# Quantum computation with Josephson qubits using a current-biased information bus

L. F. Wei,[1,2] Yu-xi Liu,[1] and Franco Nori[1,3]
[1]*Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, 351-0198, Japan*
[2]*Institute of Quantum Optics and Quantum Information, Department of Physics, Shanghai Jiaotong University, Shanghai 200030, People's Republic of China*
[3]*Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, The University of Michigan, Ann Arbor, Michigan 48109-1120, USA*
(Received 5 August 2004; revised manuscript received 3 January 2005; published 13 April 2005)

We propose an effective scheme for manipulating quantum information stored in a superconducting nanocircuit. The Josephson qubits are coupled via their separate interactions with an information bus, a large current-biased Josephson junction treated as an oscillator with adjustable frequency. The bus is sequentially coupled to only one qubit at a time. Distant Josephson qubits without any direct interaction can be indirectly coupled with each other by independently interacting with the bus sequentially, via exciting/deexciting vibrational quanta in the bus. This is a superconducting analog of the successful ion trap experiments on quantum computing. Our approach differs from previous schemes that simultaneously coupled two qubits to the bus, as opposed to their sequential coupling considered here. The significant quantum logic gates can be realized by using these tunable and selective couplings. The decoherence properties of the proposed quantum system are analyzed within the Bloch-Redfield formalism. Numerical estimations of certain important experimental parameters are provided.

DOI: 10.1103/PhysRevB.71.134506 PACS number(s): 74.50.+r, 03.65.Ud, 03.67.Lx, 85.25.Cp

## I. INTRODUCTION

The coherent manipulation of quantum states for realizing certain potential applications, e.g., quantum computation and quantum communication, is attracting considerable interest.[1] In principle, any two-state quantum system works as a qubit, the fundamental unit of quantum information. However, only a few real physical systems have worked as qubits, because of requirements of a long coherent time and operability. Among various physical realizations, such as ions traps (see, e.g., Refs. 2–4), QED cavities (see, e.g., Refs. 5 and 6), quantum dots (see, e.g., Refs. 7 and 8) and NMR (see, e.g., Refs. 9 and 10), etc., superconductors with Josephson junctions offer one of the most promising platforms for realizing quantum computation (see, e.g., Refs. 11–31). The nonlinearity of Josephson junctions can be used to produce controllable qubits. Also, circuits with Josephson junctions combine the intrinsic coherence of the macroscopic quantum state and the possibility to control its quantum dynamics by using voltage and magnetic flux pulses. In addition, present-day technologies of integration allow scaling to large and complex circuits. Recent experiments have demonstrated quantum coherent dynamics in the time domain in both single-qubit (see, e.g., Refs. 12–14) and two-qubit Josephson systems.[15]

There are two basic types of Josephson systems used to implement qubits: charge qubits[12] and flux qubits,[13] depending on the ratio of two characteristic energies: the charging energy $E_C$ and the Josephson energy $E_J$. The charge qubit is a Cooper-pair box with a small Josephson coupling energy $E_J \ll E_C$ and a well defined number of Cooper pairs. The flux qubit operates in another extreme limit, where $E_J \gg E_C$ and the phase is well defined. A "quantronium" circuit operating in the intermediate regime of the former two has also been proposed.[14] Voltage-biased superconducting quantum interference devices (SQUIDs), which work in the charge regime and with controllable Josephson energies, form the SQUID-based charge qubits that we will consider in this work. Our results can be extended to flux and flux-charge qubits.

The key ingredient for computational speedup in quantum computation is entanglement, a property that does not exist in classical physics. Thus, manipulating coupled qubits plays a central role in quantum information processing (QIP). Heisenberg-type qubit-couplings are common for the usual solid state QIP systems, e.g., the real spin states of the electrons in quantum dots.[7,8] However, the interbit couplings for Josephson junctions involve Ising-type interactions, as superconducting qubits with two macroscopic quantum states provide pseudo-spin-$1/2$ states. Recently, either the current-current interaction, by connecting to a common inductor, or the charge-charge coupling, via sharing a common capacitor, have been proposed to directly couple two Josephson charge qubits: the $i$th and $j$th ones. Current-current interactions have been used to implement either $\sigma_y^{(i)} \otimes \sigma_y^{(j)}$-type[17] or $\sigma_x^{(i)} \otimes \sigma_x^{(j)}$-type[18] Ising couplings. While, charge-charge interactions yield a $\sigma_z^{(i)} \otimes \sigma_z^{(j)}$-type[15,16] coupling. Compared to single-qubit operations, the two-qubit operations based on these second-order interactions are more sensitive to the environment. In addition, the capacitive coupling between qubits is not easily tunable. Thus adjusting the physical parameters for realizing two-qubit operation is not easy. In order to ensure that the quanta of the relevant $LC$ oscillator is not excited during the desired quantum operations, the time scales of manipulation in the inductively coupled circuit should be much slower than the eigenfrequency of the $LC$ circuit.[17]

Alternatively, the Josephson qubits may also be coupled together by sequentially interacting with a data bus, instead of simultaneously. This is similar to the techniques used for trapped ions,[2,3] wherein the trapped ions are entangled by

1098-0121/2005/71(13)/134506(12)/$23.00  

exciting and deexciting quanta of their shared center-of-mass vibrational mode (i.e., the data bus). This scheme allows for faster two-qubit operations and possesses longer decoherence times. Indeed, a bus design in a Josephson system has been proposed in Ref. 19 for coupling *d*-wave grain-boundary qubits. Recently, an externally connected *LC* resonator[20] and a cavity QED mode[21] were chosen as alternative data buses. However, it is not always easy to control all the physical properties, such as the eigenfrequencies and decoherence, of these data buses.

A large (e.g., up to 10 $\mu$m) current-biased Josephson junction (CBJJ)[22] is very suitable to act as information bus for coupling Josephson qubits. This is because (i) the CBJJ is an easily fabricated device[23] and may provide more effective immunities to both charge and flux noise, (ii) due to its large junction capacitance, the CBJJ can be capacitively coupled over relatively long distances, (iii) the quantum properties, e.g., quantum transitions between the junction energy levels, of the current-biased Josephson junction are well established,[24,32] and (iv) its eigenfrequency can be controlled by adjusting the applied bias current. In fact, a CBJJ itself can be an experimentally realizable qubit, as demonstrated by the recent observations of Rabi oscillations in them.[25,26] Two logic states of such a qubit are encoded by the two lowest zero-voltage metastable quantum energy levels of the CBJJ. The decoherence properties of this CBJJ qubit were discussed in detail in Ref. 27. Experimentally, the entangled macroscopic quantum states in two CBJJ qubits coupled by a capacitor were created.[28] Also, by numerical integration of the time-dependent Schrödinger equation, a full dynamical simulation of two-qubit quantum logic gates between two capacitively coupled CBJJ qubits was given in Ref. 29.

In this paper, we propose a convenient scheme to selectively couple two Josephson charge qubits. Here, a large CBJJ acts only as the information bus for transferring the quantum information between the qubits. Thus, hereafter *the CBJJ will not be a qubit*, as in Refs. 22 and 25–29. Two chosen distant SQUID-based charge qubits can be indirectly coupled by sequentially interacting these with the bus. Our proposal could be considered as a superconducting analog of the ion trap[2] QC, with the phonons (their data bus) replaced by a CBJJ. The eigenfrequency of this information bus can be easily adjusted by controlling the applied bias current. Thus, the bus can couple to any selected qubit, either resonantly or dispersively, although different qubits may possess different eigenfrequencies. The anharmonic energy levels of the bus assure that the possible transition only takes place between its ground and the first excited states. This coupling method provides a repeatable way to generate entangled states, and thus can implement elementary quantum logic gates between arbitrarily selected qubits. Our proposal shares some features with the circuits proposed in Refs. 17, 18, 20, and 22, but also has significant differences. Our proposal might be more amenable to experimental verification.

The outline of the paper is as follows. In Sec. II we propose a superconducting nanocircuit with a CBJJ acting as the data bus, and investigate its elemental quantum dynamics. The bus is biased by a dc current and is assumed to interact with only one qubit at a time. There is no direct interaction between qubits. Therefore, the elemental operations in this circuit consist of (i) the free evolution of the single qubit, (ii) the free evolution of the bus, and (iii) the coherent dynamics for a single qubit coupled to the bus. In Sec. III we show how to realize the elemental logic gates in the proposed nanocircuit: the single-qubit rotations by properly switching on/off the applied gate voltage and external flux, and the two-qubit operations by letting them couple sequentially to the bus. The vibrational quanta of the bus is excited/absorbed during the qubit-bus interactions. In Sec. IV we analyze the decoherence properties of the present qubit-bus interaction within the Bloch-Redfield formalism,[33] and give some numerical estimates for experimental implementations. Conclusions and some discussions are given in Sec. V.

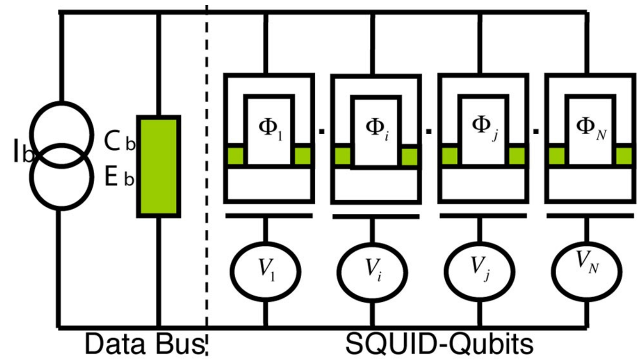


FIG. 1. SQUID-based charge qubits coupled via a large CBJJ.

## II. A SUPERCONDUCTING NANOCIRCUIT AND ITS ELEMENTARY QUANTUM EVOLUTIONS

The circuit considered here is sketched in Fig. 1. It consists of $N$ voltage-biased SQUIDs connected to a large CBJJ. The $k$th $(k=1,2,\ldots,N)$ qubit consists of a gate electrode of capacitance $C_{g_k}$ and a single-Cooper-pair box with two ultrasmall Josephson junctions of capacitance $C_{J_k}^0$ and Josephson energy $E_{J_k}^0$, forming a dc-SQUID ring. The inductances of these dc-SQUID rings are assumed to be very small and can be neglected. The SQUIDs work in the charge regime with $k_BT \ll E_J \ll E_C \ll \Delta$, in order to suppress quasiparticle tunneling or excitation. Here, $k_B$, $\Delta$, $E_C$, $T$, and $E_J$ are the Boltzmann constant, the superconducting gap, charging energy, temperature, and the Josephson coupling energy, respectively.

The connected large CBJJ biased by a dc current works in the phase regime with $E_J \gg E_C$. It acts as a tunable anharmonic $LC$ resonator with a nonuniform level spacing and works as a data bus for transferring quantum information between the chosen qubits. The mechanism for manipulating quantum information in the present approach is different from that in Refs. 17, 18, 20, and 22, although the circuit proposed here might seem similar to those there. The differences are as follows.

(1) A large CBJJ, instead of the $LC$ oscillator[17,18,20] formed by the externally connected inductance $L$ and the capacitances in circuit, works as the data bus.

(2) We modulate the applied external flux, instead of the bias current,[22] to realize the perfect coupling/decoupling between the chosen qubit and the bus.

(3) The free evolution of the bus during the operational delays will be utilized to control the dynamical phases for implementing the expected quantum gates.

After a canonical transformation[11,30] the Hamiltonian for the present circuit can be written as

$$\hat{H} = \sum_{k=1}^{N} \left[ \frac{2e^2}{C_k}(\hat{n}_k - n_{g_k})^2 - E_{J_k} \cos\left( \hat{\theta}_k - \frac{C_{g_k}}{C_k}\hat{\theta}_b \right) \right] + \hat{H}_r, \tag{1}$$

with

$$\hat{H}_r = \frac{(2\pi\hat{p}_b/\Phi_0)^2}{2\tilde{C}_b} - E_b \cos\hat{\theta}_b - \frac{\Phi_0 I_b}{2\pi}\hat{\theta}_b. \tag{2}$$

Here, $n_{g_k}=C_{g_k}V_k/(2e)$, $C_k=C_{g_k}+C_{J_k}$, $C_{J_k}=2C^0_{J_k}$, $\tilde{C}_b=C_b+\Sigma_{k=1}^N C_{J_k}C_{g_k}/C_k$, $E_{J_k}=2E^0_{J_k}\cos(\pi\Phi_k/\Phi_0)$, and $\theta_k=(\theta_{k_2}+\theta_{k_1})/2$ with $\theta_{k_1}$ and $\theta_{k_2}$ being the phase drops across two small Josephson junctions in the $k$th qubit, respectively. Also, $C_{g_k}$, $\Phi_0$, $\Phi_k$, and $V_k$ are the gate capacitance, flux quantum, external flux, and gate voltage applied to the $k$th qubit, respectively. Correspondingly, $C_b$, $\theta_b$, $E_b$, and $I_b$ are the capacitance, phase drops, Josephson energy, and the bias current of the large CBJJ, respectively. Above, the number operator $\hat{n}_k$ of excess Cooper-pair charges in the superconducting island and the phase operator $\hat{\theta}_k$ of the order parameter of the $k$th charge qubit are a pair of canonical variables and satisfy the commutation relation

$$[\hat{\theta}_k, \hat{n}_k] = i.$$

The operators $\theta_b$ and $\hat{p}_b$ are another pair of canonical variables and satisfy the commutation relation

$$[\hat{\theta}_b, \hat{p}_b] = i\hbar,$$

with $2\pi p_b/\Phi_0=2n_b e$ representing the charge difference across the CBJJ.

The CBJJ works in the phase regime. Thus, $E_{C_b}=e^2/(2\tilde{C}_b)\ll E_b$ and the quantum motion ruled by the Hamiltonian $\hat{H}_r$ equals that of a particle with mass $m=\tilde{C}_b(\Phi_0/2\pi)^2$ in a potential $U(\theta_b)=-E_b(\cos\theta_b+I_b\theta_b/I_r)$, $I_r=2\pi E_b/\Phi_0$. For the biased case $I_b<I_r$, there exists a series of minima of $U(\theta_b)$, where $\partial U(\theta_b)/\partial\theta_b=0$, $\partial^2 U(\theta_b)/\partial\theta_b^2>0$. Near these points $\theta_0=\arcsin(I_b/I_r)$, $U(\theta_b)$ approximates to a harmonic oscillator potential with a characteristic frequency

$$\omega_b = \sqrt{\frac{2\pi I_r}{\tilde{C}_b\Phi_0}}\left[1-\left(\frac{I_b}{I_r}\right)^2\right]^{1/4},$$

depending on the applied bias current $I_b$. Correspondingly, the Hamiltonian $\hat{H}_r$ reduces to

$$\hat{H}_b = \left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right)\hbar\omega_b, \tag{3}$$

with

$$\hat{a} = \frac{1}{\sqrt{2}}\left[\left(\frac{\Phi_0}{2\pi}\right)\sqrt{\frac{\tilde{C}_b\omega_b}{\hbar}}\hat{\theta}_b + i\left(\frac{2\pi}{\Phi_0}\right)\frac{\hat{p}_b}{\sqrt{\hbar\omega_b\tilde{C}_b}}\right]$$

and

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}\left[\left(\frac{\Phi_0}{2\pi}\right)\sqrt{\frac{\tilde{C}_b\omega_b}{\hbar}}\hat{\theta}_b - i\left(\frac{2\pi}{\Phi_0}\right)\frac{\hat{p}_b}{\sqrt{\hbar\omega_b\tilde{C}_b}}\right].$$

For simplicity, we have redefined the original point of the phase $\theta_b$. The approximate number of quantum metastable bound states[34] of the quantum oscillator is $N_s=(2^{3/4}/3)\sqrt{E_b/E_{C_b}}(1-I_b/I_r)^{5/4}$.

The energy scale of the quantum oscillator (3) is $\omega_b/(2\pi)\sim 10$ GHz,[25] which is of the same order of the Josephson energy in the SQUID. Therefore, the oscillating quantum of the information bus will be really excited, even if only one of the qubits is operated quantum mechanically. This is different from the case considered in Ref. 17, wherein the $LC$ oscillator shared by all charge qubits are not really excited, as the eigenfrequency of the $LC$ circuit is much higher than the typical frequencies of the qubits dynamics. For operational convenience, we assume that the bus is coupled to only one qubit at a time. The coupling between any one of the qubits (e.g., the $k$th one) and the bus can, in principle, be controlled by adjusting the applied external flux (e.g., $\Phi_k$). In this case, any direct interaction does not exist between the qubits, and the dynamics of the CBJJ can be safely restricted to the Hilbert space spanned by the two Fock states $|0_b\rangle$ and $|1_b\rangle$, which are the lowest two energy eigenstates of the harmonic oscillator of Eq. (3). Furthermore, we assume that the applied gate voltage of any chosen ($k$th) qubit works near its degeneracy point with $n_{g_k}=1/2$, and thus only two charge states $|n_k=0\rangle=|\uparrow_k\rangle$ and $|n_k=1\rangle=|\downarrow_k\rangle$, play a role during the quantum operation. All other charge states with a higher energies can be safely ignored. Therefore, the Hamiltonian

$$\hat{H}_{kb} = \hat{H}_k + \hat{H}_b + \lambda_k(\hat{a}^\dagger+\hat{a})\sigma_y^{(k)}, \tag{4}$$

with

$$\hat{H}_k = \left[\frac{\delta E_{C_k}}{2}\sigma_z^{(k)} - \frac{E_{J_k}}{2}\sigma_x^{(k)}\right], \tag{5}$$

describes the interaction between any one of the qubits (e.g., the $k$th one) and the bus, and provides the basic dynamics for the present network. Here, $\delta E_{C_k}=2e^2(1-2n_{g_k})/C_k$, $\lambda_k=E_{J_k}C_{g_k}(2\pi/\Phi_0)\sqrt{\hbar/(2\tilde{C}_b\omega_b)}/(2C_k)$, and the pseudospin operators are defined by

$$\sigma_x^{(k)} = |\uparrow_k\rangle\langle\downarrow_k| + |\downarrow_k\rangle\langle\uparrow_k|,$$

$$\sigma_y^{(k)} = -i|\uparrow_k\rangle\langle\downarrow_k| + i|\downarrow_k\rangle\langle\uparrow_k|,$$

$$\sigma_z^{(k)} = |\uparrow_k\rangle\langle\uparrow_k| - |\downarrow_k\rangle\langle\downarrow_k|.$$

Above, when the first cosine-term in Hamiltonian (1) was expanded, only the single-quantum transition process approximated to the first-order of $\hat{\theta}_b$ was considered. The higher order nonlinearities have been neglected as their effects are very weak. In fact, for the lower number states of the bus, we have $C_{g_k}\sqrt{\langle\theta_b^2\rangle}/C_k \lesssim 10^{-2}$, for the typical experimental parameters[11,15,24,27] $C_b \sim 1pF$, $\omega_b/2\pi \sim 10$ GHz, and $C_{g_k}/C_{J_k} \sim 10^{-2}$.

Notice that the coupling strength $\lambda_k$ between the qubit and the bus is tunable by controlling the flux $\Phi_k$, applied to the selected qubit, and the bias current $I_b$, applied to the information bus. For example, such a coupling can be simply turn off by setting the flux $\Phi_k$ as $\Phi_0/2$. This allows various elemental operations for quantum manipulations to be realizable in a controllable way. In the logic basis $\{|0_k\rangle,|1_k\rangle\}$, defined by

$$|0_k\rangle = \frac{|\downarrow_k\rangle + |\uparrow_k\rangle}{\sqrt{2}}, \quad |1_k\rangle = \frac{|\downarrow_k\rangle - |\uparrow_k\rangle}{\sqrt{2}},$$

and under the usual rotating-wave approximation, the above Hamiltonian (4) can be rewritten as

$$\hat{H}_{kb} = \left[\frac{E_{J_k}}{2}\tilde{\sigma}_z^{(k)} - \frac{\delta E_{C_k}}{2}\tilde{\sigma}_x^{(k)}\right] + \hbar\omega_b\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + i\lambda_k\left[\hat{a}\tilde{\sigma}_+^{(k)} - \hat{a}^\dagger\tilde{\sigma}_-^{(k)}\right], \tag{6}$$

with

$$\tilde{\sigma}_x^{(k)} = |1_k\rangle\langle 0_k| + |0_k\rangle\langle 1_k|,$$

$$\tilde{\sigma}_y^{(k)} = -i|1_k\rangle\langle 0_k| + i|0_k\rangle\langle 1_k|,$$

$$\tilde{\sigma}_z^{(k)} = |1_k\rangle\langle 1_k| - |0_k\rangle\langle 0_k|,$$

and $\tilde{\sigma}_\pm^{(k)} = (\tilde{\sigma}_x^{(k)} \pm i\tilde{\sigma}_y^{(k)})/2$. Here, the logic states $|0_k\rangle$ and $|1_k\rangle$ correspond to the clockwise and anticlockwise persistent circulating currents in the $k$th SQUID loop, respectively.

We now discuss the quantum dynamics of the above Josephson network. Without loss of generality, we assume in what follows that the bias current $I_b$ applied to the CBJJ does not change, once it is set up properly beforehand. The quantum evolutions of the system are then controlled by other external parameters: the fluxes applied to the qubits and the voltages across the gate capacitances of the qubits. Depending on the different settings of the controllable external parameters, different Hamiltonians can be induced from Eq. (6) and thus different time evolutions are obtained. Obviously, during any operational delay $\tau$ with $\Phi_{X_k} = \Phi_0/2$ and $V_k = e/C_{g_k}$, the $k$th qubit remains in its idle state because the Hamiltonian vanishes (i.e., $H_0^{(k)} = 0$) as $E_{J_k} = 0, n_{g_k} = 0$. However, the data bus still undergoes a free time evolution

$$\hat{U}_0(t) = \exp\left(\frac{-it}{\hbar}\hat{H}_b\right). \tag{7}$$

This evolution is useful for controlling the dynamical phase of the qubits to exactly realize certain quantum operations. For the other cases, the dynamical evolutions of the chosen qubit depend on the different settings of the experimental parameters.

(1) For the case where $\Phi_k = \Phi_0/2$ and $V_k \neq e/C_{g_k}$, the $i$th qubit and the bus separately evolve with the Hamiltonians $\hat{H}_1^{(k)} = -\delta E_{C_k}\tilde{\sigma}_x^{(k)}/2$ and $\hat{H}_b$ determined by Eq. (3), respectively. The relevant time-evolution operator of the whole system reads

$$\hat{U}_1^{(k)}(t) = \exp\left(\frac{-it}{\hbar}\hat{H}_1^{(k)}\right) \otimes \exp\left(\frac{-it}{\hbar}\hat{H}_b\right). \tag{8}$$

(2) If the $k$th qubit works at its degenerate point and couples to the bus, i.e., $V_k = e/C_{g_k}$ and $\Phi_k \neq \Phi_0/2$, then we have the Hamiltonian

$$\tilde{\hat{H}}_{kb} = E_{J_k}\tilde{\sigma}_z^{(k)}/2 + \hat{H}_b + i\lambda_k\left[\hat{a}\tilde{\sigma}_+^{(k)} - \hat{a}^\dagger\tilde{\sigma}_-^{(k)}\right] \tag{9}$$

from Eq. (6). The corresponding dynamical evolutions are

$$|0_b\rangle|0_k\rangle \xrightarrow{\tilde{\hat{U}}_{kb}} e^{i\Delta_k t/2}|0_b\rangle|0_k\rangle, \; \tilde{\hat{U}}_{kb} = \exp(-i\tilde{\hat{H}}_{kb}t), \; \Delta_k = E_{J_k}/\hbar - \omega_b,$$

$$|0_b\rangle|1_k\rangle \xrightarrow{\tilde{\hat{U}}_{kb}} e^{-i\omega_b t}\left\{\left[\cos\left(\frac{\Omega_k}{2}t\right) - i\frac{\Delta_k}{\Omega_k}\sin\left(\frac{\Omega_k}{2}t\right)\right]|0_b\rangle|1_k\rangle - \frac{2\lambda_k}{\hbar\Omega_k}\sin\left(\frac{\Omega_k}{2}t\right)|1_b\rangle|0_k\rangle\right\},$$

$$|1_b\rangle|0_k\rangle \xrightarrow{\tilde{\hat{U}}_{kb}} e^{-i\omega_b t}\left\{\left[\cos\left(\frac{\Omega_k}{2}t\right) + i\frac{\Delta_k}{\Omega_k}\sin\left(\frac{\Omega_k}{2}t\right)\right]|1_b\rangle|0_k\rangle + \frac{2\lambda_k}{\hbar\Omega_k}\sin\left(\frac{\Omega_k}{2}t\right)|0_b\rangle|1_k\rangle\right\}, \tag{10}$$

with $\Omega_k = \sqrt{\Delta_k^2 + (2\lambda_k/\hbar)^2}$.

Specifically, we have the time-evolution operator

$$\hat{U}_2^{(k)}(t) = \hat{A}(t)\begin{pmatrix} \cos\left(\frac{\lambda_k t}{\hbar}\sqrt{\hat{n}+1}\right) & -\frac{1}{\sqrt{\hat{n}+1}}\sin\left(\frac{\lambda_k t}{\hbar}\sqrt{\hat{n}+1}\right)\hat{a} \\ \frac{\hat{a}^\dagger}{\sqrt{\hat{n}+1}}\sin\left(\frac{\lambda_k t}{\hbar}\sqrt{\hat{n}+1}\right) & \cos\left(\frac{\lambda_k t}{\hbar}\sqrt{\hat{n}}\right) \end{pmatrix}, \tag{11}$$

with

$$\hat{A}(t) = \exp\left[-it\left(\frac{\hat{H}_b}{\hbar} + \frac{E_{J_k}\tilde{\sigma}_z^{(k)}}{2\hbar}\right)\right]$$

for the resonant case $\Delta_k=0$. This reduces Eq. (10) to the time evolutions

$$|0_b\rangle|0_k\rangle \xrightarrow{\hat{U}_2^{(k)}(t)} |0_b\rangle|0_k\rangle,$$

$$|0_b\rangle|1_k\rangle \xrightarrow{\hat{U}_2^{(k)}(t)} e^{-i\omega_b t}\left[\cos\left(\frac{\lambda_k t}{\hbar}\right)|0_b\rangle|1_k\rangle - \sin\left(\frac{\lambda_k t}{\hbar}\right)|1_b\rangle|0_k\rangle\right],$$

$$|1_b\rangle|0_k\rangle \xrightarrow{\hat{U}_2^{(k)}(t)} e^{-i\omega_b t}\left[\cos\left(\frac{\lambda_k t}{\hbar}\right)|1_b\rangle|0_k\rangle + \sin\left(\frac{\lambda_k t}{\hbar}\right)|0_b\rangle|1_k\rangle\right].$$

For another extreme case, i.e., the system works in the dispersive regime (far from the resonant point) $2\lambda_k/(\hbar|\Delta_k|)\ll 1$, we have the time evolution operator

$$\tilde{\hat{U}}_2^{(k)}(t) = \hat{A}(t)\exp\left(-i\frac{\widetilde{\hat{H}}'_{kb}t}{\hbar}\right), \tag{12}$$

with

$$\widetilde{\hat{H}}'_{kb} = \lambda_k^2(|1_k\rangle\langle 1_k|\hat{a}\hat{a}^\dagger - |0_k\rangle\langle 0_k|\hat{a}^\dagger\hat{a})/(\hbar\Delta_k).$$

It reduces to the following time evolutions:

$$|0_b\rangle|0_k\rangle \xrightarrow{\tilde{\hat{U}}_2^{(k)}(t)} \exp\left(it\frac{\Delta_k}{2}\right)|0_b\rangle|0_i\rangle,$$

$$|0_b\rangle|1_k\rangle \xrightarrow{\tilde{\hat{U}}_2^{(k)}(t)} \exp\left[-it\left(\omega_b + \frac{\Delta_k}{2} + \frac{\lambda_k^2}{\hbar^2\Delta_k}\right)\right]|0_b\rangle|1_k\rangle,$$

$$|1_b\rangle|0_k\rangle \xrightarrow{\tilde{\hat{U}}_2^{(k)}(t)} \exp\left[-it\left(\omega_b - \frac{\Delta_k}{2} - \frac{\lambda_k^2}{\hbar^2\Delta_k}\right)\right]|1_b\rangle|0_k\rangle,$$

$$|1_b\rangle|1_k\rangle \xrightarrow{\tilde{\hat{U}}_2^{(k)}(t)} \exp\left[-it\left(2\omega_b - \frac{\Delta_k}{2} - \frac{2\lambda_k^2}{\hbar^2\Delta_k}\right)\right]|1_b\rangle|1_k\rangle.$$

(3) Generally, if $\Phi_k\neq\Phi_0/2$ and $V_{g_k}\neq e/C_{g_k}$, then the Hamiltonian (6) can be rewritten as

$$\bar{\hat{H}}_{kb} = \frac{E_k}{2}\bar{\sigma}_z^{(k)} + \hat{H}_b + i\lambda_k(\hat{a}^\dagger\bar{\sigma}_-^{(k)} - \hat{a}\bar{\sigma}_+^{(k)}), \tag{13}$$

with

$$\bar{\sigma}_x^{(k)} = -\sin\eta_k\tilde{\sigma}_z^{(k)} - \cos\eta_k\tilde{\sigma}_x^{(k)},$$

$$\bar{\sigma}_y^{(k)} = -\tilde{\sigma}_y^{(k)},$$

$$\bar{\sigma}_z^{(k)} = \cos\eta_k\tilde{\sigma}_z^{(k)} - \sin\eta_k\tilde{\sigma}_x^{(k)},$$

and $\bar{\sigma}_\pm^{(k)}=(\bar{\sigma}_x^{(k)}\pm i\bar{\sigma}_y^{(k)})/2$. Here, $\cos\eta_k=E_{J_k}/E_k$, and $E_k=\sqrt{(\delta E_{C_k})^2+E_{J_k}^2}$. If the bias current $I_b$ and the flux $\Phi_k$ are set properly beforehand such that $E_{J_k}\sim\hbar\omega_b\ll\delta E_{C_k}$, then the detuning $\hbar\bar{\Delta}_k=E_k-\hbar\omega_b$ is very large (compared to the coupling strength $\lambda_k\lesssim 10^{-1}E_{J_k}$). Therefore, the time-evolution operator of the system can be approximated as

$$\bar{\hat{U}}_3^{(k)}(t) = \hat{B}(t)\exp\left\{-i\frac{\lambda_k^2 t}{\hbar^2\bar{\Delta}_k}\left[\bar{\sigma}_z^{(k)}\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right)+\frac{1}{2}\right]\right\}, \tag{14}$$

with

$$\hat{B}(t) = \exp\left[-it\left(\frac{\hat{H}_b}{\hbar} + \frac{E_k\bar{\sigma}_z^{(k)}}{2\hbar}\right)\right].$$

This implies the following evolutions:

$$|0_b\rangle|0_k\rangle \xrightarrow{\bar{\hat{U}}_3^{(k)}(t)} e^{-i\zeta_k t}\{[\cos(\xi_k t) + i\cos\eta_k\sin(\xi_k t)]\,|0_b\rangle|0_k\rangle + i\sin\eta_k\sin(\xi_k t)|0_b\rangle|1_k\rangle\},$$

$$|0_b\rangle|1_k\rangle \xrightarrow{\bar{\hat{U}}_3^{(k)}(t)} e^{-i\zeta_k t}\{[\cos(\xi_k t) - i\cos\eta_k\sin(\xi_k t)]|0_b\rangle|1_k\rangle + i\sin\eta_k\sin(\xi_k t)|0_b\rangle|0_k\rangle\},$$

$$|1_b\rangle|0_k\rangle \xrightarrow{\bar{\hat{U}}_3^{(k)}(t)} e^{-i(\zeta_k+\omega_b)t}\{[\cos(\xi'_k t) + i\cos\eta_k\sin(\xi'_k t)]|1_b\rangle|0_k\rangle + i\sin\eta_k\sin(\xi'_k t)|1_b\rangle|1_k\rangle\},$$

$$|1_b\rangle|1_k\rangle \xrightarrow{\bar{\hat{U}}_3^{(k)}(t)} e^{-i(\zeta_k+\omega_b)t}\{i\sin\eta_k\sin(\xi'_k t)|1_b\rangle|0_k\rangle + [\cos(\xi'_k t) - i\cos\eta_k\sin(\xi'_k t)]|1_b\rangle|1_k\rangle\},$$

with

$$\zeta_k = \omega_b/2 + \lambda_k^2/(2\hbar^3\bar{\Delta}_k), \quad \xi_k = E_k/(2\hbar) + \lambda_k^2/(2\hbar^2\bar{\Delta}_k)$$

and

$$\xi'_k = \xi_k + \lambda_k^2/(\hbar^2\bar{\Delta}_k).$$

In what follows we shall show that any process for manipulating the quantum information stored in the present circuit can be effectively implemented by selectively using the above elementary time evolutions $\hat{U}_0(t)$, $\hat{U}_1^{(k)}(t)$, $\hat{U}_2^{(k)}(t)$, $\tilde{\hat{U}}_2^{(k)}(t)$, and $\bar{\hat{U}}_3^{(k)}(t)$.

## III. QUANTUM MANIPULATIONS OF THE SUPERCONDUCTING NANOCIRCUIT

It is well known that any valid quantum transformation can be decomposed into a sequence of elementary one- and two-qubit quantum gates. The set of these gates is universal, and any quantum computing circuit comprises only gates from this set. Several schemes[17,18,29] have been proposed for implementing one of the universal two-qubit gates with Josephson qubits by using the direct interactions between them.

By making use of the data bus interacting sequentially with the selective qubits, Blais *et al.*[22] showed that the two-qubit gate may be effectively realized. Two important problems will be solved in our indirect-coupling approach: (i) When one of two qubits is selected to couple with the data bus, how we can let the remainder qubit decouple completely from the bus and (ii) the phase changes of the bus' and qubit's states during the operations are very complicated, how we can control these phase changes in order to precisely implement the desired quantum gate.

The scheme in Ref. 22 assumed that, when one of the two qubits is tuned to resonance with the bus, then the other qubit is hardly affected because of its different Rabi frequency. Obviously, this decoupling is not complete and thus it is not easy to assure that the bus couples only one qubit at a time. By controlling the external flux $\Phi_k$ applied to the qubits, the network proposed here provides an effective method for making the remainder qubit completely decouple from the bus. All the desired elementary operations for quantum computing can be exactly implemented by properly setting the experimentally controllable parameters, e.g., the external $\Phi_k$, the gate voltage $V_k$, the bias current $I_b$, and the duration $t$ of each selected quantum evolution, etc. Hereafter, we assume that each of the selected time evolutions can be switched on/off very quickly.

### A. Single-qubit operations

First, we show how to realize the single-qubit operations on each SQUID qubit. This will be achieved by simply turning on/off the relevant experimentally controllable parameters. For example, if $n_{g_k}\neq 1/2$ and $E_{J_k}=0$ for a time span $t$, then the time evolution $\hat{U}_1^{(k)}(t)$ in Eq. (8) is realized. This operation is the single-qubit rotation around the $x$ axis

$$\hat{R}_x^{(k)}(\varphi_k)=\begin{pmatrix}\cos\frac{\varphi_k}{2} & i\sin\frac{\varphi_k}{2}\\ i\sin\frac{\varphi_k}{2} & \cos\frac{\varphi_k}{2}\end{pmatrix}, \tag{15}$$

with $\varphi_k=\delta E_{C_k}t/\hbar$. Rotations by $\varphi_k=\pi$ and $\varphi_k=\pi/2$ produce a spin flip (i.e., a NOT-gate operation) and an equal-weight superposition of logic states, respectively.

The rotation around the $z$ axis can be implemented by using the evolution (12). This operation is conditional and dependent on the state of the bus. If the bus is in the ground state $|0_b\rangle$, the rotation reads

$$R_z^{(k)}(\phi_k)=e^{-i\varrho_k t}\begin{pmatrix}e^{-i\phi_k} & 0\\ 0 & e^{i\phi_k}\end{pmatrix}, \tag{16}$$

with $\varrho_k=\omega_b/2+\lambda_k^2/(2\hbar^2\Delta_k)$, $\phi_k=E_{J_k}t/(2\hbar)+\lambda_k^2 t/(2\hbar^2\Delta_k)$. With a sequence of $x$ and $z$ rotations, any rotation on the single qubit can be performed. For example, the Hadamard gate applied to the $k$th qubit

$$\hat{H}_g^{(k)}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

can be implemented by a three-step rotation

$$\hat{R}_z^{(k)}\left(\frac{\pi}{4}\right)\otimes\hat{R}_x^{(k)}\left(-\frac{\pi}{2}\right)\otimes\hat{R}_z^{(k)}\left(\frac{\pi}{4}\right)=\hat{H}_g^{(k)}. \tag{17}$$

Here, the relevant durations $t_1$, $t_2$, and $t_3$ are set properly to satisfy the conditions

$$\cos\left(\frac{\delta E_{C_k}t_2}{\hbar}\right)=-\sin\left(\frac{\delta E_{C_k}t_2}{\hbar}\right)=\sin\left[\frac{E_{J_k}t_1}{2\hbar}+\frac{(\lambda_k/\hbar)^2t_1}{2\Delta_k}\right]$$
$$=\sin\left[\frac{E_{J_k}t_3}{2\hbar}+\frac{(\lambda_k/\hbar)^2t_3}{2\Delta_k}\right]=\frac{1}{\sqrt{2}}.$$

### B. Two-qubit operations

Second, we show how to realize two-qubit gates by letting a pair of qubits (the $k$th and $j$th ones) interact separately with the bus. Before the quantum operation, the chosen qubits decouple from the bus. At the end of the desired gate operation the bus should be disentangled again from the qubits, and returned to its ground state. For operational simplicity, we assume that the bus resonates with the control qubit, the $k$th one, i.e., $\Delta_k=0$. We now consider the following three-step operational process.

(i) Couple the control qubit to the bus (i.e., the applied external flux $\Phi_k$ is varied to $\Phi_0$) and realize the evolution $\hat{U}_2^{(k)}(t_1)$ for the duration $t_1$:

$$\sin\left(\frac{\lambda_k t_1}{\hbar}\right)=-1. \tag{18}$$

Then, by returning the $\Phi_k$ to its initial value, i.e., $\Phi_k=\Phi_0/2$, the $k$th qubit can be decoupled from the bus exactly. Before the next step operation, there is an operational delay $\tau_1$. During this delay the state of the qubits does not evolve, while the data bus still undergoes a time evolution $\hat{U}_0(\tau_1)$.

(ii) Couple the target qubit (the $j$th one) to the bus and realize the time evolution $\hat{U}_3^{(j)}(t_2)$. This is achieved by letting the chosen qubit work near its degenerate point (i.e., $n_{g_j}\neq 1/2$) and switching on its Josephson energy (i.e., $\Phi_j\neq\Phi_0/2$). After the time $t_2$ determined by the condition

$$\cos(\xi_j t_2)=-\sin(\xi_j' t_2)=1, \tag{19}$$

we decouple the $j$th qubit from the bus and let it be in the idle state by returning its gate voltage $V_j$ to the degenerate point ($n_{g_j}=1/2$), and simultaneously switching off the relevant Josephson energy. During another operational delay $\tau_2$ before the next step operation, the bus undergoes another free evolution $\hat{U}_0(\tau_2)$.

(iii) Repeat the first step and realize the evolution $\hat{U}_2^{(k)}(t_3)$ with

$$\sin\left(\frac{\lambda_k t_3}{\hbar}\right)=1. \tag{20}$$

Diagrammatically, the above three-step operational process with two delays can be represented as follows:

$$|0_b0_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b\tau_1/2}|0_b0_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\overline{\hat{U}}_3^{(j)}(t_2)} e^{-i\chi}|0_b0_k0_j\rangle \xrightarrow{\hat{U}_2^{(k)}(t_3)} e^{-i\chi}|0_b0_k0_j\rangle,$$

$$|0_b0_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b\tau_1/2}|0_b0_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\overline{\hat{U}}_3^{(j)}(t_2)} e^{-i\chi}|0_b0_k1_j\rangle \xrightarrow{\hat{U}_2^{(k)}(t_3)} e^{-i\chi}|0_b0_k1_j\rangle,$$

$$|0_b1_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b(t_1+3\tau_1/2)}|1_b0_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\overline{\hat{U}}_3^{(j)}(t_2)} ie^{-i\chi-i\omega_b(t_1+t_2+\tau_1+\tau_2)}(\cos\eta_j|1_b0_k0_j\rangle + \sin\eta_j|1_b0_k1_j\rangle) \xrightarrow{\hat{U}_2^{(k)}(t_3)} ie^{-i\chi-i\omega_bT}(\cos\eta_j|0_b1_k0_j\rangle + \sin\eta_j|0_b1_k1_j\rangle),$$

$$|0_b1_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b(t_1+3\tau_1/2)}|1_b0_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\overline{\hat{U}}_3^{(j)}(t_2)} ie^{-i\chi-i\omega_b(t_1+t_2+\tau_1+\tau_2)}(\sin\eta_j|1_b0_k0_j\rangle - \cos\eta_j|1_b0_k1_j\rangle) \xrightarrow{\hat{U}_2^{(k)}(t_3)} ie^{-i\chi-i\omega_bT}(\sin\eta_j|0_b1_k0_j\rangle - \cos\eta_j|0_b1_k1_j\rangle),$$

with $T=t_1+t_2+t_3+\tau_1+\tau_2$ being the total duration of the process, and $\chi=\zeta_jt_2+\omega_b(\tau_1+\tau_2)/2$. Obviously, the information bus remains in its ground state $|0_b\rangle$ after the operations. If the total duration $T$ is satisfied as

$$\sin(\omega_bT)=1, \tag{21}$$

the above three-step process with two delays yields a two-qubit gate expressed by the following matrix form:

$$\hat{U}_1^{(kj)}(\eta_j)=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\eta_j&\sin\eta_j\\0&0&\sin\eta_j&-\cos\eta_j\end{pmatrix}, \tag{22}$$

which is a universal two-qubit Deutsch gate.[35]

Analogously, if the second step operation $\overline{\hat{U}}_3^{(j)}(t_2)$ in the above three-step process is replaced by the operation $\overline{\hat{U}}_2^{(j)}(t_2)$, then another two-qubit operation expressed by

$$\overline{\hat{U}}_2^{(kj)}(t_2)=\begin{pmatrix}\Gamma_j&0&0&0\\0&\Gamma_j^*&0&0\\0&0&\Lambda_je^{-i\omega_bT}&0\\0&0&0&\Lambda_j^*e^{-i\omega_bT}\end{pmatrix}, \tag{23}$$

with $\Gamma_j=\exp(i\varsigma_jt_2)$, $\Lambda_j=\exp(i\varsigma_j't_2)$, $\varsigma_j=E_{J_j}/(2\hbar)+\lambda_j^2/(2\hbar^2\Delta_j)$, $\varsigma_j'=\varsigma_j+\lambda_j^2t_2/(\hbar^2\Delta_j)$, can be implemented. This three-step operational process can similarly be represented diagrammatically as

$$|0_b0_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b\tau_1}|0_b0_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\tilde{\overline{\hat{U}}}_2^{(j)}(t_2)} \Gamma e^{-i\nu}|0_b0_k0_j\rangle \xrightarrow{\hat{U}_2^{(k)}(t_3)} \Gamma e^{-i\nu}|0_b0_k0_j\rangle,$$

$$|0_b0_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b\tau_1}|0_b0_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\tilde{\overline{\hat{U}}}_2^{(j)}(t_2)} \Gamma^* e^{-i\nu}|0_b0_k1_j\rangle \xrightarrow{\hat{U}_2^{(k)}(t_3)} \Gamma^* e^{-i\nu}|0_b0_k1_j\rangle,$$

$$|0_b1_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b(t_1+3\tau_1/2)}|1_b0_k0_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\tilde{\overline{\hat{U}}}_2^{(j)}(t_2)} \Lambda e^{-i\nu-i\omega_b(t_1+t_2+\tau_1+\tau_2)}|1_b0_k0_j\rangle \xrightarrow{\hat{U}_2^{(k)}(t_3)} \Lambda e^{-i\nu-i\omega_bT}|0_b1_k0_j\rangle,$$

$$|0_b1_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_1)\hat{U}_2^{(k)}(t_1)} e^{-i\omega_b(t_1+3\tau_1/2)}|1_b0_k1_j\rangle \xrightarrow{\hat{U}_0(\tau_2)\tilde{\overline{\hat{U}}}_2^{(j)}(t_2)} \Lambda^* e^{-i\nu-i\omega_b(t_1+t_2+\tau_1+\tau_2)}|1_b0_k1_j\rangle \xrightarrow{\hat{U}_2^{(k)}(t_3)} \Lambda^* e^{-i\nu-i\omega_bT}|0_b1_k1_j\rangle,$$

with $\nu=\omega_bt_2/2+\lambda_j^2t_2/(2\hbar^2\Delta_j)+\omega_b(\tau_1+\tau_2)/2$. Above, the durations of the first- and third-step operations have been set the same as those for realizing the two-qubit operation $\hat{U}_1^{(kj)}(\eta_j)$.

The two-qubit gate $\hat{U}_1^{(kj)}(\eta_j)$ [or $\hat{U}_2^{(kj)}(t_2)$] performed above forms a universal set. Any quantum manipulation can be implemented by using one of them, accompanied by arbitrary rotations of single qubits. Obviously, if the system works in the strong charge regime $E_{J_j}/(\delta E_{C_j})\ll 1$ and $\cos\eta_j\sim 0$, $\sin\eta_j\sim 1$, then the two-qubit gate $\hat{U}_1^{(kj)}(\eta_j)$ in Eq. (22) approximates the well-known controlled-NOT (CNOT) gate

$$\hat{U}_{\mathrm{CNOT}}^{(kj)}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}.$$

Also, if the duration $t_2$ of the evolution $\tilde{\overline{\hat{U}}}_2^{(j)}(t_2)$ and the delays $\tau_1$, $\tau_2$ are further set properly such that

$$\cos(\varsigma_jt_2)=\sin(\varsigma_j't_2)=\sin(\omega_bT)=1,$$

then the two-qubit operation $\hat{U}_2^{(kj)}$ in Eq. (23) reduces to the well-known controlled-phase (CROT) gate

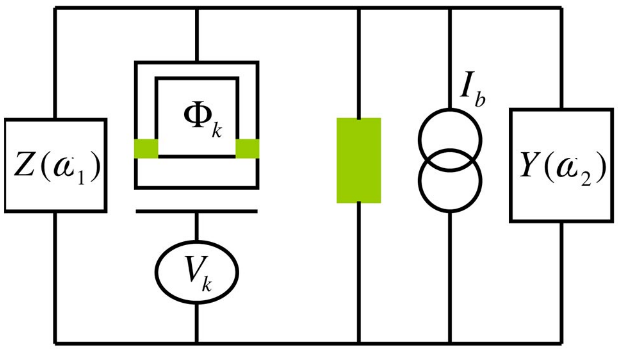


FIG. 2. Schematic diagram of a SQUID-based charge qubit with impedance $Z(\omega_1)$ coupled to a CBJJ with admittance $Y(\omega_2)$.

$$\hat{U}_{\mathrm{CROT}}^{(kj)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

## IV. DECOHERENCE OF THE QUBIT-BUS SYSTEM DUE TO THE BIASED VOLTAGE AND CURRENT NOISES

An ideal quantum system preserves quantum coherence, i.e., its time evolution is determined by deterministic reversible unitary transformations. Quantum computation requires a long phase coherent time evolution. In practice, any physical quantum system is subject to various disturbing factors which destroy phase coherence. In fact, solid-state systems are very sensitive to decoherence, as they contain a macroscopic number of degrees of freedom and interact with the environment. However, coherent quantum manipulations of the qubits are still possible if the decoherence time is finite but not too short. Hence, it is important to investigate the effects of the environmental noise on the present quantum circuit.

The typical noise sources in Josephson circuits consist of the linear fluctuations of the electromagnetic environments (e.g., circuitry and radiation noises) and the low-frequency noise due to fluctuations in various charge/current channels (e.g., the "background charge" and "critical current"). Usually, the former one behaves as Ohmic dissipation[36] and the latter one produces a $1/f$ spectrum.[37] It is well known that the problem of $1/f$ noise is still unsolved in solid-state circuits (see, e.g., Ref. 38). An efficient strategy, proposed in Refs. 39 and 40, is to suppress it by dynamical decoupling techniques using controllable pulses. Within the present work, we will consider the case of Ohmic dissipation due to linear fluctuations of the external circuit parameters: the bias current $I_b$ applied to the CBJJ and the gate voltages applied to the qubits. The effect of gate-voltage noise on a single charge qubit and that of bias-current noise on a single CBJJ has been discussed in Refs. 11 and 36 and in Ref. 27, respectively. We now study these noises together (see Fig. 2), since the interaction between a CBJJ, acting as a bus here, and a selected (e.g., the $k$th) qubit takes a central role in the present scheme for quantum manipulations. Each electromagnetic environment is treated as a quantum system with many degrees of freedom and modeled by a bath of harmonic oscillators. Furthermore, each of these oscillators is assumed to be weakly coupled to the chosen system. The Hamiltonian of a chosen ($k$th) qubit coupling to the bus, containing the fluctuations of the applied gate voltage $V_k$ and bias current $I_b$, can be generally written as

$$\hat{H} = \overline{\hat{H}}_{kb} + \hat{H}_B + \hat{V},$$

with

$$H_B = \sum_{j=1,2}\sum_{\omega_j}\left[\frac{p_{\omega_j}^2}{2m_{\omega_j}} + \frac{m_{\omega_j}\omega_j^2 x_{\omega_j}^2}{2}\right] = \sum_{j=1,2}\sum_{\omega_j}\left(\hat{a}_{\omega_j}^{\dagger}\hat{a}_{\omega_j} + \frac{1}{2}\right)\hbar\omega_j \tag{24}$$

and

$$\hat{V} = -[\sin\alpha_k\overline{\sigma}_z^{(k)} + \cos\alpha_k\overline{\sigma}_x^{(k)}](\hat{R}_1 + \hat{R}_1^{\dagger}) - (\hat{a}^{\dagger}R_2 + \hat{a}\hat{R}_2^{\dagger}), \tag{25}$$

being the Hamiltonians of the two baths and their interactions with the nondissipative qubit-bus system $\overline{\hat{H}}_{kb}$, respectively. Above, $\hat{a}_{\omega_j}$, $\hat{a}_{\omega_j}^{\dagger}$ are the Boson operators of the $j$th bath, and

$$\hat{R}_1 = \frac{eC_{gk}}{C_k}\sum_{\omega_1} g_{\omega_1}\hat{a}_{\omega_1}, \quad R_2 = \sqrt{\frac{\hbar}{2\tilde{C}_b\omega_b}}\sum_{\omega_2} g_{\omega_2}\hat{a}_{\omega_2},$$

with $g_{\omega_j}$ being the coupling strength between the oscillator of frequency $\omega_j$ and the nondissipative system. The effects of these noises can be characterized by their power spectra, which in turn depend on the corresponding "impedance" (or "inductance") and the temperature of the relevant circuits. For example, introducing the impedance $Z_t(\omega)=1/[i\omega C_t + Z^{-1}(\omega)]$ with $Z(\omega)=R_V$ being the Ohmic resistor, the corresponding voltage between the terminals of impedance $Z_t(\omega)$ can be expressed as $\delta V=\Sigma_{\omega_1}\lambda_{\omega_1}x_{\omega_1}$. Thus, the spectral density of this voltage source for Ohmic dissipation can be expressed as

$$G(\omega) = \pi\sum_{\omega_1}\frac{\lambda_{\omega_1}^2}{2m\omega_1}\delta(\omega-\omega_1) = \pi\sum_{\omega_1}|g_{\omega_1}|^2\delta(\omega-\omega_1) \sim R_V\omega. \tag{26}$$

Similarly, the spectral density for the bias-current source can be approximated as

$$F(\omega) = \pi\sum_{\omega_2}|g_{\omega_2}|^2\delta(\omega-\omega_2) \sim Y_I\omega, \tag{27}$$

with $Y_I$ being the dissipative part of the admittance of the current bias.

The well-established Bloch-Redfield formalism[33,41] offers a systematic way to obtain a generalized master equation for the reduced density matrix of the system, weakly influenced by dissipative environments. A subtle Markov approximation is also made in this theory such that the resulting master equation is local in time. Of course, in the regime of weak bath coupling and low temperatures, this theory is numerically equivalent to a full non-Markovian path-integral approach.[42] For the present qubit-bus system and in the basis spanned by the eigenstates $\{|g\rangle, |u_n\rangle, |v_n\rangle, n=1,2,\ldots\}$ of the nondissipative Hamiltonian $\hat{\bar{H}}_{kb}$, the Bloch-Redfield theory leads to the following master equations:

$$\frac{d\sigma_{\alpha\beta}}{dt} = -i\omega_{\alpha\beta}\sigma_{\alpha\beta} + \sum_{\mu,\nu}(R_{\alpha\beta\mu\nu} + S_{\alpha\beta\mu\nu})\sigma_{\mu\nu} \tag{28}$$

with

$$\begin{aligned} R_{\alpha\beta\mu\nu} = -\frac{1}{\hbar^2}\int_0^\infty d\tau \times \Big[ g_1(\tau)\Big(\delta_{\beta\nu}\sum_\kappa A_{\alpha\kappa}A_{\kappa\mu}e^{i\omega_{\mu\kappa}\tau} \\ - A_{\alpha\mu}A_{\nu\beta}e^{i\omega_{\mu\alpha}\tau}\Big) + g_1(-\tau)\Big(\delta_{\alpha\mu}\sum_\kappa A_{\nu\kappa}A_{\kappa\beta}e^{i\omega_{\kappa\nu}\tau} \\ - A_{\alpha\mu}A_{\nu\beta}e^{i\omega_{\beta\nu}\tau}\Big)\Big] \end{aligned} \tag{29}$$

and

$$\begin{aligned} S_{\alpha\beta\mu\nu} = -\frac{1}{\hbar^2}\int_0^\infty d\tau \times \Big[ g_2^\dagger(\tau)\Big(\delta_{\beta\nu}\sum_\kappa B^\dagger_{\alpha\kappa}B_{\kappa\mu}e^{i\omega_{\mu\kappa}\tau} \\ - B_{\alpha\mu}B^\dagger_{\nu\beta}e^{i\omega_{\mu\alpha}\tau}\Big) + g_2^\dagger(-\tau)\Big(\delta_{\alpha\mu}\sum_\kappa B^\dagger_{\nu\kappa}B_{\kappa\beta}e^{i\omega_{\kappa\nu}\tau} \\ - B_{\alpha\mu}B^\dagger_{\nu\beta}e^{i\omega_{\beta\nu}\tau}\Big) + g_2^-(\tau)\Big(\delta_{\beta\nu}\sum_\kappa B_{\alpha\kappa}B^\dagger_{\kappa\mu}e^{i\omega_{\mu\kappa}\tau} \\ - B^\dagger_{\alpha\mu}B_{\nu\beta}e^{i\omega_{\mu\alpha}\tau}\Big) + g_2^-(-\tau)\Big(\delta_{\alpha\mu}\sum_\kappa B_{\nu\kappa}B^\dagger_{\kappa\beta}e^{i\omega_{\beta\kappa}\tau} \\ - B^\dagger_{\alpha\mu}B_{\nu\beta}e^{i\omega_{\beta\nu}\tau}\Big)\Big] \end{aligned} \tag{30}$$

with

$$g_1(\pm\tau) = \left(\frac{eC_{g_k}}{C_k}\right)^2 \sum_{\omega_1}|g_{\omega_1}|^2[\langle n(\omega_1)+1\rangle e^{\mp i\omega_1\tau} + \langle n(\omega_1)\rangle e^{\pm i\omega_1\tau}],$$

$$g_2^\dagger(\pm\tau) = \left(\frac{\hbar}{2\tilde{C}_b\omega_b}\right)\sum_{\omega_2}|g_{\omega_2}|^2\langle n(\omega_2)+1\rangle e^{\mp i\omega_2\tau},$$

$$g_2^-(\pm\tau) = \left(\frac{\hbar}{2\tilde{C}_b\omega_b}\right)\sum_{\omega_2}|g_{\omega_2}|^2\langle n(\omega_2)\rangle e^{\mp i\omega_2\tau}.$$

Above, each one of the states $|\alpha\rangle, |\beta\rangle, \ldots$, can be equal to one of the eigenstates of $\hat{\bar{H}}_{kb}$.$\langle n(\omega_j)\rangle = 1/[\exp(\hbar\omega_j/k_BT)-1]$ is the average number of thermal photons in the mode of frequency $\omega_j$. The denotation $x_{ab} = \langle\alpha|\hat{x}|\beta\rangle$ accounts for the matrix element of operator $\hat{x}$, i.e.,

$$A_{\alpha\beta} = \langle\alpha|\hat{A}_k|\beta\rangle, \quad \hat{A}_k = \bar{\sigma}_z^{(k)}\sin\alpha_k + \bar{\sigma}_x^{(k)}\cos\alpha_k = \sigma_z^{(k)},$$

and

$$B_{\alpha\beta} = \langle\alpha|\hat{a}|\beta\rangle, \quad B^\dagger_{\alpha\beta} = \langle\alpha|\hat{a}^\dagger|\beta\rangle.$$

Also, $\omega_{\alpha\beta} = (E_\alpha - E_\beta)/\hbar$ with $E_\alpha(E_\beta)$ being one of eigenvalues of the nondissipative Hamiltonian $\hat{\bar{H}}_{kb}$, corresponding to the eigenstate $|\alpha\rangle|\beta\rangle$. The spectrum of $\hat{\bar{H}}_{kb}$ includes the ground state $|g\rangle = |-_k, 0\rangle$, corresponding to the energy $E_g = -\hbar\bar{\Delta}_k/2$, and a series of dressed doubled states

$$|u_n\rangle = \cos\theta_n|+_k, n\rangle - i\sin\theta_n|-_k, n+1\rangle,$$

$$|v_n\rangle = -i\sin\theta_n|+_k, n\rangle + \cos\theta_n|-_k, n+1\rangle$$

corresponding to the eigenvalues

$$E_{u_n} = \hbar\omega_b(n+1) - \frac{\rho_n}{2}, \quad E_{v_n} = \hbar\omega_b(n+1) + \frac{\rho_n}{2},$$

with

$$\cos\theta_n = \rho_n - \hbar\bar{\Delta}_k/\sqrt{(\rho_n - \hbar\bar{\Delta}_k)^2 + 4\lambda_k^2(n+1)}$$

and

$$\rho_n = \sqrt{(\hbar\bar{\Delta}_k)^2 + 4\lambda_k^2(n+1)}.$$

Here, $|\pm_k\rangle$ and $|n\rangle$ are the eigenstates of the operators $\bar{\sigma}_z^{(k)}$ and $\hat{H}_b$ with eigenvalues $\pm 1$ and $\hbar\omega_b(n+1/2)$, respectively.

Under the secular approximation, the evolution of the non-diagonal element $\sigma_{\alpha\beta}$ of the reduced density matrix $\sigma$ is determined by

$$\begin{aligned} \frac{d}{dt}\sigma_{\alpha\beta} + \{i[\omega_{\alpha\beta} + \mathrm{Im}(R_{\alpha\beta\alpha\beta}) + \mathrm{Im}(S_{\alpha\beta\alpha\beta})] + [\mathrm{Re}(R_{\alpha\beta\alpha\beta}) \\ + \mathrm{Re}(S_{\alpha\beta\alpha\beta})]\}\sigma_{\alpha\beta} = 0. \end{aligned} \tag{31}$$

Here, $R_{\alpha\beta\mu\nu}$ and $S_{\alpha\beta\mu\nu}$ are calculated, respectively, from $R_{\alpha\beta\mu\nu}$ and $S_{\alpha\beta\mu\nu}$ by setting $\mu=\alpha$ and $\nu=\beta$. Re($x$) and Im($x$) represent the real and imaginary parts of the complex number $x$. The formal solution of the above differential (31) reads

$$\sigma_{\alpha\beta}(t) = \sigma_{\alpha\beta}(0)\exp(-t/T_{\alpha\beta})\exp(-i\Theta_{\alpha\beta}t), \tag{32}$$

with $\Theta_{\alpha\beta} = \omega_{\alpha\beta} + \mathrm{Im}(R_{\alpha\beta\alpha\beta}) + \mathrm{Im}(S_{\alpha\beta\alpha\beta})$ being the effective oscillating frequency (the original Bohr frequency $\omega_{\alpha\beta}$ plus the Lamb shift $\Delta\omega_{\alpha\beta} = \mathrm{Im}R_{\alpha\beta\alpha\beta} + \mathrm{Im}S_{\alpha\beta\alpha\beta}$) and

$$T^{-1}_{\alpha\beta} = -[\mathrm{Re}(R_{\alpha\beta\alpha\beta}) + \mathrm{Re}(S_{\alpha\beta\alpha\beta})] \tag{33}$$

describing the rate of decoherence between the states $|\alpha\rangle$ and $|\beta\rangle$.

In the present qubit-bus system operating near the resonant point $E_k \sim \hbar\omega_b$, the decoherences relating to the lowest three energy eigenstates, i.e., $|g\rangle$, $|u_0\rangle = |u\rangle$, and $|v_0\rangle = |v\rangle$, are especially important for the desired quantum manipulations. The decoherences outside these three states are negligible. After a long but direct derivation, we obtain the decoherence rates of interest

$$T_{gu}^{-1} = \alpha_V \Bigg\{ 4(\sin\alpha_k \cos^2\theta_0)^2 \frac{2k_BT}{\hbar} + 2(\cos\alpha_k \cos\theta_0)^2 \coth\left(\frac{\hbar\omega_{ug}}{2k_BT}\right)\omega_{ug} + (\cos\alpha_k \sin\theta_0)^2\left[\coth\left(\frac{\hbar\omega_{vg}}{2k_BT}\right)-1\right]\omega_{vg} + (\sin\alpha_k \sin 2\theta_0)^2\left[\coth\left(\frac{\hbar\omega_{vu}}{2k_BT}\right)-1\right]\omega_{vu}\Bigg\} + \alpha_I \sin^2\theta_0\left\{\coth\left(\frac{\hbar\omega_{ug}}{2k_BT}\right)+1\right\}\omega_{ug}, \tag{34}$$

$$T_{gv}^{-1} = \alpha_V \Bigg\{ 4(\sin\alpha \sin^2\theta_0)^2 \frac{2k_BT}{\hbar} + 2(\cos\alpha \sin\theta_0)^2 \coth\left(\frac{\hbar\omega_{vg}}{2k_BT}\right)\omega_{vg} + (\cos\alpha \cos\theta_0)^2\left[\coth\left(\frac{\hbar\omega_{ug}}{2k_BT}\right)-1\right]\omega_{ug} + (\sin\alpha \sin 2\theta_0)^2\left[\coth\left(\frac{\hbar\omega_{vu}}{2k_BT}\right)+1\right]\omega_{vu}\Bigg\} + \alpha_I \cos^2\theta_0\left\{\coth\left(\frac{\hbar\omega_{vg}}{2k_BT}\right)+1\right\}\omega_{vg}, \tag{35}$$

and

$$T_{uv}^{-1} = \alpha_V \Bigg\{ 4(\sin\alpha \cos 2\theta_0)^2 \frac{2k_BT}{\hbar} + 2(\sin\alpha \sin 2\theta_0)^2 \coth\left(\frac{\hbar\omega_{vu}}{2k_BT}\right)\omega_{vu} + (\cos\alpha \cos\theta_0)^2\left[\coth\left(\frac{\hbar\omega_{ug}}{2k_BT}\right)+1\right]\omega_{ug} + (\cos\alpha \sin\theta_0)^2\left[\coth\left(\frac{\hbar\omega_{vg}}{2k_BT}\right)+1\right]\omega_{vg}\Bigg\} + \alpha_I\Bigg\{\sin^2\theta_0\left[\coth\left(\frac{\hbar\omega_{ug}}{2k_BT}\right)+1\right]\omega_{ug} + \cos^2\theta_0\left[\coth\left(\frac{\hbar\omega_{vg}}{2k_BT}\right)+1\right]\omega_{vg}\Bigg\}. \tag{36}$$

Above, the various Bohr frequencies read

$$\omega_{ug} = \omega_b/2 + E_k/(2\hbar) - \sqrt{(\hbar\omega_b - E_k)^2 + 4\lambda_k^2}/(2\hbar),$$

$$\omega_{vg} = \omega_b/2 + E_k/(2\hbar) + \sqrt{(\hbar\omega_b - E_k)^2 + 4\lambda_k^2}/(2\hbar),$$

and

$$\omega_{vu} = \sqrt{(\hbar\omega_b - E_k)^2 + 4\lambda_k^2}/\hbar.$$

Two dimensionless parameters $\alpha_V = \pi R_V C_{g_k}^2/[R_K C_k^2]$, $R_K = h/e^2 \approx 25.8\ \mathrm{k}\Omega$, and $\alpha_I = Y_I/(\tilde{C}_b\omega_b)$ characterize the coupling strengths between the environments and the system.

Especially, if the system works far from the resonant point (with $\lambda_k \sim 0$, achieved by switching off the Josephson energy), the above results [shown in Eqs. (34)–(36)] reduce to those[11,27,36] for the case when the qubit and the bus independently decohere. Namely, $T_{gu}^{-1}$ reduces to the rate[11]

$$T_{\uparrow\downarrow}^{-1} = 8\alpha_V k_B T/\hbar,$$

which describes the decoherence between two charge states $|\downarrow\rangle$ and $|\uparrow\rangle$ of the superconducting box with zero Josephson energy. Also, $T_{gv}^{-1}$ reduces to the decoherence rate[27]

$$T_{01}^{-1} = \alpha_I[\coth(\hbar\omega_b/2k_BT)+1]\omega_b,$$

between the ground and first excited states of the data bus. However, for the strongest coupling case (i.e., when the system works at the resonant point), we have $E_k = E_{J_k} = \hbar\omega_b$, $\cos\alpha_k = 1$, $\cos\theta_0 = \sin\theta_0 = 1/\sqrt{2}$, and $\coth[\hbar\omega_{ug}/(2k_BT)] - 1 \simeq \coth[\hbar\omega_{vg}/(2k_BT)] - 1 \sim 0$ ($<10^{-7}$, for the typical experimental parameters[12] $\lambda_k \simeq 0.1E_{J_k}$, $E_{J_k} = \hbar\omega_b \simeq 50\ \mu\mathrm{eV} \gg k_BT \simeq 3\ \mu\mathrm{eV}$). Thus, the minimum decoherence rates

$$\tilde{T}_{gu}^{-1} = (\alpha_V + \alpha_I)\omega_{ug}, \tag{37}$$

$$\tilde{T}_{gv}^{-1} = (\alpha_V + \alpha_I)\omega_{vg}, \tag{38}$$

and

$$\tilde{T}_{uv}^{-1} = \tilde{T}_{gu}^{-1} + \tilde{T}_{gv}^{-1}, \tag{39}$$

are obtained for the above three dressed states, respectively.

It has been estimated in Ref. 11 that the dissipation for a single SQUID qubit is sufficiently weak: $\alpha_V \sim 10^{-6}$ for $R_V = 50\Omega$, $C_{J_k}/C_{g_k} \sim 10^{-2}$, which allows, in principle, for $10^6$ coherent single-qubit manipulations. For a single CBJJ the dimensionless parameter $\alpha_I$ only reaches $10^{-3}$ for typical experimental parameters:[25] $1/Y_I \sim 100\Omega$, $C_b \sim 6p\mathrm{F}$, $\omega_b/2\pi \sim 10$ GHz. This implies that the quantum coherence of the present qubit-bus system is mainly limited by the bias current fluctuations. Fortunately, the impedance of the above CBJJ can be engineered[25] to be $1/Y_I \sim 560\ \mathrm{k}\Omega$. This lets $\alpha_I$ reach up to $10^{-5}$ and allow about $10^5$ coherent manipulations of the qubit-bus system.

## V. CONCLUSIONS AND DISCUSSIONS

In summary, we have proposed an effective scheme to couple any pair of selective Josephson charge qubits by letting them sequentially couple to a common CBJJ, which can be treated as an oscillator with adjustable frequency. Two logic states of the present qubit are encoded by the clockwise and anticlockwise persistent circuiting currents in the dc SQUID loop. At most one qubit can be set to interact with the bus at any moment. The interaction between the selected qubit and the data bus is tunable by controlling the flux applied to the qubit and the bias current applied to the data bus. This selective coupling provides a simple way to manipulate the quantum information stored in the connected SQUID qubits. Indeed, any pair of selective qubits without any direct interaction can be entangled by using a three-step coupling process. Furthermore, if the total duration is set up properly, the desired two-qubit universal gates, which are very similar to the CNOT and CROT gates, can be implemented via such three-step operational processes. During this operation, the mode of the data bus is unchanged, although its vibrational quantum is really excited/absorbed. After the desired quantum operation is performed on the chosen qubits, the data bus disentangles from the qubits and returns to its ground state.

In previous schemes, the distant Josephson qubits are coupled directly by either the charge-charge interaction, via connecting to a common capacitor, or by a current-current interaction, via sharing a common inductor. The present indirect coupling scheme offers some advantages: (i) the coupling strength is tunable and thus easy to be controlled for realizing the desired quantum gate, (ii) this first-order interaction is more insensitive to the environment, and thus possesses a longer decoherence time. Also, compared to previous data buses, the externally connected *LC* resonator[20] and cavity QED mode,[21] the present CBJJ bus might be easier to control for coupling the chosen qubit. For example, its eigenfrequency can be controlled by adjusting the applied dc bias current. In addition, the CBJJ is easy to fabricate using current technology[23] and may provide more effective immunities to both charge and flux noise.

By considering the decoherence due to the linear fluctuations of the applied voltage $V_k$ and current $I_b$, we have analyzed the experimental possibility of the present scheme within the Bloch-Redfield formalism. A simple numerical estimate showed that the quantum manipulations of the present qubit-bus system are experimentally possible, once the impedance $Y_I$ of the CBJJ can be engineered to have a sufficient low value, i.e., $1/Y_I$ can be enlarged sufficiently [e.g., $1/Y_I \sim 560\ \mathrm{K}\Omega$ (Ref. 25)]. Of course, this possibility, similar to those in previous schemes,[17,18,20–22] is also limited by other technological difficulties, e.g., suppress the low-frequency $1/f$ noise, and fast switch on/off the external flux to couple/decouple the chosen qubit, etc. For example, a very high sweep rate of magnetic pulse [e.g., up to $\sim 10^8$ Oe/s (Ref. 43)], is required to change half of flux quantum through a SQUID loop (with the size, e.g., 50 $\mu$m) in a sufficiently short time (e.g., the desired $\sim$40 ps ). This and other obstacles pose a challenge that motivate the exploration of novel circuit designs that might minimize some of the problems that lie ahead in the future.

## ACKNOWLEDGMENTS

This work was supported in part by the National Security Agency (NSA) and Advanced Research and Development Activity (ARDA) under Air Force Office of Research (AFOSR) contract No. F49620-02-1-0334, and by the National Science Foundation Grant No. EIA-0130383.

[1] See, e.g., M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000); C. H. Bennett and D. P. DiVincenzo, Nature (London) **44**, 247 (2000).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[3] L. F. Wei and F. Nori, Europhys. Lett. **65**, 1 (2004); L. F. Wei, S. Y. Liu, and X. L. Lei, Phys. Rev. A **65**, 062316 (2002).
[4] D. J. Wineland, M. Barrett, J. Britton, J. Chiaverini, B. L. DeMarco, W. M. Itano, B. M. Jelenkovic, C. Langer, D. Leibfried, V. Meyer, T. Rosenband, and T. Schaetz, Philos. Trans. R. Soc. London, Ser. A **361**, 1349 (2003).
[5] T. Sleator and H. Weinfurter, Phys. Rev. Lett. **74**, 4087 (1995).
[6] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[7] X. Hu and S. Das Sarma, Phys. Status Solidi B **238**, 360 (2003); Phys. Rev. A **61**, 062301 (2000); X. Hu, R. de Sousa, and S. Das Sarma, Phys. Rev. Lett. **86**, 918 (2001).
[8] T. H. Stievater, X. Li, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, C. Piermarocchi, and L. J. Sham, Phys. Rev. Lett. **87**, 133603 (2001).
[9] N. Gershenfeld and I. Chuang, Science **275**, 350 (1997).
[10] L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood, and I. L. Chuang, Nature (London) **414**, 883 (2001); L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, R. Cleve, and I. L. Chuang, Phys. Rev. Lett. **85**, 5452 (2000).
[11] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[12] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999); K. W. Lehnert, K. Bladh, L. F. Spietz, D. Gunnarsson, D. I. Schuster, P. Delsing, and R. J. Schoelkopf, Phys. Rev. Lett. **90**, 027002 (2003).
[13] J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999); **290**, 773 (2000); E. Il'ichev, N. Oukhanski, A. Izmalkov, Th. Wagner, M. Grajcar, H.-G. Meyer, A. Yu. Smirnov, Alec Maassen van den Brink, M. H. S. Amin, and A. M. Zagoskin, Phys. Rev. Lett. **91**, 097906 (2003).
[14] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
[15] Yu. A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. V. Averin, and J. S. Tsai, Nature (London) **421**, 823 (2003); T. Yamamoto, Yu. A. Pashkin, O. Astafiev, Y. Nakamura, and J. S. Tsai, *ibid.* **425**, 941 (2003).
[16] D. V. Averin and C. Bruder, Phys. Rev. Lett. **91**, 057003 (2003).
[17] Y. Makhlin, G. Schön, and A. Shnirman, Nature (London) **398**, 305 (1999).
[18] J. Q. You, J. S. Tsai, and F. Nori, Phys. Rev. Lett. **89**, 197902 (2002); a longer version of this is available in cond-mat/0306203 (unpublished); see also *New Directions in Mesoscopic Physics*, edited by R. Fazio, V. F. Gantmakher, and Y. Imry (Kluwer Academic Publishers, Dordrecht, 2003), p. 351.
[19] D. A. Lidar, L.-A. Wu, and A. Blais, Quantum Inf. Process. **1**, 155 (2002).
[20] F. Plastina and G. Falci, Phys. Rev. B **67**, 224514 (2003).
[21] J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003); Photonics Spectra **18**, 33 (2003); S. L. Zhu, Z. D. Wang, and K. Yang, Phys. Rev. A **68**, 034303 (2003); C. P. Yang Shih-I Chu, and S.

Han, *ibid.* **67**, 042311 (2003).
[22] A. Blais, A. Maassen van den Brink, and A. M. Zagoskin, Phys. Rev. Lett. **90**, 127901 (2003).
[23] R. C. Ramos, M. A. Gubrud, A. J. Berkley, J. R. Anderson, C. J. Lobb, and F. C. Wellstood, IEEE Trans. Appl. Supercond. **11**, 998 (2001).
[24] A. Wallraff, T. Duty, A. Lukashenko, and A. V. Ustinov, Phys. Rev. Lett. **90**, 037003 (2003); P. Silvestrini, V. G. Palmieri, B. Ruggiero, and M. Russo, *ibid.* **79**, 3046 (1997).
[25] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).
[26] Y. Yu, S. Y. Han, X. Chu, S.-I Chu, and Z. Wang, Science **296**, 889 (2002).
[27] J. M. Martinis, S. Nam, J. Aumentado, K. M. Lang, and C. Urbina, Phys. Rev. B **67**, 094510 (2003).
[28] A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch, P. R. Johnson, J. R. Anderson, A. J. Dragt, C. J. Lobb, and F. C. Wellstood, Science **300**, 1548 (2003).
[29] F. W. Strauch, P. R. Johnson, A. J. Dragt, C. J. Lobb, J. R. Anderson, and F. C. Wellstood, Phys. Rev. Lett. **91**, 167005 (2003).
[30] A. Shnirman, G. Schön, and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997).
[31] H. Y. Fan, Phys. Lett. A **289**, 172 (2001).
[32] J. Clarke, A. N. Cleland, M. H. Devoret, D. Esteve, and J. M. Martinis, Science **239**, 992 (1988).
[33] P. N. Argyres and P. L. Kelley, Phys. Rev. **134**, A98 (1964).
[34] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).
[35] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A **52**, 3457 (1995).
[36] U. Weiss, *Quantum Dissipative Systems*, 2nd ed. (World Scientific, Singapore, 1999).
[37] E. Paladino, L. Faoro, G. Falci, and R. Fazio, Phys. Rev. Lett. **88**, 228304 (2002).
[38] H. Gutmann, F. K. Wilhelm, W. M. Kaminsky, and S. Lloyd, cond-mat/0308107 (unpublished); G. Falci, A. D'Arrigo, A. Mastellone, and E. Paladino, cond-mat/0312442 (unpublished).
[39] K. Shiokawa and D. A. Lidar, Phys. Rev. A **69**, 030302(R) (2002).
[40] L. Faoro, and L. Viola, Phys. Rev. Lett. **92**, 117905 (2004).
[41] M. C. Goorden and F. K. Wilhelm, Phys. Rev. B **68**, 012508 (2003).
[42] L. Hartmann, I. Goychuk, M. Grifoni, and P. Hänggi, Phys. Rev. E **61**, R4687 (2000).
[43] H. Uwazumi, T. Shimatsu, and Y. Kuboki, J. Appl. Phys. **91**, 7095 (2002).