Cavity optomechanical coupling assisted by an atomic gas

H. Ian,[1,2] Z. R. Gong,[1,2] Yu-xi Liu,[2,3] C. P. Sun,[1,2] and Franco Nori[2,3,4]
[1]*Institute of Theoretical Physics, The Chinese Academy of Sciences, Beijing 100080, China*
[2]*Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama 351-0198, Japan*
[3]*CREST, Japan Science and Technology Agency (JST), Kawaguchi, Saitama 332-0012, Japan*
[4]*Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA*
(Received 6 March 2008; published 17 July 2008)

We theoretically study a cavity filled with atoms, which provides the optical-mechanical interaction between the modified cavity photonic field and a oscillating mirror at one end. We show that the cavity field "dresses" these atoms, producing two types of polaritons, effectively enhancing the radiation pressure of the cavity field upon the oscillating mirror, as well as establishing an additional squeezing mode of the oscillating mirror. This squeezing produces an adiabatic entanglement, which is absent in usual vacuum cavities, between the oscillating mirror and the rest of the system. We analyze the entanglement and quantify it using the Loschmidt echo and fidelity.

DOI: 10.1103/PhysRevA.78.013824 PACS number(s): 42.50.Wk, 85.85.+j

I. INTRODUCTION

Fabry-Perot cavities can trap incident light between the two fixed mirrors located at both ends of the cavity. A way to modify these cavities (see, e.g., Refs. [1–3]) is to introduce a mirror [4] on a flexible wall, usually a cantilever tip, at the other end. The mirror on the flexible wall is allowed to oscillate around an equilibrium position, and this oscillation is usually treated as harmonic when the flexible wall is operating at its resonance frequency [8]. This oscillation induces infinitesimal contractions and dilations of the cavity length, resulting in a radiation pressure on the mirror which is proportional to the intensity of the trapped cavity field. This mechanism facilitates an optical-mechanical coupling between the cavity field and the mirror and is now generating considerable interest. In recent years, for example, a high-precision spectrometer for detecting gravitational waves [5,6] and an interferometric measurement apparatus [7,8] have used movable cavity mirrors as sensing devices. For detecting weak signals, a number of experiments have reduced the thermal fluctuations in the mirrors, effectively lowering the temperature of the mirror [1–3].

A key variable in previous designs is the number of photons trapped inside the cavity. Since the radiation pressure on the mirror is proportional to the photon number, it is desirable to increase this photon number in order to increase the magnitude of the radiation pressure and hence to control or cool down the mirror more efficiently. Moreover, the cooling of a nanomechanical resonator or an oscillating mirror has been extensively studied recently (e.g., in Refs. [9,10]). This then naturally leads us to conceive a cavity filled by a dielectric medium to achieve this purpose. Specifically, following our previous idea in Ref. [11], we now propose that this medium can be made of a gas of two-level atoms.

Recently, Ref. [12] proposed a similar scheme to target an interesting optical effect: the cavity mode forms an optical lattice inside the cavity and arranges the free atoms that were deposited into the cavity to form a Mott-insulator-like medium with atoms trapped at the lattice sites. It was shown that, with the atoms assuming an initial Bose-Einstein condensate distribution, such an atomic condensate would act effectively as a semitransparent mirror itself and shift the cavity to function in its "superstrong coupling regime." Nonetheless, based on Monte Carlo simulations [13,14], there exist disputes for the realizability of this proposal. In addition, the relationship between the atoms, the cavity field, and the mirror as in a tripartite system has been examined from the view of quantum correlation of their thermal fluctuations in a recent article [15].

In this paper, we analyze the dynamical effect that occurs when placing an atomic medium into a Fabry-Perot cavity, but assuming that the atoms have been placed inside a transparent gas chamber. Due to the strengthened coupling, now enhanced by the mediating atoms between the cavity field and the mirror, the resulting three-component system (the gas of atoms, the cavity field, and the oscillating mirror) induces interesting phenomena worth investigating. We point out here that, in contrast with the Bose-Einstein condensate atoms in Ref. [12], which can only be realized at very low temperatures, our gas of atoms makes use of low-energy collective excitations, which avoids the stringent low-temperature requirement.

To better extract the physical features of each part of this three-component system, we assume adiabatic processes over different time scales. We employ the Born-Oppenheimer approximation to study the dynamic behavior of a micromirror by assuming it is a slow-varying part. We also study the dynamic behavior of the atomic excitations as a fast-varying process, while the reflected radiation from the mirror stays relatively constant. The complex interactions between the system components lead us to expect many interesting physical phenomena including (i) realizing an adiabatic entanglement process [16], (ii) producing squeezed modes as in optical parametric oscillators, (iii) detecting polaritons through the mechanical mode of the mirror, and (iv) detecting the mechanical mode of the mirror through the polariton spectrum.

We first describe the model in Sec. II. The resulting entanglement process is then described in Sec. III, and its quan-

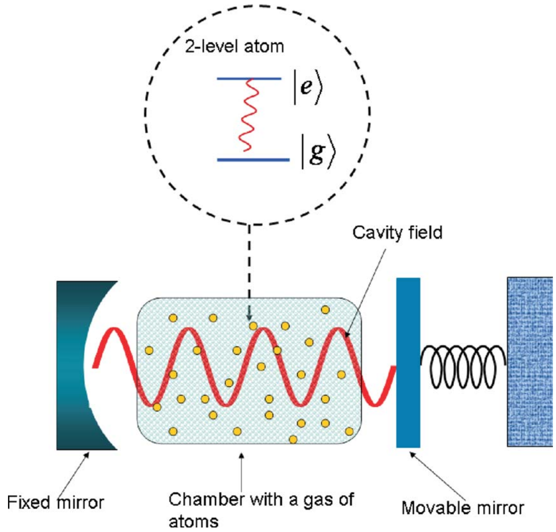

FIG. 1. (Color online) Schematic diagram illustrating the system with three main components: (i) a gas of two-level atoms, (ii) a movable mirror at one end, and (iii) a cavity field mediating the interaction between the atoms and the oscillating mirror.

tification follows in Sec. IV. The squeezed variance is derived in Sec. V, and conclusions are presented in Sec. VI.

II. ATOMIC OPTOMECHANICS

A. Exciton model

As shown in Fig. 1, the system we study here consists of a gas of two-level atoms, each with the same eigenfrequency ω_0 and a modified Fabry-Perot cavity carrying a photonic field with mode frequency Ω_C, as well as a harmonically bounded micromirror with coordinate x, momentum p, mass m, and oscillating frequency Ω_M. The system Hamiltonian, with units normalized according to $\hbar=1$ to simplify the notation, is

$$H=\omega_0\sum_j \sigma_j^z+\Omega_C a^\dagger a+\sum_j (g_j\sigma_j^+ a+g_j^*\sigma_j^- a^\dagger)+\frac{p^2}{2m}+\frac{1}{2}m\Omega_M^2x^2+\eta a^\dagger a x. \tag{1}$$

In Eq. (1), the Pauli matrix $\sigma_j^z=|e_j\rangle\langle e_j|$ denotes the internal energy of each two-level atom, while $\sigma_j^+=|e_j\rangle\langle g_j|$ and $\sigma_j^-=|g_j\rangle\langle e_j|$ in the last term of the first line denote the flip-up and flip-down operators of the jth atom. Here, a and $a^\dagger$ denote, respectively, the annihilation and creation operators of the cavity field. The last term of the second line is a radiation-pressure-type interaction on the mirror, which is proportional to the incident photon number and the coupling coefficient η of which is inversely proportional to the cavity length and proportional to the cavity field frequency ω_0. We assume that no direct interaction exists between the atoms and the mirror; the indirect interaction between them solely relies on the cavity field.

Since all the atoms have the same frequency ω_0, we can consider the gas of atoms that fills the cavity as a whole to be a Hopfield dielectric [17]. That is, the electrically insulated atoms form a dielectric medium, where the photons are repeatedly absorbed and reemited by the atoms, such that the interaction between the photons and the atoms can be completely described by a type of collective low-energy excitations (or excitons) of the ensemble of atoms as a whole. The dielectric constant of this Hopfield dielectric is in fact determined by the eigenenergy of these excitons. In terms of the atomic Pauli matrices, the exciton is described by the bosonic annihilation operator [11]

$$b=\lim_{N\to\infty}\sum_j^N \frac{g_j^*}{G}\sigma_j^- \tag{2}$$

and its Hermitian conjugate $b^\dagger$, where

$$G=\sqrt{\sum_{j=1}^N |g_j|^2}$$

can be understood as the total coupling strength. The exciton operators b and $b^\dagger$ are consistent with those of the Dicke model; a similar spin-bosonization technique has been used to study nuclear spins [18]. The resulting Hamiltonian for the system can then be written as

$$H=\omega_0 b^\dagger b+(\Omega_C+\eta x)a^\dagger a+G(b^\dagger a+ba^\dagger)+\frac{p^2}{2m}+\frac{1}{2}m\Omega_M^2x^2, \tag{3}$$

with $\eta=\Omega_C/L$. Here, L is the cavity length in mechanical equilibrium.

B. Interaction between the oscillating mirror and the polaritons

The coupling between the excitons and the cavity field can lead to the emergence of *dressed excitons*, here denoted as polaritons. In the adiabatic limit of the oscillating mirror—that is, when the mirror coordinate x stays unchanged with respect to the fast-varying field occupation number $a^\dagger a$—we can diagonalize the interaction between the excitons and the cavity field by rotating the Hilbert space of these two components through an angle

$$\theta=\arctan\left(\frac{2G}{\omega_0-\Omega_C-\eta x}\right), \tag{4}$$

for which we define a unitary transformation

$$A=a\cos\left(\frac{\theta}{2}\right)-b\sin\left(\frac{\theta}{2}\right), \tag{5}$$

$$B=a\sin\left(\frac{\theta}{2}\right)+b\cos\left(\frac{\theta}{2}\right). \tag{6}$$

The A and B operators above still obey bosonic commutation relations and can be understood as "dressed exciton modes" that mix atomic excitations b with the cavity field a. In other words, these dressed exciton modes are polaritons [11] of a phonon mode A and an optical mode B.

Under this view, the Hamiltonian of our system in Eq. (3) can be divided into two portions: the Hamiltonian H_{M} of the mirror's oscillation and the potential V from the polaritons acting on the mirror—i.e.,

$$H = H_{\mathrm{M}} + V, \tag{7}$$

$$H_{\mathrm{M}} = H_{\text{mirror oscillations}} \tag{8}$$

$$= \frac{p^2}{2m} + \frac{1}{2}m\Omega_{\mathrm{M}}^2 x^2, \tag{9}$$

$$V = V_{\text{polaritons on mirror}} \tag{10}$$

$$= \frac{1}{2}(\omega_0 + \Omega_{\mathrm{C}} + \eta x)(A^\dagger A + B^\dagger B) - \frac{1}{2}\sqrt{(\omega_0 - \Omega_{\mathrm{C}} - \eta x)^2 + 4G^2}(A^\dagger A - B^\dagger B). \tag{11}$$

The potential V in Eq. (11) quantifies the interaction between the mechanical mirror and the modes of the cavity. Without the "filling" atoms, the cavity mode is simply the photon field a, and this potential V will degenerate back to a linear radiation pressure impinging on the mirror if we do not consider the nonlinear Kerr effect that could be induced by the wave detuning due to the flexible length of the cavity [19,20].

The atoms let the linear radiation pressure be proportional to the total number $(A^\dagger A + B^\dagger \mathrm{B})$ of polaritons [$\eta x(A^\dagger A + B^\dagger B)$ in Eq. (11)] rather than the number $(a^\dagger a)$ of photons [$\eta a^\dagger a x$ in Eq. (1)]. Moreover, the atoms also impose an additional nonlinear term [the second term in Eq. (11)] for nonzero coupling constant G.

Note that this nonlinear effect, in the second term of Eq. (11), increases when increasing the number N of filled atoms because G grows with N. Thus, the gas of atoms enhances the coupling between the cavity field and the mirror. This enhanced coupling would produce squeezed states of the mirror mode and also entanglement between the mirror and the polaritons, which will be discussed in Sec. III. Without the intervening atoms, the potential V simply introduces a displacement to the mirror, producing neither squeezing nor entanglement.

III. ADIABATIC ENTANGLEMENT AND EVOLUTION UNDER SQUEEZING

A. Entanglement using the Born-Oppenheimer approximation

By considering fast-varying polariton modes and slow-varying mirror modes, we can write the wave vector at time t for our system under the Born-Oppenheimer approximation

$$|\psi(t)\rangle = \sum_{\mathbf{n}} |\mathbf{n}\rangle \otimes |\phi(\mathbf{n},t)\rangle, \tag{12}$$

where $\mathbf{n} = \{n_A, n_B\}$ denotes the collective index of energy levels of the polariton modes A and B. Thus, $A^\dagger A|\mathbf{n}\rangle = n_A|\mathbf{n}\rangle$ and $B^\dagger B|\mathbf{n}\rangle = n_B|\mathbf{n}\rangle$. Here, $|\mathbf{n}\rangle$ describes the time-independent wave vector for the polariton space in its adiabatic limit and $|\phi(\mathbf{n},t)\rangle$ the time-dependent wave vector for the mirror. The potential V in Eq. (11) then becomes an effective c number according to the eigenspectrum $\mathbf{n}$:

$$V_{\mathbf{n}} = \frac{1}{2}(\omega_0 + \Omega_{\mathrm{C}} + \eta x)(n_B + n_A) + \frac{1}{2}\sqrt{(\omega_0 - \Omega_{\mathrm{C}} - \eta x)^2 + 4G^2}(n_B - n_A). \tag{13}$$

We consider the displacement of the mirror, x, to be small around its equilibrium position $x=0$ and thus expand Eq. (13) up to second order in x:

$$V_{\mathbf{n}} = \frac{1}{2}(\omega_0 + \Omega_{\mathrm{C}})(n_B + n_A) + \frac{1}{2}\sqrt{(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2}(n_B - n_A) + \frac{\eta}{2}\left[(n_B + n_A) - \frac{(\omega_0 - \Omega_{\mathrm{C}})(n_B - n_A)}{\sqrt{(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2}}\right]x + \frac{N|g|^2\eta^2(n_B - n_A)}{[(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2]^{3/2}}x^2. \tag{14}$$

Using Eq. (14), when the polariton modes are in state $|\mathbf{n}\rangle$, the effective Hamiltonian operating on the mirror is

$$H_{\mathbf{n}} = H_{\mathrm{M}} + V_{\mathbf{n}}. \tag{15}$$

If we prepare an initial state of the system

$$|\psi(0)\rangle = \sum_{\mathbf{n}} \lambda_{\mathbf{n}}|\mathbf{n}\rangle \otimes |\phi(0)\rangle, \tag{16}$$

where $\lambda_{\mathbf{n}}$ is the expansion coefficient, then the mirror wave subvector will evolve along the path generated by the effective Hamiltonian $H_{\mathbf{n}}$:

$$|\psi(t)\rangle = \sum_{\mathbf{n}} \lambda_{\mathbf{n}}|\mathbf{n}\rangle \otimes |\phi_{\mathbf{n}}(t)\rangle, \tag{17}$$

$$|\phi_{\mathbf{n}}(t)\rangle = e^{-iH_{\mathbf{n}}t}|\phi(0)\rangle. \tag{18}$$

In other words, the final state of the mirror is determined by or dependent on the state of the polaritons in their adiabatic limit; specifically, the number distribution of the polaritons $\{n_A, n_B\}$ will decide the evolution of the mirror.

Geometrically, if the initial state $|\phi(0)\rangle$ was conceived to be represented by a point on a manifold over the Hilbert space of the mirror, then the effective Hamiltonians $H_{\mathbf{n}}$ and $H_{\mathbf{m}}$ for $\mathbf{n} \neq \mathbf{m}$ can be regarded as generators of the motion of the same vector $|\phi(0)\rangle$ toward different directions over the manifold. The evolution over time due to different generators will leave trajectories of different branches of paths on the manifold. The end points $|\phi_{\mathbf{n}}(t)\rangle$ and $|\phi_{\mathbf{m}}(t)\rangle$ of the paths are separated, and the separation depends on the discrepancy between $H_{\mathbf{n}}$ and $H_{\mathbf{m}}$ induced by different polariton distributions. The nonzero separation reflects geometrically the adiabatic entanglement of the mirror and the polaritons.

The original concept of adiabatic entanglement proposed in Ref. [16] concerns an abstract model consisting of two parts: a main system of interest and a detector apparatus external to the main system. When the main system is fast varying while the detector is slow varying, the detector is akin to a classical system and adiabatically follows the main system. Then the interaction between these two parts can be considered as a quantum measurement process and the entanglement thus emerging between these two parts, as reflected by Eq. (12), is called the adiabatic entanglement. We hence regard the three-component system discussed above (mirror, cavity field, and atomic gas) as a practical realization of the adiabatic entanglement model, where the polaritons are the main system and the mirror corresponds to the detector.

B. Evolution of squeezed coherent states of the mirror

Before quantifying the entanglement described above, we first study the dynamics of the mirror via the effective Hamiltonian $H_{\mathbf{n}}$. If we write the coordinate operator x and the momentum operator p of the mirror in their creation and annihilation operator form

$$x = \frac{1}{\sqrt{2m\Omega_{\mathrm{M}}}}(c + c^{\dagger}), \tag{19}$$

$$p = -i\sqrt{\frac{m\Omega_{\mathrm{M}}}{2}}(c - c^{\dagger}), \tag{20}$$

the effective Hamiltonian—i.e., Eq. (15)—then reads

$$H_{\mathbf{n}} = (\Omega_{\mathrm{M}} + 2\alpha_{\mathbf{n}})c^{\dagger}c + \alpha_{\mathbf{n}}(c^2 + c^{\dagger 2}) + \beta_{\mathbf{n}}(c + c^{\dagger}) + \gamma_{\mathbf{n}}, \tag{21}$$

where the coefficients depend on the polariton modes

$$\alpha_{\mathbf{n}} = \frac{G^2\eta^2(n_B - n_A)}{2m\Omega_{\mathrm{M}}[(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2]^{3/2}}, \tag{22}$$

$$\beta_{\mathbf{n}} = \frac{\eta}{\sqrt{8m\Omega_{\mathrm{M}}}}\left[(n_B + n_A) - \frac{(\omega_0 - \Omega_{\mathrm{C}})(n_B - n_A)}{\sqrt{(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2}}\right], \tag{23}$$

$$\gamma_{\mathbf{n}} = \frac{1}{2}(\omega_0 + \Omega_{\mathrm{C}})(n_B + n_A) + \frac{1}{2}\sqrt{(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2}(n_B - n_A) + \frac{N|g|^2\eta^2(n_B - n_A)}{m\Omega_{\mathrm{M}}[(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2]^{2/3}}. \tag{24}$$

The first- and second-order terms of c and $c^{\dagger}$ in Eq. (21) can be recognized as the polaritons inducing a squeezed coherent state in the mirror. The amount of displacement can be found by writing Eq. (21) as

$$H_{\mathbf{n}} = D^{\dagger}\left(\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right)H'_{\mathbf{n}}D\left(\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right), \tag{25}$$

where $D(\alpha) = \exp\{\alpha^* a - \alpha a^{\dagger}\}$ is the displacement operator. The resulting Hamiltonian in the displaced space is

$$H'_{\mathbf{n}} = (\Omega_{\mathrm{M}} + 2\alpha_{\mathbf{n}})c^{\dagger}c + \alpha_{\mathbf{n}}(c^2 + c^{\dagger 2}) - \frac{\beta_{\mathbf{n}}^2}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}} + \gamma_{\mathbf{n}}. \tag{26}$$

The amount of squeezing can be found by further diagonalizing Eq. (26) through a Bogoliubov transformation

$$C_{\mathbf{n}} = \mu_{\mathbf{n}}c - \nu_{\mathbf{n}}c^{\dagger}, \tag{27}$$

$$\mu_{\mathbf{n}} = \frac{1}{2}\left[\left(\frac{\Omega_{\mathrm{M}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right)^{1/4} + \left(\frac{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}{\Omega_{\mathrm{M}}}\right)^{1/4}\right], \tag{28}$$

$$\nu_{\mathbf{n}} = \frac{1}{2}\left[\left(\frac{\Omega_{\mathrm{M}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right)^{1/4} - \left(\frac{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}{\Omega_{\mathrm{M}}}\right)^{1/4}\right], \tag{29}$$

for which the resulting Hamiltonian becomes

$$H'_{\mathbf{n}} = \Omega_{\mathrm{M},\mathbf{n}}C_{\mathbf{n}}^{\dagger}C_{\mathbf{n}} + \zeta_{\mathbf{n}}, \tag{30}$$

where $\Omega_{\mathrm{M},\mathbf{n}}$ denotes the modified pseudoenergy splitting of the transformed mirror excitations according to $C_{\mathbf{n}}$ and $C_{\mathbf{n}}^{\dagger}$,

$$\Omega_{\mathrm{M},\mathbf{n}} = \sqrt{\Omega_{\mathrm{M}}(\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}})}, \tag{31}$$

and $\zeta_{\mathbf{n}}$ denotes the nonoperator terms

$$\zeta_{\mathbf{n}} = -\frac{(\sqrt{\Omega_{\mathrm{M}}} - \sqrt{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}})^2}{4} - \frac{\beta_{\mathbf{n}}^2}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}} + \gamma_{\mathbf{n}}. \tag{32}$$

Here, $\Omega_{\mathrm{M},\mathbf{n}}$ is called a pseudofrequency because it might become imaginary for some cases of the index $\mathbf{n}$. This reflects the fact that the distribution of polaritons has a strong influence over the time evolution of the mirror, as we have pointed out above. In the next subsection, we shall give more definite consideration for $\Omega_{\mathrm{M},\mathbf{n}}$ being real or imaginary when discussing the Loschmidt echo.

The transformation, Eq. (27), is physically equivalent to squeezing the operator c. To simplify the derivation we shall develop in the following, we define this squeezing process inversely with the operator $S_{\mathbf{n}}$:

$$c = S_{\mathbf{n}}^{\dagger}C_{\mathbf{n}}S_{\mathbf{n}}, \tag{33}$$

$$S_{\mathbf{n}} = \exp\left\{\frac{r_{\mathbf{n}}}{2}C_{\mathbf{n}}^2 - \frac{r_{\mathbf{n}}}{2}C_{\mathbf{n}}^{\dagger 2}\right\}, \tag{34}$$

where $\cosh r_{\mathbf{n}} = \mu_{\mathbf{n}}$. Over an initial coherent state $|\alpha\rangle$ with $c|\alpha\rangle = \alpha|\alpha\rangle$, we can define a special "coherent state"

$$|\alpha\rangle_{\mathbf{n}} = S_{\mathbf{n}}|\alpha\rangle \tag{35}$$

according to the operator $C_{\mathbf{n}}$—i.e.,

$$C_{\mathbf{n}}|\alpha\rangle_{\mathbf{n}} = \alpha|\alpha\rangle_{\mathbf{n}}.$$

The time evolution of the mirror, starting from an initial vacuum state, can then be computed as

$$\begin{aligned}|\phi_{\mathbf{n}}(t)\rangle = e^{-iH_{\mathbf{n}}t}|0\rangle &= D^{\dagger}\left(\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right)e^{-iH'_{\mathbf{n}}t}\left|\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right\rangle \\ &= D^{\dagger}\left(\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right)S_{\mathbf{n}}^{\dagger}(t)\left|\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}\right\rangle_{\mathbf{n}} \\ &= \left|\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}}}(e^{-i\Omega_{\mathrm{M},\mathbf{n}}t} - 1)\right\rangle,\end{aligned} \tag{36}$$

where $S_{\mathbf{n}}^{\dagger}$ is the Hermitian conjugate of the squeezing operator $S_{\mathbf{n}}$ in Eq. (34) in the Heisenberg picture; more explicitly,

$$S_{\mathbf{n}}^{\dagger}(t) = \exp\left\{ \frac{r_{\mathbf{n}}(t)}{2} C_{\mathbf{n}}^{\dagger 2} - \frac{r_{\mathbf{n}}^{*}(t)}{2} C_{\mathbf{n}}^{2} \right\}, \tag{37}$$

with $r_{\mathbf{n}}(t) = r_{\mathbf{n}} \exp\{-i\Omega_{\mathrm{M},\mathbf{n}} t\}$. This derivation is similar to the technique used in Ref. [21] for computing the evolution of squeezed states. The difference is that the coupling nature of the system we consider permits the entanglement of the oscillating mirror even when initialized from a vacuum state, which avoids the difficulty of preparing a coherent superposition. The squeezed states using polaritons have been studied in Ref. [22], the phonon squeezed states in Ref. [23].

IV. QUANTIFICATION OF DECOHERENCE AND ENTANGLEMENT

A. Loschmidt echo

At the end of Sec. III A, we interpreted the adiabatic entanglement as two distinct end points of the evolution over a manifold. The metric distance between the two points naturally becomes an appropriate measure of the degree of coherence or correlation between the two quantum states. The Loschmidt echo, which has been known to characterize the decoherence of a perturbed system [24], plays the role of metric. Originally this echo was defined as the wave function overlap between the states with and without the presence of the perturbing potential. This echo exactly describes the dynamic sensitivity of the system in the context of quantum chaos.

In our case, the perturbation potential can be understood as $(H_{\mathbf{n}} - H_{\mathbf{m}})$ and the echo as

$$L_{\mathbf{n},\mathbf{m}}(t) = |\langle \phi_{\mathbf{n}}(t) | \phi_{\mathbf{m}}(t) \rangle|.$$

Using Eq. (36), we find

$$\begin{aligned} L_{\mathbf{n},m}(t) = \exp\Bigg\{ & -\sum_{i=\mathbf{m},\mathbf{n}} \frac{2\beta_i^2}{(\Omega_{\mathrm{M}} + 4\alpha_i)^2} \sin^2\left(\frac{\Omega_{\mathrm{M},i}}{2} t \right) \\ & + \left[\sum_{i=\mathbf{m},\mathbf{n}} \sin^2\left(\frac{\Omega_{\mathrm{M},i}}{2} t \right) - \sin^2(\Omega_{\mathrm{M},\mathbf{n}-\mathbf{m}} t) \right] \\ & \times \frac{\beta_{\mathbf{n}} \beta_{\mathbf{m}}}{(\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}})(\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{m}})} \Bigg\}, \end{aligned} \tag{38}$$

with

$$\Omega_{\mathrm{M},\mathbf{n}-\mathbf{m}} = \frac{1}{2}(\Omega_{\mathrm{M},\mathbf{n}} - \Omega_{\mathrm{M},\mathbf{m}}). \tag{39}$$

Note that when $\Omega_{\mathrm{M},\mathbf{n}}$ and $\Omega_{\mathrm{M},\mathbf{m}}$ are real, the echo exhibits a cycling shape, similar to the decoherence effect shown in Ref. [21], only that the oscillation is not simply sinusoidal. When the two pseudofrequencies are indeed imaginary, which occurs when

$$\alpha_{\mathbf{n}} < -\frac{\Omega_{\mathrm{C}}}{4} \tag{40}$$

for some **n** or, equivalently,

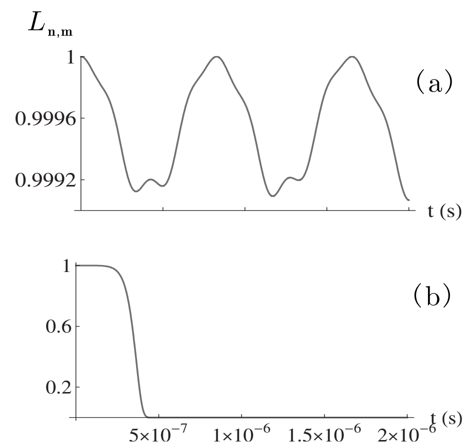

FIG. 2. Plots of the Loschmidt echo $L_{\mathbf{n},\mathbf{m}}$ over the same period of time for two mirror states under adiabatic entanglement for the operating regimes of (a) circular functions and (b) hyperbolic functions.

$$(n_A - n_B) > \frac{m\Omega_{\mathrm{M}}^2[(\omega_0 - \Omega_{\mathrm{C}})^2 + 4G^2]^{3/2}}{2G^2\eta^2}, \tag{41}$$

the sinusoidal functions will become hyperbolic and the echo will dampen with time exponentially. Whether the latter can happen depends on the difference between the excitation numbers of the polaritons. The requiring difference being large or small depends on the coupling constant G, which in turn increases with the number N of atoms in the cavity. In other words, we can operate our system in two regimes: for either periodic or hyperbolic Loschmidt echos, based on the number N of atoms.

Figure 2 plots the echo $L_{\mathbf{n},\mathbf{m}}$ between two mirror states over the same period of time for the two regimes. Without loss of generality, the parameters are all set to orders of magnitude accessible to current experiments: $\Omega_{\mathrm{M}}/2\pi = 10$ MHz, $\alpha_{\mathbf{n}}/2\pi = 10^{11}$ Hz, and $\beta_{\mathbf{n}}/2\pi = 10^{7}$ Hz. The periodic Loschmidt echo in Fig. 2(a) demonstrates the cycling of decoherence between two mirror states, whereas the hyperbolic type of echo in Fig. 2(b) shows a straight one-way decoherence. In the language of Ref. [24],

$$\Omega_{\mathrm{M}} + 4\alpha_{\mathbf{n}} = 0$$

is a critical point of dynamic sensitivity. When Eq. (41) is met, the evolution paths of the two states on the manifold always remain close to each other, giving an almost perfect echo. Once the parameters cross into the opposite side of Eq. (41), the echo gets lost almost instantly with no comeback, as shown in Fig. 2(b).

B. Fidelity

Fidelity serves as another metric for measuring the correlation between two quantum states. When seen in coordinate space, the fidelity roughly represents the overlap of the spatial wave packets of the two states (illustrated in Fig. 3 as the shaded region). Defined as the inner product of the ground states of two Hamiltonians, its physical meaning differs from that of the Loschmidt echo in that it is not a time-dependent

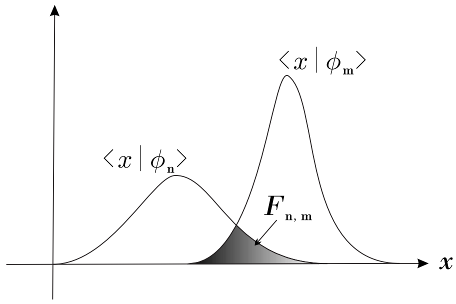

FIG. 3. Illustration of the fidelity between two quantum state vectors represented in coordinate space as Gaussian functions

measure of the distance between two evolving states, but a static estimate of the differentiating effects of two dynamic evolution generators. The fidelity has recently seen extensive applications to characterizing quantum phase transitions in strongly correlated systems [26].

For our model, we use the fidelity to estimate the effects between different polariton distributions on the mirror, under the Born-Oppenheimer approximation. From Eqs. (26), (30), and (34), the mirror ground state of the effective Hamiltonian in the adiabatic limit is the squeezed coherent vacuum state $|0\rangle_{\mathbf{n}}$ displaced by the amount $\beta_{\mathbf{n}}/(\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}})$. The fidelity, as the overlap of the ground states of two branching Hamiltonians $H_{\mathbf{n}}$ and $H_{\mathbf{m}}$ ($\mathbf{n}\neq\mathbf{m}$), can then be computed as the inner product of two coherent states:

$$F_{\mathbf{n,m}} = \left|\langle 0|S_{\mathbf{n}}^{\dagger}D^{\dagger}\left(\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}}}\right)D\left(\frac{\beta_{\mathbf{m}}}{\Omega_{\mathrm{M}}+4\alpha_{\mathbf{m}}}\right)S_{\mathbf{m}}|0\rangle\right|$$
$$= \exp\left\{-\frac{1}{2}\left(\frac{\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}}}-\frac{\beta_{\mathbf{m}}}{\Omega_{\mathrm{M}}+4\alpha_{\mathbf{m}}}\right)^2\right\}. \tag{42}$$

We hence see that the wave-packet overlap $F_{\mathbf{n},m}$ depends on various parameters and, generally, based on the relations of $\alpha_{\mathbf{n}}$ and $\beta_{\mathbf{n}}$ with Ω_{M} [cf. Eqs. (22) and (23)], the overlap decreases for increasing Ω_{M} over a normal mechanical oscillating frequency range. Figure 4 shows the plot of the fidelity with the ordinate being the mirror frequency over the range from 100 kHz to 100 MHz on a logarithmic scale for two typical values of the parameters: $\alpha_{\mathbf{n}},\alpha_{\mathbf{m}}>0$ and $\alpha_{\mathbf{n}},\alpha_{\mathbf{m}}<0$. The orders of magnitude of $\alpha_{\mathbf{n}}$ and $\beta_{\mathbf{n}}$ are set to ranges consistent with the values given in the last subsection.

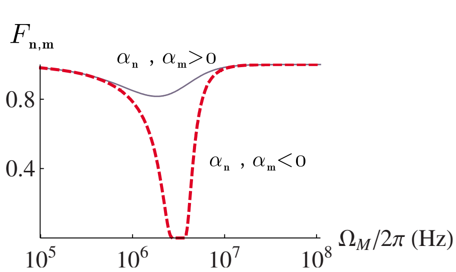

FIG. 4. (Color online) Semilogarithmic plots of the fidelity $F_{\mathbf{n},m}$ between two mirror states as a function of the mirror oscillating frequency Ω_{M} for two typical operating regimes. The solid line represents the case for $\alpha_{\mathbf{n}},\alpha_{\mathbf{m}}>0$ while the dashed line for $\alpha_{\mathbf{n}},\alpha_{\mathbf{m}}<0$.

The low-frequency range coincides with our expectation that a higher oscillating frequency of the mirror will render itself more vulnerable to the effect of the polaritons and induce its entanglement with other system components faster. When Ω_{M} further increases, the different operating regimes studied using the Loschmidt echo manifest themselves more apparently. For $\alpha_{\mathbf{n}},\alpha_{\mathbf{m}}>0$, the two ground states of the mirror always remain close to each other, corresponding to the cyclic operating region for the Loschmidt echo, and hence the fidelity retains its value close to 1, whereas for $\alpha_{\mathbf{n}},\alpha_{\mathbf{m}}<0$, it might cross into the hyperbolic operating region, where $\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}}<0$. For the latter case, the fidelity drops to 0 near the critical point $\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}}=0$, simulating the behavior of a phase transition.

V. SQUEEZED QUADRATURE VARIANCE OF THE MIRROR

The gas of atoms inside the cavity also acts like an optical parametric oscillator when regarded as a cavity dielectric. The original photon field traveling in the cavity vacuum is dressed by the atoms into two polariton modes. These two modes in their adiabatic limit act on the mirror as if confining the mirror oscillation in a nonlinear medium [cf. Eqs. (11) and (21) in which the polariton-mirror mode coupling is nonlinear]. This case occurs in traditional nonlinear optics when the signal beam and the idler beam have the same frequency and the process of optical interference is then denoted as "degenerate parametric oscillation." A mechanical version of the process was suggested in Ref. [25], where the interference took place between two nanomechanical resonators and it was shown to be the analog of a two-mode parametric down-conversion.

For our case, the procedure is half-optical (the polariton excitations) and half-mechanical (the mirror excitations). To show the similar squeezing effect in quadrature variance, we write the equations of motion of the mirror operators from Eq. (21):

$$\dot{c} = -i(\Omega_{\mathrm{M}}+2\alpha_{\mathbf{n}})c - i2\alpha_{\mathbf{n}}c^{\dagger} - i\beta_{\mathbf{n}}, \tag{43}$$

$$\dot{c}^{\dagger} = i(\Omega_{\mathrm{M}}+2\alpha_{\mathbf{n}})c^{\dagger} + i2\alpha_{\mathbf{n}}c + i\beta_{\mathbf{n}}. \tag{44}$$

The solution of the above equations, through Laplace transformation, reads

$$c(t) = \left[\cos(\Omega_{\mathrm{M},\mathbf{n}}t) - i\frac{\Omega_{\mathrm{M}}+2\alpha_{\mathbf{n}}}{\Omega_{\mathrm{M},\mathbf{n}}}\sin(\Omega_{\mathrm{M},\mathbf{n}}t)\right]c(0) - \left[\frac{i2\alpha_{\mathbf{n}}}{\Omega_{\mathrm{M},\mathbf{n}}}\sin(\Omega_{\mathrm{M},\mathbf{n}}t)\right]c^{\dagger}(0) + \frac{2\beta_{\mathbf{n}}}{\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}}}\sin^2\left(\frac{\Omega_{\mathrm{M},\mathbf{n}}t}{2}\right) - \frac{i\beta_{\mathbf{n}}}{\Omega_{\mathrm{M},\mathbf{n}}}\sin(\Omega_{\mathrm{M},\mathbf{n}}t). \tag{45}$$

We recognize that, unlike a typical optical parametric oscillator, even when the mirror is set initially to a vacuum state $|0\rangle$, the expectation value $\langle 0|c(t)|0\rangle$ will be nonzero over time because of the perturbation from the polaritons. As long as the numbers n_A and n_B of polaritons are not both zero at the same time, the inhomogeneous term $i\beta_{\mathbf{n}}$ on the right-hand side of Eqs. (43) and (44) would become nonzero and the motion of the mirror would be initiated by the incident polaritons, which is consistent with the vacuum-state entanglement we discussed in the last section. Compared to Ref. [9] for generating a squeezed entangled state of a mechanical resonator, the requirement of preparing different initial Fock and coherent states is lifted.

When the criterion (41) is met, the variance $\langle \Delta x^2 \rangle$ in the coordinate quadrature (19) demonstrates a squeezing effect:

$$\langle \Delta x^2 \rangle = \frac{2\cosh^2(\Omega_{\mathrm{M},\mathbf{n}}t)}{m\Omega_{\mathrm{M}}} + \frac{2\sinh^2(\Omega_{\mathrm{M},\mathbf{n}}t)}{m(\Omega_{\mathrm{M}}+4\alpha_{\mathbf{n}})}, \tag{46}$$

where $\Omega_{\mathrm{M},\mathbf{n}}$ is meant in the equation above to be the real magnitude of the pseudofrequency $\Omega_{\mathrm{M},\mathbf{n}}$.

VI. CONCLUSION AND REMARKS

We have studied a cavity system composed of atoms, a cavity field, and a movable mirror and showed that the collective excitations of the atoms are dressed by the cavity field and transformed into polaritons, causing their entanglement with the cavity mirror. The mirror state, in the adiabatic limit of the polaritons, is distinctly squeezed, according to the number distribution of two polariton modes; and its variance in coordinate space is also squeezed.

Before we conclude this paper, we make note of a recent article by Paz and Roncaglia [27] in which the entanglement dynamics between two resonators at finite temperatures is classified into "sudden-death" [28], "sudden-death-and-revival," and "no-sudden-death" regions according to the amount of fluctuations the resonators experience compared to their squeezing rate. Note that the squeezing rate in our model, Eq. (34), defined through Eq. (27), is also related to the choice of operating regimes determined by Eq. (41). Therefore, we conclude that entanglement operates in regions of different characteristics not only in finite-temperature environments, but also in zero-temperature settings, as shown by our model.

ACKNOWLEDGMENTS

C.P.S. acknowledges support by the NSFC under Grant No. 90503003 and the NFRPC under Grants No. 2006CB921205 and No. 2005CB724508. F.N. gratefully acknowledges partial support from the National Security Agency (NSA), Laboratory Physical Science (LPS), (U.S.) Army Research Office (USARO), National Science Foundation (NSF) under Grants No. EIA-0130383 and No. JSPS-RFBR 06-02-91200, and the Core-to-Core (CTC) program supported by the Japan Society for the Promotion of Science (JSPS).

[1] C. H. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004).
[2] S. Gigan, H. R. Boehm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Baeuerle, M. Aspelmeyer, and A. Zeilinger, Nature (London) **444**, 67 (2006).
[3] D. Kleckner and D. Bouwmeester, Nature (London) **444**, 75 (2006).
[4] Hereafter, any reference to the term "mirror" refers solely and specifically to this movable mirror, which is of our main interest here, not the fixed mirror at the opposite end of the cavity.
[5] B. Meers and N. MacDonald, Phys. Rev. A **40**, 3754 (1989).
[6] J. M. Aguirregabiria and L. Bel, Phys. Rev. A **36**, 3768 (1987).
[7] I. Tittonen, G. Breitenbach, T. Kalkbrenner, T. Muller, R. Conradt, S. Schiller, E. Steinsland, N. Blanc, and N. F. de Rooij, Phys. Rev. A **59**, 1038 (1999).
[8] O. Arcizet, P.-F. Cohadon, T. Briant, M. Pinard, A. Heidmann, J.-M. Mackowski, C. Michel, L. Pinard, O. Francais, and L. Rousseau, Phys. Rev. Lett. **97**, 133601 (2006); O. Arcizet, T. Briant, A. Heidmann, and M. Pinard, Phys. Rev. A **73**, 033819 (2006).
[9] F. Xue, Y. D. Wang, Y. X. Liu, and F. Nori, Phys. Rev. B **76**, 205302 (2007).
[10] J. Q. You, Y. X. Liu, and F. Nori, Phys. Rev. Lett. **100**, 047001 (2008).
[11] L. He, Y. X. Liu, S. Yi, C. P. Sun, and F. Nori, Phys. Rev. A **75**, 063818 (2007).
[12] D. Meiser and P. Meystre, Phys. Rev. A **73**, 033417 (2006); **74**, 065801 (2006).
[13] J. K. Asboth and P. Domokos, Phys. Rev. A **76**, 057801 (2007).
[14] D. Meiser and P. Meystre, Phys. Rev. A **76**, 057802 (2007).
[15] C. Genes, D. Vitali, and P. Tombesi, Phys. Rev. A **77**, 050307 (2008).
[16] C. P. Sun, X. F. Liu, D. L. Zhou, and S. X. Yu, Phys. Rev. A **63**, 012111 (2000).
[17] J. J. Hopfield, Phys. Rev. **112**, 1555 (1958).
[18] Z. Song, P. Zhang, T. Shi, and C. P. Sun, Phys. Rev. B **71**, 205314 (2005).
[19] S. Mancini and P. Tombesi, Phys. Rev. A **49**, 4055 (1994).
[20] Z. R. Gong, H. Ian, Yu-xi Liu, C. P. Sun, and F. Nori, e-print arXiv:0805.4102.
[21] Y. D. Wang, Y. B. Gao, and C. P. Sun, Eur. Phys. J. B **40**, 321 (2004).
[22] X. Hu and F. Nori, Phys. Rev. B **53**, 2419 (1996).
[23] X. Hu and F. Nori, Phys. Rev. Lett. **76**, 2294 (1996); **79**, 4605 (1997).
[24] F. M. Cucchietti, D. A. R. Dalvit, J. P. Paz, and W. H. Zurek, Phys. Rev. Lett. **91**, 210403 (2003).
[25] F. Xue, Yu-xi Liu, C. P. Sun, and F. Nori, Phys. Rev. B **76**, 064305 (2007).
[26] P. Zanardi, H. T. Quan, X. G. Wang, and C. P. Sun, Phys. Rev. A **75**, 032109 (2007).
[27] J. P. Paz and A. J. Roncaglia, Phys. Rev. Lett. **100**, 220401 (2008).
[28] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).