Model of coherent emission from disordered arrays of driven Josephson vortices

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(Rceived 28 February 2008; revised manuscript received 23 April 2010; published 25 May 2010)

We propose a mechanism of coherent emission from driven vortices in stacked intrinsic Josephson junctions. In contrast to super-radiance, which occurs only for highly ordered vortex lattices, we predict resonant radiation emission from weakly correlated vortex arrays. Our analytical results for the terahertz wave intensity, resonance frequencies, and the dependence of terahertz emission power on dissipation are in good agreement with the ones obtained by recent simulations.

DOI: 10.1103/PhysRevB.81.174531
PACS number(s): 74.50.+r, 74.25.Gz

I. INTRODUCTION

It has been experimentally observed1–3 and confirmed both analytically4,5 and numerically6,7 that moving Josephson vortices (JVs) emit sub-terahertz electromagnetic radiation. Terahertz radiation has applications in physics, astronomy, chemistry, biology, and medicine.8 This motivates recent proposals9 for terahertz filters,10,11 detectors,12 quantum devices,13,14 and emitters5,6 based on highly anisotropic layered superconductors (e.g., Bi2Sr2CaCu2O8+, see Ref. 5), that can be modeled as coupled intrinsic Josephson junctions (IJJs).

The ultimate challenge in this field is to produce coherent terahertz radiation. It is commonly believed that this goal can be achieved by controlling super-radiance from highly ordered vortex lattices.15,16 A vortex lattice is deemed necessary because the constructive interference of Josephson plasma waves from individual JVs is strongly suppressed by small amounts of disorder. Unfortunately, driven periodic lattices are often very unstable (especially in the presence of impurities, defects, and pinning centers), and moving JVs form either a mixture of coexisting different lattices17 or even disordered arrays.6 Moreover, a broad radiation spectrum by individual vortices results in a broad spectrum of the emitted radiation. Which is suppressed by vortex disorder, in our approach radiation coherence is preserved even if the vortex distribution can become appreciably modulated by the radiation itself for wavelengths shorter than \( \lambda_{\text{EM}} \). The shorter relevant length scale \( \lambda_{\text{G}} \) determines the cross section of the nonlinear vortex core (where the linear approximation \( \sin \varphi \approx \varphi \) with the gauge-invariant phase difference across the junction having a vortex is not valid). According to this picture, for \( \lambda_{\text{G}} \ll \lambda_{\text{EM}} \) and for a sufficiently high vortex density, radiation is emitted through a linear mechanism, as from a JV lattice whereas the vortex-radiation coupling occurs mainly in the inner JV cores. Under these conditions, the magnetic interaction among vortices is much weaker than their interaction with the emitted radiation. Our approach explains the spatial modulation of the JV density numerically found in Refs. 6 and 7. Our analytical estimates, based on a one-dimensional (1D) sine-Gordon model, prove to be in good agreement with their simulations and explain their results.

Let us now summarize a central idea of our approach. Consider a moving JV lattice emitting radiation. This radiation will bounce back and forth the sample edges, like in a laser cavity. This radiation accumulates and creates a standing wave with a wavelength about \( \lambda_{\text{EM}} \). This standing wave modulates the JV density which is now in resonance with the standing wave. This positive feedback enhances the radiation of vortices. Namely, the JV motion emits radiation, which is weaker at first. This radiation bounced inside the sample (acting as a cavity) locks the collective motion of the JVs. This collective motion produces stronger emission. The JVs then interact more strongly with the electromagnetic standing wave, compared with the now much weaker vortex-vortex interaction. Thus, the triangular vortex lattice, produced by...
the vortex-vortex interaction, is finally replaced by a more
organized but still modulated by the radiation and vortex
structure.

II. NONLOCAL SINE-GORDON MODEL

Layered superconductors can be considered as stacks of
strongly interacting IJJs. As the superconducting layers are
only a few nanometers thick, i.e., the interlayer distance \( s \) is
much smaller than the magnetic field penetration depth \( \lambda_{ab} \),
the currents flowing through different junctions are coupled.
On neglecting, for the time being, external drives and inter-
dissipation, a system of stacked IJJs is well described by the
coupled sine-Gordon equations,\(^{19}\)

\[
\left( 1 - \frac{\lambda^2}{s^2} \Delta_x^2 \right) \left( \frac{\varphi^{(n)}}{\omega_p^n} + \sin \varphi^{(n)} \right) - \lambda^2 \varphi_{xx}^{(n)} = 0, \tag{1}
\]

where \( \varphi^{(n)} \) is the gauge-invariant phase difference across the
nth junction. Here, \( \omega_p \) is the Josephson plasma frequency, \( \lambda \)
the London penetration depth \( \lambda = \lambda_{ab} \gamma \) along the layers,
and the operator \( \Delta_x^2 \) is defined by \( \Delta_x^2 = f^{(n+1)} - 2 f^{(n)} - f^{(n-1)} \).

A full analysis of this set of equations is a complicated
problem which requires numerical simulation. However, if
we restrict ourselves to the case of moderate magnetic fields,
when JV cores do not overlap, we can reduce Eq. (1) to an
effective 1D problem. Indeed, as was shown in Ref. \(^{20}\) [see Eqs. (21) and (22) and Fig. 2 there], the phase difference \( \varphi \)
decreases very fast away from the junction where a vortex
located. Thus, a reasonable strategy could consist in neglect-
ing the nonlinear couplings between junctions at a distance
of some \( s \) from the vortex center; on solving the linearized
Eq. (1) for such junctions, one would end up with a few
coupled nonlinear equations for a few junctions in the vicinity
of the vortex center. The case when the nonlinearity was
restricted to one junction only has been considered in Ref. \(^{5}\).
The coupled junction system of Eq. (1) boils down to a 1D
Josephson junction described by a nonlocal sine-Gordon
equation. We assume below that retaining the nonlinear
coupling between more junctions can lead to the same nonlocal
1D sine-Gordon equation with additional noisellike weak per-
turbations.

For simplicity, let us consider the pair of adjacent junc-
tions \( j \) and \( (j+1) \) locating a moving JV. We then reduce the
description of the IJJ stack to a 1D problem by assuming the
nonlinear coupling to be important only for the paired junc-
tions and linearizing Eq. (1) for all other junctions (i.e., for
\( n \neq j, j+1 \)). It is interesting to note that the importance of
the interaction between two neighboring junctions is numerically
well established.\(^{21}\) Moreover, Koshelev\(^{22}\) recently reduced
the multijunction system to two coupled junctions, and this
model reproduced the simulation data in Ref. \(^{21}\) and
interpreted the experimental results of Ref. \(^{3}\).

Following the approach in Refs. \(^{5, 23, 24}\), the equation
for the averaged phase difference across a junction pair
\( \varphi = (\varphi^{(j+1)} + \varphi^{(j)})/2 \) can be written as

\[
\frac{\varphi^{(j)}}{\omega_p^n} + \sin \varphi = \frac{\gamma s}{2\pi} \int dx' K_0 \left( \frac{|x-x'|}{\lambda} \right) \varphi_{x'}(x') + P[\varphi] \sin \varphi, \tag{2}
\]

where \( K_0 \) is the modified Bessel function and

\[ P[\psi] = 1 - \cos \psi \]

with \( \psi = (\varphi^{(j+1)} - \varphi^{(j)})/2 \). The length

\[
\lambda_G = \frac{\gamma s}{2} = \frac{\lambda_{EM}^2}{2\lambda} \tag{3}
\]

defines the size of the JV core. Again, the contribution of the
next-to-neighbor junctions to the dynamics of the tagged JV
weakens fast\(^{20}\) with their distance from the vortex center,
thus, allowing all other nonlinear Eq. (1) to be replaced by an
effective nonlinear medium.

An additional equation for \( \varphi \) can be derived for a pair of
JVs in two adjacent junctions so that the equations for \( \varphi \) and
\( \psi \) form a closed set.\(^{25}\) However, when extending Eq. (2) to
describe the collective motion of \( N \) traveling JVs (randomly
distributed along \( N_l \) stacked IJJs of length \( L \)), the phase \( \varphi \)
may be regarded as a mean-field superposition of the \( n = N/N_l \) individual JVs phases, \( \varphi^{(j)} \), contained in one layer,
only.

In our one-IJJ description, we assume that \( \psi \) is relatively
small; this may be the case, for instance, due to the random
superposition of the vortex dynamics in different junctions.
Anyway, the good agreement between the analytical results
reported here and earlier numerical simulations, validates a
posteriori our assumption. Thus, the functional \( P[\varphi] \) can be
modeled as a spatial perturbation \( \delta + \epsilon P(x) \), where the real
function \( P(x) \) can be either periodic or random in \( x \), depend-
ing on the operating conditions. The constant \( \epsilon \) is a measure
of the strength of the perturbation while the small offset \( \delta \)
can be conveniently eliminated by rescaling the dimensional
parameters \( \omega_p \) and \( \lambda_G \) as appropriate. Here, we assimilate
such a perturbation as an effective quenched Gaussian disor-
der along the IJJs; that is, \( P(x) \) is modeled as a random,
delta-correlated function with

\[
\langle P(x) \rangle = 0, \quad \langle P(x)P(x') \rangle = 2\delta(x-x') \tag{4}
\]

and \( \langle \cdots \rangle \) denoting the average over different disorder real-
izations. The constant \( \epsilon \) will be taken as a perturbation pa-
ter and only effects to leading order in \( \epsilon \) will be consid-
ered. Moreover, deviations from the Gaussian statistics,
implicit in the definition of \( P[\varphi] \), are assumed to be negli-
gible within this approximation.

A. Josephson vortex array

We now introduce dimensionless units by expressing \( x \) and
\( t \) in units of the characteristic length \( \lambda_{EM} \) and the recip-
rocal of the plasma frequency \( \omega_p \), respectively, that is,

\[
x \rightarrow \tilde{x} = x/\lambda_{EM}, \quad t \rightarrow \tilde{t} = \omega_p t.
\]

As a consequence, the system characteristic lengths \( \lambda \) and
\( \lambda_G \) get rescaled as follows:
\[ \lambda \to \tilde{\lambda} = \lambda/\lambda_{EM}, \quad \lambda_{G} \to \tilde{\lambda}_{G} = \lambda_{EM}/\lambda = 1/\tilde{\lambda}, \]

and, of course, \( \lambda_{EM} \to \tilde{\lambda}_{EM} = 1 \). Correspondingly, \( \varphi(x,t) \) is given in units of the magnetic-flux quantum \( \Phi_0 \) and all speeds in units of \( \omega_p \). Hereafter, for the sake of simplicity, we shall only use dimensionless variables and, therefore, omit the “tilde” notation altogether.

The field \( \varphi(x,t) \), corresponding to a dense distribution of JVs traveling with speed \( V \ll 1 \), can be expanded as \( \varphi(x,t) = p(x - V t) - \kappa \sin[p(x - V t)] + \cdots \), \( p = 2\pi p \) and \( \rho = n/L \) denotes the linear JV density with number \( n \) of vortices located along the length \( L \). The linear term in Eq. (5) corresponds to the phase difference of a uniform vortex spatial distribution whereas the periodic correction accounts for a residual phase modulation on the lattice scale \( 1/\rho \), with amplitude \( \kappa \). As in the expansion (5) for \( \varphi(x,t) \), the nonlocal field (2) can be approximated to an effective sine-Gordon equation, where

\[ \frac{1}{\pi \lambda} \int K_0 \left( \frac{|x' - x|}{\lambda} \right) \varphi_{xx}(x') dx' \to c_p^2 \varphi_{xx}, \]

\[ c_p^2 = \left[ 1 + (p\lambda)^2 \right]^{1/2}, \]

and, consistently,

\[ \kappa = (\gamma_v/c_p)^2 \]

with

\[ \gamma_v = (1 - V^2/c_p^2)^{-1/2}. \]

In the regime considered in Refs. 6 and 7, where \( \lambda \gg 1 \) and \( \gamma_v \approx 1 \), the parameter \( c_p \) can be further approximated to \( (p\lambda)^{-1/2} \).

The validity condition for truncating the expansion (5) to its first order, \( \kappa \ll 1 \), or equivalently \( c_p p \gg 1 \), implies a direct core-core interaction; that is, \( 1/\lambda > 1/p \). We recall that here \( 1/\lambda \) represents the size of a vortex core in dimensionless units. Accordingly, for \( \lambda > 1 \), \( c_p \) must be regarded as the maximum velocity of a vortex array in a layered superconductor, to be compared with the maximum dimensionless velocity \( 1/\lambda \) of a single vortex (i.e., \( \omega_p \lambda_{G} \) in dimensional units\(^{2,3} \)).

In the opposite limit, \( p/\lambda \gg 1 \), vortices only weakly interact on the magnetic length scale \( \lambda_{EM} \) (rescaled here to 1); the limiting velocity \( c_p \) grows larger than \( 1/\lambda \) and the JV array becomes unstable.

**B. Radiation mechanism**

The emission of radiation by fast-moving JVs also takes place on the magnetic length scale \( \lambda_{EM} \). The effective phase difference \( \varphi \) associated with an array of JVs moving along an IJJ, thus, obeys the perturbed local sine-Gordon equation,

\[ \varphi_n - c_p^2 \varphi_{xx} + \sin \varphi = -\beta \varphi_{x} - f + \epsilon P(x) \sin \varphi. \]  

Note that, in leading order, \( \varphi \approx p(x - V t) \), as can be seen from Eq. (5). Here, for completeness, we have restored the viscous term \(-\beta \varphi\), and the current-induced drive \( f \), that allow us to control the net JV speed \( V \) (see, e.g., Ref. 24).

Like in the more conventional single sine-Gordon–soliton perturbation schemes\(^{27,28} \), we consider the Ansatz,

\[ \varphi(x,t) \to \varphi(x,t) + \chi(x,t), \]

which, inserted in Eq. (7), yields\(^{26} \)

\[ \chi_n - c_p^2 \chi_{xx} + \left( \cos \varphi \right) \chi = -\beta \chi_x + \epsilon P(x) \sin \varphi, \]

where \( \varphi = 0 \) is the ground state and only terms \( O(\epsilon) \) have been kept. The wave number \( q \) and the angular frequency \( \omega \) of the unperturbed plasmon modes (i.e., for \( \beta = \epsilon = 0 \)) form a continuum spectrum,\(^{27} \) with

\[ \omega^2 = 1 + c_p^2 q^2. \]

However, as for the field [Eq. (5)] with \( p \gg 1 \), the radiation-vortex coupling \( (\cos \varphi) \chi \) becomes negligible, and the plasma-wave dispersion relation can be approximated to \( \omega \approx c_p q \). On introducing the spatial Fourier components of \( P(x) \) and \( \chi(x,t) \), defined by

\[ P(x) = \frac{2}{\pi} \int_{-\infty}^{\infty} P(k) e^{i k x} dk, \quad \chi(x,t) = \frac{2}{\pi} \int_{-\infty}^{\infty} \chi_q(t) e^{i q x} dq, \]

Eq. (9) can be rewritten as\(^{29} \)

\[ \frac{d}{dt} B(q) - \frac{\beta}{2} B(q) = \frac{\epsilon}{2l} e^{i (\omega - p V)t} P(q - p) - (p \to -p), \]

where

\[ B(q) = (\chi_q - i q |\lambda_q| e^{i \omega t} \]

is directly related to the spectral density of the array emission power,

\[ W(q) = \frac{4}{\pi} \frac{d}{dt} |B(q)|^2, \]

that is,

\[ W(q) = \frac{2 \epsilon^2}{\pi} \left[ \frac{|P(q - p)|^2 (\beta/2)}{(\beta/2) + (\omega - p V)^2} + (p \to -p) \right]. \]

Here we use that \( P(x) \) is a real function so that \( P^*(k) = P(-k) \). The notation \( (p \to -p) \) denotes the symmetric term obtained by replacing \( p \to -p \) in the first term inside the square brackets. This means that two waves propagate in opposite directions with the same frequency \( \omega \); for \( |P(q - p)| = |P(q + p)| \), they generate standing plasma oscillations, like those reported in Refs. 6 and 7. To simplify our notation, hereafter we restrict ourselves to JVs driven in one assigned direction, say \( V > 0 \).

The spectral emission power [Eq. (11)] is key to our analysis of a resonant IJJ. The spectrum \( W(q) \) can be easily specialized for any choice of \( P(x) \). In the case of quenched Gaussian disorder, see Eq. (4),

\[ \langle |P(q)|^2 \rangle = \frac{1}{8} \]

so that on disorder-averaging Eq. (11), we obtain the IJJ spectral emission power per unit of length,
This spectrum holds for $\beta \ll \omega$ or, equivalently, for $V \gg \beta/p$, and has a sharp resonance maximum,

$$w(\omega) = \frac{\epsilon^2}{4\pi} \left[ \frac{\beta^2/2}{(\beta^2/2 + (\omega - pV)^2)} + (\omega \rightarrow -\omega) \right]. \tag{13}$$

C. Vortex dynamics

Subject to a drive $f$ produced by an externally applied electrical current, the vortices in an IJJ flow with an average speed $V$ and, simultaneously, their cores interact with the electromagnetic waves they radiate. For the relatively weak vortex-core repulsion, $1/\lambda \approx 1/p$, simulated in Refs. 6 and 7, we expect that the vortex array can be modulated, both in space and time, by the resonant plasma modes.

To express the average speed $V$ of a JV array with $p/\lambda \gg 1$ as a function of the drive $f$, from Eq. (7) we derive the energy balance equation per unit of length of radiating IJJ,

$$w(V) + \beta(pV)^2 = pf. \tag{16}$$

Equation (16) tells us that the rate at which the drive pumps energy into the system (right-hand side), must be equilibrated by the radiative, $w(V)$, and the viscous loss, $\beta(pV)^2$, of the soliton array $\varphi(x,t)$ (left-hand side).

For $V \gg \beta/p$, the total emission power of the radiating sine-Gordon solitons,

$$w(V) = \frac{\epsilon^2}{2} \tag{17}$$

is computed by integrating the spectral emission power [Eq. (11)]; solving the ensuing Eq. (16) with respect to $V$, we obtain

$$V(f) = f \pm (f^2 - 2\beta \epsilon^2)^{1/2} / 2\beta p, \tag{18}$$

where only the rising branch with the + sign is stable.\(^{29}\) Therefore, the observable velocity-drive characteristic $V(f)$ is expected to show a step at

$$f_{\text{th}} = (2\beta)^{1/2} \epsilon \tag{19}$$

and to grow linearly with $f$ for $f > f_{\text{th}}$, when the radiation loss becomes negligible, namely,

$$V = \frac{f}{\beta p}. \tag{20}$$

Note that, at variance with an emitting JV lattice,\(^{29,30}\) no multiple hysteretic steps in the $V(f)$ are predicted. Indeed, the condition $f > f_{\text{th}}$ simply implies that the effective phase $\varphi$ is not pinned by disorder;\(^{31}\) for $f \leq f_{\text{th}}$, instead, the JV array can move only by creeping, namely, through the nucleation and the subsequent migration of array defects.\(^{32,33}\) Creeping is likely responsible for the smooth low-current $J-V$ characteristics shown in Fig. 4 of Ref. 6. Moreover, in the linear regime [Eq. (20)], the wavelengths $\lambda_c$ of the emitted radiation are expected to be much shorter than the length $L$ of the IJJ (see below) so that corrections due to the appropriate standing-wave periodic boundary conditions are on the order of $\lambda_c/L$.

A vortex is sensitive to the radiation field only when the wavelengths $\lambda_c$ excited in the IJJ are larger than its size. For the parameter choice of Refs. 6 and 7, this can only occur on the JV core scale $\lambda_G$ because $\lambda_G \approx \lambda_c$.

The interaction between the radiation standing wave, say, $\chi(x,t) = \chi_0 \cos(qx)\cos(\omega t + \phi)$ \tag{21} and a single JV solution of the nonlocal sine-Gordon Eq. (2),

$$\varphi(x) = \pi + 2 \arctan(\lambda x) \tag{22}$$

(both in dimensionless units) is well described by the non-relativistic quasiparticle approach of Ref. 15. The JV center of mass with coordinate $X(t)$ is subject to an oscillating sinusoidal trap,

$$\ddot{X} = -\beta \dot{X} + \chi_0 \frac{q^2}{\lambda} e^{-q\lambda} \cos(qX) \cos(\omega t + \phi) \tag{23}$$

with an amplitude which is exponentially suppressed at short wavelengths, i.e., for $q/\lambda \gg 1$. However, for sufficiently large trap amplitudes, the vortices in each layer get spatially distributed with wave vector $q$.

III. COMPARISON WITH NUMERICAL RESULTS

The results in Refs. 6 and 7 can be easily analyzed within the above theoretical framework. To make contact with their numerical data, one must express all lengths in units of $\lambda$, with $\lambda = 200 \mu m$; the velocities in units of the light speed in the dielectric $c = c_0 / \sqrt{\epsilon_r}$, where $c_0$ is the speed of light in vacuum and $\epsilon_r = 10$ is the simulated dielectric constant; the forces in units of $J/J_c$, where $J_c$ is the critical JJ current and $J$ is the superconducting current across the IJJs; and the angular frequencies in units of the plasma gap frequency $\nu_p = \omega_p / 2\pi = \lambda = 0.47 \times 10^{12}$ Hz. Moreover, the actual layer JV density is $\rho = n/L = 0.6 \pm 5 \times 10^9 \mu m^{-1}$ with $L = 100 \mu m$, the layer thickness is $s = 15 \AA$, and the penetration length ratio $\gamma = 500$. For this choice of numerical parameters, the length scales we introduced in the previous section read, respectively, $\lambda_G = 0.38 \mu m$, $\lambda_{EM} = 8.7 \mu m$, and $1/p = 0.27 \mu m$.

First, we note that the simulations of Refs. 6 and 7 correspond to the physical condition where $\lambda_G \ll \lambda_r$. As for the resonant modes $\chi_0 \approx \epsilon$, see Eq. (14), the amplitude of the driving force in Eq. (23) turns out to scale like $\epsilon(\lambda/\lambda_G)^{1/2}$, which is strong enough to drag a JV against the disorder field [Eq. (4)] and the array of restoring forces. This explains the disordered spatial distribution of the emitting JVs, which, far from forming any ordered lattice, seem rather to get trapped by the plasma standing waves. In spite of the coherent nature of the plasma radiation, the vortex distributions in each IJJ can differ from one another because of the intrinsic disorder brought about by the layer-layer coupling.
In Fig. 1, we compare the current-voltage characteristics from simulation, reported in Fig. 4 of Ref. 6, with the force-velocity (f-V) curve of Eq. (20). In the units of Ref. 6,

\[
\frac{J}{J_c} = 2\pi\beta\left(\frac{\rho\lambda V}{c}\right) = 2\pi\beta\frac{\nu}{\nu_p},
\]

where \(\nu = \rho \lambda V/c\) is the flux-flow voltage across a IJJ layer and \(\nu_p = \nu_0 \Phi_0\) with \(\Phi_0 = h/2e\) denoting the flux quantum. The agreement is quite good in the linear regime whereas the depinning threshold [Eq. (19)] is clearly visible for \(J/J_c = 0.2\), which, in our units, corresponds to setting \(\epsilon = 1\).

The resonant plasma radiation is investigated in Refs. 6 and 7 on the linear branch of the J-V characteristics. Three plots of the plasma standing waves, two in Ref. 6 and one in Ref. 7, are shown for different \(J/J_c\); from there we read out the corresponding resonance wavelengths \(\lambda_c\). Furthermore, the resonance frequencies \(\nu_c = \pm 2\pi\omega_0\) are either given explicitly in the text or shown in the figure (see, e.g., Fig. 5 of Ref. 6). The product \(\nu_c\) appears to define a Whart velocity, denoted here by \(c_S\), independent of the simulation parameters \(J/J_c\) and \(\beta\), that is,

\[
\frac{c_S}{c} = 0.04.
\]

In view of our radiation mechanism [Eq. (9)], the ratio \(c_S/c\) can be identified with \(c_p\) in Eq. (6); accordingly, for \(\lambda \gg 1\) and \(\gamma_\nu = 1\),

\[
\frac{c_S}{c} \approx \frac{1}{\sqrt{\lambda p}}
\]

is predicted to be on the order of 0.036, which is reasonably close to the result in Refs. 6 and 7, given the accuracy of the data available.

IV. CONCLUSIONS

We propose a mechanism of coherent radiation from the moving Josephson vortices in layered superconductors. We show that due to the two-scale structure of Josephson vortices, they radiate terahertz radiation on a characteristic scale \(\lambda_{EM}\) which is much larger than the Josephson vortex-core size \(\lambda_c \sim \gamma_\nu\). Among all emitted waves, only standing modes in the sample (working as a cavity) survive. These standing modes produce modulation of the density of JVs. This, in

\[
V = \frac{\nu_c}{\rho}.
\]

This equation holds for all different choices of the simulation parameters presented in Refs. 6 and 7. Note that the measured JV speeds are relatively small, \(V \ll c\), as assumed in our nonrelativistic treatment of Eq. (9), where \(\gamma_\nu = 1\). Moreover, when combined with Eq. (24), this equation yields the dependence of \(\nu_c\) on the simulation control parameters \(J/J_c\) and \(\beta\). The ensuing law

\[
\frac{\nu_c}{\nu_p} = \frac{1}{2\pi\beta J_c}
\]

closely matches all spectral resonance peaks reported in Ref. 6, as shown in Fig. 2. Note that combining Eqs. (24) and (28) yields the simple \(\beta\)-independent relation

\[
\frac{\nu_c}{\nu_p} = \frac{V}{\nu_p}.
\]

Finally, we notice from Eqs. (14) and (20) that \(w^{\max}\) is proportional to \(\epsilon/\beta\) and \(V\) is proportional to \(f/\beta\); as a consequence, one would expect that on decreasing \(\beta\) the IJJ spectral emission band shifts to lower \(J/J_c\) while growing in intensity, both inversely proportional to \(\beta\). This is exactly the dependence displayed in Fig. 6 of Ref. 6.
Thus cause the suppression of terahertz radiation. In this frequency domain. Indeed, the now prevailing interpretation is that JVs ought to be considered as perturbing degrees of freedom, which destroy the layer coherence and thus cause the suppression of terahertz radiation. In this study, however, we reach the conclusion that, under appropriate conditions, applied magnetic fields do help amplify and tune terahertz emission. This interesting result is also consistent with the recent systematic studies in Ref. 18.

ACKNOWLEDGMENTS

F.N. acknowledges partial support from the National Security Agency (NSA), Laboratory for Physical Sciences (LPS), Army Research Office (ARO), National Science Foundation (NSF) under Grant No. 0726909. F.N. and S.S. acknowledge partial support from JSPS-RFBR under Grant No. 06-02-91200. S.S. acknowledges partial support from the UK EPSRC via Grants No. EP/D072581/1 and No. EP/F005482/1.