Excitation of surface Josephson plasma waves in layered superconductors

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This is a theoretical study of the resonant suppression of the specular reflection of terahertz waves in layered superconductors due to the excitation of surface Josephson plasma waves (SJPWs). Here, we consider in detail the specific case of SJPW excitations by evanescent electromagnetic waves via the attenuated total reflection of incident waves in a dielectric prism. We also derive the dispersion relation for surface waves propagating along the vacuum-superconductor interface parallel to the **ab** plane. We show that, due to the excitation of SJPWs, the reflectivity of the incident wave depends resonantly on both its frequency and incident angle. We find the optimal conditions for the best matching of the incident wave and SJPWs, as well as for the total suppression of the specular reflection.

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I. INTRODUCTION

The unusual optical properties of layered superconductors, including reflectivity and transmissivity, caused by the excitation of Josephson plasma waves (JPWs), were studied in, e.g., Refs. 1–5. Most of the previous work on this problem has focused on propagating waves in the frequency range *above* the gap of the JPW spectrum, i.e., above the Josephson plasma frequency: $\omega > \omega_J$. A similar gapped spectrum also appears in solid state plasmas.[6] In such situations, the surface electromagnetic waves[6–10] with frequencies *below* the gap can propagate along the sample boundary.

In layered superconductors, there has been a recent prediction[11] of the existence of surface Josephson plasma waves (SJPWs) in the terahertz and subterahertz frequency range, below the Josephson plasma frequency. In general, surface waves play a very important role in many fundamental resonance phenomena, such as the Wood anomalies in the reflectivity[7–9,12] and transmissivity[10,13–19] of periodically corrugated metal and semiconductor samples. Therefore, it is important to describe how to excite surface waves in layered superconductors and to study the resonances associated with these surface waves.

In this paper, we show that the surface Josephson plasma waves at the boundary between the vacuum and a layered superconductor can be excited via the so-called "attenuated-total-reflection method" (Otto configuration[7–9,20,21]) in a frequency range below ω_J, i.e., by an *evanescent* wave in the vacuum gap between the superconductor and a dielectric prism. The dispersion relation for surface waves propagating along the vacuum-superconductor interface is derived for the geometry when the sample surface coincides with the **ab** plane, i.e., the SJPW propagates along the superconducting layers (see Fig. 1). Due to the resonant excitation of the SJPW, the reflectivity of the incident wave depends sharply on its frequency and incident angle. This resonance effect can be useful for *filtering and detecting* terahertz and subterahertz radiation using layered superconductors. We find the optimal conditions for the best matching of the incident wave and the SJPWs, as well as for the total suppression of the specular reflection.

II. DISPERSION RELATION FOR SURFACE JOSEPHSON PLASMA WAVES

Consider a plane-shaped interface (the xy plane) separating the vacuum ($z>0$ in Fig. 1) and a layered superconductor ($z \leqslant 0$). We study a linear surface transverse-magnetic monochromatic electromagnetic wave propagating along the x axis with the electric, $\mathbf{E}=\{E_x,0,E_z\}$, and magnetic, $\mathbf{H}=\{0,H,0\}$, fields proportional to $\exp[i(qx-\omega t)]$ and decaying into both the vacuum and layered superconductor away from the interface $z=0$. When $q>\omega/c$, the Maxwell equations yield an exponential decay of the wave amplitude into the vacuum,

$$H^{\text{vac}}, E_x^{\text{vac}}, E_z^{\text{vac}} \propto \exp(iqx - i\omega t - k_v z), \quad z>0, \tag{1}$$

with the decay constant

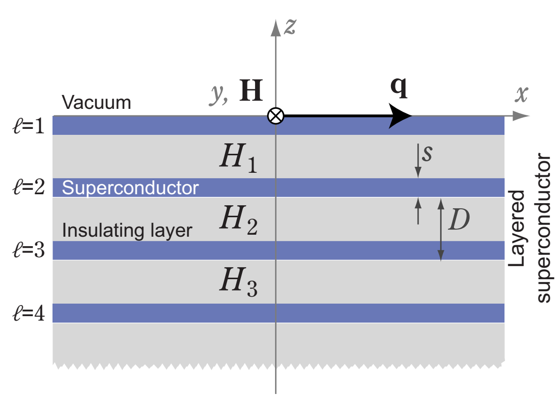

FIG. 1. (Color online) Geometry for studying surface waves.

$$k_v = \sqrt{q^2 - \frac{\omega^2}{c^2}} > 0.$$

Moreover, the Maxwell equations provide the ratio of amplitudes for the tangential electric and magnetic fields at the interface $z=+0$ (i.e., right above the sample surface):

$$\frac{E_x^{\mathrm{vac}}}{H^{\mathrm{vac}}} = \frac{ic}{\omega}k_v = \frac{ic}{\omega}\sqrt{q^2 - \frac{\omega^2}{c^2}}. \qquad (2)$$

The spectrum of the SJPWs depends on the relative orientation of the crystallographic axes and the sample surface. Below, we consider the case when the crystallographic **ab** plane coincides with the xy plane and the **c** axis is along the z axis. Superconducting layers are numbered by the integer $l=1,2,3,\ldots$, as shown in Fig. 1. The thickness s of the superconducting layers is small compared with the spatial period D.

The electromagnetic field inside the layered superconductor, $z<0$, is defined by the distribution of the gauge-invariant phase difference $\varphi_l(x,t)$ of the order parameter between the lth and $(l+1)$th layers. As is known,[4,22–28] the phase difference $\varphi_l(x,t)$ is described by a set of coupled sine-Gordon equations,

$$\left(1 - \frac{\lambda_{ab}^2}{D^2}\partial_l^2\right)\left(\frac{\partial^2\varphi_l}{\partial t^2} + \omega_r\frac{\partial\varphi_l}{\partial t} + \omega_J^2 \sin\varphi_l\right) - \lambda_c^2\omega_J^2\frac{\partial^2\varphi_l}{\partial x^2} = 0. \qquad (3)$$

Here, λ_{ab} and $\lambda_c = c/\omega_J\sqrt{\varepsilon_s}$ are the London penetration depths across and along the layers, respectively, the operator ∂_l^2 is defined by $\partial_l^2 f_l = f_{l+1} + f_{l-1} - 2f_l$,

$$\omega_r = \frac{4\pi\sigma_c}{\varepsilon_s}$$

is the relaxation frequency, σ_c is the quasiparticle conductivity across the layers, and

$$\omega_J = \sqrt{\frac{8\pi eDJ_c}{\hbar\varepsilon_s}}$$

is the Josephson plasma frequency. The latter is determined by the maximal Josephson current J_c, the interlayer dielectric constant ε_s, and the interlayer spacing D. The spatial variations in the z direction of fields inside the very thin superconducting layers are neglected.

After linearization ($\sin\varphi_l \to \varphi_l$), the electric and magnetic field components of the wave within the superconductor can be expressed via φ_l as

$$H_l^s = i\mathcal{H}_0\frac{1-\Omega^2-ir\Omega}{q\lambda_c}\varphi_l, \quad \Omega = \frac{\omega}{\omega_J}, \quad r = \frac{\omega_r}{\omega_J}, \qquad (4)$$

$$E_{x,l}^s = \mathcal{H}_0\frac{\Omega\lambda_{ab}^2}{\lambda_c\sqrt{\varepsilon_s}}\left(\frac{1-\Omega^2-ir\Omega}{q\lambda_c}\right)\left(\frac{\varphi_l - \varphi_{l-1}}{D}\right), \qquad (5)$$

$$E_{z,l}^s = i\mathcal{H}_0\frac{\Omega}{\sqrt{\varepsilon_s}}\varphi_l, \quad \mathcal{H}_0 = \frac{\Phi_0}{2\pi D\lambda_c}, \qquad (6)$$

where $\Phi_0 = \pi c\hbar/e$ is the magnetic flux quantum.

The dimensionless damping $r = 4\pi\sigma_c\lambda_c/c$ is controlled by the sample temperature T, $r \sim \exp(-\Delta/T)$ (Δ is the modulus of the order parameter), and can be reduced to negligibly small values, $r \ll 1$. As was shown in Refs. 29 and 30, the intralayer quasiparticle conductivity, σ_{ab}, should also be included when ω is far enough from the Josephson plasma frequency. The contribution of the in-plane conductivity to the dissipation can be easily incorporated in our analysis. However, for the frequency range considered here (close to ω_J, $|1-\Omega| \ll 1$), this contribution is strongly suppressed and can be safely omitted because the relative value of the term with σ_{ab} is

$$|1-\Omega|\frac{\sigma_{ab}}{\sigma_c}\left(\frac{\lambda_{ab}}{\lambda_c}\right)^2 \sim |1-\Omega| \ll 1.$$

Here, we use the standard values $\sigma_{ab}/\sigma_c = 10^5$, $\lambda_c/\lambda_{ab} = 200$, for Bi-2212 compounds.

The linearized version of the coupled sine-Gordon equations (3), together with Eqs. (4)–(6), has a solution of the form

$$\varphi_l,\ H_l^s,\ E_{x,l}^s,\ E_{z,l}^s \propto \exp(iqx - i\omega t - k_s lD) \qquad (7)$$

inside a layered superconductor and give the relation between the decay constant k_s ($\mathrm{Re}(k_s) > 0$), wave number q, and dimensionless frequency Ω,

$$\sinh^2\left(\frac{k_s D}{2}\right) = \frac{D^2}{4\lambda_{ab}^2}\left(1 + \frac{q^2\lambda_c^2}{1-\Omega^2-ir\Omega}\right). \qquad (8)$$

The dispersion relation, $q(\omega)$, for the surface Josephson plasma wave can be obtained by matching the in-plane fields H and E_x at the vacuum-superconductor interface. Thus, in order to find the spectrum of the surface JPW, we should derive the ratio E_x^s/H^s at $z=0$ and use the impedance matching, $E_x^{\mathrm{vac}}/H^{\mathrm{vac}} = E_x^s/H^s$.

The difference between the magnetic field $H^s(z=0)$ at the sample surface and its value H_1^s between the first and second superconducting layers is defined by the x component of the supercurrent density $J_x(l=1)$. In the London approximation, the value of J_x is proportional to the x component of the vector potential, $A_x(l=1)$, and, therefore, to the electric field $E_{x,1}^s$. As a result, we obtain the relation

$$\frac{H^s(z=0) - H_1^s}{D} \approx \frac{A_x(l=1)}{\lambda_{ab}^2} \approx \frac{-ic}{\lambda_{ab}^2\omega}E_{x,1}^s. \qquad (9)$$

Moreover, for $l=1$, Eq. (7) implies that

$$H^s(z=0) - H_1^s = H^s(z=0)[1 - \exp(-k_s D)]. \qquad (10)$$

Using Eqs. (9) and (10), we obtain the ratio between the electric and magnetic fields at $z=-0$ (i.e., right below the sample surface),

$$\frac{E_x^s(z=0)}{H^s(z=0)} = i\frac{\Omega\lambda_{ab}^2\omega_J}{cD}[1-\exp(-k_s D)] = 2ib\Omega Z\left(\sqrt{1+\frac{1}{Z}} - 1\right), \qquad (11)$$

with

$$b = \frac{\lambda_{ab}^2 \omega_J}{cD}, \quad Z = \Gamma^2 \left[1 + \frac{\kappa^2}{\varepsilon_s (1 - \Omega^2 - ir\Omega)} \right],$$

$$\Gamma = \frac{D}{2\lambda_{ab}}, \quad \kappa = \frac{cq}{\omega_J}. \tag{12}$$

Matching the impedances in Eqs. (11) and (2), we obtain the *dispersion relation* for the surface Josephson plasma waves,

$$\sqrt{\kappa^2 - \Omega^2} = 2b\Omega^2 Z \left(\sqrt{1 + \frac{1}{Z}} - 1 \right). \tag{13}$$

For $Bi_2Sr_2CaCu_2O_{8+\delta}$ superconductors, one can use the following values of the parameters: $b \approx 0.7$, $\omega_J / 2\pi = 1$ THz, $D = 20$ Å, and $\lambda_{ab} = 2000$ Å.

For simplicity, below we consider surface waves in the continuum limit ($lD \to -z$), when the damping length k_s^{-1} in the superconductor is large compared with the interlayer spacing D:

$$k_s D \ll 1. \tag{14}$$

Under such a condition, the value of Z in Eq. (13) is small. So, in the *continuum limit*, the dispersion relation takes the form

$$\kappa^2 = \Omega^2 + 4b^2 \Omega^4 \Gamma^2 \left(1 + \frac{\kappa^2}{\varepsilon_s (1 - \Omega^2 - ir\Omega)} \right). \tag{15}$$

III. EXCITATION OF SURFACE JOSEPHSON PLASMA WAVES: RESONANT ELECTROMAGNETIC ABSORPTION

Here, we describe how to excite a surface Josephson plasma wave by a wave incident from a dielectric prism onto a superconductor separated from the prism by a thin vacuum gap (see Fig. 2). In the absence of the superconductor, the incident wave completely reflects from the bottom of the prism, if the incident angle θ exceeds the limit angle θ_t for total internal reflection. However, *the evanescent wave penetrates under the prism* a distance about a wavelength. The wave vector of the evanescent mode is along the bottom surface of the prism and its value is higher than ω / c. This feature is the same as for surface waves. So, it is natural to expect the spatial-temporal matching (coincidence of both the frequencies and wave vectors) of the evanescent mode and the surface Josephson plasma wave for a certain incident angle. When the resonant excitation of SJPWs by the incident wave occurs, this results in a *strong suppression of the reflected wave*. This is the well known attenuated-total-reflection method for *generating surface waves*. Below, we present a detailed description of this method for SJPWs propagating along the superconducting layers. The geometry is shown in Fig. 2.

We consider an electromagnetic wave with the electric, $\mathbf{E}^d = \{E_x^d, 0, E_z^d\}$, and magnetic, $\mathbf{H}^d = \{0, H^d, 0\}$, fields incident from the dielectric prism. The prism has permittivity ε and is separated from the layered superconductor by a vacuum interlayer of thickness h. The wave frequency ω is assumed to be below the Josephson plasma frequency ω_J.

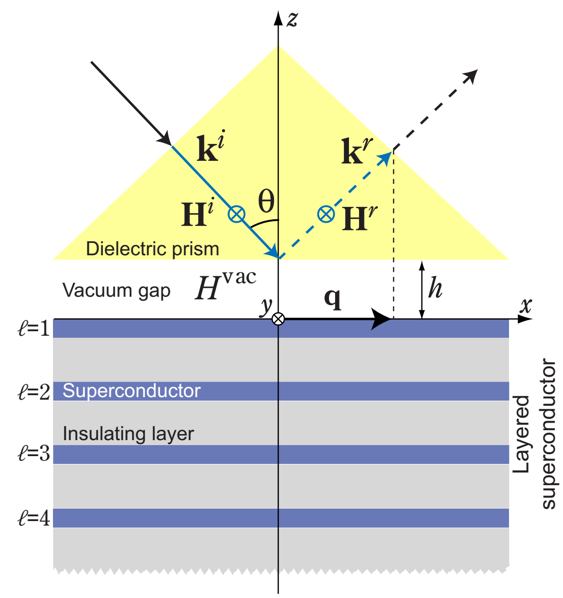

FIG. 2. (Color online) Geometry considered here (Otto configuration): a dielectric prism is separated from a layered superconductor by a vacuum gap of thickness h. An electromagnetic wave with incident angle $\theta > \theta_t$ can excite SJPWs that satisfy the following resonant condition: $\omega \sin \theta / c = q$. Here $\mathbf{k}^i$ and $\mathbf{k}^r$ are the wave vectors of the incident and reflected waves associated with the magnetic field amplitudes $\mathbf{H}^i$ and $\mathbf{H}^r$. The resonant excitation of SJPWs by the incident wave produces a strong suppression of the reflected wave. This method for producing surface waves is known as the "attenuated-total-reflection" method.

The magnetic field H^d in the dielectric prism can be represented as a sum of incident and reflected waves with amplitudes H^i and H^r, respectively,

$$H^d = H^i \exp[iqx - ik_d(z - h)] + H^r \exp[iqx + ik_d(z - h)]. \tag{16}$$

Here and below, we omit the time-dependent multiplier, $\exp(-i\omega t)$. The plane $z = 0$ corresponds to the vacuum-superconductor boundary. The tangential q and normal k_d components of the wave vector, for waves in the prism, are defined by

$$q = k\sqrt{\varepsilon} \sin \theta, \quad k_d = \sqrt{k^2 \varepsilon - q^2} = k\sqrt{\varepsilon} \cos \theta, \tag{17}$$

with $k = \omega / c$. The condition of total internal reflection of the wave in the dielectric prism is supposed to be fulfilled, i.e.,

$$\sin^2 \theta > \frac{1}{\varepsilon}. \tag{18}$$

The magnetic field,

$$H^{\rm vac} = H^i [h^+ \exp(iqx + k_v z) + h^- \exp(iqx - k_v z)], \tag{19}$$

of the evanescent mode in the vacuum gap is generated by the wave from the dielectric. Here, h^+ (h^-) are the dimensionless amplitudes of the evanescent waves that exponentially increase (decrease) with the spatial increment rate,

$$k_v = \sqrt{q^2 - k^2} = k\sqrt{\varepsilon \sin^2 \theta - 1}. \tag{20}$$

Using Maxwell's equations, one can express the x components, E_x^d and E_x^{vac}, of the electric field in the dielectric prism and in the vacuum gap via the magnetic field amplitudes,

$$E_x^d = \frac{k_d}{k\varepsilon} H^i\{h^r \exp[iqx + ik_d(z-h)] - \exp[iqx - ik_d(z-h)]\}, \quad h^r = H^r/H^i,$$

$$E_x^{\text{vac}} = -i\frac{k_v}{k} H^i[h^+ \exp(iqx + k_v z) - h^- \exp(iqx - k_v z)]. \tag{21}$$

In the layered superconductor, the electromagnetic field is described by Eqs. (7)–(11).

Using the conditions of continuity of the tangential components of the electric and magnetic fields at the dielectric-vacuum and vacuum-layered superconductor interfaces, one obtains a set of four linear algebraic equations for four unknown wave amplitudes, h^r, h^+, h^-, and H^s. Solving this set gives the *reflection coefficient*

$$R \equiv h^r = \frac{R_F(k_v/k - a) + (k_v/k + a)C(h,\theta)}{(k_v/k - a) + (k_v/k + a)R_F C(h,\theta)}, \tag{22}$$

of the wave from the bottom of the prism. Here,

$$R_F = \frac{k_d - ik_v\varepsilon}{k_d + ik_v\varepsilon} \equiv \exp(-i\psi) \tag{23}$$

is the *Fresnel reflection coefficient* and

$$C(h,\theta) = \exp(-2k_v h) \tag{24}$$

is the parameter that provides the *coupling* between waves in the dielectric prism and the layered superconductor. Also,

$$a \equiv a(\Omega,\theta) = 2b\Omega Z\left(\sqrt{1 + \frac{1}{Z}} - 1\right) \tag{25}$$

is the *effective surface impedance* of the superconductor [see Eq. (11)]. Below, we assume the coupling parameter C to be small. However, even when $C \ll 1$, the coupling of the waves in the dielectric prism and superconductor plays a very important role in the excitation of SJPWs and anomalies in the reflection properties (Wood's anomalies). First, the dispersion relation of the surface Josephson plasma waves is modified, involving the radiation leakage through the dielectric prism. The new spectrum of the SJPWs is defined by the denominator in Eq. (22). Actually, the region where the coupling $C \ll 1$ (when the radiation leakage of the excited SJPW through the prism does not dominate) corresponds to the strongest excitation of the surface waves by the incident waves. Furthermore, the coupling results in breaking the total internal reflection of the electromagnetic waves from the dielectric-vacuum interface. Due to this coupling, the reflection coefficient R in Eq. (22) differs from the Fresnel one R_F, its modulus becoming less than unity. Moreover, as we show below, the reflection of waves with any frequency $\omega < \omega_J$ can be completely suppressed, for the specific incident angle θ and depth h of the vacuum gap. This provides a way to control and filter the terahertz radiation.

To study the phenomenon of attenuated total reflection, we consider the case which is most suitable for observations, when the following inequalities are satisfied:

$$\frac{b^2\Gamma^2\varepsilon \sin^2\theta}{\varepsilon_s} \ll 1 - \Omega^2 \ll \frac{\varepsilon \sin^2\theta}{\varepsilon_s}. \tag{26}$$

Here, the left inequality corresponds to the continuum limit for the field distribution in the z direction, whereas the right one allows neglecting unity in the square brackets in Eq. (12). Also, we assume the dissipation parameter r in Eq. (12) to be small compared with $1-\Omega^2$,

$$r \ll 1 - \Omega^2. \tag{27}$$

For this frequency region, the complex parameter $a(\Omega,\theta)$, Eq. (25), can be presented as

$$a(\Omega,\theta) \equiv a' + ia'' = 2b\Gamma\sqrt{\frac{\varepsilon \sin\theta}{\varepsilon_s(1-\Omega^2)}}\left(1 + \frac{ir}{2(1-\Omega^2)}\right). \tag{28}$$

When the inequalities in Eqs. (26) and (27) are satisfied, the expression for the reflectivity coefficient R can be significantly simplified. First, the phase ψ of the Fresnel reflectivity coefficient R_F, Eq. (23), is small. In the vicinity of the SJPW spectrum, at $k_v/k \simeq a'$,

$$\psi \simeq \frac{4b\Gamma\varepsilon}{\sqrt{\varepsilon_s(\varepsilon-1)(1-\Omega^2)}} \ll 1. \tag{29}$$

Second, the main changes of the reflectivity coefficient R in Eq. (22) occur in the region of the incident angles θ close to the limit angle θ_t for total internal reflection,

$$\vartheta \equiv \theta - \theta_t \ll 1, \quad \sin^2\theta_t = \frac{1}{\varepsilon}. \tag{30}$$

Third, the parameter a' in Eq. (28) is almost independent of the angle ϑ in the essential region where ϑ changes, whereas it depends very strongly on the frequency detuning $(1-\Omega)$. Using the properties mentioned above, the reflection coefficient R can be rewritten in the form

$$R = \frac{X(\Omega,\vartheta) - iB(\Omega)[C_{\text{opt}}(\Omega) - C(h,\vartheta)]}{X(\Omega,\vartheta) - iB(\Omega)[C_{\text{opt}}(\Omega) + C(h,\vartheta)]}, \tag{31}$$

with

$$X(\Omega,\vartheta) \simeq \sqrt{2}(\varepsilon-1)^{1/4}\sqrt{\vartheta} - \frac{2b\Gamma}{\sqrt{\varepsilon_s(1-\Omega^2)}}, \tag{32}$$

$$B(\Omega) \simeq \frac{16b^2\Gamma^2\varepsilon}{\varepsilon_s\sqrt{\varepsilon-1}(1-\Omega^2)}, \tag{33}$$

$$C_{\text{opt}}(\Omega) \simeq \frac{r\sqrt{\varepsilon-1}\sqrt{\varepsilon_s}}{16b\Gamma\varepsilon\sqrt{1-\Omega^2}}. \tag{34}$$

Equations (31) and (32) show that the modulus of the reflectivity $R(\theta)$ has a *sharp resonance minimum* at

$$\vartheta = \vartheta_{\rm res} \simeq \frac{2b^2\Gamma^2}{\varepsilon_s\sqrt{\varepsilon-1}(1-\Omega^2)}. \tag{35}$$

The minimum value of R is

$$|R|_{\min} \simeq \frac{|C_{\rm opt}(\Omega) - C(h,\vartheta_{\rm res})|}{C_{\rm opt}(\Omega) + C(h,\vartheta_{\rm res})}. \tag{36}$$

It is clearly seen that this value depends strongly on the frequency detuning $(1-\Omega)$, dissipation parameter r, and the coupling between the waves in the dielectric prism and the layered superconductor, i.e., on the thickness h of the vacuum gap. This offers several important applications of the predicted anomaly of the reflectivity in the terahertz range. For instance, if the coupling parameter $C(h,\vartheta_{\rm res})$ is equal to the optimal value $C_{\rm opt}$, i.e., the thickness h takes on the optimal value,

$$h_{\rm opt} = \frac{c}{\omega}\frac{\sqrt{\varepsilon_s(1-\Omega^2)}}{4b\Gamma}\ln\left(\frac{16b\Gamma\varepsilon\sqrt{1-\Omega^2}}{r\sqrt{\varepsilon_s(\varepsilon-1)}}\right), \tag{37}$$

the reflection coefficient R at $\vartheta=\vartheta_{\rm res}$ vanishes. This means that a complete suppression of the reflectivity can be achieved by an appropriate choice of the parameters, due to the resonant excitation of the surface Josephson plasma waves.

We emphasize that Eqs. (35) and (37) describe the conditions for the *best matching* of the incident wave and SJPWs. Under such conditions, the amplitude $H^s_{\max}$ of the excited surface wave is much higher than the amplitude H^i of the incident wave:

$$\frac{|H^s_{\max}|}{H^i} \sim \left(\frac{1-\Omega^2}{r}\right)^{1/2}\left(\frac{(1-\Omega^2)\varepsilon_s}{b^2\Gamma^2\varepsilon}\right)^{1/4} \gg 1. \tag{38}$$

Thus, we can achieve a high concentration of energy in the terahertz SJPW. This proposed experimental setup could provide an unusual terahertz *resonator* or *cavity*.

For these optimal conditions, the total energy coming to the layered superconductor from the dielectric prism is transformed into Joule heat due to the quasiparticle resistance. Thus, if the conditions for the total suppression of the reflectivity are satisfied, the energy flux (i.e., the z component of the Poynting vector of the incident wave) is completely absorbed. The dependence of the *absorptivity coefficient A* on the wave frequency and the incident angle is described by a resonance curve,

$$A(\Omega,\vartheta) = 1 - |R(\Omega,\vartheta)|^2 \simeq \frac{4B^2(\Omega)C(h,\vartheta)C_{\rm opt}(\Omega)}{X^2(\Omega,\vartheta) + B^2(\Omega)[C_{\rm opt}(\Omega)+C(h,\vartheta)]^2}. \tag{39}$$

The half-width $\delta\vartheta$ of the resonance line is much less than $\vartheta_{\rm res}$,

$$\frac{\delta\vartheta}{\vartheta_{\rm res}} \simeq \frac{16b\Gamma\varepsilon[C_{\rm opt}(\Omega)+C(h,\vartheta_{\rm res})]}{\sqrt{\varepsilon_s(\varepsilon-1)(1-\Omega^2)}} \ll 1. \tag{40}$$

If the total suppression of the reflectivity occurs, Eq. (40) can be simplified,

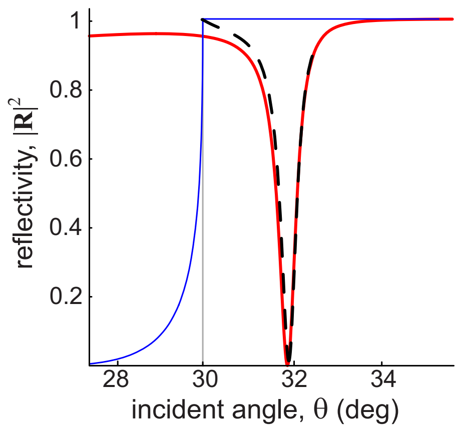

FIG. 3. (Color online) The dependence of the reflectivity coefficient $|R|^2$ on the incident angle θ, calculated for the parameters $b=0.7$, $\Gamma=0.005$, $r=10^{-6}$, $1-\Omega^2=1.2\times10^{-5}$, $\varepsilon_s=20$, and $\varepsilon=4$. The thickness of the vacuum gap is one wavelength, $hk=2\pi$. The solid red curve presents the results of numerical calculations using Eqs. (22)–(24). The dashed black curve (that almost overlaps the red curve) describes the analytically obtained asymptotic dependence [Eqs. (31)–(34)]. The vertical line at $\vartheta=30°$ corresponds to the limiting angle of the total internal reflection. The blue thin solid curve presents the Fresnel reflectivity coefficient.

$$\frac{\delta\vartheta}{\vartheta_{\rm res}} \simeq \frac{4b^2\Gamma^2 r}{\varepsilon_s\sqrt{\varepsilon-1}} \ll 1. \tag{41}$$

IV. NUMERICAL CALCULATIONS

Inequalities (26) are not necessary for the observation of the total suppression of the reflectivity and the resonant increase of the electromagnetic absorption. Departing from the strong inequalities (26), toward the region of parameters where $B\sim1$, we perform numerical calculations. Figure 3 demonstrates the resonant suppression of the reflectivity for the parameter $B(\Omega)\approx1.9$. Nevertheless, the asymptotic formulas in Eqs. (31) and (34) describe rather well the resonant behavior of the reflectivity R.

Figure 4 shows the sharp decrease of the reflectivity in the $(\theta,(1-\Omega))$ plane, due to the resonant excitation of the surface Josephson plasma waves. Obviously, the suppression of the reflectivity can be observed by changing the frequency at a given incident angle, as is demonstrated in Fig. 5.

We also numerically calculate the total magnetic field distribution (Fig. 6). The interference pattern is seen in the nonresonant case, when the amplitudes of the incident and reflected waves practically coincide. For the resonant conditions, the reflected wave is suppressed and there is no interference of waves in the far-field zone (prism region). Otherwise, the near-field "torch" structure of the SJPW is clearly seen in the vacuum gap.

V. CONCLUSION

In this paper, we have theoretically studied the excitation of surface Josephson plasma waves in layered superconduct-

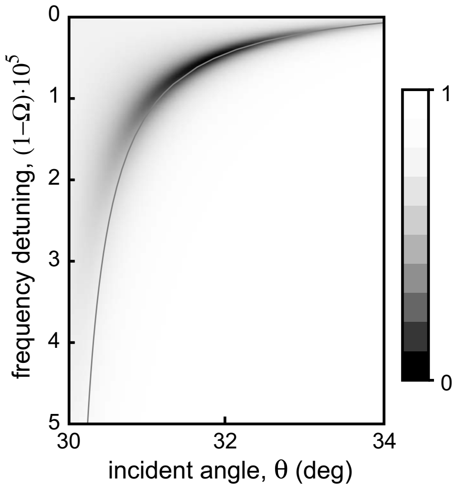

FIG. 4. The reflectivity coefficient in the plane $(\theta,(1-\Omega))$ shown in gray levels for the same values of the parameters as in Fig. 3. The dispersion relation of the dielectric-vacuum-layered superconductor is presented by the solid curve.

ors in the attenuated-total-reflection geometry (Fig. 2). It is shown that the reflection coefficient of the superconductor can be completely suppressed due to the resonant excitation of SJPWs. The conditions for the resonance and the shape of the resonant curve $R(\theta,\Omega)$ are derived analytically and calculated numerically. The suppression of the reflectivity $|R|^2$ is accompanied by the resonant increase of the electromagnetic absorption in the layered superconductor. It should be noted that this process can result in a transition of the superconductor into the resistive or even into the normal state. Thus, a novel kind of resonance phenomenon can be observed due to the excitation of the SJPW. Moreover, this strongly selective interaction of SJPWs, with the incident wave having a certain frequency and direction of propagation, can be used for designing terahertz detectors and filters.

We would like to note that the nonlinear regime of the electromagnetic wave propagation can be easily achieved during the resonant excitation of SJPW. Indeed, under the resonance conditions, the electromagnetic field in the superconductor is significantly increased with respect to the amplitude of the incident wave. Therefore, the value of the gauge-invariant phase of the order parameter increases also. A simple evaluation made by means of Eqs. (4) and (38) gives, for φ in the resonance region,

$$\varphi_{\max} \sim \frac{H^i}{\mathcal{H}_0}[(1-\Omega^2)\varepsilon_s r^2 b^2 \Gamma^2 \varepsilon]^{1/4} \sim \frac{H^i}{\mathcal{H}_0} \times 10^5, \quad (42)$$

at $b=0.7$, $\Gamma=0.005$, $r=10^{-6}$, $1-\Omega^2=1.2\times 10^{-5}$, $\varepsilon_s=20$, and $\varepsilon=4$. Under such conditions, the nonlinear regime can be observed when $H^i \sim 10^{-3}$ Oe.

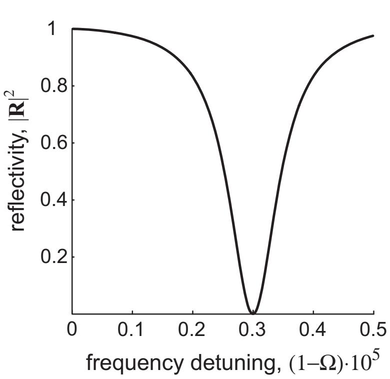

FIG. 5. The frequency dependence of the reflectivity coefficient $|R|^2$ obtained for $\theta=31.867°$. Other parameters are the same as in Fig. 3.

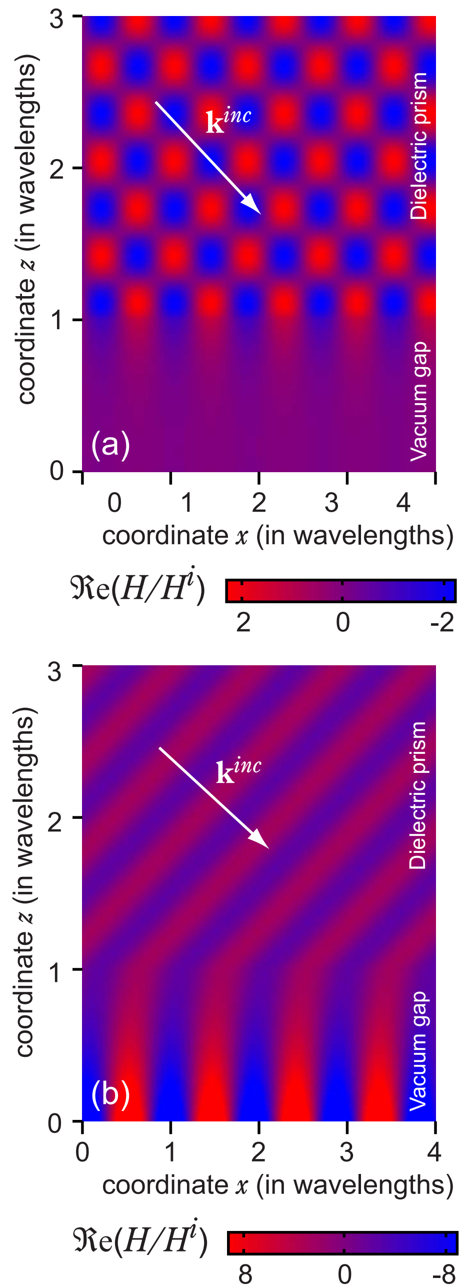

FIG. 6. (Color online) The magnetic field distribution (a) for the nonresonant case, $\theta \neq \theta_{\rm res}$, and (b) for the resonant condition, $\theta = \theta_{\rm res} = 31.867°$. Other parameters are the same as in Fig. 3.

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