Surface plasma waves across the layers of intrinsic Josephson junctions

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(Received 10 June 2008; published 8 August 2008)

We predict surface electromagnetic waves propagating across the layers of intrinsic Josephson junctions. We find the spectrum of the surface waves, and study the distribution of the electromagnetic field inside and outside the superconductor. The profile of the amplitude oscillations of the electric-field component of such waves is peculiar: initially, it increases toward the center of the superconductor and, after reaching a crossover point, decreases exponentially.

DOI: 10.1103/PhysRevB.78.054502 PACS number(s): 74.78.Fk, 74.50.+r, 74.72.Hs

I. INTRODUCTION

A very recent burst of interest in layered high T_c is due to the discovery of a generation of superconductors based on FeAs layers, $LaO_{1-x}F_xFeAs$,[1] and other such systems that possess a similar structure. The conventional representative of layered high T_c, $Bi_2Sr_2CaCu_2O_{8+x}$ (Bi2212), and $Tl_2Ba_2CaCu_2O_{8+x}$ (Tl2212) have a structure of superconducting CuO_2 layers with Josephson coupling between them. The layered structures of high T_c favor the propagation of so-called Josephson plasma waves,[2,3] propagating with frequencies above the Josephson plasma frequency ω_p. The gap structure of the Josephson plasma excitation spectra has been experimentally observed from measurements of the Josephson plasma resonance.[3] Josephson plasma waves can exhibit remarkable features, including the slowing down of light and self-focusing effects,[4] and are linked to applications in the THz frequency range.[5]

It was recently predicted that the layered structure of high-T_c superconductors allows the propagation of surface waves.[6] Such waves propagate below the Josephson plasma frequency ω_p and propagate in the vicinity of the superconducting surface *along* the layers. In this paper we show that there exist surface electromagnetic TM waves propagating *across* the superconducting layers.

The electric, $\mathbf{E}=\{E_x,0,E_z\}$, and magnetic, $\mathbf{H}=\{0,H,0\}$, components of the electromagnetic waves are proportional to $\exp[i(qx-\omega t)]$, and decay both in the vacuum and inside the layered superconductor. Such surface waves across the layers are strongly influenced by an external magnetic field $\mathbf{h}_0$ applied along the superconducting layers. Here we describe the propagation across the layers of such surface waves and estimate the influence of an external magnetic field on their spectrum.

II. MODEL AND RESULTS

Consider an interface between the vacuum ($z>0$ in Fig. 1) and a layered superconductor ($z\leq 0$). Let the c axis of the superconductor be along the x axis so that the vacuum-superconductor interface lies in the xy plane and an external magnetic field $\mathbf{h}_0$ is applied along the y axis, parallel to the superconducting layers (see Fig. 1). The electromagnetic field inside the layered superconductor ($z<0$) is determined by the distribution of the gauge-invariant phase difference $\varphi(x,z,t)$ of the order parameter between neighboring layers. It is described by a set of coupled sine-Gordon equations,[7] which, in the continuum limit (see, e.g., Ref. 8), can be written as

$$\left(1-\lambda_{ab}^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2\varphi}{\partial t^2}+\omega_p^2\sin\varphi\right)-\lambda_c^2\omega_p^2\frac{\partial^2\varphi}{\partial z^2}=0. \quad (1)$$

Here we neglect the relaxation terms caused by the quasiparticle conductivity; λ_{ab} and $\lambda_c=c/\omega_p\sqrt{\varepsilon}$ are the magnetic penetration depths across and along layers, respectively, while $\omega_p=(8\pi eDj_c/\hbar\varepsilon)^{1/2}$ is the Josephson plasma frequency. The latter is determined by the critical Josephson current j_c, the interlayer dielectric constant ε, and the spatial period of the layered structure D. The gradient of the superconducting phase is related to the magnetic field $h(z)$, directed along y, as (e.g., Ref. 9)

$$-\frac{\partial\varphi}{\partial z}=\frac{2\pi D}{\Phi_0}\left(1-\lambda_{ab}^2\frac{\partial^2}{\partial x^2}\right)h(z). \quad (2)$$

Using Eq. (2) as a boundary condition at $z=0$, we solve Eq. (1) to obtain the dependence of the superconducting phase $\varphi(z)$ on the distance z from the interface under a homogeneous stationary magnetic field h_0,

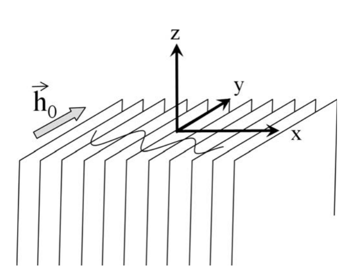

FIG. 1. Interface between vacuum ($z>0$) and a layered superconductor ($z<0$) in an external magnetic field $\mathbf{h}_0$. A layered high-T_c superconductor has a structure of superconducting layers coupled via intrinsic Josephson junctions.

$$\varphi_0(z) = -4 \arctan\left[\exp\left(\frac{z-z_0}{\lambda_c}\right)\right], \quad z<0, \tag{3}$$

where a positive constant $z_0>0$ is defined by the boundary condition

$$-\left.\frac{\partial\varphi_0(z)}{\partial z}\right|_{z=0} = \frac{2\pi D}{\Phi_0}h_0,$$

so that

$$z_0 = \lambda_c \operatorname{arccosh}\left(\frac{h_c}{h_0}\right), \quad \text{where} \quad h_c = \frac{\Phi_0}{\pi D\lambda_c}.$$

Here we study the case of relatively small fields when h_0 is less than the critical value h_c and Josephson vortices do not penetrate the superconductor.

A. Surface waves at $h_0 \lesssim h_c$

We take into account the t and x dependences of superconducting phase $\varphi(x,z,t)$ as small variations around the stationary configuration $\varphi_0(z)$ given by Eq. (3). Assuming

$$\varphi(x,z,t) = \varphi_0(z) + \varphi_w(x,z,t),$$

as a sum of the static and wave terms, we linearize Eq. (1) to obtain

$$\left(1-\lambda_{ab}^2\frac{\partial^2}{\partial x^2}\right)\left[\frac{\partial^2\varphi_w}{\partial t^2} + \omega_p^2\varphi_w \cos\varphi_0(z)\right] - \lambda_c^2\omega_p^2\frac{\partial^2\varphi_w}{\partial z^2} = 0.$$

Substituting $\varphi_w(x,z,t)=\xi(z)\exp(iqx-i\omega t)$, we derive an ordinary differential equation for $\xi(z)$,

$$-\frac{\lambda_c^2}{(1+q^2\lambda_{ab}^2)}\frac{d^2\xi}{dz^2} + \left[1-\Omega^2 - \frac{2}{\cosh^2[(z-z_0)/\lambda_c]}\right]\xi = 0, \tag{4}$$

where we have introduced $\Omega \equiv \omega/\omega_p$. Here we are interested in a solution decaying inside the layered superconductor: $\xi(z)\to 0$ at $z\to -\infty$. Equation (4) has the form of a one-dimensional (1D) Schrödinger equation for a particle with energy

$$E(\Omega) = \Omega^2 - 1,$$

in a potential

$$U(z) = -\frac{2}{\cosh^2[(z-z_0)/\lambda_c]}.$$

The bound states corresponding to the waves decaying at $z \to -\infty$ can exist for negative energies $E(\Omega)<0$, i.e., for $\Omega<1$. One can write an exact solution of Eq. (4) in terms of the hypergeometric function,

$$\xi(z) = [1-\zeta(z)^2]^{\epsilon/2}F\left[\epsilon - s, \epsilon + s + 1, \epsilon + 1, \frac{1+\zeta(z)}{2}\right],$$

where

$$\zeta(z) = \tanh\left(\frac{z-z_0}{\lambda_c}\right),$$

and

$$s = \frac{1}{2}[-1+\sqrt{1+8(1+q^2\lambda_{ab}^2)}],$$

$$\epsilon = \sqrt{(1-\Omega^2)(1+q^2\lambda_{ab}^2)}.$$

We have studied the behavior of the spectrum of surface Josephson plasma waves by means of the WKB approximation valid for

$$Q \equiv q\lambda_{ab} \gg 1.$$

If the inequalities

$$0 < (1-\Omega^2) < \frac{2h_0^2}{h_c^2}$$

are satisfied, there exists a classical turning point $z=z_t$. According to Eq. (4), this point is defined by the equation $E(\Omega)=U(z_t)$ that leads to

$$1-\Omega^2 = \frac{2}{\cosh^2[(z_t-z_0)/\lambda_c]}.$$

The "wave function" $\xi(z)$ oscillates in the region $z_t<z<0$ and exponentially decays at $-\infty<z<z_t$. After the procedure of matching the wave functions at the turning point by the connecting formulas known from quantum mechanics, we obtain the quasiclassical expression for $\xi(z)$. For the classically allowed region $z_t<z<0$, we have

$$\xi(z) \simeq \frac{A}{[E(\Omega)-U(z)]^{1/4}} \times\cos\left[\frac{\sqrt{1+Q^2}}{\lambda_c}\int_{z_t}^{z} dz'\sqrt{E(\Omega)-U(z')} - \frac{\pi}{4}\right], \tag{5}$$

and the underbarrier wave function for $-\infty<z<zt$,

$$\xi(z) \simeq \frac{A/2}{[U(z)-E(\Omega)]^{1/4}}\exp\left[\frac{\sqrt{1+Q^2}}{\lambda_c}\int_{z_t}^{z} dz'\sqrt{U(z)-E(\Omega)}\right].$$

The waves $\xi(z)\exp(iqx-i\omega t)$, corresponding to the solution [Eq. (5)], are Josephson plasma waves running along the interface of the layered superconductor and *across* its layers. From Eq. (2) we obtain the relation of $\xi(z)$ to amplitudes of the electromagnetic field components in the layered superconductor. For the magnetic-field distribution inside the sample, we obtain

$$H(z) = -\frac{h_c\lambda_c}{2(1+q^2\lambda_{ab}^2)}\frac{d\xi}{dz}. \tag{6}$$

From the ac Josephson relation, Maxwell equations, and substituting $\lambda_c=c/\omega_p\sqrt{\epsilon}$ and $h_c=\Phi_0/\pi D\lambda_c$, we obtain the amplitudes of the electric-field components:

$$E_x(z) = \frac{\Phi_0}{2\pi cD}(-i\omega)\xi(z) = -i\frac{h_c\Omega}{2\sqrt{\varepsilon}}\xi(z),$$

and

$$E_z(z) = \frac{\lambda_{ab}^2 q\Omega}{\lambda_c\sqrt{\varepsilon}} H(z).$$

Using the Maxwell equations in vacuum, we obtain the dependence of the electromagnetic field components outside the superconductor. This gives an exponential decay for positive z,

$$H^{\mathrm{vac}}, E_x^{\mathrm{vac}}, E_z^{\mathrm{vac}} \propto \exp(iqx - i\omega t - k_v z), \quad z > 0,$$

with the decay constant $k_v = \sqrt{q^2 - \omega^2/c^2} > 0$ for $q > \omega/c$. In the WKB regime, when $Q \gg 1$, we obtain

$$k_v = \frac{1}{\lambda_{ab}}\sqrt{Q^2 - \frac{\lambda_{ab}^2\Omega^2}{\lambda_c^2\varepsilon}} \simeq \frac{Q}{\lambda_{ab}},$$

as $\lambda_{ab}/\lambda_c\varepsilon \ll 1$. Because of the large $\lambda_c/\lambda_{ab} \gg 1$ and $Q \gg 1$, the surface wave decays very quickly in vacuum on the scale $\sim\lambda_{ab}/Q$, which is much smaller than λ_c.

The ratio of amplitudes for the tangential electric and magnetic fields at the interface $z=+0$, above the surface of superconductor, is

$$\frac{E_x^{\mathrm{vac}}}{H^{\mathrm{vac}}} = \frac{ic}{\omega}k_v = \frac{ic}{\omega}\sqrt{q^2 - \omega^2/c^2}. \tag{7}$$

In order to derive the dispersion relation for surface Josephson plasma waves, we calculate the ratio $E_x(0)/H(0)$ in the superconductor using Eqs. (6) and (5), and then equate this ratio to the vacuum impedance [Eq. (7)]. This gives

$$\frac{\lambda_{ab}\Omega^2\sqrt{1+Q^2}}{\lambda_c\varepsilon\sqrt{\left(Q^2 - \frac{\lambda_{ab}^2\Omega^2}{\lambda_c^2\varepsilon}\right)\left[\frac{2h_0^2}{h_c^2} + E(\Omega)\right]}} = \tan\left[-\frac{\sqrt{1+Q^2}}{\lambda_c}\int_{z_t}^{0} dz'\sqrt{E(\Omega) - U(z')} + \frac{\pi}{4}\right],$$

with $Q = q\lambda_{ab}$. Because $\lambda_{ab}/\lambda_c\varepsilon \ll 1$, this relation can be simplified while disregarding the vacuum contribution. Thus,

$$\frac{\sqrt{1+Q^2}}{\lambda_c}\int_{z_t}^{0} dz'\sqrt{E(\Omega) - U(z')} = \pi\left(n + \frac{1}{4}\right), \quad n = 1,2,3\ldots. \tag{8}$$

A set of dispersion curves for $n=1,\ldots,10$ is shown in Fig. 2 for two different values of the external magnetic field, $h_0/h_c=0.5$ [Fig. 2(a)] and $h_0/h_c=0.9$ [Fig. 2(b)].

The dispersion relation [Eq. (8)] corresponds to surface waves of an unusual nature. The electromagnetic field does not decrease monotonically into the superconductor. Instead, the number of oscillations of $\xi(z)$ with increasing amplitude occurs before the exponential decrease. An example of the oscillating field $E_x(z)$ distribution in surface Josephson plasma waves with parameters $h_0/h_c=0.5$, $n=10$, and $Q=50$ is shown in Fig. 3.

B. Surface waves at small h_0

Equation (8) does not describe all branches of the spectrum of surface Josephson plasma waves. For relatively

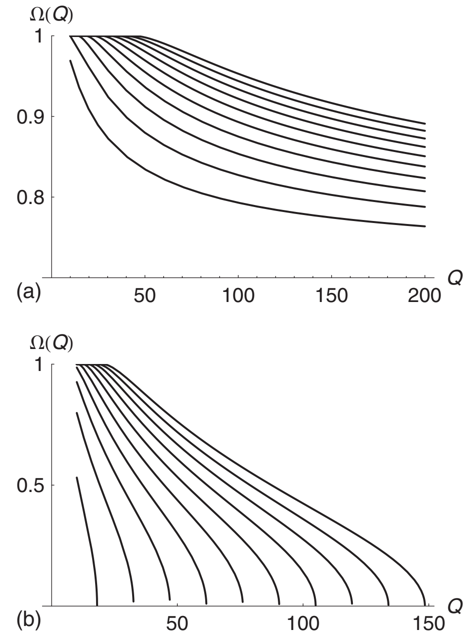

FIG. 2. (a) Dispersion curves for surface Josephson plasma waves at an external magnetic field $h_0/h_c=0.5$. The branches correspond to $n=1,2,\ldots,10$ from bottom to top. The spectrum is limited from below by the value $\Omega > (1-2h_0^2/h_c^2)^{1/2} \simeq 0.71$. (b) Dispersion curves for surface Josephson plasma waves at an external magnetic field $h_0/h_c=0.9$. The branches correspond to $n=1,2,\ldots,10$, from left to right.

small magnetic fields and small frequency Ω, when the inequality

$$E(\Omega) = \Omega^2 - 1 < -\frac{2h_0^2}{h_c^2}$$

is fulfilled, the whole superconducting area $z<0$ is classically disallowed. In this case, the quasiclassical wave function $\xi(z)$ exponentially decreases starting from the boundary,

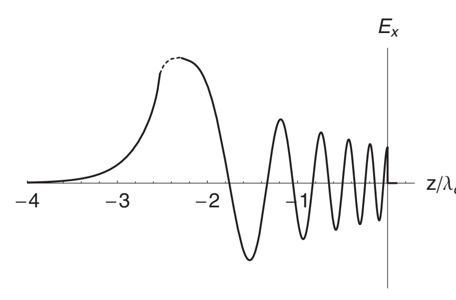

FIG. 3. Spatial distribution of the field $E_x(z)$ of surface Josephson plasma waves calculated for $h_0/h_c=0.5$, $n=10$, and $Q=50$. The dashed curve corresponds to the classical turning point $z=z_t$, where the WKB approximation fails. $z<0$ corresponds to a plasma wave in the layered superconductor while $z>0$ corresponds to vacuum. As seen from the figure, the amplitude of the surface wave decays very quickly in vacuum on a scale much smaller than λ_c. Notice that the number of nodes of the distribution $E_x(z)$ corresponds to the integer parameter n in Eq. (8).

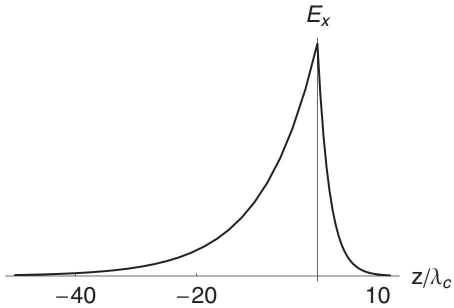

FIG. 4. Spatial distribution of the field $E_x(z)$ of surface Josephson plasma waves calculated for $h_0=0$ and $Q=0.001$. $z<0$ corresponds to a plasma wave in the layered superconductor while $z>0$ corresponds to the vacuum.

$$\xi(z) \simeq \frac{A}{[U(z)-E(\Omega)]^{1/4}} \times \exp\left[\frac{\sqrt{1+Q^2}}{\lambda_c}\int_0^z dz'\sqrt{U(z')-E(\Omega)}\right].$$

A typical distribution of the wave amplitude for $Q=0.001$ and $h=0$ is shown in Fig. 4. The corresponding dispersion relation can be written as

$$\sqrt{Q^2-\frac{\lambda_{ab}^2\Omega^2}{\lambda_c^2\varepsilon}} = \frac{\lambda_{ab}\Omega^2\sqrt{1+Q^2}}{\lambda_c\varepsilon\sqrt{-E(\Omega)-2h_0^2/h_c^2}}. \quad (9)$$

This dispersion curve is shown in Fig. 5 for several values of the magnetic field, $h_0/h_c=0,0.1,\ldots,0.7$. We have used the parameters: $\lambda_c/\lambda_{ab}=500$ and $\varepsilon=20$. Note that Eq. (9) is valid not only in the WKB approximation and is applicable even in the absence of an external magnetic field.

III. CONCLUSION

We have predicted Josephson plasma waves propagating along the surface of anisotropic high T_c and *across* its superconducting layers. There exist different modes of such waves ordered by the number of nodes n of the amplitude of the electric-field component inside the superconductor. The profile of the surface waves is unusual: first, the amplitude of the oscillations increases inside the superconductor and, after reaching the last node, decreases exponentially.

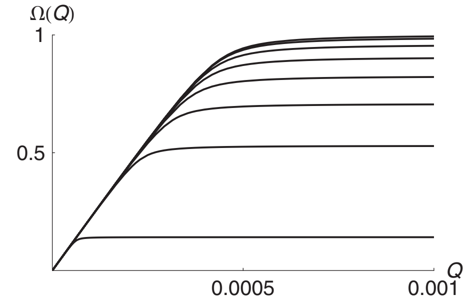

FIG. 5. Dispersion curve for surface Josephson plasma waves at different values of an external magnetic field given by Eq. (9). The branches correspond to magnetic-field values $h_0/h_c=0,0.1,0.2,\ldots,0.7$, numbered from top to bottom.

ACKNOWLEDGMENTS

We gratefully acknowledge partial support from the National Security Agency (NSA), Laboratory Physical Science (LPS), U.S. Army Research Office (USARO), National Science Foundation (NSF) Grant No. EIA-0130383, Core-to-Core (CTC) program supported by the Japan Society for the Promotion of Science (JSPS), and JSPS-Russian Foundation for Basic Research (RFBR) via Contract No. 06-02-91200. Also, we acknowledge support from the Ministry of Science, Culture and Sports of Japan via the Grant-in-Aid for Young Scientists Grant No. 18740224, the EPSRC-GB via Contracts No. EP/D072581/1, EP/E042589/1 and No. EP/F005482/1, and ESF network program "Arrays of Quantum Dots and Josephson Junctions."

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