

Control of photon propagation via electromagnetically induced transparency in lossless mediaLiang He,^{1,2} Yu-xi Liu,^{2,3} S. Yi,¹ C. P. Sun,^{1,2} and Franco Nori^{2,3,4}¹*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China*²*Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama 351-0198, Japan*³*CREST, Japan Science and Technology Agency (JST), Kawaguchi, Saitama 332-0012, Japan*⁴*Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA*

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We study the influence of a lossless material medium on the coherent storage and quantum-state transfer of a quantized probe light in an ensemble of Λ -type atoms. The medium is modeled as uniformly distributed two-level atoms with the same energy level spacing, coupling to a probe light. This coupled system can be simplified to a collection of two-mode polaritons which couple to one transition of the Λ -type atoms. We show that, when the other transition of Λ -type atoms is controlled by a classical light, electromagnetically induced transparency can also occur for the polaritons. In this case the coherent storage and quantum transfer for photon states are achievable through novel dark states with respect to the polaritons. By calculating the corresponding dispersion relation, we find that the ensemble of three-level atoms with Λ -type transitions may serve as quantum memory for it slows or even stops light propagation through the mechanism of electromagnetically induced transparency.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) [1] is a typical quantum coherent effect, in which the propagation of a probe field in a Λ -type atom ensemble can be well controlled by a classical light [2,3]. Most recently, the EIT phenomenon was suggested as an active mechanism [4–6] to slow down and even stop the photon propagation, so that the photon state can be stored or released coherently. These investigations [4–6] are mainly motivated by the fast development of quantum information science and technology [7]. This is because, with the help of quantum storage, one could complete a series of quantum logical operations within the decoherence time.

In this paper we study the EIT mechanism for quantum information processing in the presence of a lossless medium. This is motivated by two reasons. First, we notice that buffer gases, with different atom species, are used in some of the recent EIT experiments [8,9]. Usually one introduces a buffer gas to lengthen the ground-state coherence lifetime of confined EIT atoms. For the EIT effect in a Λ sample with a buffer gas, the probe field has a low group velocity when it has a small detuning with respect to resonance [8,9]. These coherent phenomena essentially result from the gaseous medium: the buffer gas. To see the coherent effect of the buffer gas, one sets up atoms in the “buffer gas” to be resonant with the probe light (in this case the “buffer gas” no longer only acts as a buffer to cool down the EIT atoms); the “buffer gas” just plays the role of a coherent medium and the photon will be coupled with collective excitations of the buffer atoms to form quasiparticles, called polaritons.

Second, the study of EIT for photon-state storage should be extended to solid-state systems for applications in scalable quantum computing. Here, EIT atoms with Λ -type transitions may be realized using solid-state devices, such as artificial atoms based on quantum dots, which are usually

embedded in a solid-state substrate. To make such solid-state devices as coherent storage units based on the EIT mechanism, one should consider the EIT process in the medium of the substrate.

In our study, we first model the medium as a collection of N two-level atoms, weakly coupled to the quantized probe field [10]. The “weak” interaction between the atoms and the probe field is assumed to excite a few atoms, such that the collective excitations of the atoms behave as bosons. In turn, the photons of the probe field are dressed by the collective excitations, forming polaritons [11] of two modes. According to Hopfield’s original paper on quantum polariton [11] and also according to others [10], such a polariton can be regarded as a macroscopically averaged electromagnetic field or a displacement field.

We then show that, when one of the two polariton modes is resonant with the three-level Λ -type atoms, there still exists a dark state which decouples from the upper energy level of the Λ atom. Utilizing the dark state, we can adiabatically manipulate the quantum state of the photon such that the photon state is coherently transferred to the atomic collective excitation state. We further calculate the susceptibility of light propagation in the EIT atomic ensemble embedded in a medium.

In the usual case, due to inhomogeneous broadening, ground-state decoherence, loss, etc., the broadened energy levels of atoms in the EIT ensemble can behave as energy bands and thus limit the transparency due to the off-resonance of some atoms. Here, the coherent processes induced by the two-level lossless medium can only result in a frequency split of the effective light field, which also causes the off-resonance with respect to the fixed energy levels of EIT atoms. However, by making use of the Hopfield model [10,11], the split can be estimated quantitatively and then one can restore the resonance for the EIT by the effective light field in the medium.

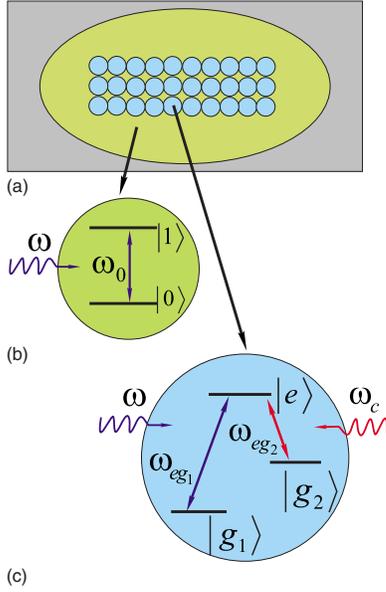


FIG. 1. (Color online) (a) Schematic diagram of the system under consideration. (b) The medium [yellow background in (a)] is modeled by N two-level atoms; each medium atom has an identical transition frequency ω_0 . (c) The level structure of the three-level Λ -type atoms.

The remaining part of this paper is organized as follows. In Sec. II, we study the coupled system of quantum light plus medium atoms and describe this using two-mode polaritons. Section II is devoted to study the EIT effect of a single three-level atom induced by polaritons. In Sec. IV, for an ensemble of three-level atoms, we construct the many-atom dark states based on the spectra-generating algebra method. The influence of the medium on quantum-state transfer is discussed in Sec. V. In Sec. VI, we study the propagation of dressed light. Finally, conclusions are presented in Sec. VII.

II. MICROSCOPIC HOPFIELD MODEL FOR MATERIAL MEDIA INTERACTING WITH A SINGLE-MODE CAVITY FIELD

The system under consideration, shown schematically in Fig. 1, includes a single-mode cavity, a lossless medium, and M identical three-level Λ -type atoms. The medium is modeled by N two-level atoms with equal level spacing ω_0 , and for the j th medium atom, the ground and excited states are denoted, respectively, as $|0\rangle_j$ and $|1\rangle_j$. The three-level Λ -type atoms have two lower states $|g_1\rangle$ and $|g_2\rangle$ plus an upper state $|e\rangle$. A single-mode cavity field ω is assumed as the probe field to induce a transition between levels $|e\rangle$ and $|g_1\rangle$. Finally, a classical control field ω_c is introduced to couple $|e\rangle$ and $|g_2\rangle$.

To better understand the effect of the medium, we shall only consider, in this section, the interaction between the single-mode probe field and the medium, which is described by the Hamiltonian

$$H_{L-M} = \hbar\omega a^\dagger a + \hbar\omega_0 \sum_{j=1}^N |1\rangle_j \langle 1| + \hbar \sum_{j=1}^N g_j (a + a^\dagger) (\sigma_+^{(j)} + \sigma_-^{(j)}), \quad (1)$$

where a^\dagger (a) is the creation (annihilation) operator for the quantum probe field and $\sigma_+^{(j)} = |1\rangle_j \langle 0|$, $\sigma_-^{(j)} = |0\rangle_j \langle 1|$, and $\sigma_z^{(j)} = |1\rangle_j \langle 1| - |0\rangle_j \langle 0|$ are the quasispin Pauli operators for the j th atom. Here g_j is the electric-dipole coupling strength between the probe field and the j th atom. For simplicity, we shall assume throughout this paper that $g_j \equiv g_{\text{medium}}$ is independent of individual atoms.

To simplify Hamiltonian (1), we define the collective quasi-spin-wave operators as

$$B_k^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_+^{(j)} \exp\left(\frac{2\pi i k j}{N}\right),$$

$$B_k = \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_-^{(j)} \exp\left(-\frac{2\pi i k j}{N}\right),$$

where $k=0, \dots, N-1$. In the large- N limit with low-excitation conditions, it was proven that the above collective quasi-spin-wave operators B_k and $B_{k'}^\dagger$ satisfy the bosonic commutation relations [12,13]

$$[B_k, B_{k'}^\dagger] = \delta_{kk'} \quad (2)$$

and

$$\sum_{j=1}^N |1\rangle_j \langle 1| = \sum_k B_k^\dagger B_k.$$

The commutation relation in Eq. (2) suggests that the low-excitation behavior of the medium can be described by N bosonic operators, i.e., exciton operators. The low-energy part of Eq. (1) then reduces to Hopfield's Hamiltonian [11]

$$H_{L-M} = \hbar\omega a^\dagger a + \hbar\omega_0 B_0^\dagger B_0 + \hbar G (a + a^\dagger) (B_0 + B_0^\dagger), \quad (3)$$

where $G = g_{\text{medium}} \sqrt{N} \propto \sqrt{N/V}$, with V being the effective volume of the probe field, has a finite Van Hove limit. We remark that, to obtain Eq. (3), we have neglected $N-1$ free exciton modes B_1, B_2, \dots, B_{N-1} , as they are decoupled from the probe field.

Equation (3) can be solved using polariton operators employed by several authors [10,11]. Following the procedure given in Ref. [10], we define the polariton operators as

$$c_k = x_1^k a + y_1^k a^\dagger + x_2^k B_0 + y_2^k B_0^\dagger, \quad (4)$$

with $k=1, 2$. Here c_k and $c_{k'}^\dagger$ satisfy the usual bosonic commutation relation $[c_k, c_{k'}^\dagger] = \delta_{kk'}$ and $[c_k, c_{k'}] = 0$. Assuming that the Hamiltonian, Eq. (3), is diagonalized by c_1 and c_2 , namely,

$$H_{L-M} = \hbar\Omega_1 c_1^\dagger c_1 + \hbar\Omega_2 c_2^\dagger c_2,$$

the coefficients x_l^k and y_l^k ($l, k=1, 2$) are then obtained via the equation

$$[c_k, H_{L-M}] = \hbar \Omega c_k.$$

Explicitly, we have

$$x_j^k = \frac{1}{2}(v_j^k + u_j^k), \quad y_j^k = \frac{1}{2}(v_j^k - u_j^k),$$

where

$$u_1^k = \frac{\omega}{\Omega_k} v_1^k, \quad u_2^k = \frac{\Omega_k^2 - \omega^2}{2G\Omega_k} v_2^k,$$

$$v_2^k = \frac{\Omega_k^2 - \omega^2}{2G\omega_0} v_1^k, \quad v_1^k = \sqrt{\frac{4G^2\Omega_k\omega_0}{(\Omega_k^2 - \omega^2)^2 + 4\omega_0\omega G^2}},$$

and the eigenfrequencies

$$\Omega_k^2 = \frac{1}{2}\{\omega_0^2 + \omega^2 + (-1)^k \sqrt{(\omega_0^2 - \omega^2)^2 + 16\omega_0\omega G^2}\}. \quad (5)$$

The above results are identical to those obtained using the Hopfield's approach, as shown in Ref. [11], where the effect of the medium is phenomenologically modeled by many harmonic oscillators. Our results then indicate that those phenomenological harmonic oscillators essentially originate from the low-energy collective excitations of the medium atoms. As a matter of fact, the same as previous treatments for the effect of the medium [10], our approach also relies on the weak-coupling assumption, which suggests that the medium effect can be equivalently studied according to either the two-level model or the harmonic oscillators.

III. EFFECT OF MATERIAL MEDIA ON THE DARK STATE OF A THREE-LEVEL ATOM

In this section, we shall assume that there is only one three-level Λ -type atom embedded in the medium. As explained in Sec. II, this atom couples with a single-mode cavity field and a classical light field. However, due to the effect of the medium, the single cavity mode is effectively replaced by the two-mode polariton, resulting in the two-color EIT model shown in Fig. 2. The corresponding Hamiltonian takes the form

$$\begin{aligned} h = & \hbar \sum_{i=1}^2 \Omega_i c_i^\dagger c_i + \hbar \omega_{eg_1} |e\rangle\langle e| + \hbar(\omega_{eg_1} - \omega_{eg_2}) |g_2\rangle\langle g_2| \\ & + \hbar g \sum_{i=1}^2 u_i (c_i + c_i^\dagger) (|e\rangle\langle g_1| + \text{H.c.}) \\ & + \hbar \xi (e^{-i\omega_c t} |e\rangle\langle g_2| + \text{H.c.}), \end{aligned} \quad (6)$$

where $u_i = u_i^i$, g and ξ are the Rabi frequencies for, respectively, the single-mode cavity field and the classical light field, and ω_{eg_1} (ω_{eg_2}) is the atomic transition frequency from the level $|e\rangle$ to the level $|g_1\rangle$ ($|g_2\rangle$) as shown in Fig. 1.

We first assume that the classical field and one of two polariton modes—say, c_1 —satisfy the two-photon resonance condition: i.e., $\Omega_1 = \omega_c + \omega_{eg_1} - \omega_{eg_2}$. In the interaction picture and taking the rotating-wave approximation, the Hamiltonian, Eq. (6), becomes (using $\hbar = 1$)

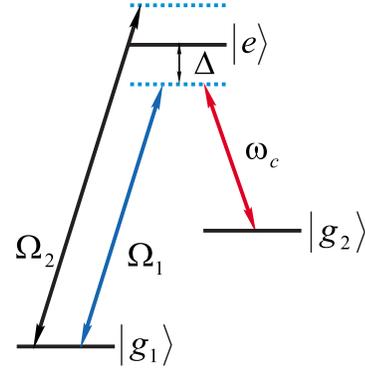


FIG. 2. (Color online) The coupled system of the two-mode polaritons and a three-level atom. Ω_1 and Ω_2 are, respectively, the frequencies of the c_1 and c_2 modes. ω_c is the frequency of the classical control field and its detuning with the transition frequency ω_{eg_2} is Δ .

$$\begin{aligned} h_I = & \tilde{\Omega}_2 c_2^\dagger c_2 + \Delta |e\rangle\langle e| + \xi |e\rangle\langle g_2| + (\xi |e\rangle\langle g_2| \\ & + g(u_1 c_1 + u_2 c_2) |e\rangle\langle g_1| + \text{H.c.}), \end{aligned} \quad (7)$$

where $\tilde{\Omega}_2 = \Omega_2 - \omega_c - (\omega_{eg_1} - \omega_{eg_2})$ and $\Delta = \omega_{eg_1} - \Omega_1 = \omega_{eg_2} - \omega_c$. We note that h_I possesses an invariant subspace spanned by the states $|e, n_1, n_2\rangle$, $|g_2, n_1, n_2\rangle$, $|g_1, n_1 + 1, n_2\rangle$, and $|g_1, n_1, n_2 + 1\rangle$; here, n_1 and n_2 are the number of polaritons for modes c_1 and c_2 , respectively. The matrix representation of the h_I in this invariant subspace is then

$$\begin{aligned} h_I = & (n_1 \tilde{\Omega}_1 + n_2 \tilde{\Omega}_2) I \\ & + \begin{pmatrix} \Delta & \xi & gu_1 \sqrt{n_1 + 1} & gu_2 \sqrt{n_2 + 1} \\ \xi & 0 & 0 & 0 \\ gu_1 \sqrt{n_1 + 1} & 0 & 0 & 0 \\ gu_2 \sqrt{n_2 + 1} & 0 & 0 & \tilde{\Omega}_2 \end{pmatrix}, \end{aligned} \quad (8)$$

with I being the identity matrix. Here h_I has a zero eigenvalue corresponding to the eigenstate

$$|\psi_0\rangle = \cos \theta |g_1, n_1 + 1, n_2\rangle - \sin \theta |g_2, n_1, n_2\rangle, \quad (9)$$

where θ is determined by $\tan \theta = \xi / (gu_1 \sqrt{n_1 + 1})$. We immediately notice that $|\psi_0\rangle$ is a dark state formed by polaritons and the two lower atomic levels, in contrast with that formed directly by photons. Furthermore, $|\psi_0\rangle$ can be factorized as

$$|\psi_0\rangle = |\phi_0\rangle \otimes |n_2\rangle = (\cos \theta |g_1, n_1 + 1\rangle - \sin \theta |g_2, n_1\rangle) \otimes |n_2\rangle, \quad (10)$$

where $|\phi_0\rangle$ superposes different polariton number states. Considering now the $n_1 = 0$ subspace, if we manipulate the Rabi frequency ξ of the classical field adiabatically, such that θ varies from 0 to $\pi/2$, the information of a single-polariton state is then stored into the atomic state.

IV. COLLECTIVE ATOMIC EXCITATION DRESSED BY POLARITONS

As the adiabatic manipulation described in the previous section is only accompanied by a single-polariton transfer, it

cannot be used to transfer or store a general state which is a superposition of multiple-polariton number states. To fulfill this purpose, an ensemble of atoms is needed to serve as the quantum data bus or quantum memory. We therefore consider in this section M identical three-level Λ -type atoms interacting with the polariton modes. The same as the single three-level atom case, we assume that only the c_1 mode satisfies the two-photon resonance condition. The Hamiltonian of the system, in the interaction picture, is (using $\hbar=1$)

$$H_I = \tilde{\Omega}_2 c_2^\dagger c_2 + \Delta \sum_{j=1}^M \sigma_{ee}^{(j)} + \xi \sum_{j=1}^M \sigma_{eg_2}^{(j)} + g \left[(u_1 c_1 + u_2 c_2) \sum_{j=1}^M \sigma_{eg_1}^{(j)} + \text{H.c.} \right], \quad (11)$$

where $\sigma_{\mu\nu}^{(j)} = |\mu\rangle_j \langle \nu|$ ($\mu, \nu = e, g_1, g_2$) is a flip operator of the j th atom. To further simplify the notation, we define the collective quasispin operators [12]

$$S = \sum_{j=1}^M \sigma_{ee}^{(j)}, \quad T_+ = \sum_{j=1}^M \sigma_{eg_2}^{(j)}, \quad A^\dagger = \frac{1}{\sqrt{M}} \sum_{j=1}^M \sigma_{eg_1}^{(j)}, \quad (12)$$

where $A^\dagger(A)$ characterizes the collective atomic excitations. We note that, in the large- M limit with low atomic excitations, the operators A^\dagger and A satisfy the bosonic commutation relation $[A, A^\dagger] = 1$. The Hamiltonian, Eq. (11), can now be rewritten as

$$H_I = \tilde{\Omega}_2 c_2^\dagger c_2 + \Delta S + (\xi T_+ + g u_1 \sqrt{M} c_1 A^\dagger + g u_2 \sqrt{M} c_2 A^\dagger + \text{H.c.}). \quad (13)$$

Following the procedure developed in Ref. [12], we introduce another atomic collective excitation operator

$$C = \frac{1}{\sqrt{M}} \sum_{j=1}^M \sigma_{g_1 g_2}^{(j)}. \quad (14)$$

In the large- M and low-excitation limit, the collective operators defined in Eqs. (12) and (14) satisfy the basic commutation relations,

$$[A, S] = A, \quad [C, S] = 0, \quad [A, A^\dagger] = 1, \quad [C, C^\dagger] = 1,$$

$$[T_+, C^\dagger] = A^\dagger, \quad [T_-, A^\dagger] = C^\dagger, \quad [S, T_\pm] = \pm T_\pm. \quad (15)$$

The effective Hamiltonian, Eq. (13), is a function of the operators $A, A^\dagger, C, C^\dagger$, and T_\pm that generate a close algebra \mathcal{L} , corresponding to a noncompact group $U(\mathcal{L})$. This means that the composite system, consisting of photons, the medium, and Λ -type atoms, possesses a dynamic symmetry of $U(\mathcal{L})$. Using the symmetry analysis [12], we construct a dark-state operator of the polariton operator c_1 and atomic operator C :

$$D = c_1 \cos \theta - C \sin \theta,$$

which satisfies $[H_I, D] = 0$ and $[D, D^\dagger] = 1$. Furthermore, we introduce the state

$$|\mathbf{0}\rangle = |v\rangle \otimes |0\rangle_{c_1} \otimes |0\rangle_{c_2},$$

where $|v\rangle = |g_1, g_1, \dots, g_1\rangle$ is the collective ground state with all atoms in their ground states and $|0\rangle_{c_1}$ and $|0\rangle_{c_2}$ are the vacuum of the polariton modes. We note that $|\mathbf{0}\rangle$ is an eigenstate of H_I with zero eigenvalue, and consequently, a degenerate class of H_I with zero eigenvalue can be constructed as follows:

$$|D_n\rangle = \frac{1}{\sqrt{n!}} D^{\dagger n} |\mathbf{0}\rangle,$$

which can be used as a quantum memory. There also exist other eigenstates with zero eigenvalue; however, as shown in Appendix A, the adiabatic evolution does not mix them with the dark state $|D_n\rangle$.

V. QUANTUM ADIABATIC MANIPULATIONS IN THE PRESENCE OF MATERIAL MEDIA

In earlier work [4–6, 12], the EIT system was proposed as an efficient quantum memory by adiabatic quantum manipulation. In the presence of the medium, we explore the possibility of implementing such quantum manipulation by taking into account the coupling between the quantum light field and the medium.

The goal of the EIT-based quantum memory is to transmit the information of the quantum light to the low-excitation state of the Λ -type atomic ensemble. To see the key point of our studies here, we would like to recall the basic physical process of the EIT-based quantum storage. If there is no interaction between the quantum light and the medium ($G=0$), the state used for quantum storage can be expressed as

$$|\Psi(\theta)\rangle = \sum_n c_n |d_n(\theta)\rangle \otimes |0\rangle_B,$$

where $|0\rangle_B$ is the vacuum state of the collective excitation of the medium atoms and

$$|d_n(\theta)\rangle = (a^\dagger \cos \theta - C^\dagger \sin \theta)^n |0\rangle_C \otimes |0\rangle_a$$

is a dark state formed by the probe light and the collective excitations of three-level atoms. Here, $|0\rangle_C$ is the vacuum state defined by C and $|0\rangle_a$ is the photon vacuum state. Therefore, a perfect quantum storage of photon states by an atomic ensemble can be realized by the following adiabatic evolution:

$$\begin{aligned} |\Psi(\theta=0)\rangle &= |0\rangle_C \otimes \left(\sum_n c_n |n\rangle_a \right) \otimes |0\rangle_B \rightarrow \left| \Psi \left(\theta = \frac{\pi}{2} \right) \right\rangle \\ &= \left(\sum_n (-1)^n c_n |n\rangle_C \right) \otimes |0\rangle_a \otimes |0\rangle_B. \end{aligned}$$

As shown in the previous section, after we turn on the coupling between the medium and quantum light, the dark state $|d_n(\theta)\rangle$ is replaced by $|D_n(\theta)\rangle$, a dark state formed by the resonant mode c_1 of the polariton and the collective excitations of three-level atoms. In this case, an ideal quantum-state transfer should realize the process

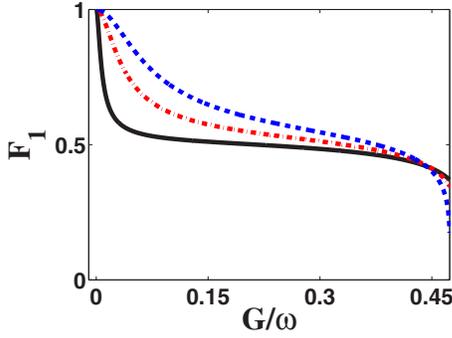


FIG. 3. (Color online) The coupling strength G dependence of the one-photon-state transmission efficiency F_1 for $\omega_0=0.99\omega$ (black solid line), $\omega_0=0.95\omega$ (red dash-dotted line), and $\omega_0=0.9\omega$ (blue dashed line).

$$|0\rangle_C \otimes \sum_n c_n |n\rangle_a \otimes |0\rangle_B \rightarrow \sum_n c_n |n\rangle_C \otimes |0\rangle_{c_1} \otimes |0\rangle_{c_2}.$$

However, as we shall show below, quantum-state transfer can only be partially achieved when $G \neq 0$. Without loss of generality, assuming that the photon state to be transferred is a Fock state, namely, the initial state of the system is

$$|\Psi(t=0)\rangle = |0\rangle_C \otimes |n\rangle_a \otimes |0\rangle_B.$$

We note that state $|n\rangle_a \otimes |0\rangle_B$ can be expanded using the Fock states of polaritons, which gives

$$\begin{aligned} |\Psi(0)\rangle &= S_{n0}|0\rangle_C \otimes |n\rangle_{c_1} \otimes |0\rangle_{c_2} + \sum_{i \neq n, j \neq 0} S_{ij}|0\rangle_C \otimes |i\rangle_{c_1} \otimes |j\rangle_{c_2} \\ &= S_{n0}|D_n(\theta=0)\rangle + |\psi'(0)\rangle, \end{aligned} \quad (16)$$

where $|\psi'(0)\rangle = \sum_{i \neq n, j \neq 0} S_{ij}|0\rangle_C \otimes |i\rangle_{c_1} \otimes |j\rangle_{c_2}$. The coefficients S_{ij} can be obtained straightforwardly; in particular, when the coupling between light and medium atoms is weak, S_{n0} is very close to unity. As the system evolves adiabatically to time t , the wave function becomes

$$|\Psi(t)\rangle = S_{n0}|D_n(\theta(t))\rangle + |\psi'(t)\rangle, \quad (17)$$

and at $\theta(t) = \pi/2$, we have

$$\left| D_n\left(\frac{\pi}{2}\right) \right\rangle = (-1)^n |n\rangle_C \otimes |0\rangle_{c_1} \otimes |0\rangle_{c_2}.$$

Therefore, the first term on the right-hand side of Eq. (17) transfers the quanta of the photon to the collective excitation of the atomic ensemble; the second term, on the other hand, represents the leakage of the quantum memory.

Furthermore, to quantify the effect of the medium, we need to calculate the efficiency of the n -photon state transfer, i.e.,

$$F_n \equiv |S_{n0}|^2.$$

The detailed results on F_n are presented in Appendix B. In Fig. 3 we plot the one-photon-state transmission efficiency F_1 versus the coupling strength G for different ratios of the frequencies of the quantum light and the collective excitation of the medium. We see that the presence of the medium notably reduces the transmission efficiency, especially when

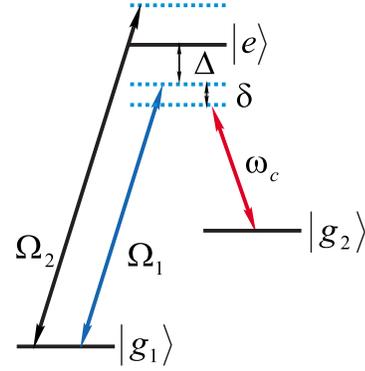


FIG. 4. (Color online) Same as Fig. 2, except that here the c_1 polariton mode and the classical light field do not satisfy the two-photon resonant condition: i.e., $\delta = \Omega_1 - \omega_c - \omega_{g_2} \neq 0$.

the quantum light is nearly resonant with the collective excitation of the medium atoms.

We notice that F_n is actually equal to 1 when $G=0$ in the resonant case and thus we cannot resort to the same calculation method about the transmission efficiency shown in Appendix B. When $\omega_0 = \omega$ together with $G=0$, there would be a singularity for the transmission efficiency if we carried out the same calculation as that in Appendix B. Physically, we can consider the cases with small detuning $\omega_0 - \omega$ and there is not an obvious jump of the transmission efficiency as shown in Fig. 3. Actually, there is a jump of F_1 from 1 to 0.5 in the resonant case when we turn on the coupling G from 0 to a small value. A strict resonant condition is never feasible in practical experiments, and thus we only consider two nearly resonant cases in Fig. 3.

VI. PROPAGATION OF THE DRESSED QUANTUM LIGHT

To consider the dynamical process of a quantum-state transfer, which is usually described by the slowing and the stopping of light, we study the dispersion and absorption properties of dressed quantum light, propagating in a Λ -type atomic ensemble. To achieve our goal, we consider the case when the c_1 mode and the classical light field have a small two-photon detuning $\delta = \Omega_1 - \omega_c - \omega_{g_2} \ll \Omega_1$ (see Fig. 4). The Hamiltonian in the interaction picture now becomes

$$H_I = \tilde{\Omega}_2 c_2^\dagger c_2 + \Delta S + e^{i\delta t} \xi T_+ + gu_1 \sqrt{N} c_1 A^\dagger + gu_2 \sqrt{N} c_2 A^\dagger + \text{H.c.}$$

With the help of the basic commutation relations in Eq. (15), we can approximately write down the Heisenberg equations for the operators A , C , and c_2 ,

$$\dot{A} = -\Gamma_A A - i\Delta A - igu_1 \sqrt{N} c_1 - ie^{i\delta t} \xi C - igu_2 \sqrt{N} c_2,$$

$$\dot{C} = -\Gamma_C C - ie^{-i\delta t} \xi A,$$

$$\dot{c}_2 = -\Gamma_{c_2} c_2 - i\tilde{\Omega}_2 c_2 - igu_2 \sqrt{N} A, \quad (18)$$

where we have ignored quantum fluctuations since we only calculate the group velocity. In addition, we have phenomenologically introduced the damping rate Γ_{c_2} of the mode c_2

and the decay rates Γ_A and Γ_C for, respectively, the states $|e\rangle$ and $|g_2\rangle$. Γ_A and Γ_C can be estimated as the spontaneous emission rates of the respective levels, which are proportional to the cube of the atomic transition frequencies. We therefore have $\Gamma_A \gg \Gamma_C$. The damping rate of the polariton mode is mainly due to the spontaneous emissions of the medium atoms and the leakage of the cavity. Since the former is negligible for a lossless medium, we can further assume $\Gamma_C \gg \Gamma_{c_2}$ for a high-quality cavity.

To find a steady-state solution for the above equations of motion, it is convenient to remove the fast-oscillating factors by making the transformation $C = e^{-i\delta t} C'$, which yields

$$\dot{A} = -\Gamma_A A - i\Delta A - igu_1 \sqrt{N} c_1 - i\xi C' - igu_2 \sqrt{N} c_2,$$

$$\dot{C}' = -\Gamma_C C' - i\xi A + i\delta C',$$

$$\dot{c}_2 = -\Gamma_{c_2} c_2 - i\tilde{\Omega}_2 c_2 - igu_2 \sqrt{N} A. \quad (19)$$

The steady-state solution can be obtained by letting $\dot{A} = \dot{C}' = \dot{c}_2 = 0$, from which we find the mean value of A as

$$\langle A \rangle = \frac{-igu_1 \alpha \beta \langle c_1 \rangle}{\alpha \beta (\Gamma_A + i\Delta) + \beta \xi^2 + \alpha g^2 u_2^2 N}, \quad (20)$$

with $\alpha = \Gamma_C - i\delta$ and $\beta = \Gamma_{c_2} + i\tilde{\Omega}_2$.

It is noted here that dressed quantum light or propagating polaritons nearly on resonance with the Λ -type atom can be described by

$$E(t) = \varepsilon e^{-i\Omega_1 t} + \text{H.c.} = u_1 \sqrt{\frac{\omega}{2V\epsilon_0}} c_1 e^{-i\Omega_1 t} + \text{H.c.},$$

where ϵ_0 is the permittivity of free space and V is the effective mode volume, which, for simplicity, is chosen to be equal to the interaction volume. In this case, the time-independent part of the polariton field strength is $u_1 \sqrt{\omega}/2V\epsilon_0$. We remark here that the Hopfield polariton field can be understood as a displacement field or macroscopic electromagnetic field corresponding to the polarization

$$\langle P \rangle = \langle p \rangle e^{-i\Omega_1 t} + \text{H.c.} = \epsilon_0 \chi \langle \varepsilon \rangle e^{-i\Omega_1 t} + \text{H.c.}, \quad (21)$$

where χ is the susceptibility. After neglecting the effect of the nonresonance polariton, the average polarization

$$\langle p \rangle = \frac{\mu}{V} \left\langle \sum_{j=1}^N \sigma_{eg_2}^{(j)} \right\rangle = \frac{\mu \sqrt{N}}{V} \langle A \rangle \quad (22)$$

can also be expressed in terms of the average of the exciton operator A . Combining Eqs. (20)–(22), we obtain

$$\chi = \frac{i2g^2 N \alpha \beta}{\omega [\alpha \beta (\Gamma_A + i\Delta) + \beta \xi^2 + \alpha g^2 u_2^2 N]}. \quad (23)$$

The real and imaginary parts χ_1 and χ_2 of the complex susceptibility $\chi = \chi_1 + i\chi_2$ can be explicitly expressed as

$$\chi_1 = \frac{(\delta \Gamma_{c_2} - \Gamma_C \tilde{\Omega}_2) \Theta + (\Gamma_C \Gamma_{c_2} + \tilde{\Omega}_2 \delta) \Xi}{\Theta^2 + \Xi^2} F,$$

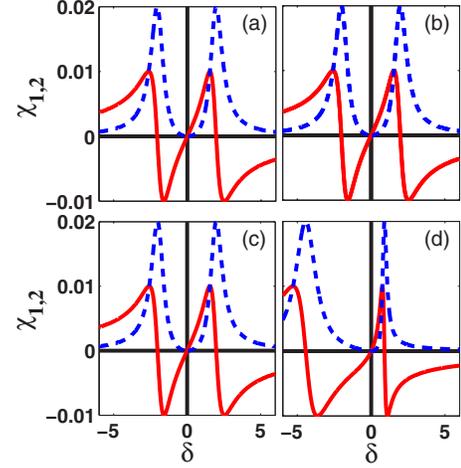


FIG. 5. (Color online) χ_1 (red solid lines) and χ_2 (blue dashed lines) versus the two-photon detuning δ (in units of Γ_A , i.e., here $\Gamma_A = 1$). (a) $\omega_0 = 0.9\omega$ and $G = 0$, (b) $\omega_0 = 0.9\omega$ and $G = 0.1\omega$, (c) $\omega_0 = \omega$ and $G = 0.1\omega$, and (d) $\omega_0 = \omega$ and $G = 0.001\omega$. Other parameters are $\Gamma_A = 1$, $\Gamma_C = 10^{-4}$, $\Gamma_{c_2} = 10^{-6}$, $g\sqrt{N} = 100$, and $\omega = 10^6$.

$$\chi_2 = \frac{(\Gamma_C \Gamma_{c_2} + \tilde{\Omega}_2 \delta) \Theta - (\delta \Gamma_{c_2} - \Gamma_C \tilde{\Omega}_2) \Xi}{\Theta^2 + \Xi^2} F,$$

where $F = 2g^2 N / \omega$ and

$$\Theta = \Gamma_C (\Gamma_A \Gamma_{c_2} - \Delta \tilde{\Omega}_2) + \xi^2 \Gamma_{c_2} + \delta (\Delta \Gamma_{c_2} + \tilde{\Omega}_2 \Gamma_A) + g^2 u_2^2 N \Gamma_C,$$

$$\Xi = -\delta (\Gamma_A \Gamma_{c_2} - \Delta \tilde{\Omega}_2) + \tilde{\Omega}_2 \xi^2 + \Gamma_C (\Delta \Gamma_{c_2} + \tilde{\Omega}_2 \Gamma_A) - \delta g^2 u_2^2 N.$$

It is well known that χ_1 and χ_2 are related to the dispersion and absorption, respectively. In Fig. 5, χ_1 and χ_2 are plotted versus the two-photon detuning δ .

Figure 5(a) shows the case (i) where there is no coupling between the quantum light and the medium. The result is obviously the same as that of conventional EIT effects. Figures 5(b) and 5(c) demonstrate almost the same dispersion and absorption properties as that in the case without the influence of the medium shown in Fig. 5(a). Figure 5(b) describes case (ii) that the frequency of the quantum light ω and the collective excitation frequency of the medium ω_0 are largely detuned in comparison with the parameters $g^2 u_2^2 N$, δ , and Δ , i.e.,

$$|\omega - \omega_0| \gg \delta, \Delta, g^2 u_2^2 N.$$

Figure 5(c) describes case (iii) that the coupling strength G between the quantum light medium is larger than the parameters $g^2 u_2^2 N$, δ , and Δ , i.e., $G \gg \delta, \Delta, g^2 u_2^2 N$.

The above phenomenon, predicted by our numerical calculations, can be well explained. In the configuration, illustrated in Fig. 4, when the mode c_1 of the polariton is nearly resonant with respect to the transition between $|e\rangle$ and $|g_1\rangle$, the role of the mode c_2 can be neglected if this mode is off-resonance with respect to the transitions from $|e\rangle$ to $|g_1\rangle$ and $|g_2\rangle$. Then the system will be reduced to the conventional EIT model, where the mode c_1 of the polariton plays the same role as that of the quantum light, taken as the probe

field. This case must lead to the same result as the conventional EIT case about the dispersion and absorption even in the presence of the medium. Now we can show that both cases (ii) and (iii) can give rise to the condition that the frequencies of the mode c_1 and c_2 —i.e., Ω_1 and Ω_2 —are largely detuned as mentioned above. We note that

$$|\Omega_2 - \Omega_1| = (\Omega_2 + \Omega_1)^{-1} \sqrt{(\omega_0^2 - \omega^2)^2 + 16\omega\omega_0 G^2}.$$

In case (ii) with $|\omega - \omega_0| \gg \delta, \Delta, g^2 u_2^2 N$, combining the condition

$$\sqrt{(\omega_0^2 - \omega^2)^2 + 16\omega\omega_0 G^2} > |\omega_0^2 - \omega^2|$$

with the condition $\Omega_2 + \Omega_1$, being of the order of $\omega + \omega_0$, we can find that $|\Omega_2 - \Omega_1|$ is of the order of $|\omega - \omega_0|$. This implies that $|\Omega_2 - \Omega_1| \gg \delta, \Delta, g^2 u_2^2 N$, which shows that the large-detuning condition is satisfied. The same analysis can also be applied to case (iii) if we note the condition

$$\sqrt{(\omega_0^2 - \omega^2)^2 + 16\omega\omega_0 G^2} > 4\sqrt{\omega\omega_0} G.$$

Due to $|\tilde{\Omega}_2| \approx |\Omega_2 - \Omega_1|$, if

$$|\Omega_2 - \Omega_1| \gg \delta, \Delta, g^2 u_2^2 N,$$

we can neglect all terms which do not have a factor of $\tilde{\Omega}_2$ in the denominator and numerator of Eq. (23). After calculations, we can obtain the same expression of the susceptibility as that in the conventional EIT case [14,15].

For the result illustrated in Fig. 5(d), where the parameters are assumed to satisfy the condition $|\omega - \omega_0|/\omega \ll 1$ and $G/\omega \ll 1$, we can obtain the results, about the dispersion and the absorption, which are different from those in the conventional EIT. The phenomenon is the deformed transparent window which is assisted by the collective excitation of the medium. Indeed, if $|\omega - \omega_0|/\omega \ll 1$ and $G/\omega \ll 1$, $|\tilde{\Omega}_2| \approx |\Omega_2 - \Omega_1|$ is smaller than or of the same order of δ and Δ . So the term related to the mode c_2 of the polariton, i.e., $g^2 u_2^2 N(\Gamma_C - i\delta)$, has a dominant effect on the susceptibility χ .

VII. CONCLUSIONS

In conclusion, we have studied the influence of a lossless medium on an EIT system. We find that even in the presence of the medium, the whole system still has dark states. This implies that, in some cases, the EIT system in the medium can still serve as a quantum memory. We also calculate the dispersion and absorption properties of the dressed quantum light. We find that the result obtained here is quite different from that of the conventional EIT. If the coupling strength between the quantum light and the medium is sufficiently strong, the ensemble of the three-level atoms with Λ -type transitions can easily become transparent if the usual EIT approach is applied.

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APPENDIX A: DYNAMIC SYMMETRY ANALYSIS OF THE SYSTEM

Starting from the dark state $|D_n\rangle$, we can use the spectrum-generating algebra method [16] to build other eigenstates of the whole system. We now introduce the bright-state polariton operator

$$B = c_1 \sin \theta + C \cos \theta,$$

which satisfies

$$[B, B^\dagger] = 1, \quad [B, D^\dagger] = 0, \quad [B, D] = 0.$$

It is straightforward to obtain the commutation relations

$$[H_I, B^\dagger] = \varepsilon B^\dagger, \quad [H_I, A^\dagger] = \Delta A^\dagger + \varepsilon B^\dagger + g u_2 \sqrt{N} c_2^\dagger,$$

$$[H_I, c_2^\dagger] = \tilde{\Omega}_2 c_2^\dagger + g u_2 \sqrt{N} A^\dagger,$$

with $\varepsilon = \sqrt{g^2 u_1^2 N \xi^2}$. We can introduce three independent bosonic operators

$$Q_i = \eta_1^i A + \eta_2^i B + \eta_3^i C, \quad i = 1, 2, 3,$$

which satisfy

$$[Q_i, Q_j^\dagger] = \delta_{ij}, \quad [Q_i, Q_j] = 0,$$

to diagonalize the Hamiltonian H_I , i.e.,

$$[Q_i, H_I] = \varepsilon_i Q_i.$$

Based on the above commutation relations, we can construct the eigenstates

$$|e(m_1, m_2, m_3, n)\rangle = \frac{Q_1^{\dagger m_1} Q_2^{\dagger m_2} Q_3^{\dagger m_3}}{\sqrt{m_1! m_2! m_3!}} |D_n\rangle$$

of the whole system, corresponding to eigenvalues

$$E \equiv E(m_1, m_2, m_3) = m_1 \varepsilon_1 + m_2 \varepsilon_2 + m_3 \varepsilon_3,$$

with $m_1, m_2, m_3 = 0, 1, 2, \dots$. The above equations show that there exists a larger class of states

$$\mathbf{S}: \{|e(\{m_i\}, n)\rangle \equiv |D(\{m_i\}, n)\rangle | n = 0, 1, \dots; E = 0\},$$

with zero eigenvalue $E \equiv E(m_1, m_2, m_3) = 0$; here, $\{m_i\} \equiv m_1, m_2, m_3$. But we show below that these states with zero eigenvalue do not mix with each other under adiabatic manipulation. Any state

$$|\phi(t)\rangle = \sum_{m_1 m_2 m_3 n} c_{m_1 m_2 m_3 n} |D(m_1, m_2, m_3, n)\rangle,$$

with zero eigenvalue, evolves according to

$$i \frac{d}{dt} c_{m_1 m_2 m_3 n}(t) = \sum_{m_1' m_2' m_3' n'} D_{m_1 m_2 m_3 n}^{m_1' m_2' m_3' n'} c_{m_1' m_2' m_3' n'} + F,$$

where F is a certain functional of the eigenstates with non-zero eigenvalues, which can be neglected under the adiabatic conditions [17,18] and

$$iD_{m_1 m_2 m_3 n}^{m'_1 m'_2 m'_3 n'} = \langle D(m'_1, m'_2, m'_3, n) | \partial_t | D(m_1, m_2, m_3, n) \rangle.$$

We note that $\partial_\theta B = D$ and $\partial_\theta D = B$, and we have

$$\partial_t |D(m, n)\rangle = \dot{\theta} \partial_\theta |D(m_1, m_2, m_3, n)\rangle,$$

where $\partial_\theta |D(m_1, m_2, m_3, n)\rangle$ includes six terms $|e(m_1 \mp 1, m_2, m_3, n \pm 1)\rangle$, $|e(m_1, m_2 \mp 1, m_3, n \pm 1)\rangle$, and $|e(m_1, m_2, m_3 \mp 1, n \pm 1)\rangle$, which are all eigenstates with non-zero eigenvalues. This implies the exact result

$$\langle D(m'_1, m'_2, m'_3, n) | \partial_t | D(m_1, m_2, m_3, n) \rangle = 0,$$

showing that there is no mixing among the states with zero eigenvalue during the adiabatic evolution.

APPENDIX B: CALCULATION OF THE TRANSMISSION EFFICIENCY

Starting from the Hamiltonian (3), we now calculate the transmission efficiency in the coordinate representation. We recall the relation between the operators a^\dagger , a , B^\dagger , and B and the corresponding coordinate operators and moment operators x_1 , x_2 , p_1 , and p_2 , i.e.,

$$x_1 = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a), \quad (\text{B1})$$

$$p_1 = i\sqrt{\frac{m\hbar\omega}{2}}(a^\dagger - a), \quad (\text{B2})$$

$$x_2 = \sqrt{\frac{\hbar}{2m\omega}}(B^\dagger + B), \quad (\text{B3})$$

$$p_2 = i\sqrt{\frac{m\hbar\omega}{2}}(B^\dagger - B), \quad (\text{B4})$$

where m is the mass of the oscillator. We have here assumed the masses of the two oscillators to be the same. Therefore, we have the Hamiltonian

$$H_{L-M} = \frac{1}{2m}(p_1^2 + p_2^2) + \frac{1}{2}(Ax_1^2 + Bx_2^2 + Cx_1x_2) \quad (\text{B5})$$

for two coupled harmonic oscillators. Here, we neglect the zero-point energy and

$$A = m\omega^2, \quad B = m\omega_0^2, \quad C = 4Gm\sqrt{\omega\omega_0}. \quad (\text{B6})$$

Let $\omega \neq \omega_0$ and define the canonical coordinates

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos \frac{\alpha}{2} & -\sin \frac{\alpha}{2} \\ \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad (\text{B7})$$

where

$$\tan \alpha = \frac{C}{B-A}. \quad (\text{B8})$$

Then we diagonalize H_{L-M} with two decoupled harmonic oscillators

$$H_{L-M} = \frac{1}{2m}(p_{y_1}^2 + p_{y_2}^2) + \frac{1}{2}(K_1 y_1^2 + K_2 y_2^2), \quad (\text{B9})$$

with

$$K_1 = \frac{A+B-K}{2}, \quad K_2 = \frac{A+B+K}{2}, \quad (\text{B10})$$

and

$$K = (B-A) \sqrt{1 + \frac{C^2}{(B-A)^2}}. \quad (\text{B11})$$

Therefore, the eigenenergy of the above system is

$$E = \hbar\Omega_1 \left(n_1 + \frac{1}{2} \right) + \hbar\Omega_2 \left(n_2 + \frac{1}{2} \right), \quad (\text{B12})$$

for $n_1, n_2 = 0, 1, 2, \dots$, and the corresponding eigenstate $|n_1\rangle_{c_1} \otimes |n_2\rangle_{c_2}$ is expressed as

$$\begin{aligned} \psi_{n_1 n_2}(y_1, y_2) &= \mathcal{N}_{n_1}^{(b_1)} \mathcal{N}_{n_2}^{(b_2)} \exp \left\{ -\frac{1}{2} b_1^2 y_1^2 - \frac{1}{2} b_2^2 y_2^2 \right\} \\ &\quad \times H_{n_1}(b_1 y_1) H_{n_2}(b_2 y_2), \end{aligned}$$

in terms of the n th-order Hermite polynomial $H_n(\xi)$ with

$$\begin{aligned} \mathcal{N}_{n_1}^{(b_1)} &= \left[\frac{b_1}{\sqrt{\pi 2^{n_1} n_1!}} \right]^{1/2}, \quad \mathcal{N}_{n_2}^{(b_2)} = \left[\frac{b_2}{\sqrt{\pi 2^{n_2} n_2!}} \right]^{1/2}, \\ b_1 &= \left(\frac{mK_1}{\hbar} \right)^{1/4}, \quad b_2 = \left(\frac{mK_2}{\hbar} \right)^{1/4}. \end{aligned} \quad (\text{B13})$$

When there is no coupling between the medium and quantum light, the Hamiltonian of the medium and quantum light is

$$H_{\text{uncoupled}} = \frac{1}{2m}(p_1^2 + p_2^2) + \frac{1}{2}(Ax_1^2 + Bx_2^2). \quad (\text{B14})$$

Therefore the eigenenergies and eigenstates $|n_1\rangle_a \otimes |n_2\rangle_B$ are given by

$$E = \hbar\omega \left(n_1 + \frac{1}{2} \right) + \hbar\omega_0 \left(n_2 + \frac{1}{2} \right) \quad (\text{B15})$$

and

$$\begin{aligned} \phi_{n_1 n_2}(x_1, x_2) &= \mathcal{N}_{n_1}^{(a_1)} \mathcal{N}_{n_2}^{(a_2)} \exp \left\{ -\frac{1}{2} a_1^2 x_1^2 - \frac{1}{2} a_2^2 x_2^2 \right\} \\ &\quad \times H_{n_1}(a_1 x_1) H_{n_2}(a_2 x_2), \end{aligned}$$

respectively, for $n_1, n_2 = 0, 1, 2, \dots$, and

$$\begin{aligned} \mathcal{N}_{n_1}^{(a_1)} &= \left[\frac{a_1}{\sqrt{\pi 2^{n_1} n_1!}} \right]^{1/2}, \quad \mathcal{N}_{n_2}^{(a_2)} = \left[\frac{a_2}{\sqrt{\pi 2^{n_2} n_2!}} \right]^{1/2}, \\ a_1 &= \left(\frac{mA}{\hbar} \right)^{1/4}, \quad a_2 = \left(\frac{mB}{\hbar} \right)^{1/4}. \end{aligned}$$

Now we can calculate the transmission efficiency of the n -photon state:

$$\begin{aligned}
 F_n &= |S_{n0}|^2 = |c_2 \langle 0| \otimes c_1 \langle n| \otimes \langle v| |v\rangle \otimes |n\rangle_a \otimes |0\rangle_B|^2 \\
 &= \left| \int \int \psi_{n0}[y_1(x_1, x_2), y_2(x_1, x_2)] \phi_{n0}[x_1, x_2] dx_1 dx_2 \right|^2 \\
 &= \left| \int \int \exp\left\{-\frac{1}{2}(a_1^2 x_1^2 + a_2^2 x_2^2 + b_1^2 y_1^2 + b_2^2 y_2^2)\right\} \right. \\
 &\quad \times H_n(a_1 x_1) H_0(a_2 x_2) H_n(b_1 y_1) H_0(b_2 y_2) \\
 &\quad \left. \times \mathcal{N}_n^{(b_1)} \mathcal{N}_0^{(b_2)} \mathcal{N}_n^{(a_1)} \mathcal{N}_0^{(a_2)} dx_1 dx_2 \right|^2. \quad (\text{B16})
 \end{aligned}$$

To explicitly calculate the integral in the above formula we can define a new pair of coordinates,

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos \frac{\beta}{2} & -\sin \frac{\beta}{2} \\ \sin \frac{\beta}{2} & \cos \frac{\beta}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad (\text{B17})$$

where

$$\tan \beta = \frac{R}{Q-P},$$

$$P = b_1^2 \cos^2 \frac{\alpha}{2} + b_2^2 \sin^2 \frac{\alpha}{2} + a_1^2,$$

$$Q = b_1^2 \sin^2 \frac{\alpha}{2} + b_2^2 \cos^2 \frac{\alpha}{2} + a_2^2,$$

$$R = 2 \left[b_2^2 \cos \frac{\alpha}{2} \sin \frac{\alpha}{2} - b_1^2 \cos \frac{\alpha}{2} \sin \frac{\alpha}{2} \right]. \quad (\text{B18})$$

Therefore, the integral

$$\begin{aligned}
 &\int \int \exp\left\{-\frac{1}{2}(a_1^2 x_1^2 + a_2^2 x_2^2 + b_1^2 y_1^2 + b_2^2 y_2^2)\right\} \\
 &\quad \times H_n(a_1 x_1) H_0(a_2 x_2) H_n(b_1 y_1) H_0(b_2 y_2) dx_1 dx_2
 \end{aligned}$$

is transformed to

$$\begin{aligned}
 &\int \int \exp\left\{-\frac{1}{2}W_1 z_1^2 - \frac{1}{2}W_2 z_2^2\right\} \\
 &\quad \times H_n(a_1 x_1) H_0(a_2 x_2) H_n(b_1 y_1) H_0(b_2 y_2) J dz_1 dz_2, \quad (\text{B19})
 \end{aligned}$$

where

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial z_1} & \frac{\partial x_1}{\partial z_2} \\ \frac{\partial x_2}{\partial z_1} & \frac{\partial x_2}{\partial z_2} \end{vmatrix} = 1$$

is the Jacobi determinant and

$$W_1 = \frac{P+Q-S}{2}, \quad W_2 = \frac{P+Q+S}{2}, \quad (\text{B20})$$

where

$$S = (Q-P) \sqrt{1 + \frac{R^2}{(Q-P)^2}}$$

and

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \cos \frac{\beta}{2} & \sin \frac{\beta}{2} \\ -\sin \frac{\beta}{2} & \cos \frac{\beta}{2} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix},$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos \frac{\alpha-\beta}{2} & -\sin \frac{\alpha-\beta}{2} \\ \sin \frac{\alpha-\beta}{2} & \cos \frac{\alpha-\beta}{2} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}.$$

We note that $H_n(b_1 y_1)$, $H_0(b_2 y_2)$, $H_n(a_1 x_1)$, and $H_0(a_2 x_2)$ can be expressed as a polynomial of z_1 and z_2 , so that the integral (B19) can be calculated with the help of the integral formula

$$\int_{-\infty}^{+\infty} t^n e^{-\rho t^2} = \frac{1}{2} [1 + (-1)^n] \rho^{-(1+n)/2} \Gamma\left[\frac{1+n}{2}\right],$$

where $n=0, 1, \dots, \rho > 0$, and $\Gamma[n]$ is the gamma function.

For the simplest case, when $n=1$, we have

$$\begin{aligned}
 F_1 &= \left| \frac{4}{\pi} \sqrt{\frac{a_1^3 a_2 b_1^3 b_2}{W_1 W_2}} \left[\frac{1}{W_1} \cos \frac{\alpha-\beta}{2} \cos \frac{\beta}{2} \right. \right. \\
 &\quad \left. \left. - \frac{1}{W_2} \sin \frac{\alpha-\beta}{2} \sin \frac{\beta}{2} \right] \right|^2. \quad (\text{B21})
 \end{aligned}$$

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