

# Correlation-induced suppression of decoherence in capacitively coupled Cooper-pair boxes

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Charge fluctuations from gate bias and background traps severely limit the performance of a charge qubit in a Cooper-pair box (CPB). Here we present an experimentally realizable method to control the decoherence effects of these charge fluctuations using two strongly capacitively coupled CPBs. This coupled-box system has a low-decoherence subspace of two states. Our results show that the interbox Coulomb correlation can help significantly suppress decoherence of this two-level system, making it a promising candidate as a logical qubit, encoded using two CPBs.

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## I. INTRODUCTION

Various superconducting nanocircuits have been proposed as quantum bits (qubits) for a quantum computer.<sup>1-5</sup> In the meantime, it has long been recognized that background charge fluctuations can severely limit the performance of microelectronic devices, particularly those based on the manipulation of electrical charge, such as single electron transistors<sup>6</sup> and superconducting Cooper-pair boxes (CPBs).<sup>7-9</sup> The struggle to suppress or even eliminate noise from charge fluctuations in superconducting devices has been a prolonged battle with limited success. Here, instead of focusing on perfecting materials, we propose an alternative experimentally realizable method to suppress the effects of these charge fluctuations using two strongly (capacitively) coupled CPBs.

Cooper-pair boxes are one of the prominent candidates for qubits in a quantum computer. Recent experiments<sup>11</sup> have revealed quantum coherent oscillations in two CPBs coupled capacitively and demonstrated the feasibility of a conditional gate as well as creating macroscopic entangled states. Scalable quantum-computing schemes (see, e.g., Ref. 12) have also been proposed based on charge qubits. Clearly, effective suppression of charge noise is essential to the practical implementation of scalable quantum computing in a charge-based scheme. It has been shown<sup>9</sup> that while operating at the degeneracy point (where the two lowest charge states have the same energy in the absence of Josephson coupling), the charge qubit has a long decoherence time of  $\tau \approx 500$  ns. However, when the charge qubit is operated away from the degeneracy point, it experiences strong dephasing by the charge fluctuations, and the decoherence time of the system is greatly reduced.<sup>7,9,10</sup>

Two separate CPBs generally experience uncorrelated charge fluctuations as they are most strongly affected by their own gate biases and the nearest fluctuating charge traps. However, if the two boxes are strongly coupled capacitively (with no tunnel coupling so that the two-box states do not approach those of a single large box in the strong coupling limit), the fluctuations affecting one box will affect the other

through Coulomb interaction. In the limit of extremely strong interbox coupling (corresponding to a very large mutual capacitance between the two CPBs), the two boxes would experience an identical charge environment, so that, in principle, a decoherence-free subspace<sup>13,14</sup> could be established for coupled-box states. However, in reality this limit involves many degenerate charge states for the electrostatic energy of the coupled boxes, so that logical qubit encoding is impossible. Can we still achieve a decoherence-suppressed logical qubit in two capacitively coupled boxes? Below we show that there exists an intermediate parameter regime where a strong interbox Coulomb correlation induces a significant suppression of decoherence in certain two-box states, so that considerable benefit can be reaped by encoding a logical qubit in terms of these states.

## II. CHARACTERIZATION OF TWO COUPLED COOPER-PAIR BOXES

Consider two capacitively coupled CPBs (see Fig. 1). Each CPB is individually biased by an applied gate voltage  $V_i$  and coupled to the leads by a symmetric dc superconducting quantum interference device (SQUID). The dc SQUID is pierced by a magnetic flux  $\Phi_i$ , which provides a tunable effective Josephson coupling

$$E_{Ji}(\Phi_i) = 2E_{Ji}^0 \cos\left(\frac{\pi\Phi_i}{\Phi_0}\right), \quad (1)$$

where  $\Phi_0 = h/2e$  is the flux quantum. The system Hamiltonian is

$$H_S = \sum_i [E_{ci}(n_i - n_{xi})^2 - E_{Ji}(\Phi_i) \cos \varphi_i] + E_m(n_L - n_{xL})(n_R - n_{xR}), \quad (2)$$

with  $i=L,R$  for left and right. Here the charging energy  $E_{ci}$  of the  $i$ th superconducting island and the mutual capacitive coupling  $E_m$  are given by<sup>15</sup>

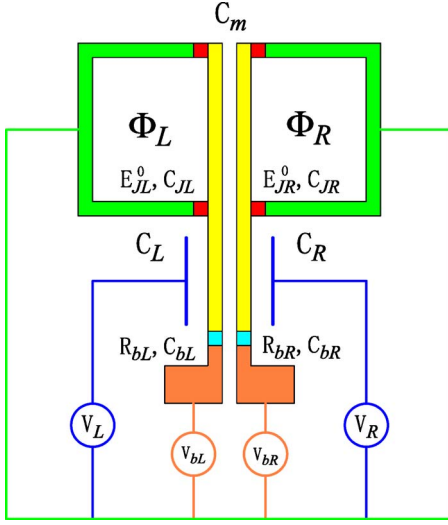


FIG. 1. (Color online) Strongly coupled Cooper-pair boxes. A bias voltage  $V_i$  is applied to the  $i$ th charge box through a gate capacitance  $C_i$ , and a symmetric dc SQUID (with Josephson coupling energy  $E_{Ji}^0$  and capacitance  $C_{Ji}$  for each junction) is coupled to the box. Also, each box is connected to a detector via a probe junction (or a less invasive point contact). When a measurement is performed, the probe junction is biased with an appropriate voltage  $V_{bi}$ . The two boxes are closely spaced long superconducting islands with sufficiently large mutual capacitance  $C_m$ , and the barrier between them is strong enough to prohibit the interbox Cooper-pair tunneling.

$$E_{ci} = \frac{2e^2 C_{\Sigma i}}{\Lambda},$$

$$E_m = \frac{4e^2 C_m}{\Lambda}, \quad (3)$$

with  $\Lambda$  given by

$$\Lambda = C_{\Sigma i} C_{\Sigma j} - C_m^2, \quad (4)$$

where

$$C_{\Sigma i} = C_m + C_i + C_{Ji} \quad (5)$$

is the total capacitance of the  $i$ th island. The offset charge is

$$2en_{xi} = Q_{Vi} + Q_{0i}, \quad (6)$$

where  $Q_{0i}$  is the background charge, and

$$Q_{Vi} = C_i V_i + C_{bi} V_{bi} \quad (7)$$

is induced by both the gate voltage  $V_i$  and the probe voltage  $V_{bi}$ . The average phase drop  $\varphi_i$  across the two Josephson junctions in the dc SQUID is conjugate to the Cooper pair number  $n_i$  on the box. Both CPBs operate in the charging regime  $E_{ci} \gg E_{Ji}$  and at low temperatures  $k_B T \ll E_{ci}$ . The states of the two coupled boxes can thus be expanded on the basis of the charge eigenstates  $|n_L n_R\rangle \equiv |n_L\rangle |n_R\rangle$ .

When the two CPBs are strongly coupled, the total Hamiltonian can be rewritten in terms of the total charge on the coupled boxes and the charge difference across the boxes. Assuming, for simplicity,

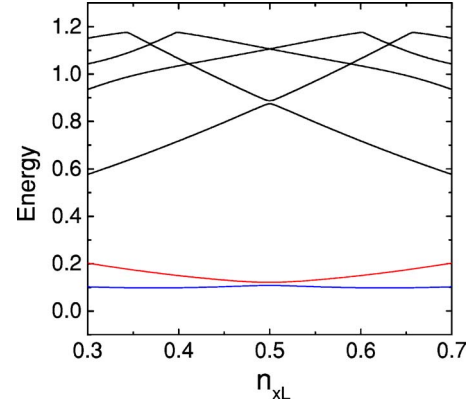


FIG. 2. (Color online) Dependence of the energy levels of the coupled-box system on the reduced offset charge  $n_{xL}$  for  $n_{xR}=0.5$ . Here  $\Delta E_i = E_{ci}/4$ , and  $E_{Ji} = E_{ci}/10$ , with  $i=L,R$ . The energy is in units of  $E_c$ . We choose ten two-box basis states  $|m, n\rangle$  that have the lowest electrostatic energy. The two lowest levels remain nearly unchanged in the vicinity of the degeneracy point  $(n_{xL}, n_{xR}) = (\frac{1}{2}, \frac{1}{2})$ .

$$C_{\Sigma L} = C_{\Sigma R} = C_{\Sigma}, \quad (8)$$

so that

$$E_{cL} = E_{cR} = E_c,$$

$$E_{JL} = E_{JR} = E_J, \quad (9)$$

we have

$$H_S = \left( E_c - \frac{\Delta E}{2} \right) (n_L + n_R - n_{xL} - n_{xR})^2$$

$$+ \frac{\Delta E}{2} (n_L - n_R - n_{xL} + n_{xR})^2 - E_J (\cos \varphi_L + \cos \varphi_R), \quad (10)$$

where

$$\Delta E = \frac{2e^2}{C_m + C_{\Sigma}} = E_c - \frac{1}{2} E_m > 0. \quad (11)$$

Notice that when  $C_m$  is much larger than  $C_i$  and  $C_{Ji}$ ,  $E_c$  essentially represents the charging energy of individual Josephson junctions, while  $\Delta E$  represents the charging energy of the large capacitor  $C_m$ , so that  $\Delta E \ll E_c$ .

At the double degeneracy point of  $(n_{xL}, n_{xR}) = (\frac{1}{2}, \frac{1}{2})$ , the two lowest energy states are given by

$$|\pm\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle) + |\delta\psi_{\pm}\rangle, \quad (12)$$

where

$$|\delta\psi_{\pm}\rangle = O\left(\frac{E_J}{E_m}\right) [\alpha_{\pm} (|00\rangle \pm |11\rangle) + \dots], \quad (13)$$

with a splitting of  $E_J^2/2(E_c - \Delta E)$  (see Fig. 2). The symmetry in these states indicates that they are well insulated from pure dephasing and relaxation due to charge noise, as we will

show below. It is thus quite natural to adopt these two coupled-box states  $|\pm\rangle$  to encode a logical qubit. Below we calculate the dephasing and relaxation properties of the  $|\pm\rangle$  states and discuss how they can be coherently manipulated.

### III. CORRELATION-INDUCED COHERENCE-PRESERVING SUBSPACE

To clarify the origin of the correlated environments for the two coupled CPBs, we study the fluctuations<sup>16</sup> of the reduced offset charge  $n_{xi}$ , which could originate from the gate voltage  $V_i$ , probe voltage  $V_{bi}$ , and background charge  $Q_{0i}$ . The interaction Hamiltonian between the charge noise and the coupled CPBs takes the form

$$H_I = -2 \left( E_c - \frac{\Delta E}{2} \right) (n_L + n_R) (\delta n_{xL} + \delta n_{xR}) - \Delta E (n_L - n_R) (\delta n_{xL} - \delta n_{xR}). \quad (14)$$

We can use the language of a two-level system to describe each of the CPBs around the degeneracy point

$$(n_{xL}, n_{xR}) = \left( \frac{1}{2}, \frac{1}{2} \right),$$

and rewrite the system Hamiltonian in terms of the Pauli matrices

$$H_S = \sum_i H_i + \frac{1}{4} E_m \sigma_{zL} \sigma_{zR},$$

$$H_i = [\varepsilon_i(n_{xi}) + \varepsilon_m(n_{xj})] \sigma_{zi} - \frac{1}{2} E_{Ji}(\Phi_i) \sigma_{xi}, \quad (15)$$

with  $|\uparrow\rangle_i \equiv |0\rangle_i$ ,  $|\downarrow\rangle_i \equiv |1\rangle_i$ , and  $i, j=L, R$  ( $i \neq j$ ). Here

$$\varepsilon_i(n_{xi}) = E_{ci} \left( n_{xi} - \frac{1}{2} \right),$$

$$\varepsilon_m(n_{xj}) = \frac{1}{2} E_m \left( n_{xj} - \frac{1}{2} \right), \quad (16)$$

The interaction Hamiltonian Eq. (14) between the CPB system and the environment can now be projected onto the single-box two-level basis

$$H_I = \sum_i \left( E_{ci} \delta n_{xi} + \frac{1}{2} E_m \delta n_{xj} \right) \sigma_{zi} = E_c (\sigma_{zL} + \sigma_{zR}) (\delta n_{xL} + \delta n_{xR}) - \Delta E (\sigma_{zL} \delta n_{xR} + \sigma_{zR} \delta n_{xL}). \quad (17)$$

with  $i, j=L, R$ , and  $i \neq j$ . Though each CPB is directly coupled to its own charge environment, the interisland Coulomb interaction in terms of  $E_m$  ensures that the environment is partly shared between the two islands, causing the CPBs to experience correlated noises. Indeed, notice that in Eq. (17) the first part of the Hamiltonian should not lead to dephasing between the  $|\pm\rangle$  states since it affects both identically. When each of the two environments is modeled by a thermal bath of simple harmonic oscillators described by the annihilation

(creation) operator  $b_{ji}$  ( $b_{ji}^\dagger$ ), the Hamiltonian of the whole system, including the two baths, is

$$H = H_S + H_B + H_I,$$

$$H_B = \sum_n (\hbar \omega_{nL} b_{nL}^\dagger b_{nL} + \hbar \omega_{nR} b_{nR}^\dagger b_{nR}),$$

$$H_I = \frac{1}{2} \sum_i \left( \sigma_{zi} + \frac{E_m}{2E_{ci}} \sigma_{zj} \right) X^{(i)}, \quad (18)$$

with  $i, j=L, R$  ( $i \neq j$ ). Here, the bath operator

$$X^{(i)} = 2E_{ci} \delta n_{xi} \quad (19)$$

is given by

$$X^{(i)} = \sum_n \lambda_{ni} x_n^{(i)} \equiv \sum_n \hbar K_n^{(i)} (b_{ni}^\dagger + b_{ni}), \quad (20)$$

with

$$K_n^{(i)} = \frac{\lambda_{ni}}{\sqrt{2m_{ni}\hbar\omega_{ni}}}, \quad (21)$$

where  $m_{ni}$  and  $\omega_{ni}$  denote the mass and frequency of the  $n$ th harmonic oscillator in the bath coupled to the box  $i$ , while  $\lambda_{ni}$  characterizes the coupling strength between the  $i$ th oscillator and the box.

We first focus on pure dephasing between CPB states with  $E_{Ji}(\Phi_i)=0$ , which can be solved analytically.<sup>14</sup> For correlated noises studied here, the reduced off-diagonal density matrix elements for the two lowest energy eigenstates of the coupled-box system decay as

$$\rho_{ab} \sim \exp[-\eta(t)], \quad (22)$$

where the damping factor is given by

$$\eta(t) = \sum_{i=L,R} \left( 1 - \frac{E_m}{2E_{ci}} \right)^2 \Gamma_i(t), \quad (23)$$

and  $a, b=+, -$  ( $a \neq b$ ). Here  $+$  and  $-$  denote the two lowest eigenstates. Also,

$$\Gamma_i(t) = \frac{1}{\pi} \int_0^\infty d\omega S_i(\omega) \left( \frac{\sin(\omega t/2)}{\omega/2} \right)^2, \quad (24)$$

and the power spectrum of the  $i$ th bath is

$$S_i(\omega) = J_i(\omega) \coth \left( \frac{\hbar\omega}{2k_B T} \right), \quad (25)$$

where

$$J_i(\omega) = \pi \sum_n [K_n^{(i)}]^2 \delta(\omega - \omega_{ni}). \quad (26)$$

When  $t \rightarrow \infty$ ,  $\Gamma_i(t)$  tends to  $t S_i(\omega)|_{\omega \rightarrow 0}$ . For the symmetric case we are considering,

$$\Delta E = E_c - \frac{1}{2} E_m, \quad (27)$$

so that

$$\eta(t) = \left( \frac{\Delta E}{E_c} \right)^2 \sum_{i=L,R} \Gamma_i(t). \quad (28)$$

In the limit of strong interbox coupling,  $\Delta E \ll E_c$ , pure dephasing can then be strongly suppressed as compared to a single CPB. For example, for  $\Delta E = E_c/10$ , the prefactor takes the value 1/100, so that the dephasing time is two orders of magnitude longer than when the boxes are only weakly coupled. This is in strong contrast to the corresponding single CPB expression for pure dephasing

$$\eta(t) = \Gamma(t). \quad (29)$$

For the  $1/f$  noise arising from the background charge fluctuations, the power spectrum is

$$S_{fi}(\omega) = \left( \frac{2E_{ci}}{\hbar e} \right)^2 \frac{\alpha_i}{\omega}, \quad (30)$$

and  $\Gamma_i(t)$  can be written as<sup>7</sup>

$$\Gamma_i(t) = \frac{1}{\pi} \int_{\omega_c}^{\infty} d\omega S_{fi}(\omega) \left( \frac{\sin(\omega t/2)}{\omega/2} \right)^2, \quad (31)$$

where the cut-off frequency  $\omega_c \ll 1/\tau$ , with  $\tau$  being the decoherence time of the system.

The above calculation focuses on the pure dephasing of the coupled boxes and is applicable to parameter regimes away from the double degeneracy point

$$(n_{xL}, n_{xR}) = \left( \frac{1}{2}, \frac{1}{2} \right).$$

At the degeneracy point, we can estimate the effects of the charge fluctuations by directly projecting the system-environment coupling Hamiltonian (14) into the  $|\pm\rangle$  basis. The matrix elements are

$$\begin{aligned} \langle + | H_I | + \rangle &= \langle - | H_I | - \rangle = -2 \left( E_c - \frac{1}{2} \Delta E \right) (\delta n_{xL} + \delta n_{xR}), \\ \langle - | H_I | + \rangle &= \Delta E (\delta n_{xL} - \delta n_{xR}). \end{aligned} \quad (32)$$

From the first equation of Eq. (32)

$$\langle + | H_I | + \rangle - \langle - | H_I | - \rangle = 0, \quad (33)$$

so that there is no pure dephasing between  $|+\rangle$  and  $|-\rangle$  states as the charge fluctuations affect both identically. On the other hand, the second equation of Eq. (32), i.e., the transition matrix element, dictates that charge fluctuation does lead to relaxation between these two states. Using the spin-boson model above, one can calculate this transition rate straightforwardly. Here we emphasize that compared to a single CPB, the system-reservoir interaction strength is  $\Delta E$  instead of  $E_c$ , just like in Eq. (28) for pure dephasing. Furthermore, charge fluctuations that couple to the two boxes equally will not lead to relaxation because the coupling here is proportional to  $\delta n_{xL} - \delta n_{xR}$ .

In short, a pair of capacitively coupled CPBs can have strongly suppressed pure dephasing and relaxation around the degeneracy point because of the reduced interaction strength. Therefore  $|\pm\rangle$  are perfect candidates to encode a logical qubit. Moreover, as shown in Fig. 2, the two lowest levels are well separated from the higher levels in the coupled CPBs and the leakages from the qubit states to the higher-level states can be negligibly small. In contrast, for the single CPB qubit in the charge-flux regime where the charging energy is reduced,<sup>9</sup> the two lowest levels are not well separated from the higher levels<sup>10</sup> and appreciable leakages are expected.

#### IV. DISCUSSION AND CONCLUSION

Coherence-preserving quantum states can be prepared as follows. First, consider an initial point on the  $n_{xL}-n_{xR}$  plane close to (0,0). Here the system ground state is  $|00\rangle$ . Then, shifting adiabatically (e.g., along the  $n_{xL}=n_{xR}$  direction) to the region around the degeneracy point, we arrive at the coherence-preserving ground state  $|+\rangle$ . Now, using a two-frequency microwave to interact with the system for a period of time (basically a Raman process), as in the case of trapped ions,<sup>17</sup> one can obtain any superposition of  $|\pm\rangle$  states, so that an arbitrary single qubit operation is feasible. Readout of the logical-qubit states can be achieved by various approaches. For instance, one can rotate the logical qubit states to the charge eigenstates  $|01\rangle$  and  $|10\rangle$ , so that simple charge detection using, for example, single electron transistors, can determine the state of the coupled CPBs. In the case of probe junction detection (Fig. 1 and Ref. 11), when appropriate bias voltages  $V_{bi}$  are applied to the probe junctions, the measured current  $I_i$  through the  $i$ th probe junction is proportional to the probability for the  $i$ th box to have a Cooper pair in it.

Decoherence in two coupled qubits<sup>18,19</sup> and during a conditional gate<sup>20</sup> have attracted much attention recently. It has been shown that a decoherence-free subspace exists for two physical qubits coupled to the same bath.<sup>19</sup> Recently, Zhou *et al.*<sup>21</sup> proposed an encoded qubit using a pair of closely spaced CPBs sharing a common lead, and the two boxes were assumed to couple to an identical bath. In their proposed setup, fluctuations originating from the gate voltage may be identical because of the common lead. However, the background charge fluctuations<sup>22</sup> cannot be so since these fluctuations originate from local charge traps near each box. As shown in Eq. (17), an identical bath could only be achieved in the presence of interbox interaction and in the limiting case of  $E_{ci} = \frac{1}{2} E_m$ . Unfortunately, at this limit, the two-level-system description for the individual CPB breaks down. Thus the proposed ideal single-bath scenario can never be achieved in the presence of background charge fluctuations. Nevertheless, as shown in our study of the coupled CPBs here, though the ideal single-bath case cannot be realized to obtain a decoherence-free subspace, the strong interbox coupling does enable a coherence-preserving logical qubit where the correlated baths lead to suppression of decoherence in the coupled CPBs. Reference 23 also proposes to use interbit couplings to reduce decoherence in a model Hamiltonian.

We emphasize that the encoding idea presented here is based on actively employing the interbox interaction to correlate different environments experienced by the individual physical qubits. This goes beyond the decoherence-free subspace concept, where symmetry alone is used (passively) to combat noise from a naturally existing common environmental reservoir to all the qubits.

In conclusion, we have shown that in two strongly capacitively coupled CPBs, the charge fluctuations experienced by the two boxes are strongly correlated. The interbox Coulomb correlation creates a two-box subspace of two states in which pure dephasing and relaxation are strongly suppressed due to the correlated noises. These two coupled CPBs can therefore

be used to encode a logical qubit that possesses superior coherence properties. We have also discussed how such logical qubits can be manipulated and measured.

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