PHYSICAL REVIEW A **77**, 062339 (2008)

# Quantum entanglement via two-qubit quantum Zeno dynamics

Xiang-Bin Wang,[1,2,3] J. Q. You,[1,4] and Franco Nori[1,5]

[1]*Advanced Study Institute, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan*

[2]*CREST and Imai QCI project of ERATO-SORST, Japan Science and Technology Agency (JST), Kawaguchi, Satima 332-0012, Japan*

[3]*Department of Physics, Tsinghua University, Beijing 100084, China*

[4]*Department of Physics and Surface Physics Laboratory (National Key Laboratory), Fudan University, Shanghai 200433, China*

[5]*Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA*

(Received 4 January 2007; published 27 June 2008)

We study the two-particle quantum Zeno dynamics with a type of nondeterministic collective measurement whose outcome indicates whether the two-particle state has been collapsed to $|11\rangle$. Such a threshold detection, when used continuously, can lead to nontrivial quantum dynamics. We show that such type of dynamics can be used to produce quantum entanglement almost deterministically. We then numerically show the robustness of the method and we find that the operational errors of the small-angle rotations do not accumulate. We also propose a possible implementation using superconducting flux qubits.

DOI: 10.1103/PhysRevA.77.062339 PACS number(s): 03.67.Lx, 03.65.Ud, 03.65.Xp

## I. INTRODUCTION

Due to the quantum Zeno effect [1–3], quantum decay can be suppressed if a particle is continuously observed in the same basis. However, if we continuously observe a particle with a slowly changing basis, the particle's state will keep following the measurement basis. As we shall show, applying this fact to a multiqubit system, one can make nontrivial quantum-state steering.

The quantum Zeno effect of a *single* particle has attracted considerable interest in the past. Recently, the *multi*-particle dynamics due to the quantum Zeno effect has also been studied [4,5]. In particular, Ref. [4] proposed to realize a controlled-NOT (CNOT) gate through the quantum Zeno effect of two optical qubits. Here we study a *two*-qubit quantum Zeno dynamics with threshold detection, which checks whether the two-particle state has been collapsed to $|11\rangle$. As we shall show below, such measurements can suppress the coefficient of the state $|11\rangle$ and can lead to nontrivial two-particle and multi-particle entangled states, e.g., Bell states, Greenberger-Horne-Zeilinger (GHZ) states, cluster states, and so on.

Quantum entanglement is an important resource for quantum information processing (QIP). Two-qubit joint operations are crucial for tasks such as creating and manipulating quantum entanglement in QIP. Indeed, CNOT gates and single-qubit unitary transformations are sufficient for generating any quantum entanglement and for universal quantum computing. However, implementing a CNOT gate experimentally seems to be a daunting task. This is a huge barrier to scalable quantum computing, which requires numerous CNOT gates. To avoid this difficulty, it has been proposed [6–10] to replace the CNOT gate with a Bell measurement. Indeed, it is possible to replace CNOT gates by quantum teleportation [7], where the only collective operation is a Bell measurement. However, so far it is unknown how to do a projective Bell measurement without using a CNOT gate. In other words, the complete projective Bell measurement seems to be as difficult to implement experimentally as a CNOT gate. In practice, a nondeterministic collective measurement is often used because it is easier to implement. However, many of these proposals can only realize probabilistic QIP.

An elegant alternative, one-way quantum computation using cluster states [11], is promising. In that approach, cluster states are first produced and afterwards used for quantum computing through individual measurements only. Efforts have been made towards the efficient generation of cluster states via nondeterministic two-qubit measurements. Also, there are proposals for generating cluster states using solid-state qubits (see, e.g. [12–16]).

Nondeterministic collective measurements, as already demonstrated in a number of experiments, can be used to produce entangled states including cluster states probabilistically. Indeed, cluster-state quantum computation has recently been demonstrated with such a technique [17].

Here we present an alternative approach. We show that one can actually produce, almost *deterministically*, quantum entanglement, such as a cluster state via nondeterministic measurements, which we name "threshold measurements" or "$J$–measurements." These indicate whether the measured two-qubit state is $|11\rangle$. Although the measurement outcome itself is nondeterministic, by using the quantum Zeno effect (see, e.g., [1–3]), a certain quantum subspace is almost inhibited from decay if it is measured continuously; therefore providing an almost deterministic result.

Consider the following two-qubit nondeterministic collective measurement composed of two projectors:

$$J_1 = |1\rangle\langle 1| \otimes |1\rangle\langle 1|, \quad J_0 = \mathcal{I} - J_1, \tag{1}$$

where $\mathcal{I}$ is the four-dimensional identity operator. We call this type of measurement "$J$–measurement." This $J$–measurement is different from a parity measurement or singlet-triplet measurement [8,18,19]. Our measurement is a *threshold measurement* on whether both qubits are in state $|1\rangle$.

As shown below, technically, a $J$–measurement does not need to control exactly the interaction strength or duration, neither does it assume any synchronization difficulty. One only needs to turn on the "threshold detector" to see whether the current is larger than the threshold value. For clarity, we assume that a two-qubit state is monitored by a $J$-measurement detector: If the detector clicks, the state is collapsed to $|11\rangle$; if the detector does not click, the state is projected into the subspace

$$J_0 = \{|00\rangle, |01\rangle, |10\rangle\}. \tag{2}$$

If the state is initially in the subspace $J_0$, the quantum Zeno effect will inhibit the state to evolve to $|11\rangle$ if the $J$ measurement is performed frequently. We shall show that, by only using *single*-qubit operations and $J$-measurements, one can almost deterministically produce large cluster states *without* using any other separate conditional dynamics or quantum entangler.

As mentioned earlier, our work is somewhat related to prior works, in particular Ref. [4]. Both Ref. [4] and our work propose to apply the two-qubit quantum Zeno effect for quantum information processing (QIP). However, these two approaches are quite different in many aspects. First, the roles of the quantum Zeno effect in these two works are different. In Ref. [4], one needs both a quantum entangler (a coupled optical fiber or a beam splitter) and the quantum Zeno effect (through a two-photon absorption by an atomic gas). In our design, we *only* need a threshold measurement and a single-qubit rotation. Except for these, we need neither a separate quantum entangler nor a two-qubit quantum unitary. In other words, in our design, the quantum Zeno effect has a more crucial role: it *produces* the quantum entanglement rather than assisting the separate quantum entangler for QIP. Second, the calculations and the results are different. In our design, since we only use the threshold measurement, there are additional results from the dynamics due to the quantum Zeno effect itself. The effects of the errors caused by discrete measurements are studied in detail in our work. These results are not limited to any specific physical system. Third, the proposed physical systems for experimental realization are different. Reference [4] studies an optical system with two-photon absorption by atoms, while here we consider solid-state qubits with our threshold detection. Of course, different systems and approaches have different advantages in various aspects, such as technical overheads, costs, robustness, scalability, and so on.

## II. QUANTUM ENTANGLEMENT THROUGH A QUANTUM ZENO EFFECT BASED ON *J*–MEASUREMENTS

Let us define

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

We now show how to drive the two-qubit state $|00\rangle$ to the maximally entangled state $|\psi^{+}\rangle$ by repeating the following $W$ operation (on the two qubits): (i) Rotate each individual qubit by the same small angle $\theta$, and then (ii) perform a $J$–measurement.

After a number of $W$ operations, the state $|00\rangle$ can be driven into $|\psi^{+}\rangle$ with probability $1-O(\sin\theta)$. We do not have to require a constant $\theta$ for each application of $W$, but to simplify the presentation we now assume a constant positive $\theta$ for each step. The initial state is

$$|\chi_0\rangle = |00\rangle = a_0|00\rangle + \sqrt{2}b_0|\psi^{+}\rangle = a_0|00\rangle + b_0(|01\rangle + |10\rangle), \tag{3}$$

with $a_0=1$ and $b_0=0$. After the first $W$ operation, the initial state becomes

$$|\chi_1\rangle = a_1|00\rangle + b_1(|01\rangle + |10\rangle), \tag{4}$$

with probability

$$N_1 = 1 - \sin^4\theta \approx 1. \tag{5}$$

Here

$$a_1 = \cos^2\theta / N_1,$$

and

$$b_1 = \sin\theta\cos\theta / N_1.$$

Thus, the probability amplitude of $|\psi^{+}\rangle$ increases after each step. Through the iterative application of $W$, the state $|\chi_0\rangle$ will, sooner or later, be projected into $|\psi^{+}\rangle$. Therefore, we only need to show that after less than $k_1 \sim O(1/\sin\theta)$ applications of $W$, the two-qubit quantum state $|\chi_0\rangle$ is mapped into $|\psi^{+}\rangle$ with high probability. In this case, the total probability that the state $|\chi_0\rangle$ is projected into $|11\rangle$ during the whole process is only $O(\sin\theta)$. Therefore, given a sufficiently small $\theta$, the failure probability is negligible and the result is almost deterministic.

Let us consider now the state $|\chi_i\rangle$ obtained after $W$ is applied $i$ times to $|\chi_0\rangle$:

$$|\chi_i\rangle = W^i|\chi_0\rangle = a_i|00\rangle + b_i(|01\rangle + |10\rangle). \tag{6}$$

Assume that $a_i, b_i \geq 0$. After applying $W$ one more time we obtain

$$|\chi_{i+1}\rangle = W|\chi_i\rangle = a_{i+1}|00\rangle + b_{i+1}(|01\rangle + |10\rangle) \tag{7}$$

with

$$a_{i+1} = [a_i(1-\sin^2\theta) - 2b_i\sin\theta\cos\theta]/N_{i+1};$$

$$b_{i+1} = [b_i(1-\sin^2\theta) + a_i\sin\theta\cos\theta]/N_{i+1}$$

with

$$N_{i+1} = 1 - a_i^2\sin^4\theta - 4b_i^2\sin^2\theta\cos^2\theta \sim 1 - O(\sin^2\theta).$$

The amplitude difference between $|00\rangle$ and $|\psi^{+}\rangle$ changes after each step. We define

$$\begin{aligned}\delta_{i+1} &= b_{i+1} - a_{i+1} - (b_i - a_i)\\ &= [(a_i + 2b_i)\sin\theta\cos\theta + (a_i - b_i)\sin^2\theta]/N_{i+1}.\end{aligned}$$

After $k_1$ applications of $W$, we obtain

$$b_{k_1} - a_{k_1} = b_0 - a_0 + \sum_{i=1}^{k_1}\delta_i.$$

Our goal now is to know how large $k_1$ must be so that

$$a_{k_1} \sim 0,$$

i.e.,

$$(b_{k_1} - a_{k_1}) \sim 1/\sqrt{2}.$$

If all $\{a_i, b_i; i \leq k_1\}$ are non-negative, then

$$\delta_{i+1} \geq \sin\theta \cos\theta,$$

therefore

$$b_{k_1} - a_{k_1} \geq -1 + k_1 \sin\theta \cos\theta. \tag{8}$$

Given this, we conclude that there exists a positive number

$$k_1 \sim O(1/\sin\theta), \tag{9}$$

such that after $W$ is applied $k_1$ times, $a_{k_1}$ must be almost zero, provided that $\theta$ is sufficiently small. From the above derivation and a similar derivation, we draw the following lemma:

*Lemma*. Iterating the $W$ operation can map the state $|00\rangle$ into $|\psi^+\rangle$, and also map the state $|\psi^+\rangle$ into $-|00\rangle$ in the same number of steps. Together with single-qubit unitary operations, any state $\alpha|00\rangle + \beta|\psi^+\rangle$ can be mapped into $|\psi^+\rangle$ with less than $k_1 = O(1/\sin\theta)$ iterations of $W$.

Iterating the $W$ operation can also map the initial state $|10\rangle$ into the maximally entangled state $|\psi^+\rangle$. This can be seen as follows: Consider now the initial state

$$|\chi_0'\rangle = |10\rangle = \frac{1}{\sqrt{2}}(|\psi^+\rangle - |\psi^-\rangle). \tag{10}$$

The state $|\psi^-\rangle$ is invariant under identical individual rotations. Also, $|\psi^+\rangle$ can be mapped into $-|00\rangle$ (see the Lemma above). Therefore, we obtain the state

$$|\chi_{k_1}'\rangle \approx -\frac{1}{\sqrt{2}}(|00\rangle + |\psi^-\rangle) \tag{11}$$

after $k_1$ iterations of $W$. After applying a local phase flip, the state is changed into

$$|\chi'\rangle \approx \frac{1}{\sqrt{2}}(|00\rangle + |\psi^+\rangle). \tag{12}$$

Again using our Lemma above we conclude that this state can also be mapped into $|\psi^+\rangle$.

### III. QUANTUM DYNAMICS OF THE *W* OPERATOR

We now study more precisely the properties of $W$ using its matrix representation. Given any initial state $|\gamma\rangle$, after a $W$ operation, the (un-normalized) state in the $J_0$ space becomes:

$$|\gamma_1\rangle = M(\theta)|\gamma\rangle = J_0 R(\theta) \otimes R(\theta)|\gamma\rangle, \tag{13}$$

where $M(\theta)$ is the matrix representation of $W$. The probability that the qubit is projected into the $J_0$ subspace is $|\langle\gamma_1|\gamma_1\rangle|^2$. In matrix representation,

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and $J_0 = I_3 \oplus 0$ ($I_3$ is the $3 \times 3$ identity matrix). Since we are only interested in the case when the initial state $\gamma \in J_0$, the matrix representation for a $W$ operation in $J_0$ space is simplified to

$$M(\theta) = \begin{pmatrix} \cos^2\theta & -\sin\theta\cos\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta & -\sin^2\theta \\ \sin\theta\cos\theta & -\sin^2\theta & \cos^2\theta \end{pmatrix}. \tag{14}$$

In this matrix representation, the ket states are represented by

$$[|00\rangle, |10\rangle, |01\rangle] = \left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right]. \tag{15}$$

Hereafter $M(\theta)$ is simply denoted by $M$. After $N$ iterations of $W$, the evolution operator in the $J_0$ subspace is $M^N$. We now test our results numerically. First, we iterate $W$ for $k_1 = 100$ times with $\theta = \pi/(200\sqrt{2})$. We then obtain the numerical matrix

$$M^{100} = \begin{pmatrix} 0.0039 & -0.7028 & -0.7028 \\ 0.7028 & 0.4980 & -0.5020 \\ 0.7028 & -0.5020 & 0.4980 \end{pmatrix}. \tag{16}$$

This shows that if we start from the initial state $|00\rangle$, after 100 iterations of $W$, we obtain the maximally entangled state $|\psi^+\rangle$ with probability 98.8% and a fidelity larger than 99.99%. Iterating $W$ 1000 times with $\theta = \pi/(2000\sqrt{2})$, we obtain a highly entangled state: with 99.9% probability and a fidelity larger than $1 - 10^{-6}$.

### IV. INTELLIGENT EVOLUTION

If the initial state is $|10\rangle$, after iterating the $W$ operator, we can also obtain the maximally entangled state $|\psi^-\rangle$. Also, we want to have an "intelligently designed" evolution which will produce different maximally entangled states depending on whether the initial state is $|00\rangle$ or $|10\rangle$, since this type of evolution is crucial in expanding a cluster state, as shown below. After $k_1 = 100$ iterations of $W$, we perform a phase flip operation $P = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ to the first qubit, and apply the $W$ operation $k_2 = 50$ times to obtain the final evolution matrix

$$M^{50} P M^{100} = \begin{pmatrix} 0.0027 & 0.0011 & -0.9958 \\ -0.7008 & -0.6994 & 0.0027 \\ 0.7047 & -0.7033 & -0.0012 \end{pmatrix}. \tag{17}$$

As shown below, such an "intelligent" evolution can expand a cluster state deterministically. In the above three-stage operations, $W$ was iterated $k_1$ times, then a phase flip $P$ was applied, and finally $k_2$ iterations of $W$. If $\theta$ is *very* small, the constraints $k_1\theta = \pi/(2\sqrt{2})$ and $k_2\theta = \pi/(4\sqrt{2})$ will produce almost perfect results (i.e., with both the probability and the fidelity almost equal to 1). Now we show this explicitly. Suppose that after $k_1$ iterations of $W$, the initial state $|00\rangle$ is mapped into the maximally entangled state $|\psi^+\rangle$. This requires $m_{11}$ (the matrix element of the first row and the first column of the matrix $M^{k_1}$) to be exactly 0. Here,

$$M^{k_1} = [\cos^2\theta I_3 + r(\theta)]^{k_1}, \tag{18}$$

and

$$r(\theta) = \begin{pmatrix} 0 & -\sin\theta\cos\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & 0 & -\sin^2\theta \\ \sin\theta\cos\theta & -\sin^2\theta & 0 \end{pmatrix}. \quad (19)$$

Therefore

$$M^{k_1} = \sum_{n=0}^{k_1} C_{k_1}^n r^n \cos^{2k_1-2n}\theta, \quad (20)$$

with $C_{k_1}^n = \binom{n}{k_1} = k_1(k_1-1)\cdots(k_1-n+1)/n!$. Any term of the form $k_1^l \sin^j\theta$ is discarded in the summation if $j>l$ because $\theta$ is very small. Therefore, we obtain

$$m_{11} \approx \cos(\sqrt{2}k_1 \sin\theta) \quad (21)$$

which becomes 0 when

$$k_1\theta = \frac{\pi}{2\sqrt{2}}. \quad (22)$$

Consider now another initial state

$$|10\rangle = (|\psi^+\rangle - |\psi^-\rangle)/\sqrt{2}. \quad (23)$$

The $|\psi^-\rangle$ part is invariant under $W$. According to our Lemma above, after $k_1$ iterations of $W$, the state $|10\rangle$ must be changed to

$$(|\psi^-\rangle - |00\rangle)/\sqrt{2}. \quad (24)$$

After the phase-flip $P$ is applied, the state becomes

$$|\chi\rangle = -(|\psi^+\rangle + |00\rangle)/\sqrt{2}. \quad (25)$$

Let us recall now the evolution property for the initial state $|00\rangle$ under iterations of $W$. According to our Lemma, after $k_2$ iterations of $W$ with $k_2\theta=\pi/(4\sqrt{2})$, the state $|\chi\rangle$ becomes $-|\psi^+\rangle$. This means, if we start from $|\chi\rangle$, we only need

$$k_2\theta = \frac{\pi}{4\sqrt{2}} \quad (26)$$

in order to obtain $-|\psi^+\rangle$. Based on these facts we conclude the following theorem:

*Theorem*. The operator $W^{k/2}\, PW^k$ can change the initial states $(|00\rangle,|10\rangle)$ into $(|\psi^-\rangle,-|\psi^+\rangle)$ if $k\theta=\pi/(2\sqrt{2})$, and the $\theta$ for every step is very small.

Our $W$ operation is not limited to produce two-qubit entanglement, as shown below; it can also be used to *expand* a cluster state almost deterministically.

## V. QUANTUM ENTANGLEMENT EXPANSION

As is known [11], one can build a large cluster state from the product state $|+\rangle|+\rangle\cdots|+\rangle$, where $|+\rangle=(|0\rangle+|1\rangle)/\sqrt{2}$, with a controlled-phase (C-Phase) gate applied to the nearest qubits from the left to the right. A C-Phase will change any state $|i\rangle|j\rangle$ into $(-1)^{ij}|i\rangle|j\rangle$, $(i,j\in 0,1)$. For example, consider the two-qubit case: The state $|+\rangle|+\rangle$ is changed into $(|0\rangle|+\rangle+|1\rangle|-\rangle)/\sqrt{2}$, which can be transformed into $|\psi^+\rangle$ by a single-qubit flip operation. In general, an $n$-qubit cluster state can be written in the following bipartite form:

$$|C_n\rangle = |E\rangle|0\rangle + |E'\rangle|1\rangle, \quad (27)$$

where $|E\rangle$ and $|E'\rangle$ span the subspace of the first $(n-1)$ qubits, $|0\rangle$ and $|1\rangle$ span the subspace of the $n$th qubit. We can expand this to an $(n+1)$-qubit cluster state using a C-Phase gate with an ancilla qubit $|+\rangle$. Explicitly, after the C-Phase gate, the expanded cluster state becomes

$$|C_{n+1}\rangle = |E\rangle|0\rangle|+\rangle + |E'\rangle|1\rangle|-\rangle. \quad (28)$$

The few lines above are known results on how to produce a cluster state with C-Phase gates [11]. Below we show how to expand a cluster state in the form of Eq. (28) by our $W$ operations. Here, we do not need any C-Phase gate since the $W$ operation is sufficient for such type of expansion. We first take a Hadamard transform of the last qubit of the initial $n$-qubit cluster state in Eq. (27) and we set the ancilla state to be $|0\rangle$. The entire state of the $(n+1)$ qubits is now

$$|D\rangle = (|E\rangle|+\rangle + |E'\rangle|-\rangle) \otimes |0\rangle$$
$$= \frac{1}{\sqrt{2}}[|E\rangle(|00\rangle + |10\rangle) + |E'\rangle(|00\rangle - |10\rangle)]. \quad (29)$$

According to our theorem, the operator $W^{k/2}\, PW^k$ leads to the following transformation

$$|00\rangle \to |\psi^-\rangle; \quad |10\rangle \to -|\psi^+\rangle \quad (30)$$

if $k\theta=\pi/(2\sqrt{2})$. This means that, after applying $W^{k/2}PW^k$, the state of $(n+1)$ qubits becomes

$$\frac{1}{\sqrt{2}}[|E\rangle(|\psi^-\rangle - |\psi^+\rangle) + |E'\rangle(|\psi^-\rangle + |\psi^+\rangle)]$$
$$= -|E\rangle|10\rangle + |E'\rangle|01\rangle. \quad (31)$$

After applying a phase flip and a Hadamard transform to the last two qubits, and a bit-flip to the $n$th qubit, we obtain an $(n+1)$-qubit entangled state identical to that of Eq. (28). This means that the $J$−measurement can be used to produce and expand a cluster state almost deterministically, if the rotation angle $\theta$ of every step is sufficiently small.

## VI. ROBUSTNESS ANALYSIS

In practice, any protocol always has errors. In our protocol, there are many iterations. Since our scheme measures the qubits frequently, one natural question raised here is as follows: If there are small errors in each iteration, will these errors accumulate and finally lead to the failure of this scheme? Here we make a partial investigation of this problem. In each step, we need to rotate *both* qubits by a small angle $\theta$. Intuitively, there could be operational errors in doing the rotation. Say, sometimes the rotated angle is larger than $\theta$, and sometimes it is smaller than $\theta$. Here we do numerical simulations to determine the final effects of such operational errors with two assumptions: (i) In each step, the rotated angles of each qubits are the same. (ii) There are only occasional errors in the rotation. Say, at step $i$, the rotation angle can be $\theta_i=\theta+\epsilon_i$ which is different from $\theta$, but each $\epsilon_i$ is random.

TABLE I. Numerical results of the $P_s$ values as defined in Eq. (32) given different operational errors. These numerically test the robustness of our results with respect to random operational errors in the rotation. Here $\epsilon_M$ is the largest possible error of the rotation angle in every step (in percentage), $k$ indicates the number of iterations of the operator $W^{k/2}\ PW^k$.

| $k\backslash\epsilon_M\rightarrow$ | 0% | 5% | 10% | 20% | 50% |
|---|---|---|---|---|---|
| 50 | 97.17% | 97.18% | 97.13% | 97.07% | 96.69% |
| 100 | 98.58% | 98.58% | 98.58% | 98.57% | 98.50% |
| 1000 | 99.87% | 99.86% | 99.86% | 99.85% | 99.85% |

To have a quantitative evaluation of the robustness of our protocol, we define $P_s$ as the probability of obtaining a perfect result, averaged over the results from the initial states of $|00\rangle$ and $|01\rangle$. Explicitly

$$P_s = \frac{1}{2}(|\langle\psi^-|W^{k/2}PW^k|00\rangle|^2 + |\langle\psi^+|W^{k/2}PW^k|10\rangle|^2). \quad (32)$$

Here we have taken into account both the probability that the measurement outcome goes beyond the $J_0$ subspace and the probability that the final state is not $|\psi^+\rangle$, although the outcome is in the subspace $J_0$. Contrary to one's intuition, the more iterations of $W$ are taken, the less the outcome state is affected by the operational errors, as shown by the numerical test in Table I. From the numerical results there we can see that even for a not-so-large number of steps, e.g., $k=50$, fairly good results can be obtained under quite large operational errors (50%). As shown in the table, in the case when the largest error in every step is bounded by 50%, the average fidelity is larger than 96%.

Above we have presented our results on the two-qubit quantum Zeno effect and its application in generating and expanding quantum entanglement. We find that good fidelity can be achieved even if we only use fewer than 100 steps with operational errors (occasional error) up to 50% in every step. The final question remaining is how to physically implement the $J$−measurement, which is a two-qubit threshold measurement.

## VII. IMPLEMENTATION

There have been a number of proposals [20–22] for two-qubit measurements based on quantum dots or superconducting qubits. There [20–22], not only two-qubit measurement schemes are given, but also their feasibility, including decoherence. On the other hand, threshold detections for a single-qubit have been experimentally demonstrated already [23–25].

Compared with the one-qubit threshold measurement, the two-qubit threshold measurement does not need extra precise control of interaction or synchronization. Here we consider an implementation scheme for two-qubit threshold detection, using Josephson-junction circuits (see, e.g., [24–28]).

Consider a circuit with one large junction, denoted by "0" and two parallel flux qubits, each one consisting of three smaller junctions, as shown in Fig. 1. If the current across junction 0 is larger than a certain critical value $I_{T0}$, it switches from the superconducting state to the normal state. The direction of the current contributed by any qubit in the circuit depends on its state, say, $|1\rangle$ for the "up" current and $|0\rangle$ for the "down" one. The current contributed from those three-junction flux qubits is significantly less than $I_{T0}$. However, with an appropriate bias current, the current contributed by those flux qubits determines whether the large junction, 0, will be switched to the nonsuperconducting state with a nonzero voltage $V$. The current is determined by the quantum state of those flux qubits in the circuit. Suppose that the state $|1\rangle,|0\rangle$ of each individual qubit contributes a current $\pm I_D$, respectively. If the bias current is set to be, e.g., $I_b=I_{T0}-I_D$, by monitoring the voltage $V$, we can conclude whether the state of those flux qubits has been projected to the state $|11\rangle$. Of course, the bias current $I_b$ and the magnetic flux $\Phi_e$ can be tuned. Consider the case where there are only two qubits. There are two subspaces, $J_0=\{|00\rangle,|01\rangle,|10\rangle\}$ and $J_1=|11\rangle$. A state in subspace $J_1$ ($J_0$) will cause (not cause) junction "0" to switch from the superconducting to the normal state, given a certain bias current $I_b$ and an external field $\Phi_e$. Thus, when the current $I_b$ is biased, we can conclude whether the quantum state of those observed qubits belongs to subspace $J_0$ or $J_1$, by monitoring the voltage $V$. If no bias current is applied, there is no measurement. But if the bias current slightly below $I_{T0}$ is applied, a "$J$" measurement is performed.

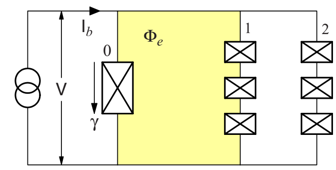


FIG. 1. (Color online) The so called $J$−measurement can be implemented by a Josephson-junction circuit with flux qubits. Junction "0" is a larger junction. Flux qubit 1 and flux qubit 2 each consists of three small junctions. $\Phi_e$ is the flux of the external magnetic field threading the loop connecting junction "0" and qubit 1.

The Hamiltonian for a flux qubit is [28]

$$H = I_p\left(\Phi_e - \frac{1}{2}\Phi_0\right)\sigma_z + \Delta\sigma_x, \quad (33)$$

where $I_p$ is the maximum persistent supercurrent of the flux qubit, $\Delta$ is the tunneling amplitude of the barrier and $\Delta \ll I_p\Phi_0$, with $\Phi_0$ being the flux quantum. Initially we can set $\Phi_e \ll \Phi_0/2$ so that the state $|00\rangle$ is produced for the two flux qubits. We then shift $\Phi_e$ to $\Phi_0/2$ very fast and apply $I_b$ frequently. After a time period of $\pi/(2\sqrt{2}\Delta)$, the entangled state $|\psi^+\rangle$ is produced if $V=0$ is verified throughout the period. This procedure can be extended so as to experimentally produce large cluster states. For existing technologies of superconducting qubits, the detection time is around 1 ns, while the decoherence time can be several $\mu$s (see, e.g., [29]), which indicates that thousands of $J$−measurements could be done within the decoherence time. In the future, it would be interesting to study the effects of decoherence on this circuit.

## VIII. CONCLUDING REMARKS

We have studied the two-qubit quantum Zeno effect with threshold detection, a type of nondeterministic collective measurement: the $J-$measurement which distinguishes two subspaces $J_1=\{|11\rangle\}$, and $J_0=\{|00\rangle,|01\rangle,|10\rangle\}$. We show that the two-qubit quantum Zeno effect can be used to produce and expand quantum entanglement, such as cluster states, which are a useful resource for quantum computing. These give insights on the quantum Zeno effect and its possible application in QIP. The method presented here can also be used to produce other types of entangled states, including the Greenberger-Horne-Zeilinger states and the so-called "*W* states" [30]. We also discussed the possible implementation of the $J-$measurement with superconducting qubits.

## ACKNOWLEDGMENTS

We are grateful to Hans Briegel and Sahel Ashhab for enlightening discussions. F.N. gratefully acknowledges partial support from the National Security Agency (NSA), Laboratory Physical Science (LPS), Army Research Office (ARO), National Science Foundation (NSF) Grant No. EIA-0130383, JSPS-RFBR 06-02-91200, and Core-to-Core (CTC) program supported by the Japan Society for Promotion of Science (JSPS). X.B.W. was supported in part by the National Fundamental Research Program (NFRPC) of China Grants No. 2007CB807900 and No. 2007CB807901, NSFC Grant No. 60725416, and China High Tech. Program Grant No. 2006AA01Z420. J.Q.Y. was supported by the National Natural Science Foundation of China (NSFC) Grants No. 10534060 and No. 10625416, and the NFRPC Grant No. 2006CB921205.

[1] D. J. Griffiths, *Introduction to Quantum Mechanics* (Prentice Hall, Upper Saddle River, NJ, 1994); K. Koshino and A. Shinimu, Phys. Rep. **412**, 191 (2005).
[2] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1977); A. Peres, Am. J. Phys. **48**, 931 (1985); Wayne M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990); A. G. Kofman and G. Kurizki, Nature (London) **405**, 546 (1990); E. Block and P. R. Berman, Phys. Rev. A **44**, 1466 (1991); C. Search and P. R. Berman, Phys. Rev. Lett. **85**, 2272 (2000); S. A. Gurvitz, L. Fedichkin, D. Mozyrsky, and G. P. Berman, *ibid.* **91**, 066801 (2003).
[3] W. H. Zurek, Phys. Rev. Lett. **53**, 391 (1984); A. Barenco *et al.*, SIAM J. Comput. **26**, 1541 (1997); L. Vaidman, L. Goldenberg, and S. Wiesner, Phys. Rev. A **54**, R1745 (1996); L.-M. Duan and G.-C. Guo, *ibid.* **57**, 2399 (1998); A. Beige, D. Braun, B. Tregenna, and P. L. Knight, Phys. Rev. Lett. **85**, 1762 (2002); P. Facchi and S. Pascazio, *ibid.* **89**, 080401 (2002); J. Pachos and H. Walther, *ibid.* **89**, 187903 (2002); D. Dhar, L. K. Grover, and S. M. Roy, *ibid.* **96**, 100405 (2006); F. Helmer, M. Mariantoni, E. Solano, and F. Marquardt, e-print arXiv:0712.1908.
[4] J. D. Franson, B. C. Jacobs, and T. B. Pittman, Phys. Rev. A **70**, 062302 (2004).
[5] G. Gordon and G. Kurizki, Phys. Rev. Lett. **97**, 110503 (2006); F. F. Fanchini and R. d. J. Napolitano, Phys. Rev. A **76**, 062306 (2007).
[6] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London) **409**, 46 (2001).
[7] D. Gottesman and I. L. Chuang, Nature (London) **402**, 390 (1999); D. W. Leung, Int. J. Quantum Inf. **2**, 33 (2004).
[8] A. Ekert and R. Josza, Philos. Trans. R. Soc. London, Ser. A **356**, 1769 (1998); T. B. Pittman, M. J. Fitch, B. C. Jacobs, and J. D. Franson, Phys. Rev. A **68**, 032316 (2003); J. D. Franson, M. M. Donegan, M. J. Fitch, B. C. Jacobs, and T. B. Pittman, Phys. Rev. Lett. **89**, 137901 (2002).
[9] N. Linden and S. Popescu, Phys. Rev. Lett. **87**, 047901 (2001); R. Josza and N. Linden, Proc. R. Soc. London, Ser. A **459**, 2011 (2003); E. Biham, G. Brassard, D. Kenigsberg, and T. Mor, Theor. Comput. Sci. **320**, 15 (2004); H. F. Hofmann and S. Takeuchi, Phys. Rev. Lett. **88**, 147901 (2002).
[10] M. A. Nielsen, Phys. Lett. A **308**, 96 (2003); Phys. Rev. Lett. **93**, 040503 (2004).
[11] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001); R. Raussendorf and H. J. Briegel, *ibid.* **86**, 5188 (2001); R. Raussendorf, D. E. Browne, and H. J. Briegel, J. Mod. Opt. **49**, 1299 (2002); Phys. Rev. A **68**, 022312 (2003).
[12] M. Borhani and D. Loss, Phys. Rev. A **71**, 034308 (2005).
[13] Y. S. Weinstein, C. S. Hellberg, and J. Levy, Phys. Rev. A **72**, 020304(R) (2005).
[14] T. Tanamoto, Y. X. Liu, S. Fujita, X. Hu, and F. Nori, Phys. Rev. Lett. **97**, 230501 (2006).
[15] J. Q. You, X. B. Wang, T. Tanamoto, and F. Nori, Phys. Rev. A **75**, 052319 (2007).
[16] Z.-Y. Xue and Z. D. Wang, Phys. Rev. A **75**, 064303 (2007).
[17] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, Nature (London) **434**, 169 (2005).
[18] D. E. Browne and T. Rudolph, Phys. Rev. Lett. **95**, 010501 (2005); T. Rudolph and S. Virmani, New J. Phys. **7**, 228 (2005).
[19] Y. L. Lim, A. Beige, and L. C. Kwek, Phys. Rev. Lett. **95**, 030505 (2005).
[20] H.-A. Engel and D. Loss, Science **309**, 586 (2005).
[21] S. D. Barrett and T. M. Stace, Phys. Rev. B **73**, 075324 (2006).
[22] R. Ruskov, A. N. Korotkov, and A. Mizel, Phys. Rev. B **73**, 085317 (2006).
[23] T. Hayashi, T. Fujisawa, H.-D. Cheong, Y.-H. Jeong, and Y. Hirayama, Phys. Rev. Lett. **91**, 226804 (2003).
[24] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
[25] K. Katz *et al.*, Science **312**, 1498 (2006).
[26] J. Q. You and F. Nori, Phys. Today **58**(11), 42 (2005).
[27] J. Q. You, J. S. Tsai, and F. Nori, Phys. Rev. Lett. **89**, 197902 (2002); Phys. Rev. B **68**, 024510 (2003).
[28] J. B. Majer, F. G. Paauw, A. C. J. ter Haar, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. Lett. **94**, 090501 (2005); G. S. Paraoanu, *ibid.* **97**, 180406 (2006).
[29] P. Bertet, I. Chiorescu, G. Burkard, K. Semba, C. J. P. M. Harmans, D. P. DiVincenzo, and J. E. Mooij, Phys. Rev. Lett. **95**, 257002 (2005).
[30] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).