Testing Bell's inequality in a constantly coupled Josephson circuit by effective single-qubit operations

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(Received 1 August 2005; published 26 September 2005)

In superconducting circuits with interbit untunable (e.g., capacitive) couplings, ideal local quantum operations cannot be exactly performed on individual Josephson qubits. Here we propose an effective dynamical decoupling approach to overcome the "fixed-interaction" difficulty for effectively implementing elemental logical gates for quantum computation. The proposed single-qubit operations and local measurements should allow testing Bell's inequality with a pair of capacitively coupled Josephson qubits. This provides a powerful approach, besides spectral-analysis [Nature **421**, 823 (2003); Science **300**, 1548 (2003)], to verify the existence of macroscopic quantum entanglement between two fixed-coupling Josephson qubits.

DOI: 10.1103/PhysRevB.72.104516 PACS number(s): 03.65.Ud, 03.67.Lx, 85.25.Dq

I. INTRODUCTION

Nonlocality (i.e., entanglement) is one of the most profound features of quantum theory and plays an important role in quantum information processing, including quantum computation and quantum communication.[1] Mathematically, an entangled pure state of a composite system is defined as a state that cannot be factorized into a direct tensor product of the states associated with individual subsystems. The presence of multipartite entanglement is necessary for implementing quantum algorithms that are exponentially faster than classical ones.[1] Also, entanglement may offer some information transfer modes, e.g., teleportation and quantum cryptography.[1] Therefore, generating and verifying entanglement between qubits are of great practical importance.[2]

The existence of entanglement can be verified by using various experimental methods, e.g., quantum tomographic techniques, Bell-state analysis, quantum jump measurements, etc. (see, e.g., Refs. 3 and 4). Also, spectral analysis has been used to probe the existence of two-qubit entanglement in coupled Josephson qubits.[5,6] However, the degree of entanglement between the two always-present interacting qubits changes rapidly[7] and, at certain times, two-qubit states can be almost separable. Thus, verifying the instantaneously generated entangled state in coupled systems,[8] and using it to realize quantum information processing, e.g., teleportation and quantum memory, are very important challenges.

Historically, Bell's inequality always served as one of the most important witnesses of entanglement: Its violation implies that entanglement must be shared by the separate parts. Numerous experimental tests of Bell's inequality have been made with entangled photons separated far apart (e.g., up to 500 m) (Ref. 9) and entangled *closely spaced* trapped ions (e.g., *separated a few micrometers apart*).[10] A Bell-like inequality has also been tested via single-neutron interferometry by measuring the correlations between two entangled degrees of freedom (comprising spatial and spin components) of *single* neutrons.[11] The results from these experiments strongly violate the tested Bell's inequalities, and thus agree with quantum mechanical predictions. Very recently, preliminary proposals have been explored for testing Bell's inequalities with switchable Josephson qubits.[12]

Almost all proposals for manipulating quantum information rely on the execution of both single-qubit and two-qubit gates. In some cases (e.g., trapped ions, QED cavities, and quantum dots[13]), single-qubit operations are easy to realize by applying certain controllable local fields. However, the interbit couplings are fixed and uncontrollable in current experimental Josephson circuits[4–6] and NMR systems.[14] Nevertheless, qubits in NMR are individually addressable, because the coupling constants J_{ij} between the ith and jth qubits are sufficiently weak [i.e., much smaller than the differences $\Delta\omega_{ij}=|\omega_i-\omega_j|$ between the eigenfrequencies of the qubits, e.g., $J_{ij}/\Delta\omega_{ij}\lesssim 10^{-4}$ (Ref. 14)]. However, the usual capacitive coupling in Josephson circuits[4,5] is relatively strong, thus making it difficult to perform local single-qubit operations on individual qubits. The method[15] of physically arranging qubits (i.e., using two or three physical qubits to encode a logical qubit) cannot be directly used in the present two-qubit system.

Here we develop an approach to overcome the serious "fixed-interaction" difficulty and effectively implement desired single-qubit operations on selected qubits. This dynamical decoupling method can be used to generate long-lived entangled states in coupled Josephson qubits. The long-lived entanglement obtained here, assisted by the proposed local single-qubit operations, should allow us to test Bell's inequality with a pair of capacitively coupled Josephson qubits. Its violation would provide another robust physical evidence of statistically nonlocal correlations at the macroscopic scale.

II. DYNAMICAL DECOUPLING

We consider the nanocircuit sketched in Fig. 1. This is similar to that in Ref. 5, just replacing the left Josephson

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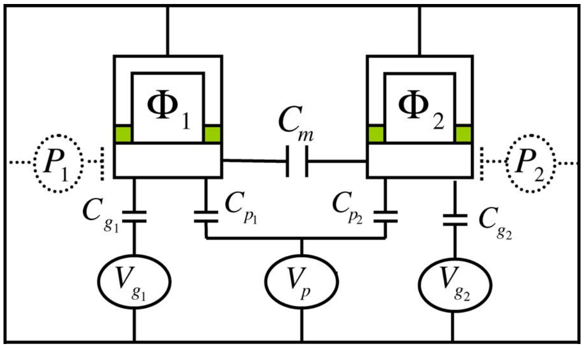

FIG. 1. (Color online) Two capacitively coupled charge qubits. The quantum states of two Cooper-pair boxes (i.e., qubits) are manipulated by controlling the applied gate voltages V_{g_1}, V_{g_2}, and external magnetic fluxes Φ_1, Φ_2 (penetrating the SQUID loops). P_1 and P_2 (dashed line parts) read out the final qubit states.

junction there by a superconducting quantum interference device (SQUID) loop with controllable Josephson energy. The two Cooper-pair boxes are coupled via the capacitance C_m. The qubits work in the charge regime, with $k_B T \ll E_J^{(j)} \ll E_C^{(j)} \ll \Delta$ $(j=1,2)$, where both quasiparticle tunneling and excitations are effectively suppressed and the number n_j (with $n_j=0,1,2,\ldots$) of Cooper pairs in the boxes is a good quantum number. Here, k_B, T, Δ, $E_C^{(j)}$, and $E_J^{(j)}$ are the Boltzmann constant, temperature, superconducting gap, the charging, and Josephson energies of the jth qubit, respectively. Following Refs. 4 and 5, the system is assumed to work near the coresonant point and its quantum dynamics can be restricted to the subspace spanned by the four lowest charge states: $|00\rangle$, $|10\rangle$, $|01\rangle$, and $|11\rangle$. Thus, the Hamiltonian of this circuit is

$$\hat{H} = \frac{1}{2}\sum_{j=1,2}[E_C^{(j)}\sigma_z^{(j)} - E_J^{(j)}\sigma_x^{(j)}] + E_{12}\sigma_z^{(1)}\sigma_z^{(2)}. \qquad (1)$$

Here, $E_{12}=E_m/4$ is effective interbit coupling with $E_m=4e^2C_m/C_\Sigma$. The effective charge energy $E_C^{(j)}$ is $E_C^{(j)}=E_{C_j}(-1/2+n_{g_j})+E_m(-1/4+n_{g_k}/2)$ $(j,k=1,2)$ with $n_{g_j}=(C_{g_j}V_{g_j}+C_{p_j}V_p)/(2e)$ and $E_{C_j}=4e^2C_{\Sigma_k}/C_\Sigma$. The effective Josephson energy of the SQUID is $E_J^{(j)}=2\varepsilon_J^{(j)}\cos(\pi\Phi_j/\Phi_0)$ with Josephson energy $\varepsilon_J^{(j)}$ of the single junction. Above, $C_\Sigma=C_{\Sigma_1}C_{\Sigma_2}-C_m^2$, and C_{Σ_j} is the sum of all capacitances connected to the jth box. The pesudospin operators are defined as $\sigma_z^{(j)}=|0_j\rangle\langle 0_j|-|1_j\rangle\langle 1_j|$ and $\sigma_x^{(j)}=|0_j\rangle\langle 1_j|+|1_j\rangle\langle 0_j|$.

First, let us consider the circuit working at the coresonance point (i.e., $n_{g_1}=n_{g_2}=0.5$, yielding $E_C^{(1)}=E_C^{(2)}=0$), and the applied fluxes are set as $\Phi_j=0$, $\Phi_k=\Phi_0/2$, $k\neq j$ (yielding $E_J^{(j)}=2\varepsilon_J^{(j)}$, $E_J^{(k)}=0$). In this case, the circuit has the Hamiltonian

$$\hat{H}_1 = -\varepsilon_J^{(j)}\sigma_x^{(j)} + E_{12}\sigma_z^{(1)}\sigma_z^{(2)}. \qquad (2)$$

The corresponding time-evolution operator reads

$$\hat{U}_1(t) = \exp\left(-\frac{it}{\hbar}\hat{H}_1\right) = \exp\left[\frac{it}{\hbar}\varepsilon_J^{(j)}\sigma_x^{(j)}\right]\hat{U}_{\mathrm{int}}(t), \qquad (3)$$

where the operator $\hat{U}_{\mathrm{int}}(t)$ is determined by

$$i\hbar\frac{\partial \hat{U}_{\mathrm{int}}(t)}{\partial t} = \hat{H}_{\mathrm{int}}(t)\hat{U}_{\mathrm{int}}(t), \qquad (4)$$

with

$$\hat{H}_{\mathrm{int}}(t) = E_{12}\exp\left[-\frac{it}{\hbar}\varepsilon_J^{(j)}\sigma_x^{(j)}\right]\sigma_z^{(1)}\sigma_z^{(2)}\exp\left[\frac{it}{\hbar}\varepsilon_J^{(j)}\sigma_x^{(j)}\right].$$

Considering $\zeta_j=E_{12}/(2\varepsilon_J^{(j)})\ll 1$ (e.g., $\zeta_j\lesssim 1/4$ in the circuit[5]), one can make the following perturbation expansion:

$$\begin{aligned}\hat{U}_{\mathrm{int}}(t) &= 1 + \left(-\frac{i}{\hbar}\right)\int^t dt'\hat{H}_{\mathrm{int}}(t') \\ &\quad + \left(-\frac{i}{\hbar}\right)^2\int^t\int^{t'} dt'dt''\hat{H}_{\mathrm{int}}(t')\hat{H}_{\mathrm{int}}(t'') + \cdots \\ &= 1 - \left(\frac{it}{\hbar}\right)\frac{E_{12}^2}{2\varepsilon_J^{(j)}}\sigma_x^{(j)}\otimes I^{(k)} + \hat{O}(\zeta_j^2). \end{aligned} \qquad (5)$$

Neglecting the higher-order terms of ζ_j, since it is small, the Hamiltonian of the system can be effectively rewritten as

$$\hat{H}_{\mathrm{eff}}^{(j)} = -\left[\varepsilon_J^{(j)} + \frac{E_{12}^2}{2\varepsilon_J^{(j)}}\right]\sigma_x^{(j)}\otimes\hat{I}^{(k)}. \qquad (6)$$

Above, the first-order expansion term $\hat{U}_{\mathrm{int}}^{(1)}(t)=(-i/\hbar)\int^t dt'\hat{H}_{\mathrm{int}}(t')$ practically does not contribute to the time evolution, due to its small probability (proportional to ζ_j^2). Under this approximation, the fixed interaction between the qubits has been effectively eliminated, except resulting in a shift of the relatively strong Josephson energy. Thus, the system effectively undergoes an evolution

$$\hat{R}_x^{(j)}(\varphi_j) = \exp[i\varphi_j\sigma_x^{(j)}], \quad \varphi_j = \frac{\varepsilon_J^{(j)}t}{\hbar}(1+2\zeta_j^2), \qquad (7)$$

which reduces to the single-qubit $\sigma_x^{(j)}$-rotation (i.e., qubit flip) on the jth qubit, if the duration is set by $\cos\varphi_j=0$.

The robustness of this dynamical decoupling can be verified by testing the difference of the corresponding physical effects, e.g., the transition probabilities P between two selected states, due to the present approximate time-evolution

$$\hat{U}_{\mathrm{appr}}(t) = R_x^{(j)}(\varphi_j)\otimes I^{(k)} \qquad (8)$$

and the exact one

$$\begin{aligned}\hat{U}_{\mathrm{ex}}(t) &= \exp(-it\hat{H}_1/\hbar) = A(t)\sigma_x^{(j)}\otimes\hat{I}^{(k)} + B(t)(|0_j0_k\rangle\langle 0_j0_k| \\ &\quad + |1_j1_k\rangle\langle 1_j1_k|) + B^*(t)(|1_j0_k\rangle\langle 1_j0_k| + |0_j1_k\rangle\langle 0_j1_k|),\end{aligned} \qquad (9)$$

respectively. Here,

$$A(t) = i\rho_j(t), \quad B(t) = [1-\rho_j^2(t)]^{1/2}\exp[-i\xi_j(t)], \qquad (10)$$

with

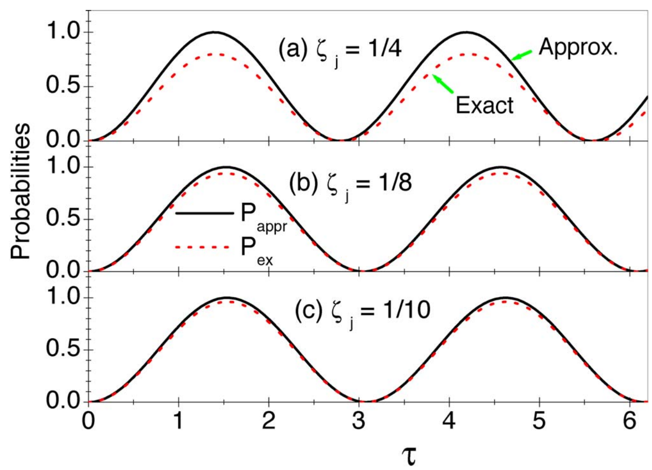

FIG. 2. (Color online) Probabilities $P_{\rm appr}$ (solid lines) and $P_{\rm ex}$ (dotted lines) for the transition $|1_j0_k\rangle \leftrightarrow |0_j0_k\rangle$ versus $\tau=\varepsilon_J^{(j)}t/\hbar$, due to the approximate and exact time evolutions, respectively. The difference between them decreases with decreasing interbit coupling ζ_j.

$$\rho_j(t) = \nu_j^{-1}\sin(\varepsilon_J^{(j)}\nu_j t/\hbar),$$

$$\nu_j = [1 + (E_{12}/\varepsilon_J^{(j)})^2]^{1/2},$$

$$\xi_j(t) = \arctan[2\zeta_j\nu_j^{-1}\tan(\varepsilon_J^{(j)}\nu_j t/\hbar)].$$

Figure 2 shows that the probabilities for the transition $|1_j0_k\rangle \leftrightarrow |0_j0_k\rangle$ due to the evolutions $\hat{U}_{\rm appr}(t)$ (solid lines) and $\hat{U}_{\rm ex}(t)$ (dotted lines) possess the same oscillating period. Also, the difference between the probabilities decreases when deceasing the coupling strength ζ_j: The largest differences are less than 0.06 and 0.04 for coupling strengths $\zeta_j=1/8$ and $\zeta_j=1/10$, respectively.

Similarly, if the system works far from the coresonance point (e.g., $n_{g_j}<0.25$),[4] then both $E_J^{(j)}$ and the fixed coupling E_{12} are the small perturbative quantities, compared to the charging energy $E_C^{(j)}$. Thus, the Hamiltonian (1) can be effectively approximated by

$$\hat{H}_2 = \sum_{j=1,2} E_j\sigma_z^{(j)} + E_{12}\sigma_z^{(1)}\otimes\sigma_z^{(2)}, \tag{11}$$

with $E_j=E_C^{(j)}[1+\varsigma_j^2/(1-\varsigma_{12}^2)]$, and $\varsigma_j=E_J^{(j)}/(2E_C^{(j)})$, $\varsigma_{12}=E_{12}/E_C^{(j)}$. The evolution corresponding to this effective Hamiltonian results in a two-qubit operation

$$R_z^{(12)}(\chi) = \exp[-i\chi_{12}\sigma_z^{(1)}\sigma_z^{(2)}]\otimes\prod_{j=1,2}\exp[-i\chi_j\sigma_z^{(j)}], \tag{12}$$

where $\chi=\{\chi_j,\chi_{12}\}$ and $\chi_j=E_jt/\hbar$, $\chi_{12}=E_{12}t/\hbar$. For the simplest case where $E_J^{(j)}=0$ and thus $\varsigma_j=0$, we have $E_j=E_C^{(j)}$. By using a refocusing technique, like in NMR,[14] we can effectively realize another important single-qubit operation

$$R_z^{(j)}(\phi_j) = [R_z^{(12)}(\chi)\sigma_x^{(k)}]^2 = \exp[-i\phi_j\sigma_z^{(j)}], \tag{13}$$

with $\phi_j=2\chi_j$. The inverse of this operation, i.e., the gate $R_z^{(j)}(-\phi_j)=\exp[i\phi_j\sigma_z^{(j)}]$ can be obtained by changing the signs of E_j via controlling the applied gate voltage.

The single-qubit gates $R_x^{(j)}(\varphi_j)$ and $R_z^{(j)}(\phi_j)$ do not commute, and thus constitute a universal single-qubit gate set, which can assist the realization of two-qubit gates to implement any unitary operation on this circuit. For example, a Hadamard-like operation

$$R_j(\theta_j) = R_z^{(j)}(\theta_j/2)R_x^{(j)}(\pi/4)R_z^{(j)}(-\theta_j/2) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i\exp(i\theta_j) \\ -i\exp(-i\theta_j) & 1 \end{pmatrix}, \tag{14}$$

can be implemented, which will take an important role for testing Bell's inequality.

III. TESTING BELL'S INEQUALITY BY USING EFFECTIVE SINGLE-QUBIT LOCAL OPERATIONS

By using the above dynamical decoupling procedure, we now show that Bell's inequality may be tested with fixed-coupling Josephson qubits.

First, the desired entanglement between these SQUID-based qubits can be created in a repeatable way. Initially, the system works sufficiently far from the coresonance point and remains at the state $|\psi_0\rangle=|00\rangle$, $\Phi_j=0$. Now, a pair of gate voltage pulses brings the system to the coresonance point[5] and lets the system undergo the evolution $\hat{U}_3(t)=\exp(-it\hat{H}_3/\hbar)$, with

$$\hat{H}_3 = -\varepsilon_J\sum_{j=1,2}\sigma_x^{(j)} + E_{12}\sigma_z^{(1)}\otimes\sigma_z^{(2)}. \tag{15}$$

For simplicity, here we assume that $\varepsilon_J^{(1)}=\varepsilon_J^{(2)}=\varepsilon_J$. We analytically derive the time-dependent degree of entanglement or *concurrence*[7] $C_E(t)$ of this circuit

$$C_E(t) = \frac{1}{2}\sqrt{P^2(t)+Q^2(t)}, \tag{16}$$

with

$$P(t) = \cos^2\vartheta(t) - \cos\varrho(t) + \sin^2\vartheta(t)\left(\frac{1}{1+\tilde{\zeta}^2} - \frac{1}{1+\tilde{\zeta}^{-2}}\right), \tag{17}$$

$$Q(t) = \frac{\sin^2[2\vartheta(t)]}{\sqrt{1+\tilde{\zeta}^{-2}}} - \sin\varrho(t), \tag{18}$$

and

$$\vartheta(t) = \gamma(t)(1+\tilde{\zeta}^2)^{1/2}, \quad \varrho(t) = 2\tilde{\zeta}\gamma(t),$$

$$\gamma(t) = 2\varepsilon_J t/\hbar, \quad \tilde{\zeta} = E_{12}/(2\varepsilon_J).$$

Figure 3 shows this evolution, showing some plateaus near the times t_e when $\sin\vartheta(t_e)=0$. At these times, the system is in the following compact entangled state

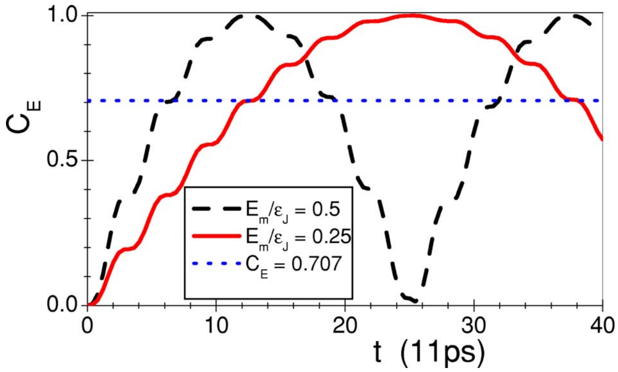

FIG. 3. (Color online) Dynamical evolution of the degree of entanglement $C_E(t)$ for the circuit (Fig. 1) with couplings: $E_m/\varepsilon_J=1/2$ (dashed line), $1/4$ (solid line), respectively. Here, $\varepsilon_J=30$ μeV. The $C_E=0.707$ (dotted line) gives the threshold for the violation of Bell's inequality for the entangled state (8), whose degree of entanglement slowly changes during several short time intervals (see the plateaus in the figure).

$$|\psi_e\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{19}$$

with

$$C_E(t_e) = 2|\alpha_+\alpha_-| = |\sin(E_{12}t_e/\hbar)|, \tag{20}$$

and

$$\alpha_\pm = [1 \pm \exp(\pm i t_e E_{12}/\hbar)]/2.$$

These states are very adjacent to the eigenstates of $\hat{H}_3$, and almost do not evolve for several very short time intervals. Thus, periodically, their degrees of entanglement are almost unchanged, shown by the short plateaus in Fig. 3. The maximally entangled states (corresponding to the top plateaus in Fig. 3) occur when the pulse durations t_e are set properly such that the condition $\cos(E_{12}t_e/\hbar)=0$ is further satisfied.

Next, using dynamically generated single-qubit operations, the controllable variables $\{\theta_j\}$ can be encoded into the generated entangled states, keeping the degree of entanglement unchanged. The change of concurrence of the two-qubit entangled state can be effectively suppressed by continuously applying controllable single-qubit operations. This is similar to the approaches for suppressing decoherence in open quantum systems by using the quantum Zeno effect and the "bang-bang" decoupling method.[16] Thus, the new entangled state

$$|\psi_e'\rangle = \prod_{j=1,2} \hat{R}_j(\theta_j)|\psi_e\rangle = \sum_{m,n=0,1} a_{mm}|mn\rangle, \tag{21}$$

with

$$a_{00} = [\alpha - \beta \exp(i\theta_1 + i\theta_2)]/2,$$

$$a_{10} = [-i\alpha \exp(-i\theta_1) - i\beta \exp(i\theta_2)]/2,$$

$$a_{01} = [-i\alpha \exp(-i\theta_2) - i\beta \exp(i\theta_1)]/2,$$

$$a_{11} = [\alpha \exp(-i\theta_1 - i\theta_2) - \beta]/2,$$

has the same degree of entanglement as that of the $|\psi_e\rangle$ generated above.

Finally, the correlations between the classical variables $\{\theta_j\}$ can be measured by simultaneously detecting the populations of qubits in the excited $|1\rangle$ or ground states $|0\rangle$.[17] Experimentally, the above steps can be repeated many times for the same classical variables and then the correlation function $E_e(\theta_1,\theta_2)$ can be measured as

$$E_e(\theta_1,\theta_2) = \frac{N_{\rm same}(\theta_1,\theta_2) - N_{\rm diff}(\theta_1,\theta_2)}{N_{\rm same}(\theta_1,\theta_2) + N_{\rm diff}(\theta_1,\theta_2)}, \tag{22}$$

with $N_{\rm same}(\theta_1,\theta_2)(N_{\rm diff}(\theta_1,\theta_2))$ being the number of events with two qubits found in the same (different) logic states. Theoretically, the above projected measurements can be expressed via

$$\hat{P}_T = |11\rangle\langle 11| + |00\rangle\langle 00| - |10\rangle\langle 10| - |01\rangle\langle 01| = \hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}, \tag{23}$$

and the correlation in the outcomes can be calculated as

$$E(\theta_1,\theta_2) = \langle\psi_e'|\hat{P}_T|\psi_e'\rangle = \pm\sin(t_e E_{12}/\hbar)\sin(\theta_1+\theta_2). \tag{24}$$

For the sets of angles: $\{\theta_j,\theta_j'\}=\{-\pi/8, 3\pi/8\}$, the Clauser, Horne, Shimony, and Holt (CHSH) (Ref. 9) function

$$\begin{aligned} f(|\psi_e'\rangle) &= |E(\theta_1,\theta_2) + E(\theta_1',\theta_2) + E(\theta_1,\theta_2') - E(\theta_1',\theta_2')| \\ &= 2\sqrt{2}|\sin(t_e E_{12}/\hbar)| \end{aligned} \tag{25}$$

is larger than 2 for

$$|\sin(t_e E_{12}/\hbar)| > \frac{1}{\sqrt{2}}. \tag{26}$$

Therefore, properly setting the pulse duration t_e to prepare the desired entangled state (whose plateaulike concurrence is larger than 0.707), the CHSH-type Bell's inequality[9]

$$f_e(|\psi_e'\rangle) < 2 \tag{27}$$

can be effectively tested by experimentally measuring the CHSH function: $f_e(|\psi_e'\rangle)=|E_e(\theta_1,\theta_2)+E_e(\theta_1',\theta_2)+E_e(\theta_1,\theta_2') - E_e(\theta_1',\theta_2')|$.

IV. DISCUSSIONS AND CONCLUSIONS

The simplest dynamical-decoupling process proposed above consists of two $\sigma_x^{(j)}$ pulses and two delays

$$\hat{U}_d(\tau) = \exp[-iE_{12}\tau\sigma_z^{(1)}\sigma_z^{(2)}]. \tag{28}$$

The duration of the $\sigma_x^{(j)}$ pulse is calculated[5] as $t_x=(2l+1)t_0$, $l=0,1,2,\ldots$; and $t_0=\pi\hbar/[2\varepsilon_J^{(j)}(1+2\zeta_j^2)]\simeq 31$ ps. Consequently, the longest time delay τ between the two σ_x pulses could be estimated[5] as $\tau\sim 270$ ps (for $l=0$), ~ 200 ps (for $l=1$), ~ 145 ps (for $l=2$), and ~ 98 ps (for $l=3$), etc. Thus, it is easy to experimentally check this simplest proposal for eliminating the fixed interbit coupling by using the pulse sequence: $\hat{U}_d(\tau)\sigma_x^{(j)}\hat{U}_d(\tau)\sigma_x^{(j)}$ or $\sigma_x^{(j)}\hat{U}_d(\tau)\sigma_x^{(j)}\hat{U}_d(\tau)$. After these operations the two qubits should return to their initial states. Furthermore, a universal two-qubit controlled-$\sigma_z^{(j)}$ gate could be implemented by using the operational sequence: $\hat{U}_d(-\pi/4E_{12})R_z^{(k)}(\pi/4)R_z^{(j)}(\pi/4)$.

Similar to other theoretical schemes (see, e.g., Ref. 18), the realizability of the present proposal also faces the technological challenge of rapidly switching on/off the Josephson energy of the qubit by using fast magnetic pulses.[19] This experimental difficulty could be relaxed by increasing the durations of the applied pulses. Especially, the decoherence time of the two-qubit capacitively coupled Josephson circuit[20] could be increased by decreasing the coupling capacitance C_m. In principle, the lifetime of the generated entangled state (19) adequately allows us to perform the required operations for testing Bell's inequality (27), since such a state is very adjacent to the eigenstates of the circuit's Hamiltonian $\hat{H}_3$. In fact, the decay time of a two-qubit excited state is long (up to $\sim$0.6 ns), even for very strong interbit coupling (e.g., $E_m \simeq 2\varepsilon_J^{(j)}$ in the recent experiment[5]). In addition, the influence of the environmental noise and operational imperfections is not fatal, as the nonlocal correlation $E(\theta_i,\theta_j)$ in Bell's equality is *statistical*—i.e., its fluctuations could be effectively suppressed by averaging over several repeatable experiments.

In summary, we propose an effective dynamical-decoupling approach to overcome the fixed-interaction difficulty in superconducting nanocircuits. The dynamically generated single-qubit operations may be used to test Bell's inequality, providing another way to verify the existence of entanglement between two capacitively coupled Josephson qubits. The proposed approach can be easily modified to manipulate quantum entanglement in other "fixed-interaction" solid-state systems, e.g., the capacitively (inductively) coupled Josephson phase (flux) circuits, and the Ising (Heisenberg)-spin chain.

ACKNOWLEDGMENTS

We thank Dr. J. S. Tsai, Dr. Y. Pashkin, and Dr. X. Hu for useful discussions. This work was supported in part by the National Security Agency (NSA) and Advanced Research and Development Activity (ARDA) under Air Force Office of Research (AFOSR) Contract No. F49620-02-1-0334, and by the National Science Foundation Grant No. EIA-0130383.

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