

**Robust and scalable optical one-way quantum computation**

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We propose an efficient approach for deterministically generating scalable cluster states with photons. This approach involves unitary transformations performed on atoms coupled to optical cavities. Its operation cost scales linearly with the number of qubits in the cluster state, and photon qubits are encoded such that single-qubit operations can be easily implemented by using linear optics. Robust optical one-way quantum computation can be performed since cluster states can be stored in atoms and then transferred to photons that can be easily operated and measured. Therefore, this proposal could help in performing robust large-scale optical one-way quantum computation.

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**I. INTRODUCTION**

In principle, linear optical elements (e.g., beam splitters, phase shifters, etc.) combined with single-photon sources and detectors can be used for efficient quantum information processing [1]. Experimental progress in optical systems has demonstrated control of photonic qubits, quantum gates, and small quantum algorithms (e.g., [2–5]). Optical quantum computation (QC) has been suggested [6,7] using cluster states [8–14]. One-way optical QC using a four-photon cluster state has been demonstrated experimentally [4,5,15]. In spite of this progress, scalable optical one-way QC still remains elusive because of the difficulty in generating cluster states with a large number of qubits.

Photon cluster states are excellent candidates for one-way QC because of the fast and easy implementation of single-qubit operations on photons, and also because photonic qubits have negligible decoherence. However, it is difficult to generate cluster states with photons because of the absence of significant interactions between photons. In general, there are two types of methods for generating cluster states with photons: (1) by introducing an effective interaction between photons through measurements [6,7]; (2) by using a nonlinear optical process, parametric down-conversion [4,5,16–18]. Generating cluster states through measurements [1] is probabilistic. Because of its intrinsically probabilistic nature, the method based on down-conversion is exponentially inefficient for generating large cluster states [4]; therefore it can prepare cluster states of only a few qubits. In short, neither of these methods is efficient in generating cluster states with a large number of photon qubits.

In this paper, we propose an approach that is different from the previously proposed methods for generating cluster states with photons. This method is deterministic and efficient in generating scalable photon cluster states. The cost of the approach scales linearly with the number of qubits in a cluster state. The standard encoding of photon qubits allows easy-to-implement single-qubit operations using passive linear optics.

Our approach for generating cluster states with photons is as follows: First, generate a cluster state in atoms trapped in the periodic potential of an optical lattice, and then transfer the cluster state from the atomic system to photons through

the coupling between the atoms and optical cavities. Note that atomic cluster states can be easily obtained in experiments [19,20] on optical lattices through next-neighbor interactions; this can be achieved in a *single* operational step and is *independent* of the size of the systems [19].

For robust and scalable one-way QC, large cluster states need to be generated and stored for performing single-qubit operations and readout. Cluster states can be easily *generated and stored with atoms*, but it is difficult to perform measurements on atoms. In contrast, it is easy to perform *measurements* on photons, but it is difficult to store quantum states using photons. Thus, this hybrid proposal uses the best from atomic and photonic qubits, to provide robust one-way QC. Namely, to generate and store cluster states in an atomic system, then transfer to photons the states that are subjected to measurements, and then perform single-qubit operations and measurements on photonic qubits.

In this work, we focus on transferring to a photonic system a cluster state originally generated in an atomic system. As we discussed, this is a crucial step in our proposal. The method presented employs five-level atoms coupled to optical cavities. This method has the following advantages: (1) Neither the cavity-mode frequencies nor the atomic level spacings need to be adjusted during the operation process. (2) No measurement is required. Our approach for generating photon cluster states is based on unitary transformations, that is, a deterministic method. (3) There is no time limitation for moving atoms in or out of the cavities; therefore it should be relatively easy to manipulate the system in experiments. (4) We choose the traditional encoding of a photonic qubit in each cavity, by using a photon in either a left-circularly polarized mode or a right-circularly polarized mode of the cavity. With this encoding, single-qubit operations are easy to implement by using polarization rotators [1]. Even though we consider here natural atoms, in the future, this proposal could be extended to artificial atoms [21–24].

The structure of this work is as follows: In Secs. II and III, we explain how to generate cluster states on photonic systems using five-, four-, and three-level atoms interacting with cavities, respectively. We close with a conclusion.

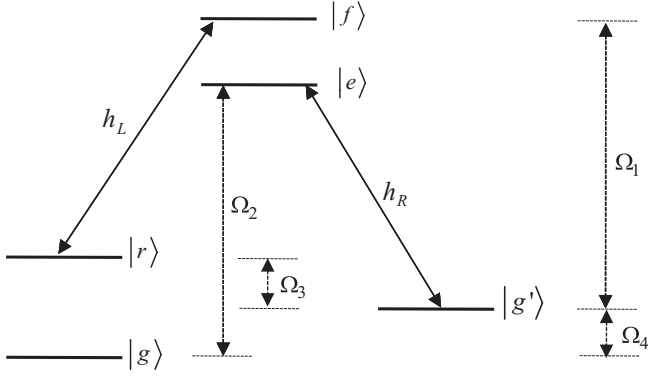


FIG. 1. Energy diagram of an atom with five levels. The transitions  $|g'\rangle \leftrightarrow |e\rangle$  and  $|r\rangle \leftrightarrow |f\rangle$  are resonantly coupled to the left- ( $\sigma_L^-$ ) and right- ( $\sigma_R^+$ ) circularly polarized modes of the cavity with coupling strengths  $h_L$  and  $h_R$ . The transitions  $|g'\rangle \leftrightarrow |f\rangle$ ,  $|g\rangle \leftrightarrow |e\rangle$ ,  $|g'\rangle \leftrightarrow |r\rangle$ , and  $|g\rangle \leftrightarrow |g'\rangle$  are driven by laser fields with Rabi frequencies  $\Omega_1$ ,  $\Omega_2$ ,  $\Omega_3$ , and  $\Omega_4$ , respectively.

## II. TRANSFERRING A CLUSTER STATE FROM ATOMS IN AN OPTICAL LATTICE TO POLARIZED PHOTONS: FIVE-LEVEL ATOMIC SYSTEM

Consider a system composed of  $n$  atoms and  $n$  cavities. Each atom has a five-level structure as depicted in Fig. 1, and is placed in a two-mode cavity. The two modes in each cavity are left-circularly polarized ( $\sigma_L^-$ ) and right-circularly polarized ( $\sigma_R^+$ ), respectively. For each atom, the two lowest energy levels  $|g\rangle$  and  $|g'\rangle$  represent the two logic states of a qubit; while for each cavity, the two logic states of a qubit are represented by the occupation of a photon (subscript  $p$ ) in the left- and right-circularly polarized modes of a cavity as

$$|0\rangle_p = |0\rangle_L |1\rangle_R, \quad |1\rangle_p = |1\rangle_L |0\rangle_R, \quad (1)$$

where  $|k\rangle_L |m\rangle_R$  represents the state of the cavity with  $k$  or  $m$  photons in the left- or right-circularly polarized mode.

Let us assume that an  $n$ -qubit cluster state  $|\Psi_C\rangle$  was prepared in the  $n$ -atom system, and each two-mode cavity is in the vacuum state  $|\emptyset\rangle_p = |0\rangle_L |0\rangle_R$ . Thus, the initial state of the whole system is  $|\Psi_C\rangle_a \otimes |\emptyset\rangle_p^{\otimes n}$ , where the subscripts  $a$  and  $p$  represent the atomic and the photonic systems, respectively. The task is to perform the state transfer

$$|\Psi_C\rangle_a |\emptyset\rangle_p^{\otimes n} \longrightarrow |g\rangle_a^{\otimes n} |\Psi_C\rangle_p, \quad (2)$$

that is, transfer the cluster state from the atomic system to the photons inside the optical cavities. Then photons leaking out of the cavities would be in the same cluster state as originally prepared in the atomic system.

Alternatively, we can also transfer an atomic single-qubit state of the cluster state to a photonic qubit (see Sec. IV below). This state transfer can be achieved through the following transformation (performed on each two-mode cavity with an atom inside):

$$\begin{aligned} |g\rangle|0\rangle_L|0\rangle_R &\longrightarrow |g\rangle|0\rangle_L|1\rangle_R, \\ |g'\rangle|0\rangle_L|0\rangle_R &\longrightarrow |g\rangle|1\rangle_L|0\rangle_R. \end{aligned} \quad (3)$$

Below, we will show the procedure to implement this transformation.

### A. Transformation of a state from atoms to polarized photons

To achieve this transformation, two interactions between the atom with the two cavity modes are needed. One is the resonant interaction between the  $|r\rangle \leftrightarrow |f\rangle$  transition and the  $\sigma_L^-$  mode. The other one is the resonant interaction between the  $|g'\rangle \leftrightarrow |e\rangle$  transition and the  $\sigma_R^+$  mode. In the interaction picture, the Hamiltonians for these are

$$H_L = h_L (a_L |f\rangle \langle r| + \text{H.c.}) \quad (4)$$

and

$$H_R = h_R (a_R |e\rangle \langle g'| + \text{H.c.}), \quad (5)$$

where  $h_L$  ( $h_R$ ) is the coupling strength of the atom with the  $\sigma_L^-$  ( $\sigma_R^+$ ) mode of the cavity;  $a_L$  ( $a_R$ ) is the annihilation operator of the  $\sigma_L^-$  ( $\sigma_R^+$ ) mode of the cavity.

The transformation in Eq. (3) can be achieved in four steps as follows:

Step (i). Apply a pulse to the atom for a time interval  $\tau_1$ , which is resonant with the  $|g'\rangle \leftrightarrow |f\rangle$  transition; then wait for a time interval  $\tau_L$ . We denote by  $\Omega_1$  the Rabi frequency of the pulse and  $\phi_1$  the phase of the pulse. The time evolution for this step is as follows:

(ia) During the time interval  $\tau_1$ , the pulse applied to the atom leads to the transformation

$$\begin{aligned} |g'\rangle|0\rangle_L|0\rangle_R &\rightarrow \left[ \cos\left(\frac{\Omega_1}{2}\tau_1\right) |g'\rangle - ie^{-i\phi_1} \right. \\ &\quad \left. \times \sin\left(\frac{\Omega_1}{2}\tau_1\right) |f\rangle \right] \otimes |0\rangle_L|0\rangle_R. \end{aligned} \quad (6)$$

For  $\frac{\Omega_1}{2}\tau_1 = \pi/2$ ,  $\phi_1 = -\pi/2$ , the state  $|g'\rangle|0\rangle_L|0\rangle_R$  is transformed into  $|f\rangle|0\rangle_L|0\rangle_R$ .

(ib) During the waiting time  $\tau_L$ , the  $|r\rangle \leftrightarrow |f\rangle$  transition of the atom is resonant with the  $\sigma_L^-$  mode of the cavity. The Hamiltonian describing this process is  $H_L$  in Eq. (4). The operator  $U_L = \exp(-iH_L\tau_L)$  performs the transformation

$$\begin{aligned} |f\rangle|0\rangle_L|0\rangle_R &\rightarrow \cos(h_L\tau_L) |f\rangle|0\rangle_L|0\rangle_R \\ &\quad - i \sin(h_L\tau_L) |r\rangle|1\rangle_L|0\rangle_R. \end{aligned} \quad (7)$$

For  $h_L\tau_L = \pi/2$ , the state  $|f\rangle|0\rangle_L|0\rangle_R$  evolves to  $-i|r\rangle|1\rangle_L|0\rangle_R$ .

Step (ii). Apply a pulse to the atom for a duration  $\tau_2$ . The pulse applied to the atom is resonant with the  $|g'\rangle \leftrightarrow |r\rangle$  transition. This process leads to the transformation

$$\begin{aligned} -i|r\rangle|1\rangle_L|0\rangle_R &\rightarrow -i \left[ \cos\left(\frac{\Omega_2}{2}\tau_2\right) |r\rangle - ie^{i\phi_2} \right. \\ &\quad \left. \times \sin\left(\frac{\Omega_2}{2}\tau_2\right) |g'\rangle \right] \otimes |1\rangle_L|0\rangle_R. \end{aligned} \quad (8)$$

With  $\frac{\Omega_2}{2}\tau_2 = \pi/2$  and  $\phi_2 = \pi$ , the state  $-i|r\rangle|1\rangle_L|0\rangle_R$  is transformed to  $|g'\rangle|1\rangle_L|0\rangle_R$ .

Step (iii). Apply a pulse to the atom for a time interval  $\tau_3$ , which is resonant with the  $|g\rangle \leftrightarrow |e\rangle$  transition; then wait for a time interval  $\tau_R$ . The time evolution for this step is as follows:

(iii) During the time interval  $\tau_3$ , the pulse applied to the atom leads to the transformation

$$|g\rangle|0\rangle_L|0\rangle_R \rightarrow \left[ \cos\left(\frac{\Omega_3}{2}\tau_3\right)|g\rangle - ie^{-i\phi_3} \times \sin\left(\frac{\Omega_3}{2}\tau_3\right)|e\rangle \right] \otimes |0\rangle_L|0\rangle_R. \quad (9)$$

For  $\frac{\Omega_3}{2}\tau_3 = \pi/2$  and  $\phi_3 = \pi$ , the state  $|g\rangle|0\rangle_L|0\rangle_R$  becomes  $i|e\rangle|0\rangle_L|0\rangle_R$ .

(iiib) During the waiting time  $\tau_R$ , the  $|g'\rangle \leftrightarrow |e\rangle$  transition of the atom is resonant with the  $\sigma_R^+$  mode of the cavity. The Hamiltonian for this process is  $H_R$  in Eq. (5). Then  $U_R = \exp(-iH_R\tau_R)$  transforms

$$i|e\rangle|0\rangle_L|0\rangle_R \rightarrow i \cos(h_R\tau_R)|e\rangle|0\rangle_L|0\rangle_R + \sin(h_R\tau_R)|g'\rangle|0\rangle_L|1\rangle_R. \quad (10)$$

With  $h_R\tau_R = \pi/2$ , the state  $i|e\rangle|0\rangle_L|0\rangle_R$  becomes  $|g'\rangle|0\rangle_L|1\rangle_R$ .

Step (iv). Apply a pulse to the atom for a time interval  $\tau_4$ . The pulse is resonant with the  $|g'\rangle \leftrightarrow |g\rangle$  transition. Thus we have the transformations

$$|g'\rangle|1\rangle_L|0\rangle_R \rightarrow \left[ \cos\left(\frac{\Omega_4}{2}\tau_4\right)|g'\rangle - ie^{i\phi_4} \times \sin\left(\frac{\Omega_4}{2}\tau_4\right)|g\rangle \right] \otimes |1\rangle_L|0\rangle_R \quad (11)$$

and

$$|g'\rangle|0\rangle_L|1\rangle_R \rightarrow \left[ \cos\left(\frac{\Omega_4}{2}\tau_4\right)|g'\rangle - ie^{i\phi_4} \times \sin\left(\frac{\Omega_4}{2}\tau_4\right)|g\rangle \right] \otimes |0\rangle_L|1\rangle_R. \quad (12)$$

With  $\frac{\Omega_4}{2}\tau_4 = \pi/2$  and  $\phi_4 = \pi/2$ , the state  $|g'\rangle|1\rangle_L|0\rangle_R$  is transformed to  $|g\rangle|1\rangle_L|0\rangle_R$ , and the state  $|g'\rangle|0\rangle_L|1\rangle_R$  becomes  $|g\rangle|0\rangle_L|1\rangle_R$ .

After this operation, the atom is decoupled from the cavity, and is in a stable state. One can easily check that the transformation in Eq. (3) is achieved in the four steps above.

### B. Fidelity of the transformation

Let us now study the fidelity of the state transfer operation described above. We assume that the pulses applied to the atoms can be controlled within a very short time (e.g., by increasing the pulse amplitude), such that the dissipation of the system during the pulse is negligibly small. In this case, the dissipation of the system would appear in the time evolution operations in steps (ib) and (iiib). Before any photon leaks out of each cavity, the Hamiltonians  $H_L$  and  $H_R$  become

$$H'_L = h_L(a_L|f\rangle\langle r| + \text{H.c.}) - i\gamma_f|f\rangle\langle f| - i\kappa_L a_L^\dagger a_L, \quad (13)$$

$$H'_R = h_R(a_R|e\rangle\langle g'| + \text{H.c.}) - i\gamma_e|e\rangle\langle e| - i\kappa_R a_R^\dagger a_R, \quad (14)$$

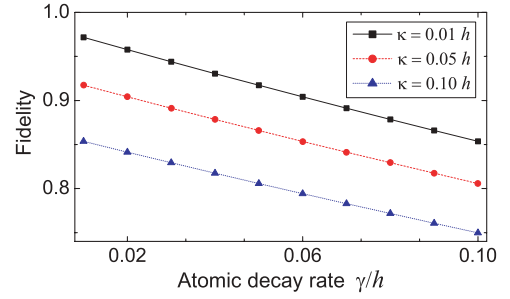


FIG. 2. (Color online) Fidelity ( $F$ ) of the transformation in Eq. (3) implemented on a five-level atom coupled with a two-mode cavity versus the spontaneous atomic decay rate  $\gamma/h$ . The solid, dashed, and dotted lines refer to the cavity decay rates  $\kappa = 0.1h$ ,  $0.05h$ , and  $0.01h$ , respectively.

where  $\gamma_f(\gamma_e)$  is the decay rate of the atomic level  $|f\rangle$  ( $|e\rangle$ ), and  $\kappa_L(\kappa_R)$  is the decay rate of the  $\sigma_L^-$  ( $\sigma_R^+$ ) mode of the cavity. For simplicity, we assume  $h_R = h, h_L = sh$ , where  $s > 1$ ,  $\gamma_e = \gamma_f = \gamma$ , and  $\kappa_L = \kappa_R = \kappa$ .

We now numerically calculate the evolution of the system governed by the Hamiltonians above. The quality of the state transfer process in Eq. (3) can be described by the fidelity of the state transfer operation

$$F = \text{Tr}[(\rho_p^{\text{id}})^{1/2} \rho_p (\rho_p^{\text{id}})^{1/2}]^{1/2}, \quad (15)$$

where  $\rho_p$  represents the photon ( $p$ ) temporal reduced density matrix, after tracing over the states of the atom, and  $\rho_p^{\text{id}}$  represents the reduced density matrix in the ideal case without considering the dissipation of the system. From Eqs. (13) and (14), one can see that the atomic spontaneous decay plays the same role as the cavity decay in the dissipation process. They both have the same effect on the fidelity, and thus the fidelity of the state transfer operation can be improved by choosing atoms that have a long energy relaxation time or by improving the coupling between the cavity and the atom. Assume  $s = 1.2$ ; then  $h_L = 1.2h_R = 1.2h$ , and the fidelity versus the spontaneous atomic decay rate  $\gamma/h$  for different cavity decay rates  $\kappa$  is shown in Fig. 2. Cluster states with a high fidelity can be obtained efficiently using existing multi-particle entanglement purification protocols [25].

### C. Linear scaling

As shown above, four steps are needed to transfer the state from an atomic qubit to a photonic qubit. Therefore an  $n$ -qubit photon cluster state can be created in  $4n$  steps. In other words, the operation cost scales *linearly* with the number of qubits in the cluster state. Hence, this approach could provide an efficient generation of scalable cluster states with photons.

### D. Atomic candidate

As a possible implementation, the  $^{87}\text{Rb}$  atom can be used as the five-level atom. The atomic levels  $|g\rangle$ ,  $|g'\rangle$ , and  $|r\rangle$  are  $|F = 1, m = -1\rangle$ ,  $|F = 1, m = 0\rangle$ , and  $|F = 2, m = 0\rangle$  of  $5^2S_{1/2}$ , respectively;  $|e\rangle$  and  $|f\rangle$  are  $|F = 1, m = 1\rangle$  and  $|F = 2, m = -1\rangle$  of  $5^2P_{1/2}$ , respectively.

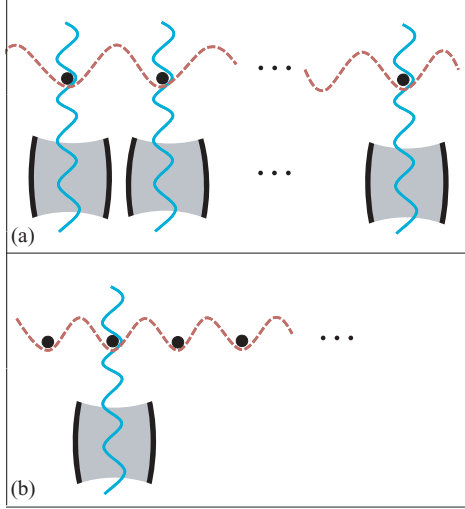


FIG. 3. (Color online) (a) Schematic diagram of the setup for translating atoms in optical lattices into an array of cavities through transverse optical lattice potentials. (b) Setup for translating an atom in an optical lattice that is subjected to measurement into a cavity through a transverse optical lattice potential. There is no order requirement for loading atoms into the cavity using this setup.

To generate an  $n$ -qubit photon cluster state, we need to send  $n$  atoms that encode the cluster state into  $n$  two-mode cavities. Atomic cluster states can be easily created on optical lattices [19], and it is also possible to load atoms into cavities through transverse optical lattice potentials [26–28]. The process of transferring the atomic cluster states to the photonic qubits must be completed in a very short time. To do this, one can prepare an array of cavities and load the atoms into the cavities through transverse optical lattice potentials. After a certain time, the photons that leak out of the cavities are in the same cluster state as the cluster state originally prepared in the atomic system. This process is illustrated in Fig. 3(a).

### III. TRANSFERRING A CLUSTER STATE FROM ATOMS IN AN OPTICAL LATTICE TO POLARIZED PHOTONS: FOUR- AND THREE-LEVEL ATOMIC SYSTEMS

As shown above, five-level atoms can be employed in performing the transformation in Eq. (3). We note that the transfer of cluster states from atomic to photonic systems can also be realized by using four-level atoms coupled to cavities. Suppose that  $n$  two-mode cavities are initially prepared in the state  $(|0\rangle_p)^{\otimes n}$ . To transfer the cluster state  $|\Psi_C\rangle$  from an atomic system to photons, one needs to perform the transformation

$$|\Psi_C\rangle_a (|0\rangle_p)^{\otimes n} \longrightarrow |g\rangle_a^{\otimes n} |\Psi_C\rangle_p. \quad (16)$$

This process can be done by applying  $n$  SWAP gates between the two coupled systems. Each SWAP gate acting on an atom in a two-mode cavity performs the transformation  $|g\rangle_a |0\rangle_p \rightarrow |g\rangle_a |0\rangle_p, |g'\rangle_a |0\rangle_p \rightarrow |g\rangle_a |1\rangle_p$ , that is,

$$\begin{aligned} |g\rangle|0\rangle_L|1\rangle_R &\longrightarrow |g\rangle|0\rangle_L|1\rangle_R, \\ |g'\rangle|0\rangle_L|1\rangle_R &\longrightarrow |g\rangle|1\rangle_L|0\rangle_R. \end{aligned} \quad (17)$$

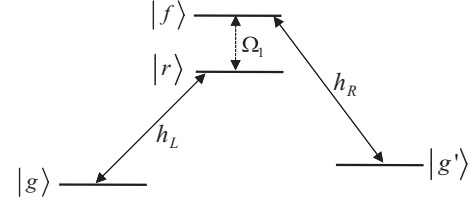


FIG. 4. Energy diagram of an atom with four levels. The transitions  $|g\rangle \leftrightarrow |r\rangle$  and  $|g'\rangle \leftrightarrow |f\rangle$  are resonantly coupled to the left- ( $\sigma_L^-$ ) and the right- ( $\sigma_R^+$ ) circularly polarized modes of the cavity with coupling strengths  $h_L$  and  $h_R$ , respectively; while the transition  $|r\rangle \leftrightarrow |f\rangle$  is driven by a classical laser field with Rabi frequency  $\Omega_1$ .

In this case, the four energy levels of the atom are shown in Fig. 4. The SWAP operations described by Eq. (17) can be implemented in three steps:

Step (i). Let the system evolve for a time  $\tau_1$  under  $H_1 = h_R(a_R|f\rangle\langle g'| + \text{H.c.})$ . The operator  $U_1 = \exp(-iH_1\tau_1)$  performs the transformation

$$\begin{aligned} |g'\rangle|0\rangle_L|1\rangle_R &\rightarrow \cos(h_R\tau_1)|g'\rangle|0\rangle_L|1\rangle_R \\ &\quad - i \sin(h_R\tau_1)|f\rangle|0\rangle_L|0\rangle_R. \end{aligned} \quad (18)$$

With  $h_R\tau_1 = \pi/2$ , the state  $|g'\rangle|0\rangle_L|1\rangle_R$  evolves to  $-i|f\rangle|0\rangle_L|0\rangle_R$ .

Step (ii). Apply a pulse to the atom for a time interval  $\tau_2$  resonant with  $|f\rangle \leftrightarrow |r\rangle$  transition. This process leads to the transformation

$$\begin{aligned} -i|f\rangle|0\rangle_L|0\rangle_R &\rightarrow -i \left[ \cos\left(\frac{\Omega_1}{2}\tau_2\right)|f\rangle - ie^{i\phi_1} \right. \\ &\quad \left. \times \sin\left(\frac{\Omega_1}{2}\tau_2\right)|r\rangle \right] \otimes |0\rangle_L|0\rangle_R. \end{aligned} \quad (19)$$

For  $\frac{\Omega_1}{2}\tau_2 = \pi/2$  and  $\phi_1 = 3\pi/2$ , the state  $-i|f\rangle|0\rangle_L|0\rangle_R$  becomes  $i|r\rangle|0\rangle_L|0\rangle_R$ .

Step (iii). The system evolves for  $\tau_3$  under  $H_3 = h_L(a_L|r\rangle\langle g| + \text{H.c.})$ . Then  $U_3 = \exp(-iH_3\tau_3)$  transforms

$$\begin{aligned} |r\rangle|0\rangle_L|0\rangle_R &\rightarrow \cos(h_L\tau_3)|r\rangle|0\rangle_L|0\rangle_R \\ &\quad - i \sin(h_L\tau_3)|g\rangle|1\rangle_L|0\rangle_R. \end{aligned} \quad (20)$$

With  $h_L\tau_3 = \pi/2$ , the state  $i|r\rangle|0\rangle_L|0\rangle_R$  becomes  $|g\rangle|1\rangle_L|0\rangle_R$ . Note that the state  $|g\rangle|0\rangle_L|1\rangle_R$  remains unchanged during the entire operation. Hence, the three-step operations above complete the SWAP operation in Eq. (17).

Compared to the use of five-level atoms, employing four-level atoms can reduce the use of the pulses. However, one would have to prepare the initial state  $|0\rangle_p$  (i.e.,  $|0\rangle_L|1\rangle_R$ ) for each cavity using auxiliary atoms, and the atoms would have to be moved out of the cavities immediately after the SWAP operation, since the prepared photon cluster states would otherwise change.

The SWAP operation in Eq. (17) can also be implemented using three-level atoms (the smallest number of levels needed for this approach). The three-level case would have the disadvantages above for using four-level atoms, and also the extra problem that adjusting the frequencies of the cavity modes would be required. Thus, five-level atoms would be the optimal choice based on the encoding of photonic qubits

above. We note that there exist proposals (e.g., [29]) in which logical states of photonic qubits are encoded as  $|0\rangle$  (no photons) and  $|1\rangle$  (one photon). Compared with this type of encoding, encoding of photon qubits with polarization mode states (in our work) can be used to perform single-qubit rotations by passive linear optical elements [1], and thus it could be used for optical QC.

#### IV. CONCLUDING REMARKS

In this work, we proposed an efficient and deterministic approach for generating scalable cluster states for optical one-way QC in a photonic system, by transferring the cluster state originally prepared in an atomic system through unitary transformations. In this proposal, we can also transfer part of the atomic cluster state to photons, for example, transfer one atomic single-qubit state of the atomic cluster state to a photonic qubit at a time, to perform a single-qubit unitary operation and then measurement (readout).

The advantage of this approach is that the remaining part of the atomic cluster state (not transferred) is always stored in the atoms. This is a hybrid method for robust one-way QC, exploiting the advantages of both atomic and photonic qubits: using atomic qubits for creating and storing a large scale cluster state and also photonic qubits for performing single-qubit rotations and measurements.

The first step is to make a large-scale cluster state on atoms ( $n$  qubits), which can be easily generated [19] with atoms, and stored there for a long time. To do one-way optical QC, one would need to perform single-qubit rotations and measurements on, say, the  $i$ th photonic qubit. This would require swapping the  $i$ th atomic single-qubit state into a photonic qubit, and measuring the photonic qubit after rotating its state [see Fig. 3(b)]. This swapping can be done deterministically using the approach described above.

Finally, the state transfer procedures introduced above can also be applied in quantum communications (e.g., transferring information between different physical systems, trying to use advantages of both), storage of quantum information, and quantum error correction.

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