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Nonlinear signal mixing in a ratchet device

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Abstract. – The nonlinear signal mixing of two, generally incommensurate, rectangular driving forces is used to control overdamped transport in Brownian ratchet devices. The interplay between the relative phase and the frequency ratio of the two driving forces is sufficient to generate an intriguing transport action that can be put to work to optimize shuttling and separation of particles in a variety of physical and technological applications. *Analytic* results for a striking multiple current reversal behavior are obtained for doubly rocked and rocked-pulsating Brownian ratchets. This tunable signal mixing can readily be implemented and exhibits an even richer behavior than those realized by the hard-to-implement, modifiable ratchet profiles.

Ratchets are nonlinear devices that, due to their intrinsic asymmetry, are capable of rectifying an external symmetric signal [1]. The simplest ratchet model is a Brownian particle diffusing in a periodic, asymmetric potential in one dimension. The input signal can be either deterministic (*e.g.* ac drive) or stochastic and time-correlated [2]; in particular, an ac signal can be injected so as to tilt periodically the ratchet potential (*rocked* ratchet [3]) or to modulate its amplitude with time (*pulsated* ratchet [4]).

Here, we study the case of a ratchet subjected simultaneously to two ac signals with periods $T_1 = 2\pi/\Omega_1$ and $T_2 = 2\pi/\Omega_2$. We consider two distinct cases: a) the two input signals are both additive and model a doubly rocked ratchet; b) one signal ac drives the ratchet, while the other one multiplicatively modulates its amplitude (rocked-pulsated ratchet). We stress that experimental realizations of both cases are relatively straightforward to implement in the laboratory (mostly affordable variations of experimental set-ups widely reviewed in the literature [1]). As an example of case a), we mention transport of magnetic flux quanta (vortices) in superconducting devices [5], whereas some molecular motor experiments [1] fall into category b). Asymmetric SQUIDS [6, 7] and Josephson junctions arrays [8] allow simple implementations of both doubly rocked and rocked-pulsated ratchets, as such devices can be conveniently

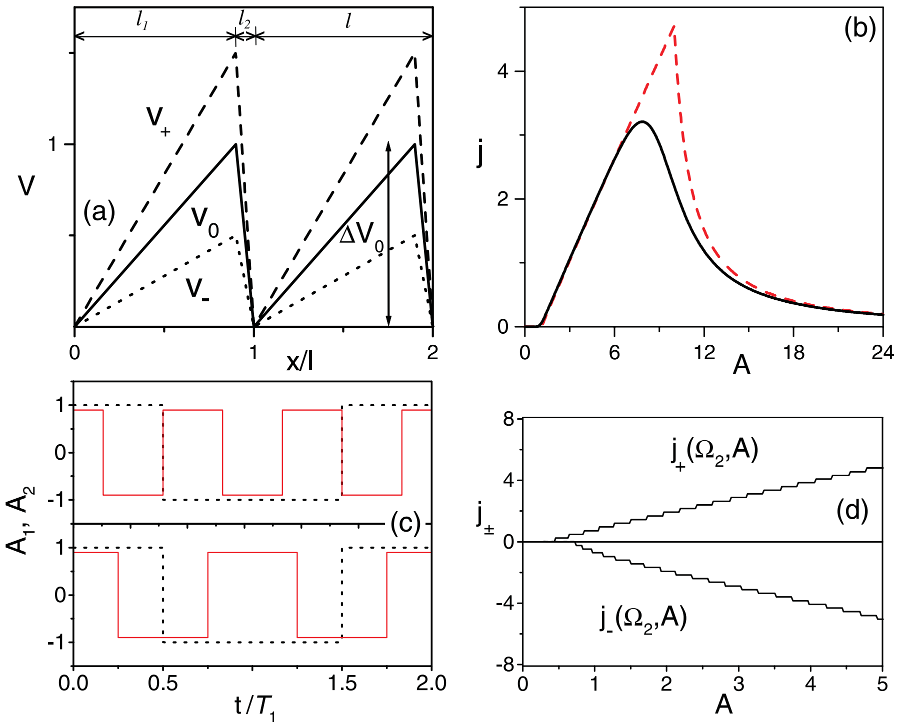

Fig. 1 – (a) Ratchet potentials. High- and low-barrier configurations $V_\pm(x)$ of the modulated potential $V(x,t)$, *i.e.* $V_\pm(x) = V_0(x)(1 \pm A_2)$ (dotted curves) with $A_2 = 0.5$. Reference ratchet potential (solid curve): $V_0(x) = qx/l_1$ for $0 < x < l_1$; $= q - q(x - l_1)/l_2$ for $l_1 < x < l = l_1 + l_2$, with $q = 1$, $l_1 = 0.9$, and $l = 1$; the barrier height ΔV_0 coincides with q. (b) Response curve $j(A)$ of the potential $V_0(x)$ driven by a rectangular force $A_1(t)$ with $A_1 = A$ in the adiabatic limit $\Omega_1 \to 0$ at zero temperature $D = 0$ (dashed curve), and low temperature $D/\Delta V_0 = 0.05$ (solid curve). (c) Input signals $A_1(t)$ (dashed), and $A_2(t)$ (solid) with $\Omega_2 = 3\Omega_1$ (upper) and $\Omega_2 = 2\Omega_1$ (lower); also: $\phi_1 = \phi_2 = 0$, $A_1 = 1$, and $A_2 = 0.9$. (d) Net currents $j_\pm(\Omega_2, A_2)$ in the tilted rocked ratchet $V_0(x) \mp |A_1|x$ for $A_1 = A_2 \equiv A$, $\Omega_2 = 1.5$, and $D = 0$; $V_0(x)$ parameters: $q = 0.4$, $l_1 = 0.7$, and $l = 1$.

driven by independent external signals (either additive or multiplicative). Finally, a variety of tunable physical systems can be effectively controlled through the combined action of two (either independent or correlated) applied signals, like colloids in arrays of optical tweezers [9], interacting binary mixtures driven on (asymmetric) period substrates [10], ferrofluids [11], dislocation transport in crystalline solids [12,13], or electron pumping in quantum dots [14].

The key result of this letter is that, no matter how we feed two periodic signals into a ratchet device, *signal mixing* determines a rich behavior of the ratchet dynamics depending on the ratio Ω_2/Ω_1 and both signal phases and amplitudes. In particular, we prove that the rectification of a primary signal by a Brownian ratchet can be controlled more effectively by applying a secondary (additive or multiplicative) signal with tunable frequency and phase, than by tinkering with the ratchet potential parameters. The latter is often difficult to implement experimentally, while tailoring the driving can be readily accomplished. Although the complexity of chaos cannot be observed in *overdamped*, adiabatically driven ratchet systems, the control of the relative phase and the frequency ratio of the driving provides a versatile way to control particle transport.

To start, let us consider the Brownian ratchet model: an overdamped Brownian particle $x(t)$ diffusing in a piecewise linear asymmetric potential $V_0(x)$ (shown in fig. 1a). Two rectangular input signals, $A_i(t) = A_i \,\mathrm{sgn}[\cos(\Omega_i t + \phi_i)]$ with $i = 1, 2$, $A_i \geq 0$, and sgn[...] denoting the sign of its argument [...], act on the particle according to the Langevin equation

$$\dot{x} = -V'(x,t) + A_a(t) + \xi(t), \tag{1}$$

where $\xi(t)$ is a stationary Gaussian white noise with $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(0) \rangle = 2D\delta(t)$, and

$$V(x,t) = V_0(x)\big[1 + A_m(t)\big]. \tag{2}$$

Equation (1) allows two distinct ways of coupling an additional control signal $A_2(t)$ to a rocked ratchet driven by $A_1(t)$: a) *doubly rocked ratchet*: $A_m(t) = 0$, $A_a(t) = A_1(t) + A_2(t)$; b) *rocked-pulsated ratchet*: $A_a(t) = A_1(t)$, $A_m(t) = A_2(t)$ with $A_2 \leq 1$. In our analytical discussion we assume that intra-well relaxation occurs on a much shorter time scale than either both periods T_1 and T_2 (fully adiabatic), or one period, T_1 or T_2 (partially adiabatic). Moreover, without loss of generality, adopting a piecewise linear substrate potential $V_0(x)$, as in fig. 1a, greatly simplifies the presentation below.

Doubly rocked ratchet. The advantage of taking the *fully adiabatic* limit (Ω_1, $\Omega_2 \to 0$) is that the output $j(\Omega_1, \Omega_2, A_1, A_2)$ of a doubly rocked ratchet is expressible analytically (see eq. (4) in ref. [3]) in terms of the current $j(A)$ of the well-studied one-frequency rocked ratchet [3], corresponding to setting $A_1 = A$, $A_2 = 0$ with $\Omega_1 \to 0$ (fig. 1b). Note that here $j(A)$ is a symmetric function of A, $j(A) = j(-A) = A[\mu(A) - \mu(-A)]/2$, where $\mu(A)$ is the nonlinear mobility of an overdamped particle running down the tilted ratchet potential $V_0(x) - Ax$. By inspecting fig. 1c, one concludes that the overall ratchet current $j(\Omega_1, \Omega_2, A_1, A_2)$ results from the interplay of the two usual one-frequency currents $j(A_1 + A_2)$ and $j(A_1 - A_2)$ driven by the ac amplitudes $A_1 + A_2$ and $A_1 - A_2$, respectively, *i.e.* for any positive integers m, n

$$j\left(\Omega_1, \Omega_2 = \Omega_1 \frac{2m-1}{2n-1}, A_1, A_2\right) = j_{\mathrm{avg}}(A_1, A_2) - \frac{(-1)^{m+n}\; p(\Delta_{n,m})}{(2m-1)(2n-1)}\; \Delta j(A_1, A_2), \tag{3}$$

while the rectified current assumes the baseline value $j_{\mathrm{avg}}(A_1, A_2) = \frac{1}{2}[j(A_1 - A_2) + j(A_1 + A_2)]$ in all other cases. Therein, $\Delta_{n,m} \equiv (2n-1)\phi_2 - (2m-1)\phi_1 \bmod 2\pi$, and

$$\Delta j(A_1, A_2) = \frac{1}{2}\big[j(A_1 - A_2) - j(A_1 + A_2)\big]. \tag{4}$$

The ϕ_1, ϕ_2 modulation is fully described by the multiplicative phase factor $p(\phi) = |\pi - \phi|/\pi - 0.5$.

Let us state a few important remarks: 1) The doubly-rocked-ratchet current (in the fully adiabatic limit) is insensitive to Ω_1, Ω_2, except at "integer-valued odd harmonics", *i.e.* $\Omega_2/\Omega_1 = (2m-1)/(2n-1)$. Its intensity coincides with the "baseline" value $j_{\mathrm{avg}}(A_1, A_2)$; spikes with decreasing amplitude $\Delta j(A_1, A_2)/(2m-1)(2n-1)$ show up at large, integer-valued odd harmonics; 2) The sign of the spike factor $\Delta j(A_1, A_2)$ is sensitive to the signal amplitudes A_1, A_2. For instance, if we choose A_1, A_2 so that $A_1 + A_2$ and $|A_1 - A_2|$ fall onto the rising (decaying) branch of $j(A)$ in fig. 1b, then $\Delta j(A_1, A_2)$ is negative (positive); 3) The current spikes at $\Omega_2/\Omega_1 = (2m-1)/(2n-1)$ depend on the initial value of ϕ_2 and ϕ_1, and for a fixed ϕ_1, their amplitude oscillates proportional to the modulation factor $p(\Delta_{n,m})$. All these properties are illustrated in fig. 2, where results from numerical simulation are displayed *vs.* the angular frequency Ω_2 at fixed Ω_1. We remark that the overall sign of our

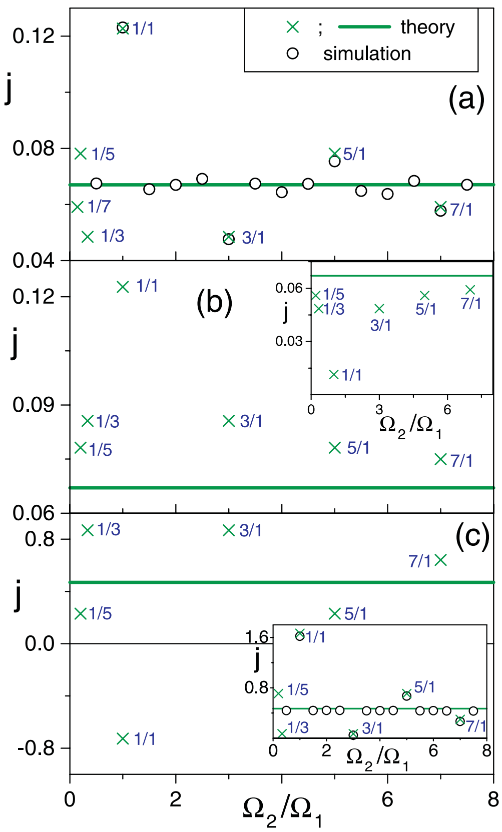

Fig. 2 – Rectified Brownian ratchet current driven by two rectangular signals $A_1(t)$, $A_2(t)$ with fixed amplitudes: (a), (b) doubly rocked ratchet, (c) rocked-pulsated ratchet. (a) Numerical simulations for $\phi_1 = \phi_2 = \pi$ and $\Omega_1 = 0.01$ (open circles) and fully adiabatic approximation (green line and green crosses). The baseline $j_{\mathrm{avg}}(A_1, A_2) = \frac{1}{2}[j(A_1 - A_2) + j(A_1 + A_2)]$ is indicated by the green line; the spikes at some selected "integer-valued odd harmonics" are marked with green crosses ($\times$); (b) Fully adiabatic approximation for $\phi_1 = \phi_2 = 3\pi/2$ (main panel) and $\phi_1 = 3\pi/2$, $\phi_2 = \pi/2$ (inset). In both cases, $A_1 = 3$, $A_2 = 2$, $D = 0.6$, and $V_0(x)$ as in fig. 1. (c) Numerical simulations in the fully adiabatic regime with $A_1 = 4$, $A_2 = 0.5$ and $\Omega_1 = 0.01$; noise level: $D = 0.4$. Main panel: $\phi_1 = \phi_2 = \pi$ (fully adiabatic approximation); inset: simulation (open circles) *vs.* the fully adiabatic approximation ($\times$) for $\phi_1 = \pi$ and $\phi_2 = 0$. $V_0(x)$ parameters are: $q = 2$, $l_1 = 0.9$, $l = 1$.

doubly rocked ratchet is always determined by the polarity of $V_0(x)$ (positive in fig. 1a), as $|\Delta j(A_1, A_2)| < |j_{\mathrm{avg}}(A_1, A_2)|$ for any choice of A_1, A_2.

In the *partially adiabatic* regime, where only one frequency tends to zero (say, $\Omega_1 \to 0$) multiple current inversions are possible (fig. 3); this is in clear contrast to the adiabatic, singly rocked case [3]. The underlying mechanism hinges on the step structure [3] of the one-frequency rocked-ratchet current in the nonadiabatic regime (see fig. 1d), where Ω_2 is small but finite. For instance, in the limit $\Omega_1 \to 0$ the net current of the doubly rocked ratchet (a) can be easily approximated to

$$j(\Omega_1 \ll \Omega_2, A_1, A_2) = \frac{1}{2}\left[j_+(\Omega_2, A_2) + j_-(\Omega_2, A_2)\right], \tag{5}$$

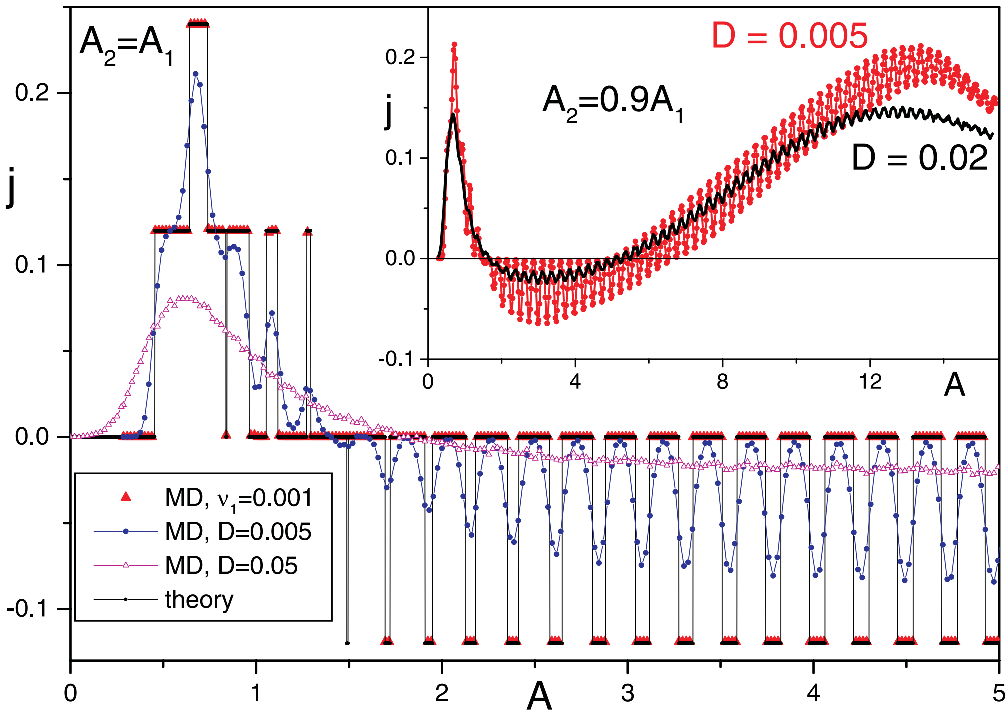

Fig. 3 – Rectified current in the doubly rocked ratchet with $A_1 = A_2 = A$, $T_1 = 10^3$, $T_1/T_2 = 240$. Simulation data for $D = 0$ (solid triangles), $5 \cdot 10^{-3}$ (dots), 0.05 (open triangles). The stepwise solid curve is our prediction for $D = 0$ (see text). Potential parameters are as in fig. 1d. Inset: as in the main panel but for $A_2 = 0.9A_1$, and $D = 0.005$ (red dots), 0.02 (black dots).

where $j_\pm(\Omega_2, A_2)$ is the average current across the static tilted ratchet potential $V_0(x) \mp |A_1|x$ driven by the rectangular signal $A_2(t)$. Note that for $\Omega_2 \gg \Omega_1$, the commensuration spikes (3) can be neglected as they decay proportional to Ω_1/Ω_2.

In order to clarify the resulting current structure (5), in fig. 3 we consider the simplified case $A_1 = A_2 \equiv A$ and $\Omega_2 \gg \Omega_1$. During one half longer period $T_1/2$, the total ac force $A_a(t)$ switches many times either between 0 and $2A$, or between 0 and $-2A$ with frequency Ω_2. The adiabatic condition requires that the higher forcing frequency Ω_2 is lower than the deterministic relaxation rate $\Omega_q = q/2l^2$, *i.e.* the Brownian particle reaches a $V_0(x)$ minimum during each half period $T_2/2$ when $A_a(t) = 0$. As a consequence, the particle moves an integer number of unit cells l during each short period T_2, thus determining the step-like structure of the currents $j_\pm$ displayed in fig. 1d. A straightforward analytical calculation of the average particle velocity to the right (left) yields $v_\pm(A) = \pm nl/T_2$ for $A_\pm^{(n+1)} < A < A_\pm^{(n)}$ and $v_\pm(A) = 0$ for $A < A_\pm^{(1)}$, with

$$A_\pm^{(n)} = \frac{1}{2}\left(\frac{(2n-1)l \pm \delta l}{2T_2} \mp f + \sqrt{\left[\frac{(2n-1)l \pm \delta l}{2T_2} \pm f\right]^2 + \frac{Q^2}{l_1 l_2} + \frac{2Q}{T_2}}\right) \tag{6}$$

with the half difference $f = Q(l_1 - l_2)/(2l_1 l_2)$ of the stopping forces and $\delta l = l_1 - l_2$. The analytical expression $[v_+(A) - v_-(A)]/2$ for the ratchet current compares very well with the simulation data displayed in fig. 3. We notice that on increasing A, the resulting ratchet current develops a negative tail made of entrained rectangular teeth of the same size. Such a negative tail persists in the presence of noise, although the teeth get gradually suppressed, thus implying, at variance with the fully adiabatic limit, a robust inverted output signal. Finally, for $A_1/A_2 \neq 1$, the ratchet characteristics grow much more complicated, though still expressible in terms of eq. (5), and may exhibit multiple current inversions (fig. 3, inset).

Rocked-pulsated ratchet. The mixing of an additive and a multiplicative signal provides a *control mechanism* of potential interest in device design. In the *fully adiabatic* limit, the ac-driven Brownian particle can be depicted as moving back and forth over two alternating ratchet potentials $V_\pm(x) = V_0(x)(1 \pm A_2)$. Both potential configurations $V_\pm(x)$ are capable of rectifying the additive driving signal $A_1(t)$; the relevant net currents $\bar{j}_\pm(A_1)$ are related to the curve $j(A)$ plotted in fig. 1b: $\bar{j}_\pm(A_1) = (1 \pm A_2)\, j[A_1/(1 \pm A_2)]$ with $D \to D/(1 \pm A_2)$. On separating the time interval $(2n-1)T_1$ into a time-uncorrelated sequence of $(2m-1)$ shorter driving cycles T_2 along $V_\pm(x)$ (we assumed $m > n$, see fig. 1c), one eventually casts the total ratchet current into the form (3) with

$$j_{\text{avg}}(A_1, A_2) = (1/2)\big[\bar{j}_-(A_1) + \bar{j}_+(A_1)\big], \qquad \Delta j(A_1, A_2) = (1/2)\big[\overline{v}_-(A_1) - \overline{v}_+(A_1)\big], \quad (7)$$

where $\overline{v}_\pm(A_1) = A_1[\mu_\pm(A_1) + \mu_\pm(-A_1)]/2$. We recall that in our notation $\mu_\pm(A)$ is the static mobility of the tilted potentials $V_\pm(x) - Ax$.

One can show that $|\Delta j(A_1, A_2)|$ may grow larger than $|j_{\text{avg}}(A_1, A_2)|$ and, therefore, a current reversal may take place for appropriate values of the model parameters, as shown by the simulation results of fig. 2c. In fact, a relatively small modulation of the ratchet potential amplitude at low temperature can easily reverse the polarity of the simply rocked ratchet $V_0(x)$. Let us consider the simplest case possible, $\Omega_1 = \Omega_2$ and $\phi_1 = \phi_2$: As the ac drive points in the "easy" direction of $V_0(x)$, namely to the right, the barrier height $V(x,t)$ is set at its maximum value $\Delta V_0(1 + A_2)$; at low temperatures, the Brownian particle cannot overcome it within a half ac-drive period $T_1/2$. In the subsequent half period the driving signal $A_1(t)$ changes sign, thus pointing against the steeper side of the $V(x,t)$ wells, while the barrier height drops to its minimum value $\Delta V_0(1 - A_2)$: Depending on the value of $\Delta V_0/D$, the particle may have a better chance to escape a potential well to the left than to the right, thus making a current reversal possible. Of course, the net current may be controlled via the modulation parameters A_2 and ϕ_2, too.

For both, the doubly rocked case a) and the rocked-pulsating case b), eq. (3) is symmetric under $m \leftrightarrow n$ exchange. This implies that, as long as the fully adiabatic approximation is tenable, each spectral spike (m, n) of the ratchet current is mirrored by a spike (n, m) of equal strength (see fig. 2). This is not true, *e.g.*, in the *partially adiabatic* regime, where the dynamics depends critically on whether Ω_1/Ω_2 or Ω_2/Ω_1 tends to zero. In the former limit additional current inversions may be observed.

The rich and intriguing effects we have investigated with this work should not be mistaken for a manifestation of harmonic mixing (HM) [15, 16], namely the mechanism where two or more linearly superimposed periodic input signals may develop a phase-dependent dc output as an effect of nonlinearity. Notice that HM may occur in a symmetric device, too. More importantly, a simple perturbation argument [15] leads to conclude that in case a) HM for a symmetric device in the fully adiabatic regime is totally suppressed by using rectangular waveforms. Moreover, rectification induced by the interplay of additive and multiplicative signals rests upon a sort of *synchronized gating* mechanism peculiar to case b). In this regard, such a mechanism can also not be considered as a HM manifestation; rather, it exhibits some similarities with the problem of polychromatic driven stochastic processes, like the control of Stochastic Resonance [17, 18].

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