Quantum information processing with superconducting qubits in a microwave field

J. Q. You[1, 2, 3] and Franco Nori[1, 2]

[1] *Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan*
[2] *Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, The University of Michigan, Ann Arbor, MI 48109-1120, USA*
[3] *National Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, Beijing 100083, China*

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We investigate the quantum dynamics of a Cooper-pair box with a superconducting loop in the presence of a nonclassical microwave field. We demonstrate the existence of Rabi oscillations for both single- and multi-photon processes and, moreover, we propose a new quantum computing scheme (including one-bit and conditional two-bit gates) based on Josephson qubits coupled through microwaves.

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Quantum computing deals with the processing of information according to the laws of quantum mechanics. Within the last few years, it has attracted considerable attention because quantum computers are expected to be capable of performing certain tasks which no classical computers can do in practical time scales. Early proposals for quantum computers were mainly based on quantum optical systems, such as those utilizing laser-cooled trapped ions [1, 2], photon or atoms in quantum elctrodynamical (QED) cavities [3, 4], and nuclear magnetic resonance [5]. These systems are well isolated from their environment and satisfy the low-decoherence criterion for implementing quantum computing. Moreover, due to quantum error correction algorithms [5], now decoherence [6] is not regarded as an insurmountable barrier to quantum computing. Because scalability of quantum computer architectures to many qubits is of central importance for realizing quantum computers of practical use, considerable efforts have recently been devoted to solid state qubits. Proposed solid state architectures include those using electron spins in quantum dots [7–9], electrons on Helium [10], and Josephson-junction (JJ) charge [11, 12, 14] and JJ phase [13, 14] devices. These qubit systems have the advantage of relatively long coherent times and are expected to be scalable to large-scale networks using modern microfabrication techniques.

The Josephson charge qubit is achieved in a Cooper-pair box [11], which is a small superconducting island weakly coupled to a bulk superconductor, while the Josephson phase qubit is based on two different flux states in a small superconducting-quantum-interference-device (SQUID) loop [13, 14]. Cooper-pair tunneling and energy-level splitting associated with the superpositions of charge states were experimentally demonstrated in a Cooper-pair box [15, 16], and recently the eigenenergies and the related properties of the superpositions of different flux states were observed in SQUID loops by spectroscopic measurements [17]. In particular, Nakamura *et al.* [18] demonstrated the quantum coherent oscillations of a Josephson charge qubit prepared in a superposition of two charge states.

In this letter, we show that the coupled system of a Cooper-pair box and a cavity photon mode undergoes Rabi oscillations and propose a new quantum computing scheme based on Josephson charge qubits [19]. The microwave-controlled approach proposed in our paper has the significant advantage that *any* two qubits (*not* necessarily neighbors) can be effectively coupled through photons in the cavity. In addition to the advantages of a superconducting device exhibiting quantum coherent effects in a macroscopic scale as well as the controllable feature of the Josephson charge qubit by *both* gate voltage *and* external flux, the motivation for this scheme is fourfold: (1) the experimental measurements [15] showed that the energy difference between the two eigenstates in a Cooper-pair box lies in the microwave region and the eigenstates can be effectively interacted by the microwave field; (2) a single photon can be readily prepared in a *high-Q* QED cavity using the Rabi precession in the microwave domain [20]. Moreover, using a QED cavity, [21] produced a reliable source of photon number states on demand. In addition, the cavity in [21] was tuned to ~ 21 GHz, which is close to the 20 GHz microwave frequency used in a very recent experiment [22] on the Josephson charge qubit. Furthermore, the Q value of the cavity is 4×10^{10} (giving a very large photon lifetime of 0.3 sec); (3) our quantum computer proposal should be scalable to 10^6 to 10^8 charge qubits in a microwave cavity, since the dimension of a Cooper-pair box is $\sim 10\mu$m to 1μm; (4) the QED cavity has the advantage that *any* two qubits (*not* necessarily neighbors) can be effectively coupled through photons in the cavity. Also, we study multi-photon processes in the Josephson charge qubit since, in contrast to the usual Jaynes-Cummings model (see, *e.g.*, Chap. 10 in [23]), the Hamiltonian includes higher-order interactions between the two-level system and the nonclassical microwave field. As shown by the very recent experiment on Rabi oscillations in a Cooper-pair box [22], these higher-order interactions may be important in the Josephson charge-qubit system. The dynamics of a Josephson charge qubit coupled to a quantum resonator was studied in [24]. In contrast to our study here, the model in [24] involves: (a) only one qubit, (b) only the Rabi oscillation with a single excitation quantum of the resonator (as opposed to one or more photons), and (c) no quantum computing scheme.

We study the Cooper-pair box with a SQUID loop. In this structure, the superconducting island with Cooper-pair charge $Q = 2ne$ is coupled to a segment of a superconducting ring via two Josephson junctions (each with capacitance C_J and Josephson coupling energy E_J^0). Also, a voltage V_X is coupled to the superconducting island through a gate capacitor C; the gate voltage V_X is externally controlled and used to induce offset charges on the island. A schematic illustration of this single-qubit structure is given in [11, 14, 18]. The Hamiltonian of the system is $H = 4E_c(n - CV_X/2e)^2 - E_J(\Phi)\cos\varphi$, where $E_c = e^2/2(C + 2C_J)$ is the single-particle charging energy of the island and $E_J(\Phi) = 2E_J^0\cos(\pi\Phi/\Phi_0)$ is the effective Josephson coupling. The number n of the extra Cooper pairs on the island and average phase drop $\varphi = (\varphi_1 + \varphi_2)/2$ are canonically conjugate variables. The gauge-invariant phase drops φ_1 and φ_2 across the junctions are related to the total flux Φ through the SQUID loop by the constraint $\varphi_2 - \varphi_1 = 2\pi\Phi/\Phi_0$, where $\Phi_0 = h/2e$ is the flux quantum. This structure is characterized by two energy scales, i.e., the charging energy E_c and the coupling energy E_J^0 of the Josephson junction. In the charging regime $E_c \gg E_J^0$ and at low temperatures $k_B T \ll E_c$, the charge states $|n\rangle$ and $|n+1\rangle$ become dominant as the *controllable* gate voltage is adjusted to $V_X \sim (2n + 1)e/C$. Here, the superconducting gap is assumed to be larger than E_c, so that quasiparticle tunneling is prohibited in the system.

Here we ignore self-inductance effects on the single-qubit structure [25]. Now Φ reduces to the classical variable Φ_X, where Φ_X is the flux generated by the applied *static* magnetic field. In the spin-$\frac{1}{2}$ representation with charge states $|\uparrow\rangle = |n\rangle$ and $|\downarrow\rangle = |n + 1\rangle$, the reduced two-state Hamiltonian is given by [11, 14] $H = \varepsilon(V_X)\sigma_z - \frac{1}{2}E_J(\Phi_X)\sigma_x$, where $\varepsilon(V_X) = 2E_c[CV_X/e - (2n + 1)]$. This single-qubit Hamiltonian has two eigenvalues $E_\pm = \pm\frac{1}{2}E$, with $E = [4\varepsilon^2(V_X) + E_J^2(\Phi_X)]^{1/2}$, and eigenstates $|e\rangle = \cos\xi|\uparrow\rangle - \sin\xi|\downarrow\rangle$, and $|g\rangle = \sin\xi|\uparrow\rangle + \cos\xi|\downarrow\rangle$, with $\xi = \frac{1}{2}\tan^{-1}(E_J/2\varepsilon)$. Using these eigenstates as new basis, the Hamiltonian takes the diagonal form $H = \frac{1}{2}E\rho_z$, where $\rho_z = |e\rangle\langle e| - |g\rangle\langle g|$. Here we employ $\{|e\rangle, |g\rangle\}$ to represent the qubit.

When a *nonclassical* microwave field is applied, the total flux Φ is a quantum variable, $\Phi = \Phi_X + \Phi_f$, where Φ_f is the microwave-field-induced flux through the SQUID loop. Here we assume that a single-qubit structure is embedded in a QED microwave cavity with only a single photon mode λ. Generally, the vector potential of the nonclassical microwave field is written as $\mathbf{A}(\mathbf{r}) = \mathbf{u}_\lambda(\mathbf{r})a + \mathbf{u}_\lambda^*(\mathbf{r})a^\dagger = |\mathbf{u}_\lambda(\mathbf{r})|(e^{-i\theta}a + e^{i\theta}a^\dagger)\hat{\mathbf{A}}$, where $a^\dagger(a)$ is the creation (annihilation) operator of the cavity mode. Thus, the flux Φ_f is given by $\Phi_f = |\Phi_\lambda|(e^{-i\theta}a + e^{i\theta}a^\dagger)$, with $\Phi_\lambda = \oint \mathbf{u}_\lambda \cdot d\mathbf{l}$, where the contour integration is over the SQUID loop. Here, θ is the phase of the mode function $\mathbf{u}_\lambda(\mathbf{r})$ and its value depends on the chosen microwave field (see, *e.g.*, Chap. 2 in [23]). For instance, if a planar cavity is used and the SQUID loop of the charge qubit is perpendicular to the cavity mirrors, one has $\theta = 0$.

We shift the gate voltage V_X (and/or vary Φ_X) to bring the single-qubit system into resonance with k photons: $E \approx k\hbar\omega_\lambda$, $k = 1, 2, 3, \ldots$. Expanding the functions $\cos(\pi\Phi_f/\Phi_0)$ and $\sin(\pi\Phi_f/\Phi_0)$ into series of operators and employing the standard rotating wave approximation, we derive the total Hamiltonian of the system in this situation (with the photon Hamiltonian included)

$$H = \frac{1}{2}E\rho_z + \hbar\omega_\lambda(a^\dagger a + \frac{1}{2}) + H_{Ik}, \tag{1}$$
$$H_{Ik} = \rho_z f(a^\dagger a) + \left[e^{-ik\theta}|e\rangle\langle g|a^k g^{(k)}(a^\dagger a) + \text{H.c.}\right].$$

Here $f(a^\dagger a) = -E_J^0\sin(2\xi)\cos(\pi\Phi_X/\Phi_0)F(a^\dagger a)$, with

$$F(a^\dagger a) = \frac{1}{2!}\phi^2(2a^\dagger a + 1) - \frac{3}{4!}\phi^4[2(a^\dagger a)^2 + 2a^\dagger a + 1] + \frac{5}{6!}\phi^6[4(a^\dagger a)^3 + 6(a^\dagger a)^2 + 8a^\dagger a + 3] - \ldots,$$

where $\phi = \pi|\Phi_\lambda|/\Phi_0$, and

$$g^{(2m-1)}(a^\dagger a) = E_J^0\cos(2\xi)\sin(\pi\Phi_X/\Phi_0)G^{(2m-1)}(a^\dagger a),$$
$$g^{(2m)}(a^\dagger a) = E_J^0\cos(2\xi)\cos(\pi\Phi_X/\Phi_0)G^{(2m)}(a^\dagger a),$$

with $m = 1, 2, 3, \ldots$, and

$$G^{(1)}(a^\dagger a) = \phi - \frac{1}{2!}\phi^3 a^\dagger a + \frac{1}{4!}\phi^5[2(a^\dagger a)^2 + 1] - \ldots,$$
$$G^{(2)}(a^\dagger a) = \frac{1}{2!}\phi^2 - \frac{2}{4!}\phi^4(2a^\dagger a - 1) + \frac{15}{6!}\phi^6[(a^\dagger a)^2 - a^\dagger a + 1] - \ldots,$$
$$G^{(3)}(a^\dagger a) = -\frac{1}{3!}\phi^3 + \frac{5}{5!}\phi^5(a^\dagger a - 1) - \ldots,$$
$$G^{(4)}(a^\dagger a) = -\frac{1}{4!}\phi^4 + \frac{3}{6!}\phi^6(2a^\dagger a - 3) - \ldots,$$
$$\ldots\ldots\ldots,$$

where $g^{(k)}(a^\dagger a)$ is the k-photon-mediated coupling between the charge qubit and the microwave field. This Hamiltonian (1) is a generalization of the Jaynes-Cummings model to a solid state system. Here multiphoton processes [26] are involved for $k > 1$, in contrast with the usual Jaynes-Cummings model for an atomic two-level system interacting with a single photon mode, where only one photon is exchanged between the two-level system and the external field [23].

Rabi oscillations in multi-photon process. — The eigenvalues of the total Hamiltonian (1) are $\mathcal{E}_{\pm}(l,k) = \hbar\omega_\lambda[l + \frac{1}{2}(k+1)] + \frac{1}{2}[f(l) - f(l+k)] \pm \frac{\hbar}{2}\sqrt{\delta_{l,k}^2 + \Omega_{l,k}^2}$, and the corresponding eigenstates, namely, the dressed states are given by $|+,l\rangle = e^{-ik\theta}\cos\eta|e,l\rangle + \sin\eta|g,l+k\rangle$, and $|-,l\rangle = -\sin\eta|e,l\rangle + e^{ik\theta}\cos\eta|g,l+k\rangle$, where

$$\Omega_{l,k} = 2g^{(k)}(l+k)[(l+1)(l+2)\cdots(l+k)]^{1/2}/\hbar$$

is the Rabi frequency, $\delta_{l,k} = (E/\hbar - k\omega_\lambda) + [f(l) + f(l+k)]/\hbar$, and $\eta = \frac{1}{2}\tan^{-1}(\Omega_{l,k}/\delta_{l,k})$. Here, k is the number of photons emitted or absorbed by the charge qubit when the qubit transits between the excited state $|e\rangle$ and the ground state $|g\rangle$, and l is the number of photons in the cavity when the qubit state is $|e\rangle$.

When the system is initially at the state $|e,l\rangle$, after a period of time t, the probabilities for the system to be at states $|g,l+k\rangle$ and $|e,l\rangle$ are

$$|\langle g,l+k|\psi(t)\rangle|^2 = \frac{\Omega_{l,k}^2}{\delta_{l,k}^2 + \Omega_{l,k}^2}\sin^2\left(\frac{1}{2}\sqrt{\delta_{l,k}^2 + \Omega_{l,k}^2}\,t\right),$$

and $|\langle e,l|\psi(t)\rangle|^2 = 1 - |\langle g,l+k|\psi(t)\rangle|^2$. Obviously, they are oscillating with frequency $[\delta_{l,k}^2 + \Omega_{l,k}^2]^{1/2}$. This is the *Rabi oscillation with k photons* involved in the state transition; when $k=1$, it reduces to the usual single-photon Rabi oscillation [27]. Very recently, Nakamura *et al.* [22] investigated the temporal behavior of a Cooper-pair box driven by a strong microwave field and observed the Rabi oscillations with *multi*-photon exchanges between the two-level system and the microwave field. Different to the case studied here, the microwave field was employed there to drive the gate voltage to oscillate. Here, in order to implement quantum computing, we consider the Cooper-pair box with a SQUID loop and use the microwave field to change the flux through the loop.

Quantum computing. — Let us consider more than one single charge qubit in the QED cavity, and the cavity initially prepared at the zero-photon state $|0\rangle$. We first show the implementation of a controlled-phase-shift operation. Here a single photon process, $k=1$, is used to implement quantum computing.

(i) For all Josephson charge qubits, let $\Phi_X = \frac{1}{2}\Phi_0$, then $\cos(\pi\Phi_X/\Phi_0) = 0$, which yields $f(a^\dagger a) = 0$. Furthermore, the gate voltage for a control qubit, say A, is adjusted to have the qubit on resonance with the cavity mode ($E = \hbar\omega_\lambda$) for a period of time (where single photon is involved in the state transition), while all other qubits are kept off-resonant. The interaction Hamiltonian (in the interaction picture with $H_0 = \frac{1}{2}E\rho_z$) is given by $H_{\rm int} = e^{-i\theta}|e\rangle_A\langle g|ag^{(1)}(a^\dagger a) + {\rm H.c.}$, and the evolution of qubit A is described by $U_A(\theta,t) = \exp(-iH_{\rm int}t/\hbar)$. This unitary operation does not affect state $|g\rangle_A|0\rangle$, but transforms $|g\rangle_A|1\rangle$ and $|e\rangle_A|0\rangle$ as $|g\rangle_A|1\rangle \longrightarrow \cos(\alpha t)|g\rangle_A|1\rangle - ie^{-i\theta}\sin(\alpha t)|e\rangle_A|0\rangle$, and $|e\rangle_A|0\rangle \longrightarrow \cos(\alpha t)|e\rangle_A|0\rangle - ie^{i\theta}\sin(\alpha t)|g\rangle_A|1\rangle$, where $\alpha = g^{(1)}(1)/\hbar$. To obtain the controlled-phase-shift gate, we need the unitary operation with $\theta = 0$ and interaction time $t = \pi/2\alpha$, which transforms $|g\rangle_A|1\rangle$ ($|e\rangle_A|0\rangle$) to $-i|e\rangle_A|0\rangle$ ($-i|g\rangle_A|1\rangle$). This operation swaps the qubit state and the state of the QED cavity. A similar swapping transformation was previously used for the quantum computing with laser-cooled trapped ions [1].

(ii) While all qubits are kept off-resonant with the cavity mode and the flux Φ_X is originally set to $\Phi_X = \frac{1}{2}\Phi_0$ for each qubit, we change Φ_X to zero for only the target qubit, say B. In this case, the evolution of the target qubit B is described in the interaction picture by $U_B(t) = \exp(-iH_{\rm int}t/\hbar)$, where the Hamiltonian is $H_{\rm int} = (|e\rangle_B\langle e| - |g\rangle_B\langle g|)f(a^\dagger a)$. This Hamiltonian can be used to produce *conditional* phase shifts in terms of the photon state of the QED cavity [3]. Applying this unitary operation to qubit B for a period of time $t = \pi\hbar/2|f(1) - f(0)|$, we have [28] $|g\rangle_B|0\rangle \longrightarrow e^{i\beta}|g\rangle_B|0\rangle$, $|e\rangle_B|0\rangle \longrightarrow e^{-i\beta}|e\rangle_B|0\rangle$, $|g\rangle_B|1\rangle \longrightarrow ie^{i\beta}|g\rangle_B|1\rangle$, and $|e\rangle_B|1\rangle \longrightarrow -ie^{-i\beta}|e\rangle_B|1\rangle$, where $\beta = \pi f(0)/2|f(1) - f(0)|$.

(iii) Qubit A is again brought into resonance for $t = \pi/2\alpha$ with $\theta = 0$, as in step (i). Afterwards, a controlled two-bit gate is derived as a controlled-phase-shift gate combined with two one-bit phase gates. In order to obtain the controlled-phase-shift gate U_{AB} (which keeps two-bit states $|g\rangle_A|g\rangle_B$, $|g\rangle_A|e\rangle_B$, and $|e\rangle_A|g\rangle_B$ unaltered, but transforms $|e\rangle_A|e\rangle_B$ to $-|e\rangle_A|e\rangle_B$), one needs to further apply successively the unitary operation given in step (ii) to the control and target qubits with interaction times $t = 3\pi\hbar/4|f(0)|$ and $(2\pi - |\beta|)\hbar/|f(0)|$, respectively.

In analogy with atomic two-level systems [1, 3], one can use an appropriate *classical microwave field* [29] to produce *one-bit rotations* for the Josephson charge qubits. When the classical microwave field is on resonance with the target qubit B, the interaction Hamiltonian becomes $H_{\rm int} = \frac{\hbar\Omega}{2}[e^{-i\nu}|e\rangle_B\langle g| + {\rm H.c.}]$, with $\hbar\Omega = 2E_J^0\cos(2\xi)\sin(\pi\Phi_X/\Phi_0)(\pi|\Phi_f|/\Phi_0)$, where the value of the phase ν depends on the chosen microwave field (see, *e.g.*, Chap. 2 in [23]) and Φ_f is the flux through the SQUID loop produced by the classical microwave field. For the interaction time $t = \pi/2\Omega$, the unitary operation $V_B(\nu,t) = \exp(-iH_{\rm int}t/\hbar)$ transforms $|g\rangle_B$ and $|e\rangle_B$ as $|g\rangle_B \longrightarrow \frac{1}{\sqrt{2}}\left(|g\rangle_B - ie^{i\nu}|e\rangle_B\right)$, and $|e\rangle_B \longrightarrow \frac{1}{\sqrt{2}}\left(|e\rangle_B - ie^{-i\nu}|g\rangle_B\right)$. In terms of this one-bit rotation, the controlled-phase-shift gate U_{AB} can be converted to the controlled-NOT gate [1], $C_{AB} = V_B(-\pi/2,\pi/2\Omega)U_{AB}V_B(\pi/2,\pi/2\Omega)$. A sequence of such gates supplemented by one-bit rotations can serve as a universal element for quantum computing [30]. For microwaves of wavelength $\lambda \sim 1$ cm, the volume of a planar cavity is $\sim 1{\rm cm}^3$. For SQUID loop dimension $\sim 10\mu$m to 1μm, then 10^3 to 10^4 charge qubits may be constructed along the cavity direction. Furthermore, for a 2D array of qubits, 10^6 to 10^8 charge qubits could be placed within the cavity [31]. This number of qubits is large enough for a quantum computer.

In conclusion, we have studied the dynamics of the Cooper-pair box with a SQUID loop in the presence of a nonclassical microwave field. Rabi oscillations in the multi-photon process are demonstrated, which involve multiple photons in the transition between the two-level system and the microwave field. Also, we propose a scheme for quantum computing, which is realized by Josephson charge qubits coupled through a single photon mode in the QED cavity.

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[28] This is similar to the conditional quantum-phase gate realized in a QED cavity for two photon qubits [4]. Like the controlled-NOT gate, this conditional two-bit gate is also universal for quantum computing [4, 30]. One can entangle charge qubits and photons using this conditional quantum-phase gate [supplemented with one-bit rotations] and employ photons as flying qubits to implement quantum communications, *e.g.*, quantum teleportation.
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