



## Surface Josephson Plasma Waves in Layered Superconductors above the Plasma Frequency: Evidence for a Negative Index of Refraction

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We predict a new branch of surface Josephson plasma waves (SJPWs) in layered superconductors for frequencies higher than the Josephson plasma frequency. In this frequency range, the permittivity tensor components along and transverse to the layers have different signs, which is usually associated with negative refraction. However, for these frequencies, the bulk Josephson plasma waves cannot be matched with the incident and reflected waves in the vacuum, and, instead of the negative-refractive properties, abnormal surface modes appear within the frequency band expected for bulk modes. We also discuss the excitation of high-frequency SJPWs by means of the attenuated-total-reflection method.

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High- $T_c$  layered cuprate superconductors are important candidates for negative-refractive-index (NRI) metamaterials (see, e.g., [1,2]). Indeed, being uniaxial strongly anisotropic materials, they provide different signs of the permittivity tensor components along,  $\varepsilon_{ab}$ , and transverse,  $\varepsilon_c$ , to the layers in a wide frequency range (see, e.g., [3]), providing a possibility of NRI. These metamaterials are attracting considerable attention because they have the potential for subwavelength resolution and aberration-free imaging. Layered superconductors are very promising metamaterials because they are relatively straightforward to fabricate (compared to double negative metamaterials) and do not require negative permeability.

Experiments for the  $c$ -axis conductivity in layered superconductors prove the use of a model in which the superconducting  $\text{CuO}_2$  layers are coupled by the intrinsic Josephson effect through the layers [4]. Thus, a Josephson plasma with anisotropic current capability is produced in layered superconductors. Moreover, the physical mechanisms of the currents along and across the layers are fundamentally different. The current along the layers is similar to the current in bulk superconductors, while the current across the layers is Josephson-type.

The Josephson current along the  $c$ -axis couples with the electromagnetic field inside the insulating dielectric layers, forming Josephson plasma waves (JPWs) (see the review [5] and references therein). Thus, the propagation of electromagnetic waves through the layers is favored by the layered structure. The study of these waves is very important because of their terahertz frequency range, which is still hardly reachable for electronic and optical devices.

As in common plasma waves, JPWs propagate with frequencies above some threshold value (the Josephson plasma frequency  $\omega_J$ ). However, in the frequency range below  $\omega_J$ , the presence of the sample boundary can produce surface Josephson plasma waves (SJPWs) [6], which are analog to the surface plasmon polaritons in metals [7,8]. Such waves can propagate along the vacuum-layered superconductor interface and damp away from it.

At frequencies  $\omega$  higher than the Josephson plasma frequency  $\omega_J$ , the normal-to-the-layers components of both the group velocity and the Poynting vector of propagating JPWs have signs opposite to the sign of the normal component of the wave vector  $\mathbf{k}_s$ . This corresponds to a NRI. However, the condition  $\omega > \omega_J$  is not sufficient for NRI. This NRI effect can be observed at the vacuum-layered superconductor boundary only in a relatively narrow frequency range,  $\omega_J < \omega < \omega_1^{\text{vac}} = \omega_J[\varepsilon/(\varepsilon - 1)]^{1/2}$ , where  $\varepsilon$  is the interlayer dielectric constant. A similar limitation exists for any insulator-layered-superconductor boundary, if the dielectric constant  $\varepsilon_{\text{ext}}$  of the insulator is less than  $\varepsilon$ . The above conditions follow from the dispersion relation for the JPWs and the natural limitation for the tangential component  $q$  ( $q = k \sin\theta < k$ ,  $k = \omega/c$ ) of the wave vector for a wave incident at an angle  $\theta$  from the vacuum onto the layered superconductor. A simple analysis shows that the above inequality is compatible with the dispersion relation for JPWs only for frequencies  $\omega_J < \omega < \omega_1^{\text{vac}}$ . In other words, any wave with frequency  $\omega > \omega_1^{\text{vac}}$  incident from the vacuum cannot propagate further in a layered superconductor. JPWs with  $\omega > \omega_1^{\text{vac}}$  can only match evanescent waves in the vacuum. As for frequencies  $\omega_J < \omega < \omega_1^{\text{vac}}$ , the NRI can be ob-

served, but only for incident angles  $\theta$  higher than some critical value  $\theta_{\text{crit}}$ .

For frequencies in the interval

$$\omega_1^{\text{vac}} < \omega < \omega_2, \quad \omega_2 = \omega_J \gamma, \quad (1)$$

we predict the existence of a new branch of surface waves (here  $\gamma = \lambda_c / \lambda_{ab} \gg 1$  is the current-anisotropy parameter,  $\lambda_c = c / \omega_J \epsilon^{1/2}$  and  $\lambda_{ab}$  are the magnetic-field penetration depths along and across the layers, respectively). Despite numerous works on this issue, SJPWs with frequencies higher than  $\omega_J$  were not discussed before. Here we study them and prove that the SJPWs spectrum consists of two branches. The low-frequency branch was described in detail in [6]. Its spectrum  $\omega(q)$  follows the ‘‘vacuum light line,’’  $q = \omega/c$ , and deviates from it at frequencies close to  $\omega_J$ . The new branch of SJPWs predicted here starts at the frequency  $\omega = \omega_1^{\text{vac}}$  and follows the vacuum light line for  $\omega \ll \omega_2$ . For frequencies of the order of  $\omega_2$ , the dispersion curve  $\omega(q)$  strongly deviates from the vacuum light line and stops at the frequency  $\omega = \omega_2$  when  $q \approx 1/\lambda_{ab}$ .

Thus, SJPWs do not exist within the frequency gap  $\omega_J < \omega < \omega_1^{\text{vac}}$ . On the other hand, as shown here, this is actually the range where the NRI can be observed (for waves incident from the vacuum onto the layered-superconductor boundary). Hence, some kind of complementarity between the NRI and surface waves is established in this Letter.

*Conditions for the observation of NRI.*—Consider a layered structure consisting of superconducting and dielectric layers with thicknesses  $s$  and  $d$ , respectively (see Fig. 1). We study the transverse-magnetic JPWs propagating with wave vector  $\mathbf{k}_s = (q, 0, \kappa_s)$  and having the electric,  $\mathbf{E}^s = \{E_x^s, 0, E_z^s\}$ , and magnetic,  $\mathbf{H}^s = \{0, H^s, 0\}$ , components proportional to  $\exp[i(qx + \kappa_s z - \omega t)]$ . The coordinate system is shown in Fig. 1.

The electromagnetic field inside the layered superconductor is determined by the distribution of the gauge invariant phase difference  $\varphi(x, z, t)$  of the order parameter between neighboring layers. This phase difference can be described by a set of coupled sine-Gordon equations (see review [5]). In the continuum and linear approximation,  $\varphi(x, z, t)$  can be excluded from the set of equations for electromagnetic fields, and the electrodynamics of layered

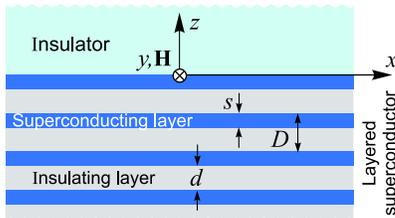


FIG. 1 (color online). Geometry for studying waves in layered superconductors. The interface  $z = 0$  divides the layered superconductor from an insulator with dielectric constant  $\epsilon_{\text{ext}}$ .

superconductors can be described in terms of an anisotropic frequency-dependent dielectric permittivity with components  $\epsilon_c(\Omega)$  and  $\epsilon_{ab}(\Omega)$  across and along the layers, respectively [2]. In Ref. [9], the effect of spatial dispersion in  $\epsilon_c$ , related to the capacitive interlayer coupling, was taken into account. Here we do not consider this effect because it is only important for a very narrow frequency range near  $\omega_J$  [9].

In the limit  $s/d \ll 1$ , the equations for  $\epsilon_c(\Omega)$  and  $\epsilon_{ab}(\Omega)$  can be written as

$$\begin{aligned} \epsilon_c(\Omega) &= \epsilon \left( 1 - \frac{1}{\Omega^2} + i\nu_c \frac{1}{\Omega} \right), \\ \epsilon_{ab}(\Omega) &= \epsilon \left( 1 - \frac{1}{\Omega^2} \gamma^2 + i\nu_{ab} \frac{1}{\Omega} \gamma^2 \right). \end{aligned} \quad (2)$$

Here we introduce the dimensionless parameters  $\Omega = \omega/\omega_J$ ,  $\nu_{ab} = 4\pi\sigma_{ab}/\epsilon\omega_J\gamma^2$ , and  $\nu_c = 4\pi\sigma_c/\epsilon\omega_J$ . The relaxation frequencies  $\nu_{ab}$  and  $\nu_c$  are proportional to the averaged quasiparticle conductivities  $\sigma_{ab}$  (along the layers) and  $\sigma_c$  (across the layers), respectively;  $\omega_J = (8\pi e D j_c / \hbar \epsilon)^{1/2}$  is the Josephson plasma frequency. The latter is determined by the critical Josephson current density  $j_c$ , the interlayer dielectric constant  $\epsilon$ , and the spatial period of the layered structure  $D = s + d \approx d$ .

Analyzing the relations for  $\epsilon_c(\Omega)$  and  $\epsilon_{ab}(\Omega)$ , we conclude that their real parts have different signs in a wide frequency range:  $\omega_J < \omega < \omega_2 = \omega_J \gamma$  (or  $1 < \Omega < \Omega_2 = \gamma \gg 1$ ). In this frequency range, the  $z$  components of the group velocity and the Poynting vector of the bulk JPWs are directed opposite to the  $z$  component of the wave-vector  $\mathbf{k}_s$ , and this, at first sight, corresponds to having a NRI. However, a more careful analysis shows that this can only be observed for a much narrower frequency interval. To verify this, one can consider the well-known dispersion relation for the normal component  $\kappa_s$  of the JPW wave vector,

$$\kappa_s^2 = \epsilon_{ab}(\Omega) \left[ k^2 - \frac{q^2}{\epsilon_c(\Omega)} \right], \quad (3)$$

that follows directly from Maxwell’s equations. Obviously, the JPWs can propagate only when  $\text{Re}(\kappa_s^2) > 0$ , where ‘‘Re’’ stands for the real part. Neglecting dissipation, the permittivity  $\epsilon_{ab}$  is negative for the frequency region  $\omega_J < \omega < \omega_2$  considered here. Consequently, JPWs can propagate if the factor  $[k^2 - q^2/\epsilon_c(\Omega)]$  in Eq. (3) is negative. Using Eq. (2) for  $\epsilon_c(\Omega)$ , one can conclude that this factor is negative only when  $1 < \Omega^2 < 1 + q^2 \lambda_c^2$ . For a wave incident, at an angle  $\theta$ , from the insulator with dielectric constant  $\epsilon_{\text{ext}}$  onto the layered superconductor, we have  $q = (\omega \epsilon_{\text{ext}}^{1/2} / c) \sin \theta$ . Thus, a NRI can be observed for waves with incident angles higher than the critical value  $\theta_{\text{crit}}$  defined by the equation,

$$\sin(\theta_{\text{crit}}) = \sqrt{\epsilon_c(\Omega) / \epsilon_{\text{ext}}}. \quad (4)$$

It is important to note that, due to the negative sign of the

permittivity  $\varepsilon_{ab}$  in Eq. (3), the incident wave penetrates the superconductor for  $\theta > \theta_{\text{crit}}$ , and totally reflects from it for  $\theta < \theta_{\text{crit}}$ , contrary to the standard case of waves incident onto the interface dividing two usual media with positive  $\varepsilon$ .

For  $\varepsilon_{\text{ext}} < \varepsilon$ , Eqs. (2) and (4), and the inequality  $\sin(\theta_{\text{crit}}) \leq 1$ , provide the conditions,

$$\omega_J < \omega < \omega_1 = \omega_J \left( \frac{\varepsilon}{\varepsilon - \varepsilon_{\text{ext}}} \right)^{1/2}. \quad (5)$$

Thus, the NRI for a layered superconductor bounded by an insulator with  $\varepsilon_{\text{ext}} < \varepsilon$  can only be observed in the frequency range  $\omega_J < \omega < \omega_1 = \omega_J \Omega_1$ . This frequency window can be expanded if one uses an insulator with large enough permittivity  $\varepsilon_{\text{ext}}$ . Only for insulators with very high permittivity  $\varepsilon_{\text{ext}} > \varepsilon$ , the negative refraction can occur in the whole frequency interval  $\omega_J < \omega < \omega_2$ . The frequency range for the existence of the bulk JPWs in superconductors with capacitive interlayer coupling was derived in [10]. If the constant of this coupling tends to zero, the frequency range obtained in [10] coincides with Eq. (5). Below we consider waves in the insulator-layered-superconductor system with  $\varepsilon_{\text{ext}} < \varepsilon$  for frequencies  $\omega > \omega_1$ .

*Surface Josephson plasma waves above  $\omega_1$ .*—When  $\omega_J < \omega < \omega_2$  and the factor in Eq. (3) is positive, the  $z$  component  $\kappa_s$  of the wave vector  $\mathbf{k}_s$  becomes imaginary. This means that the wave damps into the layered superconductor. On the other hand, for  $q > \omega \sqrt{\varepsilon_{\text{ext}}}/c$  the wave damps also into the insulator above the layered superconductor. These are the characteristic features of the surface waves discussed in this section.

Consider an interface (the  $xy$  plane) separating an insulator ( $z > 0$  in Fig. 1) and a layered superconductor ( $z \leq 0$ ). We now consider a linear surface transverse-magnetic monochromatic wave propagating along the  $x$  axis (i.e., proportional to  $\exp[i(qx - \omega t)]$ ) and decaying into both, the insulator and layered superconductor, away from the interface  $z = 0$ .

Performing the standard procedure for searching surface waves (i.e., solving the Maxwell equations for the insulator and layered superconductor with proper boundary conditions at the interface between them) we obtain the dispersion relation for the surface Josephson plasma waves:

$$\kappa(\Omega) = \Omega \left( \varepsilon_c(\Omega) \frac{\varepsilon_{\text{ext}} - \varepsilon_{ab}(\Omega)}{\varepsilon_{\text{ext}}^2 - \varepsilon_c(\Omega) \varepsilon_{ab}(\Omega)} \right)^{1/2}, \quad (6)$$

or, neglecting dissipation,

$$\kappa(\Omega) = \Omega \left( \frac{\gamma^2 - \Omega^2 + \Omega^2 \varepsilon_{\text{ext}}/\varepsilon}{\gamma^2 - \Omega^2 + \Omega^4 \varepsilon_{\text{ext}}^2/(\Omega^2 - 1)\varepsilon^2} \right)^{1/2}. \quad (7)$$

Here the dimensionless wave vector is defined as  $\kappa = cq/\omega_J \varepsilon_{\text{ext}}^{1/2}$ . Equation (7) describes two branches of the dispersion curve for the SJPWs (see Fig. 2). The first branch exists in the low-frequency range,  $0 < \omega < \omega_J$ , and it was studied before in [6]. The second (predicted

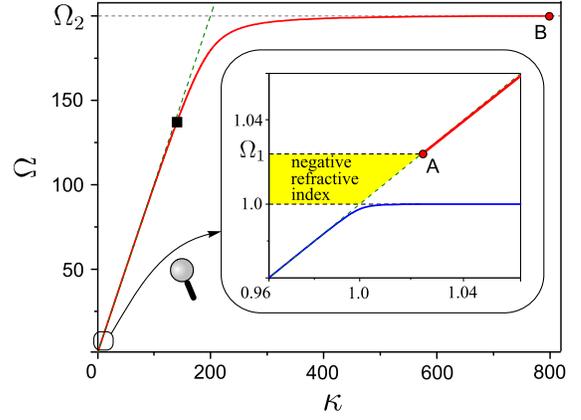


FIG. 2 (color online). The dispersion curve ( $\Omega = \omega/\omega_J$  versus  $\kappa = cq/\omega_J \varepsilon_{\text{ext}}^{1/2}$ ) for SJPWs at the vacuum-layered superconductor interface. The values of the parameters are  $\gamma = 200$ ,  $\varepsilon = 16$ . Inset: zoom-in of the spectrum near the point ( $\kappa = 1$ ,  $\Omega = 1$ ). Points A and B correspond to the beginning and end of the high-frequency branch. The green dashed line is the vacuum light line  $\Omega = \kappa$ .

here) branch starts from the light line  $\omega = cq/\varepsilon_{\text{ext}}^{1/2}$  (or  $\Omega = \kappa$ ) at  $\omega = \omega_1$  (point A in Fig. 2), then follows this line, deviates from it at  $\omega \sim \omega_2 = \gamma\omega_J$ , and stops at the point where  $q = \gamma\omega_J \varepsilon^{1/2}/c$ ,  $\omega = \omega_2$  (point B in Fig. 2).

Thus, there exists a frequency gap,  $\omega_J < \omega < \omega_1$ , in the spectrum of the SJPWs. We emphasize that the NRI should only exist within this gap. When the permittivity  $\varepsilon_{\text{ext}}$  of the

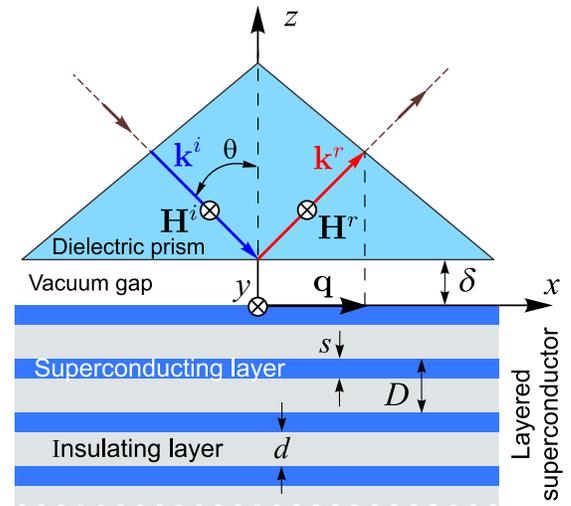


FIG. 3 (color online). A dielectric prism with permittivity  $\varepsilon_p$  is separated from a layered superconductor by a vacuum gap of thickness  $\delta$ . An electromagnetic wave with incident angle  $\theta$ , exceeding the limit angle  $\theta_t = \arcsin(\varepsilon_p^{-1/2})$  for total internal reflection, can excite surface waves that satisfy the following resonance condition:  $\omega \varepsilon_p^{1/2} \sin\theta = cq$ . Here  $\mathbf{k}^i$  and  $\mathbf{k}^r$  are the wave vectors of the incident and reflected waves associated with magnetic-field amplitudes  $\mathbf{H}^i$  and  $\mathbf{H}^r$ . The resonance excitation of surface waves by the incident wave produces a strong suppression of the reflected wave.

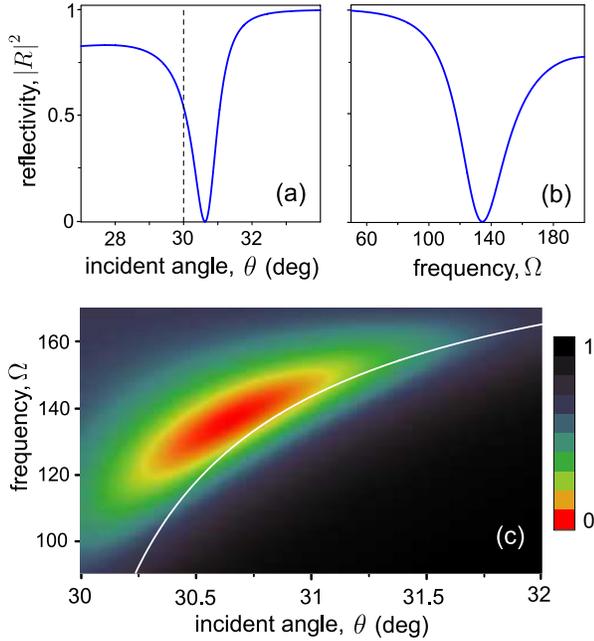


FIG. 4 (color online). (a) The reflectivity coefficient  $|R|^2$  versus the incident angle  $\theta$  calculated numerically for the parameters  $\nu_{ab} = 10^{-3}$ ,  $\gamma = 200$ ,  $\varepsilon = 16$ ,  $\varepsilon_p = 4$ , and  $\Omega = 135$ . These parameters correspond to the solid square on the dispersion curve in Fig. 2. The thickness  $\delta$  of the vacuum gap (see Fig. 3) is one wavelength,  $k\delta = 2\pi$ . The vertical dashed line at  $\theta = 30^\circ$  corresponds to the limit angle of the total internal reflection. (b) The reflectivity coefficient  $|R|^2$  versus frequency  $\Omega = \omega/\omega_J$  for  $\theta = 30.6^\circ$ . (c) Color contour plot of the reflectivity coefficient  $|R|^2$  in the plane  $(\theta, \Omega)$ . The dispersion relation for the waves in the dielectric-vacuum-layered superconductor system in Fig. 3 is presented by the solid white curve.

insulator increases, point *A* in Fig. 2 moves towards point *B*, and the gap in the SJPW spectrum increases. When  $\varepsilon_{\text{ext}} = \varepsilon(1 - 1/\gamma^2)$ , points *A* and *B* coincide, and the high-frequency branch in the SJPWs spectrum disappears.

Note that the Josephson current is small with respect to the displacement current at high frequencies,  $\Omega \gg 1$ . In this case, we can omit 1 in the denominator in Eq. (7). This corresponds to the dispersion relation for a periodic layered structure without coupling between superconducting layers. Specifically, the interlayer Josephson coupling is responsible for the appearance of a frequency gap,  $\omega_J < \omega < \omega_1$ , in the spectrum of SJPWs.

*Excitation of the SJPWs above  $\omega_1$ .*—It is known that the excitation of surface waves is accompanied by the so-called Wood anomalies of the reflectivity and transmissivity coefficients (see, e.g., [8]). These resonance anomalies can result in the complete suppression of the reflectivity by a proper choice of parameters. Here we consider the excitation of high-frequency SJPWs ( $\Omega \gg 1$ ) by a wave incident from a dielectric prism with permittivity  $\varepsilon_p$  onto a layered superconductor separated from the prism by a vacuum gap of thickness  $\delta$  (the so-called “attenuated-

total-reflection” method for producing surface waves, see Fig. 3).

The suppression of the specular reflectivity  $|R|^2 = |H^r/H^i|^2$  due to the resonant excitation of the surface waves can be observed by changing the incident angle at a given frequency or by changing the frequency at a given incident angle, as demonstrated in Figs. 4(a) and 4(b). Here  $H^i$  and  $H^r$  are the magnetic-field amplitudes of the incident and reflected waves, respectively. Note that dissipation plays a very important role in the excitation of the surface waves. In particular, the widths of the resonance curves shown in Figs. 4(a) and 4(b) are defined by the relaxation parameter  $\nu_{ab}$ . Figure 4(c) shows the sharp decrease of the reflectivity in the  $(\theta, \Omega)$  plane.

*Conclusions.*—Here we predict the existence of a new branch of SJPWs in layered superconductors for the frequency range higher than the Josephson plasma frequency, which is a very unusual phenomenon for plasmalike media. It is important that a NRI can only be observed for frequencies within the gap in the spectrum of the SJPWs. Thus, some kind of complementarity between NRI and surface waves is established in this Letter. We have also described the excitation of these SJPWs by means of the attenuated-total-reflection method.

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