

## Efficient one-step generation of large cluster states with solid-state circuits

J. Q. You,<sup>1,2</sup> Xiang-bin Wang,<sup>2,3,4</sup> Tetsufumi Tanamoto,<sup>5</sup> and Franco Nori<sup>2,6</sup>

<sup>1</sup>Department of Physics and Surface Physics Laboratory (National Key Laboratory), Fudan University, Shanghai 200433, China

<sup>2</sup>Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan

<sup>3</sup>CREST, Japan Science and Technology Agency (JST), Kawaguchi, Saitama 332-0012, Japan

<sup>4</sup>Department of Physics, Tsinghua University, Beijing 100084, China

<sup>5</sup>Corporate R & D Center, Toshiba Corporation, Saiwai-ku, Kawasaki 212-8582, Japan

<sup>6</sup>Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, University of Michigan, Ann Arbor, Michigan 48109-1040, USA

(Received 19 September 2006; published 15 May 2007)

Highly entangled states called cluster states are a universal resource for measurement-based quantum computing (QC). Here we propose an efficient method for producing large cluster states using superconducting quantum circuits. We show that a large cluster state can be efficiently generated in just one step by turning on the interqubit coupling for a short time. Because the interqubit coupling is only switched on during the time interval for generating the cluster state, our approach is also convenient for preparing the initial state for each qubit and for implementing one-way QC via single-qubit measurements. Moreover, the cluster state is robust against parameter variations.

DOI: 10.1103/PhysRevA.75.052319

PACS number(s): 03.67.Lx, 03.65.Ud, 03.67.Mn, 85.25.-j

### I. INTRODUCTION

Quantum computing (QC) with highly entangled states, known as cluster states, takes advantage of both entanglement and measurement in a remarkable way [1–6]. In sharp contrast to conventional QC, which uses unitary one- and two-qubit logic operations, this new type of QC is performed through only single-qubit projective measurements on a cluster state. This measurement-based QC is termed “one-way” because it proceeds in an inherently time-irreversible manner. Moreover, it is universal in the sense that any quantum circuit and quantum gates can be implemented on a suitable cluster state [1].

For one-way QC, the initial cluster state should be first generated. This highly entangled state provides a universal resource for QC. Ideally, it is desirable to produce a cluster state in just one step on a scalable circuit, so as to have efficient QC. However, this is challenging. Recently, a quantum-optics experiment [7] implemented one-way QC through local nondeterministic Bell measurements. Even though the cluster state was generated in one step, its generation probability was extremely low. Moreover, it is hard to implement scalable QC with optical cluster states due to the difficulty of large-scale integration in the optical devices. Alternatively, solid-state QC with cluster states were proposed [8,9] using the Heisenberg exchange interaction between electron spins in quantum dots. In these approaches, additional rotations are performed on individual qubits in order to obtain an effective Ising-like Hamiltonian for producing the cluster state. Also, several steps, instead of the ideal one step, are required to achieve a quantum-dot cluster state.

Here we propose an efficient method for one-step generation of large cluster states using superconducting quantum circuits. These circuits are based on Josephson junctions (JJs) and are regarded as promising candidates of solid-state qubits (see, e.g., [10]). We consider two scalable quantum

circuits in which an *inductive coupling* is employed to couple nearest-neighbor charge qubits in one circuit [Fig. 1(a)] and arbitrarily separated charge qubits in the other circuit [Fig. 1(b)]. Both circuits give rise to an Ising-like Hamiltonian, but the interqubit interactions are nearest-neighbor and long-range, respectively.

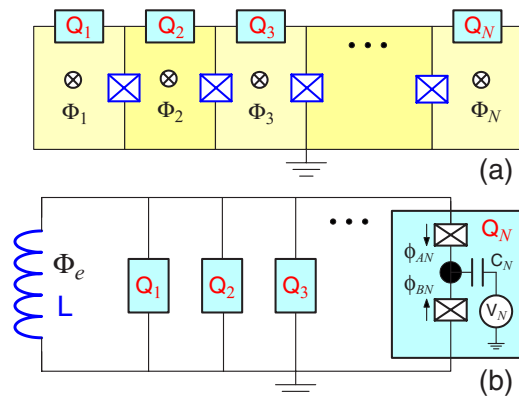


FIG. 1. (Color online) Schematic diagrams of two arrays of superconducting charge qubits (denoted as rectangular boxes). (a) Qubit array ( $Q_1, Q_2, \dots, Q_N$ ) with every nearest-neighbor charge qubits coupled by a large Josephson junction (JJ), shown as a crossed rectangle, that acts as an effective inductance  $L_j$ . The interqubit coupling is induced by the externally applied magnetic flux  $\Phi_i$  in each small superconducting loop connecting qubit  $Q_i$  and one or two large JJs. For simplicity, all large JJs are assumed to have the same Josephson coupling energy  $E_{J_0}$ . (b) Qubit array with long-range interactions, where all charge qubits are connected in parallel to a common inductance  $L$ . The interqubit coupling is induced by the external magnetic flux  $\Phi_e$  through the inductance  $L$ . As a typical example, we explicitly show the schematic diagram of charge qubit  $Q_N$ , where a superconducting island (denoted as a solid circle) is connected to two JJs (with phase drops  $\phi_{AN}$  and  $\phi_{BN}$ ) and biased by a voltage  $V_N$  through the gate capacitor  $C_N$ .

Because the decoherence time for quantum states is limited, in order to have efficient one-way QC, it is essential to generate a cluster state in a deterministic and fast way, so that only a short time is consumed. Also, it should be convenient to prepare the initial state for each qubit and to perform local single-qubit measurements. Furthermore, for a solid-state system, the produced cluster state should be robust against unavoidable parameter variations. Our proposed JJ circuits meet these requirements. First, the cluster state is generated in just one step by turning on, for a short time, the interqubit coupling. Also, this cluster-state generation is deterministic, in sharp contrast to the extremely low probability of generating nondeterministic optical cluster states [7]. Second, because the interqubit coupling is turned on only when generating the cluster state, the preparation of the initial state for each qubit can be easily achieved, before generating the cluster state, via local single-qubit operations. Moreover, due to the absence of interqubit coupling after generating the cluster state, local projective measurements used for one-way QC can also be conveniently performed via single qubits. Third, the cluster state is robust against unavoidable parameter variations because its decoherence time is not sensitive to such parameter variations. Furthermore, all qubits work at the optimal point (namely, the degeneracy point) where the quantum state has a longer decoherence time.

## II. QUBIT ARRAY WITH NEAREST-NEIGHBOR INTERACTIONS

Consider a chain of qubits described by the Hamiltonian

$$H = \hbar g(t) \sum_{i,j} \Gamma(i-j) \frac{1 \pm \sigma_x^{(i)}}{2} \frac{1 \pm \sigma_x^{(j)}}{2}, \quad (1)$$

where  $\Gamma(i-j)$  specifies the interaction range of the qubits. Similar to the quantum Ising model used for producing cluster states [1,2], this Hamiltonian is also Ising-like, but its anisotropic direction and the “magnetic” field are along the  $x$  direction, instead of the usual  $z$  direction. Below we first focus on a chain of superconducting charge qubits with nearest-neighbor interactions.

As shown in [11], two charge qubits can be coupled by a shared inductance. Because a JJ can behave like an effective inductance, one can also replace the common inductance with a large JJ [12–14]. Figure 1(a) shows an array of charge qubits with a large JJ connected to every pair of nearest-neighbor qubits. This large JJ directly couples the nearest-neighbor charge qubits. Also, the non-nearest-neighbor qubits can be coupled via the large JJs, but the interactions are negligibly small. Here we use the charge states  $|0\rangle$  and  $|1\rangle$  as the basis states, which correspond to zero and one extra Cooper pairs in the superconducting island of each qubit. The Hamiltonian of the charge-qubit array can be reduced to

$$H_A = \sum_{i=1}^N [H_i + \Lambda_{i,i+1} \sigma_x^{(i)} \sigma_x^{(i+1)}], \quad (2)$$

with  $\Lambda_{N,N+1}=0$ . The Hamiltonian  $H_i$  of the  $i$ th charge qubit is

$$H_i = \varepsilon_i(V_i) \sigma_z^{(i)} - \bar{E}_{J_i} \sigma_x^{(i)}, \quad (3)$$

where

$$\varepsilon_i(V_i) = \frac{1}{2} E_{ci} \left( \frac{C_i V_i}{e} - 1 \right),$$

$$\bar{E}_{J_i} = E_{J_i} \cos \left( \frac{\pi \Phi_i}{\Phi_0} \right). \quad (4)$$

We assume that the charge qubit works in the charging regime with  $E_{ci} \gg E_{J_i}$ . Here  $E_{ci}$  is the charging energy of the superconducting island in the  $i$ th qubit and  $E_{J_i}$  is the Josephson coupling energy of the two identical JJs coupled to the island;  $V_i$  is the gate voltage applied to the qubit and  $\Phi_i$  is the externally applied magnetic flux through a small loop connecting the  $i$ th qubit and one or two large JJs [see Fig. 1(a)]. The flux-dependent interqubit coupling is given by [12]

$$\Lambda_{i,i+1} = L_J \left( \frac{\pi^2 E_{J_i} E_{J_{i+1}}}{\Phi_0^2} \right) \sin \left( \frac{\pi \Phi_i}{\Phi_0} \right) \sin \left( \frac{\pi \Phi_{i+1}}{\Phi_0} \right), \quad (5)$$

where the large JJ acts as an effective inductance  $L_J = \Phi_0 / 2\pi I_0$ , with  $I_0 = 2\pi E_{J_0} / \Phi_0$  being its critical current.

When each charge qubit is shifted to work at the degeneracy point  $C_i V_i / e = 1$ , the Hamiltonian of the charge-qubit array becomes

$$H_A = \sum_{i=1}^N [-\bar{E}_{J_i} \sigma_x^{(i)} + \Lambda_{i,i+1} \sigma_x^{(i)} \sigma_x^{(i+1)}]. \quad (6)$$

Let

$$\frac{1}{2} \bar{E}_{J_i} = \Lambda_{i,i+1} \equiv \frac{1}{4} \hbar g, \quad i = 2, 3, \dots, N-1,$$

$$\bar{E}_{J_1} = \Lambda_{12} = \bar{E}_{J_N} \equiv \frac{1}{4} \hbar g. \quad (7)$$

These conditions can be readily satisfied by choosing suitable  $E_{J_i}$  and  $\Phi_i$  because  $\bar{E}_{J_i}$  decreases from  $E_{J_i}$  to zero and  $\Lambda_{i,i+1}$  increases from zero to  $(\pi/\Phi_0)^2 L_J E_{J_i} E_{J_{i+1}}$  for  $0 < \Phi_i/\Phi_0 < \frac{1}{2}$ . The reduced Hamiltonian can be written as

$$H_A = \hbar g \sum_{i=1}^{N-1} \frac{1 - \sigma_x^{(i)}}{2} \frac{1 - \sigma_x^{(i+1)}}{2}, \quad (8)$$

which has the form of Eq. (1) with nearest-neighbor interactions  $\Gamma(i-j) = \delta_{i+1,j}$ .

Initially, the external flux is not applied, so that no interqubit coupling is induced and one can manipulate each charge qubit separately. We first prepare all qubits in the state  $|0\rangle_i$ . This initial state can be produced by applying a gate voltage to the left ( $C_i V_i / e \sim 0$ ) of the degeneracy point and it corresponds to the ground state of the system. Then, shift the gate voltage  $V_i$  fast to the degeneracy point ( $C_i V_i / e = 1$ ) and turn on the externally applied magnetic flux  $\Phi_e$  to trigger the interqubit coupling for a period of time  $t$ . The unitary transformation generated by the Hamiltonian (8) is

$$U(t) = \exp \left[ -igt \sum_{i=1}^{N-1} \frac{1 - \sigma_x^{(i)}}{2} \frac{1 - \sigma_x^{(i+1)}}{2} \right]. \quad (9)$$

The initial state of each charge qubit can be written as  $|0\rangle_i = (|-\rangle_i + |+\rangle_i) / \sqrt{2}$ , where  $|\pm\rangle_i = (|0\rangle_i \mp |1\rangle_i) / \sqrt{2}$  are eigenstates of  $H_i = -\bar{E}_{Ji} \sigma_x^{(i)}$  with eigenvalues  $\pm \bar{E}_{Ji}$ . For the values  $gt = (2n+1)\pi$ , where  $n$  is an integer, the generated state of the charge-qubit array is a highly entangled cluster state:

$$|\phi_N\rangle = \frac{1}{2^{N/2}} \otimes_{i=1}^N (|-\rangle_i + |+\rangle_i \sigma_x^{(i+1)}), \quad (10)$$

with the convention  $\sigma_x^{(N+1)} \equiv 1$ .

The generation of cluster states in an array of *capacitively* coupled charge qubits was proposed in [15]. Because of the limitation due to the capacitive interqubit interaction, the approach in [15] is valid when each qubit works far away from the degeneracy point. This is not desirable because the decoherence time of a charge qubit becomes much shorter away from the degeneracy point. Furthermore, because the capacitive interqubit coupling is fixed [16], it is difficult to prepare the initial state for each qubit. However, the generation of cluster states proposed here employs an array of *inductively* coupled charge qubits. This proposal has obvious advantages: (1) Each charge qubit works *at* the degeneracy point when generating a cluster state, where the qubit has a *longer* decoherence time; (2) the initial state of all qubits can be easily prepared by turning off the external magnetic flux and shifting the gate voltage away from the degeneracy point; (3) when the initial state is prepared, the cluster state can be readily generated by applying the external flux  $\Phi_i$  for a period of time; and (4) after generating the cluster state, no external magnetic flux is applied and the interqubit coupling is switched off. This becomes convenient for implementing one-way QC via local single-qubit measurements on the generated cluster state.

### III. QUBIT ARRAY WITH LONG-RANGE INTERACTIONS

When multiple charge qubits are connected to a commonly shared inductance [see Fig. 1(b)], not only nearest-neighbor but also distant qubits can be coupled by this common inductance [11]. Because the common inductance for coupling the charge qubits has a large value ( $L \sim 10$  nH) [11], if the circuit is not too large, the inductances of the circuit, except  $L$ , can be neglected. The reduced Hamiltonian of the system is given by

$$H_B = \sum_{i=1}^N H_i - \sum_{i,j(j>i)}^N \Lambda_{ij} \sigma_x^{(i)} \sigma_x^{(j)}. \quad (11)$$

Here  $\bar{E}_{Ji}$  in the single-qubit Hamiltonian  $H_i$  becomes

$$\bar{E}_{Ji} = E_{Ji} \cos\left(\frac{\pi\Phi_e}{\Phi_0}\right), \quad (12)$$

with  $\Phi_e$  being the externally applied magnetic flux through the common inductance  $L$ . The interqubit coupling is

$$\Lambda_{i,j} = L \left( \frac{\pi^2 E_{Ji} E_{Jj}}{\Phi_0^2} \right) \sin^2 \left( \frac{\pi\Phi_e}{\Phi_0} \right). \quad (13)$$

Let

$$\frac{\bar{E}_{Ji}}{N-1} = \Lambda_{ij} \equiv \frac{1}{4} \hbar g, \quad (14)$$

for  $1 \leq i, j \leq N$ , and  $j > i$ . This condition can be satisfied using  $N$  identical charge qubits and a suitable  $\Phi_e$ . While fulfilling this condition and simultaneously having each charge qubit work *at* the degeneracy point, the Hamiltonian becomes

$$H_B = -\hbar g \sum_{i,j(j>i)}^N \frac{1 + \sigma_x^{(i)}}{2} \frac{1 + \sigma_x^{(j)}}{2}, \quad (15)$$

which corresponds to Eq. (1) with long-range interactions.

The initial state of each charge qubit,  $|0\rangle_i = (|-\rangle_i + |+\rangle_i) / \sqrt{2}$ , is also prepared by both turning off the external flux  $\Phi_e$  and applying a gate voltage to the left of the degeneracy point. Furthermore, we shift the gate voltage fast to the degeneracy point and apply the flux  $\Phi_e$  for a period of time  $t$ . The unitary transformation given by the Hamiltonian (15) is

$$U(t) = \exp \left[ igt \sum_{i,j(j>i)}^N \frac{1 + \sigma_x^{(i)}}{2} \frac{1 + \sigma_x^{(j)}}{2} \right]. \quad (16)$$

At  $gt = (2n+1)\pi$ , the generated cluster state is

$$|\psi_N\rangle = \frac{1}{2^{N/2}} \otimes_{i=1}^N \left( |-\rangle_i (-1)^{N-i} \prod_{j=i+1}^N \sigma_x^{(j)} + |+\rangle_i \right), \quad (17)$$

which is also a highly entangled state. In Eq. (10), the operator  $\sigma_x^{(i+1)}$  acts on the states  $|\pm\rangle$  of the  $(i+1)$ th qubit. However, for the cluster state  $|\psi_N\rangle$ , the operator  $\sigma_x^{(j)}$  acts on the states  $|\pm\rangle$  of the qubits  $j = i+1, \dots, N$ , with  $i = 1, 2, \dots, N-1$ ; this is due to the long-range nature of the interqubit coupling in Hamiltonian (15).

### IV. PARAMETER VARIATIONS AND ROBUSTNESS OF CLUSTER STATES

As in other solid-state systems, parameter variations unavoidably occur when fabricating JJ circuits. When the parameters  $E_{Ji}$  vary in the charge-qubit array in Fig. 1(a), because  $\bar{E}_{Ji} = E_{Ji} \cos(\pi\Phi_i/\Phi_0)$ , the condition

$$\bar{E}_{J1} = \bar{E}_{JN} = \frac{1}{2} \bar{E}_{Ji} = \frac{1}{4} \hbar g, \quad (18)$$

with  $i = 2, 3, \dots, N-1$ , can be satisfied by adjusting the local magnetic fluxes  $\Phi_i$ . However, if  $L_j$  and  $E_{Ji}$  vary, the condition

$$\Lambda_{i,i+1} = \frac{1}{4} \hbar g \quad (19)$$

cannot be satisfied. In order to fulfill this condition, one can connect a current source in parallel to each large JJ and bias

the JJ with a current  $I_{bi} < I_0$ . Now the interqubit coupling becomes

$$\Lambda_{i,i+1} = L_{Ji} \left( \frac{\pi^2 E_{Ji} E_{J,i+1}}{\Phi_0^2} \right) \sin \left( \frac{\pi \Phi_i}{\Phi_0} + \frac{1}{2} \gamma_i \right) \times \sin \left( \frac{\pi \Phi_{i+1}}{\Phi_0} - \frac{1}{2} \gamma_i \right), \quad (20)$$

where the effective inductance for each large JJ is

$$L_{Ji} = \frac{\Phi_0}{2\pi I_0 \cos \gamma_i}, \quad (21)$$

with

$$\gamma_i = \sin^{-1} \left( \frac{I_{bi}}{I_0} \right). \quad (22)$$

Here the condition (19) can be readily satisfied by changing the bias current  $I_{bi}$ . As for the charge-qubit array in Fig. 1(b) and the capacitively coupled charge qubits [15], the conditions for obtaining an Ising-like Hamiltonian cannot be fully satisfied for varying qubit parameters. Therefore the charge-qubit circuit in Fig. 1(a) should be advantageous for suppressing the effects of unavoidable parameter variations.

Below we further show the robustness of the cluster states against parameter variations. According to the Fermi golden rule, the relaxation rate of the  $i$ th charge qubit is

$$\Gamma_1^{(i)} \equiv \frac{1}{T_1^{(i)}} = \frac{1}{2} A_i S_i(\Omega), \quad (23)$$

where

$$A_i = \frac{\bar{E}_{Ji}^2}{\varepsilon_i^2 + \bar{E}_{Ji}^2}, \quad (24)$$

and  $S_i(\omega)$  is the power spectrum of the charge noise dominant in the charge qubit. For a typical Gaussian noise [17], the dephasing factor is

$$\eta_i(\tau) = B_i \int d\omega S_i(\omega) \frac{\sin^2(\omega\tau/2)}{2\pi(\omega/2)^2}, \quad (25)$$

with

$$B_i = \frac{\varepsilon_i^2}{\varepsilon_i^2 + \bar{E}_{Ji}^2}. \quad (26)$$

The dephasing rate  $\Gamma_\varphi^{(i)} \equiv 1/T_\varphi^{(i)}$  is defined by  $\eta_i(T_\varphi^{(i)}) = 1$ . Following the Bloch-Redfield theory (see, e.g., [18]), the decoherence rate  $\Gamma_2^{(i)} \equiv 1/T_2^{(i)}$  is

$$\Gamma_2^{(i)} = \frac{1}{2} \Gamma_1^{(i)} + \Gamma_\varphi^{(i)}. \quad (27)$$

Because all interqubit couplings are switched off after generating a cluster state, the decoherence time  $T_2$  of the cluster state is given by

$$\frac{1}{T_2} = \sum_i \frac{1}{T_2^{(i)}}. \quad (28)$$

Here all charge qubits work at the degeneracy point  $\varepsilon_i \approx 0$ , thus

$$A_i \approx 1 - \left( \frac{\varepsilon_i}{\bar{E}_{Ji}} \right)^2, \quad (29)$$

$$B_i \approx \left( \frac{\varepsilon_i}{\bar{E}_{Ji}} \right)^2.$$

Obviously,  $A_i$  and  $B_i$  are weakly affected by the variations of the parameters  $E_{Ji}$ . This indicates that the decoherence time  $T_2$  of the cluster state is not sensitive to the parameter variations. Therefore the cluster state is robust against the unavoidable parameter variations.

## V. DISCUSSION AND CONCLUSION

Each Josephson coupling energy for a charge qubit is typically  $E_J/h \sim 10$  GHz (see, e.g., [16]), which corresponds to a switching time  $\tau_1 \sim 0.1$  ns for the single-qubit operation. For the charge-qubit array in Fig. 1(a), the interqubit coupling is

$$\Lambda \sim L_J \left( \frac{\pi E_J}{\Phi_0} \right)^2,$$

where

$$L_J = \frac{\Phi_0}{2\pi I_0} = \frac{1}{E_{J0}} \left( \frac{\Phi_0}{2\pi} \right)^2.$$

Choosing, e.g.,  $E_{J0} = 5E_J$ , one obtains  $\Lambda/h \sim 0.5$  GHz. Because  $\hbar g/4 = \Lambda$ , the shortest time to generate the cluster state is  $t_s = \pi/g \sim 0.25$  ns, comparable to the switching time  $\tau_1$  of the single-qubit operation. For the array of charge qubits coupled by a common inductance  $L$  [see Fig. 1(b)], the interqubit coupling is

$$\Lambda \sim L \left( \frac{\pi E_J}{\Phi_0} \right)^2.$$

Using  $L = 10$  nH, one has  $\Lambda/h \sim 1.1$  GHz. The corresponding shortest time for generating the cluster state is  $t_s \sim 0.11$  ns  $\approx \tau_1$ . Let  $T_2$  be the decoherence time of a qubit. The decoherence time of  $N$  weakly coupled qubits can be estimated as  $T_2^{(N)} \sim T_2/N$ . For a charge qubit with  $T_2 \sim 0.5$   $\mu$ s at the degeneracy point [19], considering an array with  $N = 100$  charge qubits, one obtains  $T_2^{(N)} \sim 5$  ns. This decoherence time is longer than the shortest time  $t_s$  for generating the cluster state. Here the common inductance is chosen to be large (e.g.,  $L = 10$  nH), but the interqubit couplings are still weak, and those couplings are turned off after generating a cluster state. Thus the effects of  $L$  on the decoherence of the cluster state are small. Moreover, these decoherence effects can be further reduced when  $L$  is replaced by a large JJ acting as an effective inductance.

For the usual quantum Ising model, where its anisotropic direction and the ‘‘magnetic’’ field are both along the  $z$  direc-

tion, the basis states used for representing a cluster state are the eigenstates  $|0\rangle_i$  and  $|1\rangle_i$  of  $\sigma_z^{(i)}$ . To implement one-way QC, single-qubit projective measurements are performed [1] on the basis states  $|\pm\rangle_i$ , namely, the eigenstates of  $\sigma_x^{(i)}$ . In our proposed charge-qubit arrays the reduced Hamiltonian is also Ising-like, but its anisotropic direction and the “magnetic” field are along the  $x$  direction, instead of the  $z$  direction [1]. Now the cluster state is represented using basis states  $|\pm\rangle_i$ , instead of  $|0\rangle_i$  and  $|1\rangle_i$ . Correspondingly, the single-qubit projective measurements are performed on the basis states  $|0\rangle_i$  and  $|1\rangle_i$ . In charge qubits, these states correspond to zero and one extra Cooper pairs in the superconducting island of each qubit. The local single-qubit projective measurements on these basis states can be implemented using, e.g., either a probe junction [16] connected to the superconducting island or a single-electron transistor [20] coupled to the charge qubit. Because the single-electron transistor has high efficiency for reading out the quantum state, it is more advantageous to use it for performing local single-qubit projective measurements. Also, because such a transistor is coupled to each charge qubit via a small capacitance, it only produces weak

backaction on the qubit state in the absence of quantum measurement.

In conclusion, we propose an efficient method for producing large cluster states using superconducting circuits. We consider two charge-qubit arrays where either nearest-neighbor or arbitrarily separated qubits are inductively coupled. The initial cluster state can be efficiently generated in just one step by turning on the interqubit coupling for a short time. Also, our approach is convenient for preparing the initial state of the system and for implementing one-way QC via single-qubit measurements on the cluster state because the interqubit coupling is switched off in both cases.

#### ACKNOWLEDGMENTS

We would like to thank L. F. Wei for discussions. This work was supported in part by the NSA, LPS, ARO, and NSF. J.Q.Y. was supported by the NFRPC Grant No. 2006CB921205 and the National Natural Science Foundation of China Grant Nos. 10474013, 10534060, and 10625416.

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