Efficient Quantum Circuits for One-Way Quantum Computing

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While Ising-type interactions are ideal for implementing controlled phase flip gates in one-way quantum computing, natural interactions between solid-state qubits are most often described by either the XY or the Heisenberg models. We show an efficient way of generating cluster states directly using either the imaginary SWAP (iSWAP) gate for the XY model, or the $\sqrt{\text{SWAP}}$ gate for the Heisenberg model. Our approach thus makes one-way quantum computing more feasible for solid-state devices.

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Significant theoretical and experimental efforts have been devoted to study how qubits interact with external perturbations and among themselves (see, e.g., [1–7]). However, in general, qubit interactions are still difficult to control precisely. Furthermore, turning on interqubit interactions can open new decoherence channels. For instance, the Heisenberg exchange interaction between electrons is electrostatic in nature; turning it on makes the spin system vulnerable to charge fluctuations in the environment [8]. To improve the reliability of a solid-state quantum circuit, it is thus generally desirable to have as few interqubit interaction operations as possible. In other words, universal quantum gates should minimize the number of two-qubit operations.

Many solid-state qubits have interqubit interactions described by various kinds of exchange Hamiltonians: XY [2,3], XXZ, or the isotropic Heisenberg exchange model [4–7], *rather than* the Ising model [1] (e.g., Table I). For Ising interactions, two-qubit gates [such as the controlled NOT (CNOT) and controlled phase flip (CPF)] are obtained by turning on the spin-spin interaction just *once*. For non-Ising interactions, these two-qubit gates are more difficult to implement: both CNOT and CPF gates require turning on the two-qubit interaction, as iSWAP or $\sqrt{\text{SWAP}}$ gates, *at least twice*, in addition to several single-qubit gates [11,12]. Two-qubit gates, such as iSWAP or $\sqrt{\text{SWAP}}$, are universal. An important open problem is how to reliably implement those universal gates in real systems.

One-way quantum computing (QC) is a novel measurement-based approach [13–17], which starts with the creation of a highly entangled cluster state using CPF gates and has been extensively discussed in Ref. [16]. Here, we propose optimized one-way QC quantum circuits that are tailored to the interqubit interaction actually present in solid-state nanostructures. In particular, we show that the relatively cumbersome and expensive CPF and CNOT gates (typically studied in QC) can be replaced by a *single-application* of an iSWAP (XY model), $\sqrt{\text{SWAP}}$ (Heisenberg model), or generalized $\sqrt{\text{SWAP}}$ gate (XXZ model) without additional overheads. This change of the underlying two-qubit gates makes our new quantum circuit simpler, faster, and more robust. We also show that iSWAP gates are particularly useful in the construction of large cluster states, which indicates that one-way QC may be more easily realized in systems such as cavity-coupled flux or spin qubits. Furthermore, we show that a measurement, combined with either the XY or the XXZ interaction, can further improve gate efficiencies in solid-state quantum computing schemes. With studies of solid-state quantum coherent manipulations still limited by relatively crude control technology, the improvements we identify in this study will greatly enhance the chance of experimental success in the demonstration of one-way QC in nanostructures.

(J, J^z; t)-gate.—We first derive the basic two-qubit gates for the general exchange interactions. The XY, XXZ, and Heisenberg models are described by the Hamiltonian $H = \sum_{i<j} H^{(ij)}$ with

$$H^{(ij)} = J_{ij}(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y) + J_{ij}^z\sigma_i^z\sigma_j^z, \quad (1)$$

where σ_i^α ($\alpha = x, y, z$) are the Pauli matrices acting on the i-th qubit with qubit basis $|0\rangle = |\downarrow\rangle$ and $|1\rangle = |\uparrow\rangle$. For simplicity, we take $J = J_{ij}$ and $J^z = J_{ij}^z$. The XY model then corresponds to $J^z = 0$ and the Heisenberg model to $J^z = J$. In the case of two qubits, $H^{(12)} = J(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y) + J^z\sigma_1^z\sigma_2^z$ leads to a two-qubit evolution described

TABLE I. Examples of interqubit interactions.

Two-qubit interaction	Qubit system
Ising	charge [1]
XY	flux [2,9], charge-flux [2], phase [2], (charge [2], flux [2], spin [3]) in cavity
XXZ	electrons on helium [10]
Heisenberg	spin [4], donor atom [7]

by $U^{(12)}(t) = e^{-itH^{(12)}}$ ($\hbar = 1$) so that

$$|01\rangle \rightarrow A|01\rangle + iB|10\rangle, \qquad |10\rangle \rightarrow A|10\rangle + iB|01\rangle, \quad (2)$$

with $A \equiv e^{2iJ^z t}\cos 2Jt$ and $B \equiv -e^{2iJ^z t}\sin 2Jt$, while $|00\rangle$ and $|11\rangle$ are unchanged (an overall phase factor $e^{-iJ^z t}$ has been omitted). Hereafter, we call this very general operation of turning on $H^{(12)}$ for a time period t, the $(J, J^z; t)$-gate. The iSWAP gate is obtained when $J^z = 0$ and $t = \tau_{i\text{SWAP}} = \pi/(4J)$, and the $\sqrt{\text{SWAP}}$ gate is obtained when $J = J^z$ and $t = \tau_{\sqrt{\text{SWAP}}} = \pi/(8J)$. The conventional CNOT or CPF gate requires two iSWAP gates for the XY model [11] or two $\sqrt{\text{SWAP}}$ gates for the Heisenberg model [4,11,12], plus additional single-qubit rotations. For example, the XY-model CNOT gate is usually described [11] by $U^{(12)}_{\text{CNOT}} = e^{i\pi/4\sigma_1^z} e^{-i\pi/4\sigma_2^x} e^{-i\pi/4\sigma_2^z}[i\text{SWAP}]_{12} \times e^{i\pi/4\sigma_1^x}[i\text{SWAP}]_{12} e^{-i\pi/4\sigma_2^z}$, where $[i\text{SWAP}]_{12} \equiv U^{(12)}(\tau_{i\text{SWAP}})$.

Generation of cluster states using $(J, J^z; t)$*-gates.*— Cluster states [13] are generated by a two-body evolution of the form $S_{ij} \equiv (1 + \sigma_i^z + \sigma_j^z - \sigma_i^z\sigma_j^z)$ acting on a product state $\Pi_i|+\rangle_i$, where $|\pm\rangle_i = (|0\rangle_i \pm |1\rangle_i)/\sqrt{2}$. The difficulty of applying this approach to natural non-Ising spin models is that neighboring interactions generally do *not* commute: $[H^{(i,i-1)}, H^{(i,i+1)}] \neq 0$, so that $\exp(-iHt) \neq \Pi_{ij}\exp[-iH^{(ij)}t]$. In order to create cluster states using these non-Ising spin interactions, *pairwise* bonding between qubits are needed [18]. Specifically, for a d-dimensional (d-D) qubit array, cluster states are generated in $2d$ steps. First, two-qubit cluster states are created by performing CPF operations between pairs of nearest-neighbor qubits. These qubit pairs are then connected to each other via another set of CPF operations, and a 1-D chain cluster state is generated. Afterwards, two chains are connected resulting in a ladder structure. Two ladder cluster states can then be connected into 2-D cluster states, and so on.

Can we further streamline this process of cluster state generation? An important step in optimizing a quantum circuit, for a particular type of interaction, is to identify the fastest route to a desired entanglement. When we closely inspect the various spin interactions, we find that CNOT or CPF gates are generally not the best two-qubit gates to generate cluster states (except in the case of Ising interactions). Instead, a more efficient approach is to replace the CPF gate [in the generation of pair cluster states (the first step above)] by a single application of the $(J, J^z; t)$-gate in the general XXZ model, together with single-qubit rotations. The initial two-qubit state here needs to be $(|0\rangle_1 + e^{i\theta_1}|1\rangle_1)(|0\rangle_2 + e^{i\theta_2}|1\rangle_2)$, with $\theta_2 - \theta_1 = \pi$ or 0. If $\theta_2 - \theta_1 = \pi$, the duration of the $(J, J^z; t)$-gate is $t = \pi/[4(J + J^z)]$; if $\theta_2 - \theta_1 = 0$, $t = (\pi/4 + m_s\pi/2)/(J - J^z)$, where m_s is any integer. After appropriate single-qubit rotations, a two-qubit cluster state $|\Psi\rangle_{12}^C \equiv (|0\rangle_1|+\rangle_2 + |1\rangle_1|-\rangle_2)$ is generated (for simplicity, we omit normalization coefficients). For isotropic Heisenberg exchange interactions, where the $(J, J^z; t)$-gate takes the form of $\sqrt{\text{SWAP}}$, we need to prepare the initial state $|+\rangle_1|-\rangle_2$. Applying $\sqrt{\text{SWAP}}$ then leads to $[\sqrt{\text{SWAP}}]_{12}|+\rangle_1|-\rangle_2 = |0\rangle_1 \times \{|0\rangle_2 - i|1\rangle_2\} + i|1\rangle_1\{|0\rangle_2 + i|1\rangle_2\}$. After two single-qubit rotations, $\exp[i\pi(\sigma_2^z - \sigma_1^z)/4]$, $|\Psi\rangle_{12}^C$ is obtained. For XY interactions, the pulse sequence is even simpler: A cluster state $|\Psi\rangle_{12}^C$ of two qubits is simply created by applying the iSWAP gate $[\text{SWAP}]_{12}|+\rangle_{y1}|+\rangle_{y2}$, where $|\pm\rangle_{yi} \equiv (|0\rangle_i \pm i|1\rangle_i)/\sqrt{2}$ is an eigenstate of σ^y.

Our new approach here can save more than half the time over the conventional method during the first step of cluster state generation. For example, when using the two-qubit spin Hamiltonian [12] $H_s^{(ij)} = J\vec{\sigma}_i \cdot \vec{\sigma}_j + (\vec{B}_i \cdot \vec{\sigma}_i + \vec{B}_j \cdot \vec{\sigma}_j)/2$, with $|\vec{B}_i|/2 = J$ for simplicity, a time $t_{\text{CPF}} = \pi/J$ is needed for generating a two-qubit cluster state including single-qubit rotations, using the conventional method. However, using our new method, it takes $\tau_{\sqrt{\text{SWAP}}} + \pi/(4J) = (3/8)(\pi/J)$, which amounts to a ~ 2.7 *speed up* in time for generating a two-qubit cluster state. For spin qubits [5] based on quantum dots, with $J \sim 50\ \mu$eV, the time required for generating a two-qubit cluster state would be ~ 15 psec. For a flux qubit [2] in the rotating frame, the Hamiltonian is $\tilde{H}_{\text{fq}} = H_0 + H_{xy}$, where $H_0 = \sum_{i=1}^2 (\Omega^R/2)(\sigma_i^x\cos\phi_i + \sigma_i^y\sin\phi_i)$, $\quad H_{xy} = J(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y)$, and Ω^R is the half-amplitude of the applied classical field. The time required to generate a two-qubit cluster state (CS) previously [19] was $t_{\text{CS}}^{\text{old}} = (11\pi)/(4\Omega^R) + \pi/(4J) \sim 3$ ns ($\Omega^R \sim J \sim 0.5$ GHz). In the method proposed here, we just need $t_{\text{CS}}^{\text{new}} = \tau_{i\text{SWAP}} \sim 0.25$ ns, which is *over 1 order of magnitude faster.*

The reduction in the number of quantum gates naturally increases the robustness of cluster state generation. Consider a simple case where there are phase errors in each of the one- and two-qubit gates, such that $\theta \rightarrow \theta + \delta_\theta$ and $Jt \rightarrow Jt + \delta_J$, respectively ($\delta_\theta$, $\delta_J \ll 1$). The resulting two-qubit state, denoted by $|\Psi\rangle_{12}^{C(\text{error})}$, is then slightly different from the target two-qubit cluster state. The *fidelity* of this state, if generated by a single $\sqrt{\text{SWAP}}$ with single-qubit rotations starting from $|+\rangle_1|-\rangle_2$, is given by $|{}_{12}^{C}\langle\Psi|\Psi\rangle_{12}^{C(\text{error})}|^2 \sim (1 - 2\delta_\theta^2 - 4\delta_J^2)$, which is higher than the one achieved by the conventional CPF gate in Ref. [18], where $|{}_{12}^{C}\langle\Psi|\Psi\rangle_{12}^{C(\text{error})}|^2 \sim (1 - 2.5\delta_\theta^2 - 4\delta_J^2)$. When an iSWAP gate is used, starting from $|+\rangle_{y1}|+\rangle_{y2}$, the fidelity for two-qubit cluster state generation is $(1 + \cos 2\delta_J)/2 \sim (1 - \delta_J^2)$, which improves greatly over the previous result [19] of $1 - 4\delta_J^2 - [1 - \sin^2(\pi/8)]\delta_\theta^2$. For example, the fidelity increases from 0.81 (0.95) to 0.96 (0.99) for 20% (10%) errors in δ_θ and δ_J.

Generation of larger cluster states with i*SWAP gates.*— The iSWAP gate is not just an efficient substitute in the generation of pair cluster states. It can also simplify the generation of larger cluster states. Consider the case of generating a three-qubit cluster state. Starting with qubits "1" and "2" already in a cluster state, applying an iSWAP gate between qubits "2" and "3" leads to

$$[i\text{SWAP}]_{23}|\Psi\rangle_{12}^{C}|+\rangle_3 = |+\rangle_1|0\rangle_3[|0\rangle_2 - i|1\rangle_2] - i|-\rangle_1|1\rangle_3[|0\rangle_2 + i|1\rangle_2]. \quad (3)$$

Additional $\pi/2$ rotations around the z axis, $R_j^z(\pi/2) \equiv e^{-i\pi\sigma^z/4}$: $|0\rangle_j \rightarrow |0\rangle_j$, $|1\rangle_j \rightarrow i|1\rangle_j$, for $j = 2, 3$ then lead to the "twisted" cluster state shown in Fig. 1(a), which is different from the conventional three-qubit cluster state $|\Psi\rangle_{123}^{C} = |+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|-\rangle_3$ by an exchange of the indices of qubits "2" and "3." This simple example suggests that iSWAP gates can be used to expand cluster states even after the second step mentioned in the previous section. Indeed, using the iSWAP gate with only $R^z(\pi/2)$ rotations, two-qubit cluster states can be connected to make a large cluster chain as shown in Fig. 1(b). Moreover, cluster states in higher dimensions can be generated in steps, as shown in Fig. 1(c). Figure 2(a) shows the numerically obtained fidelity F^{new} of the chain cluster state of Fig. 1(b) assuming Gaussian distribution with a variance σ of phase errors. The fidelity F^{new} for N qubits is obtained as $F^{\text{new}} = |\bar{f}_N|^2$ where $\bar{f}_N$ is an average overlap defined [20] by $\bar{f}_N = \Pi_{j=1}^{N} \int (1/\sqrt{2\pi\sigma^2}) \times e^{-\delta_j^2/(2\sigma^2)}\{{}^{C}\langle\Psi|\Psi\rangle^{C(\text{error})}\}d\delta_j$. Figure 2(b) shows the fidelity increase which compares our new method with the previous one [19]. We can see that the new method provides a higher fidelity. Because the variance σ is typically related to a phase correlation function such as $\sigma^2 \approx \langle\delta_j^2(t)\rangle \propto \tau_{\text{CS}}/T_2$ with dephasing time T_2, as τ_{CS} decreases, the effect of pulse errors is reduced. Thus, the iSWAP gate combines both the power to entangle and the high reliability. Because the iSWAP gate can be decomposed into a product of a CNOT and a SWAP gates [11], it is also *fault-tolerant* [21]. Therefore, we can say that cluster states in the XY model are constructed by *fault-tolerant* iSWAP operations and $R^z(\pi/2)$ rotations.

The above property is only limited to the iSWAP [but not to the $\sqrt{\text{SWAP}}$ and the general $(J, J^z; t)$-gates]. This can be understood and illustrated by applying a $(J, J^z; t)$-gate between two qubits which are in a cluster state $|\Psi\rangle_{12}^{C}$ and a third qubit in an arbitrary superposition state $a_3|0\rangle_3 + b_3|1\rangle_3$, in an attempt to generate a three-qubit cluster state $|\Psi\rangle_{123}^{C}$. To maintain the right number of basis states in the three-qubit superposition state, we need $A = 0$ in Eq. (2). This implies that $B = \pm 1$, which allows the factorization of the three-qubit state as a three-qubit cluster state. But these conditions [$A = 0$, $B = \pm 1$ in Eq. (2)] correspond exactly to an iSWAP gate. A general $(J, J^z; t)$-gate or a $\sqrt{\text{SWAP}}$ gate would have created additional terms so that additional steps would then be needed to clean up the state.

Cluster fusion using iSWAP gates.—The power of the iSWAP gate can be further enhanced in one-way QC when combined with measurements. As an example, we show that a large $(M + N - 1)$-qubit cluster chain can be created by joining two initially separated M-qubit and N-qubit cluster chains (M and N are arbitrary integers) using one iSWAP gate and measurement, similar to the idea of "qubit fusion" described in Ref. [17].

Consider now two initially separated qubit chains that are in cluster states, $|\Psi_L\rangle = \cdots S_{12}S_{23}|+\rangle_1|+\rangle_2|+\rangle_3$ and $|\Psi_R\rangle = S_{45}S_{56}|+\rangle_4|+\rangle_5|+\rangle_6 \cdots$. We connect the end of the first chain and the beginning of the second chain by applying an iSWAP between qubits "3" and "4" (Fig. 3). The resulting state is $[i\text{SWAP}]_{34}|\Psi_L\rangle|\Psi_R\rangle = \cdots S_{12}S_{56}(2|\Theta\rangle)|+\rangle_1|+\rangle_6 \cdots$, where $|\Theta\rangle = [i\text{SWAP}]_{34}|\Psi\rangle_{23}^{C}|\Psi\rangle_{45}^{C}$. Next, we carry out a σ^x measurement on qubit "3" (or qubit "4") so that

$$|\Theta\rangle \rightarrow |+\rangle_2|0\rangle_4\{[1 - (-1)^{s_3}i]|0\rangle_5 + [1 + (-1)^{s_3}i]|1\rangle_5\} - |-\rangle_2|1\rangle_4\{[i - (-1)^{s_3}]|0\rangle_5 + [i + (-1)^{s_3}]|1\rangle_5\}. \quad (4)$$

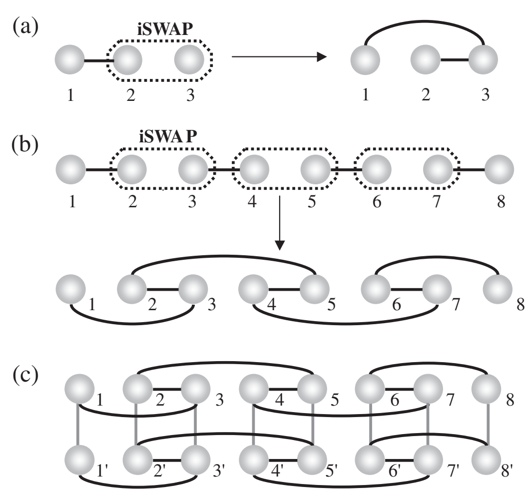

FIG. 1. Illustration of how to generate cluster states with iSWAP gates. Each circle represents a qubit. Each solid line represents a bond by cluster states. (a) A three-qubit cluster state from a two-qubit cluster state. (b) Creation of a chain cluster state. The first step is to create separated two-qubit cluster states. The second step is to apply an iSWAP gate between the 3 two-qubit cluster states. (c) The two-chain cluster states produced in (b) are vertically connected by iSWAPs, thus producing a ladder cluster state. Afterwards, several of these can be connected to produce 2-D cluster states.

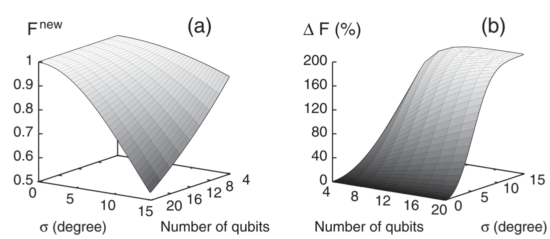

FIG. 2. Replacing CPF gates by iSWAP gates provides higher fidelity. The fidelity of a chain cluster state is plotted versus both the number of qubits and pulse phase errors δ (i.e., $Jt \rightarrow Jt + \delta$, and $\theta \rightarrow \theta + \delta$) assuming Gaussian distribution with a variance σ of phase error δ. (a) Fidelity F^{new} of this proposal. (b) Fidelity increase $\Delta F \equiv (F^{\text{new}} - F^{\text{old}})/[(F^{\text{new}} + F^{\text{old}})/2]$, where F^{old} is the fidelity of the previous method [19]. Note that F^{old} is almost zero when $\sigma \gtrsim 10°$ where the fidelity increase is $\sim$200%.

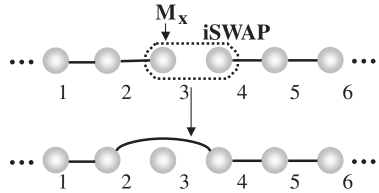

FIG. 3. Connecting two half-infinite cluster states via an *i*SWAP gate and measurement. After applying an *i*SWAP gate between qubits "3" and "4," qubit "3" is measured on the σ^x basis, following appropriate rotations in qubits "4" and "5." Qubit "3" is discarded after the measurement.

Here, $s_3 = 0$ or 1 is the result of the measurement. After applying a rotation $e^{-i\pi/4\sigma_5^z}[e^{i\pi/2\sigma_5^x}]^{s_3}$, we obtain a $(M + N - 1)$-qubit cluster state $\cdots S_{24}S_{45}|+\rangle_2|+\rangle_4|+\rangle_5 \cdots = \cdots\{|+\rangle_2|0\rangle_4|+\rangle_5 + |-\rangle_2|1\rangle_4|-\rangle_5\}\cdots$. The fidelity of this cluster fusion assuming small Gaussian pulse errors is $\sim[1 - \sigma_\theta^2(1 + s_3) - 2\sigma_J^2]$ (σ_θ and σ_J are variances for δ_θ and δ_J). Compared with a previous generation method [19] for flux qubits, the fidelity is improved to $F^{\rm new} \sim 0.96$ from $F^{\rm old} \sim 0.92$ when $\sigma_J = \sigma_\theta = 0.1$.

Connection between distant qubits.—Last but not least, general $(J, J^z; t)$-gates can generate a cluster state for two distant qubits when combined with measurements. Consider a chain of $2N$ qubits in a product of pairwise cluster states $\Pi_{j=1}^N|\Psi\rangle_{2j-1,2j}^C$. A two-qubit cluster state $|\Psi\rangle_{1,2N}^C$ is efficiently obtained as follows: (1) Apply $(J, J^z; t_1)$-gates between qubits "l" and "$l+1$" ($l = 2, 4, \ldots$) that belong to neighboring two-qubit cluster states (t_1 is determined below); (2) Perform σ^x-measurements on all intermediate qubits $2, 3, \ldots, 2N-1$. After these two steps, the $2N$-qubit state becomes $\{u_+|+\rangle_1 + v_-|-\rangle_1\}|0\rangle_{2N} + \{u_-|+\rangle_1 + v_+|-\rangle_1\}|1\rangle_{2N}$, where

$$\begin{pmatrix} u_+ & u_- \\ v_- & v_+ \end{pmatrix} = \Pi_{j=1}^{N-1}\begin{pmatrix} u_{j+} & u_{j-} \\ v_{j-} & v_{j+} \end{pmatrix}, \quad (5)$$

with $v_{j\pm} = \mp(-1)^{s_{2j}+s_{2j+1}}u_{j\mp}$ and $u_{j\pm} = 1 \pm (-1)^{s_{2j}} \times \exp\{-2i[(-1)^{s_{2j}+s_{2j+1}}J - J^z]t_1\}$. Here, s_{2j} and s_{2j+1} are measurement outcomes ($s_{2j}, s_{2j+1} = \{0, 1\}$). The unitarity of this matrix dictates that $\cos\{2[(-1)^{s_{2j}+s_{2j+1}}J - J^z]t_1\} = 0$ [this condition is generally not satisfied by the uniform Heisenberg model ($J = J^z$)]; (3) Finally, depending on the measurement outcome, rotate qubit "1" appropriately, and we obtain $|\Psi\rangle_{1,2N}^C$.

Discussions.—Here we discuss the applicability of our method for experiments. For example, to generate a 2-D cluster state, the present method ($\tau_{\rm CS}^{\rm 2D} \approx 4\tau_{i\rm SWAP} + 3\tau_{\rm rot} \sim 4$ ns) is 3 times faster than the previous method ($\tau_{\rm CS}^{\rm 2D} \approx 2\tau_{\rm CS}^{\rm old} \sim 12$ ns). In one-way QC, at least one round of measurement is necessary [13]. Since all processes must be completed within T_2, such that $\tau_{\rm CS}^{\rm 2D} + 2\tau_m < T_2$ (τ_m is a measurement time), if we take $T_2 \sim 200$ ns [22] and $\tau_m \gtrsim 50$ ns [23], there is very little time for preparing a cluster state. Thus, reducing the time for constructing cluster states is crucially important to be able to implement cluster states in solid-state qubits.

In summary, we have shown an efficient way of generating cluster states directly using either the *i*SWAP gate for the XY model, or the $\sqrt{\rm SWAP}$ gate for the Heisenberg model. The essence of our study is to identify the effects of two-qubit interactions in one-way QC, without encoding logical qubits [10]. Thus, our approach makes one-way QC more feasible for solid-state devices. In particular, the *i*SWAP gate is especially attractive for its simplicity and its ability to entangle, so that it could replace the more widely used and more cumbersome CNOT or CPF gates.

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