T = 0.045$ at two different amplitudes: circles, $f_L = 0.1 f_0$; squares, $f_L = 0.2 f_0$. The rectification is reduced at high frequencies and saturates at low frequencies.

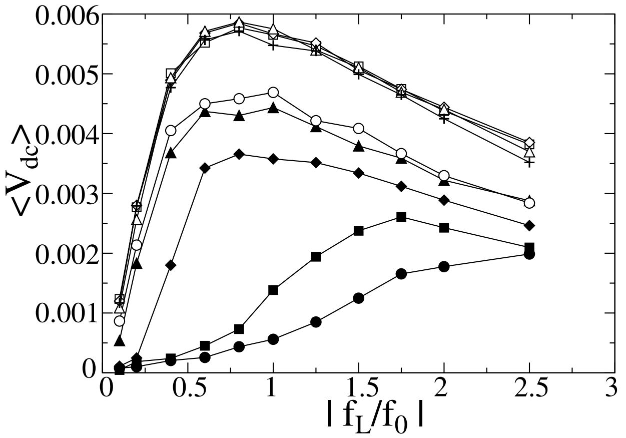

FIG. 5. Rectified average fluxon velocity $\langle v \rangle$, which can be measured as a voltage, versus the amplitude of the ac driving force. The frequency of the ac signal is filled circles, $\nu = 1$; filled squares, $\nu = 0.5$; filled diamonds, $\nu = 0.1$; filled triangles, $\nu = 0.025$; open circles, $\nu = 0.0125$; open squares, $\nu = 0.003\,125$; open diamonds, $\nu = 0.0021$; open triangles, $\nu = 0.001\,04$; pluses, $\nu = 0.000\,521$.

from the majority of the ones proposed previously on several key points: (a) it is fundamentally a 2D ratchet, as opposed to a 1D one, and more important (b) it requires collective interactions, as opposed to the mostly one-particle ratchets studied so far. In our system, we show that the asymmetric potential is created when vortices are trapped by the pinning sites and interact with unpinned vortices. The long-range interactions of the vortices leads to the formation of a periodic asymmetric potential caused by the gradient in vortex density. Mobile interstitial vortices experience this collectively produced potential, rather than interacting directly with the pinning sites in a single-particle manner. We show that there is an optimal field, temperature, and frequency for the operation of such devices. This ratchet can be created experimentally through controlled irradiation techniques and via electron-beam lithography. The use of controlled random defects in the ratchet geometry proposed here should make it possible to extend the rectification devices shown here to other systems, including colloids and other collections of charged particles.

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[1] M. O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993); **72**, 2656 (1994); J. Prost *et al., ibid.* **72**, 2652 (1994); C. R. Doering, W. Horsthemke, and J. Riordan, *ibid.* **72**, 2984 (1994); R. D. Astumian and M. Bier, *ibid.* **72**, 1766 (1994); L. P. Faucheux *et al., ibid.* **74**, 1504 (1995); R. Bartussek, P. Reimann, and P. Hänggi, *ibid.* **76**, 1166 (1996).

[2] F. Marchesoni, Phys. Rev. E **56**, 2492 (1997); Phys. Rev. Lett. **77**, 2364 (1996).

[3] C. R. Doering, Physica (Amsterdam) **254A**, 1 (1998); Nuovo Cimento Soc. Ital. Fis. **17D**, 685 (1995); C. R. Doering, L. A. Dontcheva, and M. M. Klosek, Chaos **8**, 643 (1998); T. C. Elston and C. R. Doering, J. Stat. Phys. **83**, 359 (1996).

[4] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1966), Vol. I, Chap. 46.

[5] J. Rousselet *et al.,* Nature (London) **370**, 446 (1994).

[6] I. Derenyi, C. Lee, and A. L. Barabasi, Phys. Rev. Lett. **80**, 1473 (1998).

[7] W. K. Kwok *et al.* (unpublished).

[8] I. Zapata, R. Bartussek, F. Sols, and P. Hänggi, Phys. Rev. Lett. **77**, 2292 (1996); **80**, 829 (1998); F. Falo, P. J. Martinez, J. J. Mazo, and S. Cilla, Europhys. Lett. **45**, 700 (1999); S. Weiss, D. Koelle, J. Müller, R. Gross, and K. Barthel, *ibid.* **51**, 499 (2000); E. Trías, J. J. Mazo, F. Falo, and T. P. Orlando, Phys. Rev. E **61**, 2257 (2000).

[9] C. S. Lee and B. Jankó, I. Derényi, A. L. Barabási, Nature (London) **400**, 337 (1999); J. F. Wambaugh *et al.* (unpublished) also performed independent work on 1D fluxon ratchets. Early work on asymmetric-shaped pinning traps can be found, e.g., in E. H. Brandt, J. Low Temp. Phys. **53**, 41 (1983).

[10] J. F. Wambaugh, C. Reichhardt, C. J. Olson, F. Marchesoni, and F. Nori, Phys. Rev. Lett. **83**, 5106 (1999).

[11] L. Civale *et al.,* Phys. Rev. Lett. **67**, 648 (1991); M. Konczykowski *et al.,* Phys. Rev. B **44**, 7167 (1991); L. Civale, Supercond. Sci. Technol. **10**, 11 (1997); L. M. Paulius *et al.,* Phys. Rev. B **56**, 913 (1997); W. K. Kwok *et al.,* Phys. Rev. B **58**, 14 594 (1998); D. H. Kim *et al.,* Phys. Rev. B **60**, 3551 (1999).

[12] C. Reichhardt *et al.,* Phys. Rev. B **52**, 10 441 (1995); **53**, R8898 (1996); C. J. Olson *et al.,* **56**, 6175 (1997); Phys. Rev. Lett. **80**, 2197 (1998); **81**, 3757 (1998).

[13] U. Welp *et al.* (unpublished).

[14] Because of the gradient in pinning density, the field at which $N_v = N_p$ is *lower* than the field at which the vortex density equals the maximum pinning density in the sample.