Quantum computing with many superconducting qubits

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Abstract. Two of the major obstacles to achieve quantum computing (QC) are (i) scalability to many qubits and (ii) controlled connectivity between any selected qubits. Using Josephson charge qubits, here we propose an experimentally realizable method to efficiently solve these two central problems. Since any two charge qubits can be effectively coupled by an experimentally accessible inductance, the proposed QC architecture is *scalable*. In addition, we formulate an efficient and realizable QC scheme that requires *only one* (instead of two or more) two-bit operation to implement conditional gates.

Introduction

Josephson-qubit devices [1] are based on the charge and phase degrees of freedom. The charge qubit is achieved in a Cooper-pair box [2], where two dominant charge states are coupled through coherent Cooper-pair tunneling [3]. Using Cooper-pair tunneling in Josephson charge devices [4, 5] and via spectroscopic measurements for the Josephson phase device [6, 7], it has been possible to experimentally observe energy-level splitting and related properties for state superpositions. In addition, using Josephson charge devices prepared in a superposition of two charge states [2], coherent oscillations were observed. While operating at the degeneracy point, the charge-qubit states are highly coherent [8] ($Q = 2.5 \times 10^4$), with a decoherence time of $\tau \sim 500$ ns. These important experimental results indicate that the Josephson charge and phase devices are potentially useful for solid-state qubits in quantum information processing. Important open problems would now include implementing a *two-bit coupling* and then *scaling up* the architecture to many qubits. Here, we propose a new quantum-computing (QC) scheme based on scalable charge-qubit structures. We focus on the Josephson charge qubit realized in a Cooper-pair box.

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COUPLING QUBITS

The Coulomb interaction between charges on different islands of the charge qubits would seem to provide a natural way of coupling Josephson charge qubits (e.g., to connect two Cooper-pair boxes via a capacitor). Using this type of capacitive interbit coupling, a two-bit operation [9] similar to the controlled-NOT gate was derived. However, as pointed out in [1], it is difficult in this scheme to switch on and off the coupling. Also, it is hard to make the system scalable because only neighboring qubits can interact. Moreover, implementations of quantum algorithms such as the Deutsch and Bernstein-Vazirani algorithms were studied using a system of Josephson charge qubits [10], where it was proposed that the nearest-neighbor charge qubits would be coupled by tunable dc SQUIDs. In the semiconductor literature, scalability often refers to reducing the size of the device (packing more components). In QC, scalability refers to increasing the number of qubits coupled with each other.

A suggestion for a scalable coupling of Josephson charge qubits was proposed [1, 3] using oscillator modes in a LC circuit formed by an inductance and the qubit capacitors. In this proposal, the interbit coupling can be switched and any two charge qubits could be coupled. Nevertheless, there is no efficient (that is, using one two-bit operation) QC scheme for this proposal [1, 3] in order to achieve conditional gates—e.g., the controlled-phase-shift and controlled-NOT gates. In addition, the calculated interbit coupling terms [1, 3] only apply to the case when the following two conditions are met:

(i) The quantum manipulation frequencies, which are fixed experimentally, are required to be much smaller than the eigenfrequency ω_{LC} of the LC circuit. This condition *limits* the allowed number N of the qubits in the circuit because ω_{LC} scales with $1/\sqrt{N}$. In other words, the circuits in [1, 3] are not really scalable.

(ii) The phase conjugate to the total charge on the qubit capacitors fluctuates weakly.

IMPROVED AND SCALABLE COUPLING BETWEEN ANY SELECTED QUBITS

The limitations listed above do not apply to our approach. In our scheme, a common inductance, but no LC circuit, is used to couple all Josephson charge qubits. In our proposal, both dc and ac supercurrents can flow through the inductance, while in [1, 3] only ac supercurrents can flow through the inductance and it is the LC-oscillator mode that couples the charge qubits. These yield different interbit couplings (e.g., $\sigma_y\sigma_y$ type [1, 3] as opposed to $\sigma_x\sigma_x$ in our proposal).

We employ two dc SQUIDs to connect each Cooper-pair box in order to achieve a *controllable interbit coupling*. Our proposed QC architecture is scalable in the sense that *any* two charge qubits (*not* necessarily neighbors)

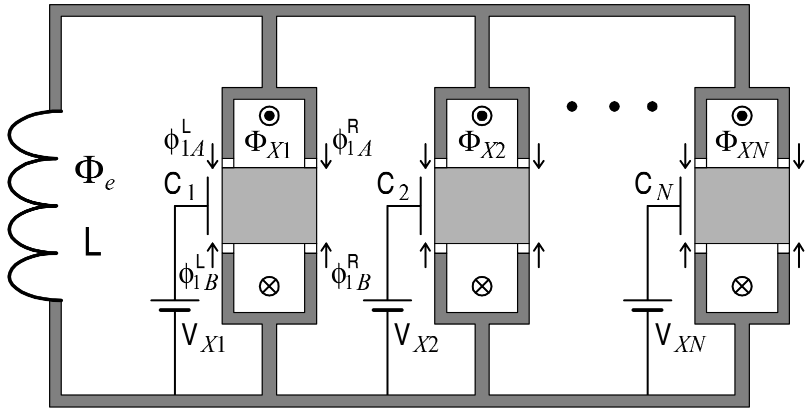

Figure 1. Schematic diagram of the proposed scalable and switchable quantum computer. Here, each Cooper-pair box is operated in the charging regime, $E_{ck} \gg E^0_{Jk}$, and at low temperatures $k_B T \ll E_{ck}$. Also, the superconducting gap is larger than E_{ck}, so that quasi-particle tunneling is strongly suppressed. All Josephson charge-qubit structures are coupled by a common superconducting inductance.

can be effectively coupled by an experimentally accessible inductance. We also formulate [11] an efficient QC scheme that requires only one (instead of two or more) two-bit operation to implement conditional gates.

This Erice summer-school presentation is based on our work in [11]. Additional work on decoherence and noise-related issues appears in, e.g., [12, 13]. Also, work more focused on entanglement and readout issues appears in [14]. Other interesting studies on charge qubits can be found in [15] for the adiabatic controlled-NOT gate, in [16] for geometric phases, and in [17, 18, 19, 20] for the dynamics of a Josephson charge qubit coupled to a quantum resonator.

Proposed scalable and switchable quantum computer

Figure 1 shows a proposed QC circuit consisting of N Cooper-pair boxes coupled by a common superconducting inductance L. For the kth Cooper-pair box, a superconducting island with charge $Q_k = 2en_k$ is weakly coupled by two symmetric dc SQUIDs and biased, through a gate capacitance C_k, by an applied voltage V_{Xk}. The two symmetric dc SQUIDs are assumed to be equal and all Josephson junctions in them have Josephson coupling energy E^0_{Jk} and capacitance C_{Jk}. The effective coupling energy is given by the SQUIDs, each one enclosing a magnetic flux Φ_{Xk}. Each SQUID provides a tunable coupling

$-E_{Jk}(\Phi_{Xk})\cos\phi_{kA(B)}$, with

$$E_{Jk}(\Phi_{Xk}) = 2E^0_{Jk}\cos(\pi\Phi_{Xk}/\Phi_0), \tag{1}$$

and $\Phi_0 = h/2e$ is the flux quantum. The effective phase drop $\phi_{kA(B)}$, with subscript $A(B)$ labelling the SQUID above (below) the island, equals the average value, $[\phi^L_{kA(B)}+\phi^R_{kA(B)}]/2$, of the phase drops across the two Josephson junctions in the dc SQUID, where the superscript L (R) denotes the left (right) Josephson junction. Above we have neglected the self-inductance effects of each SQUID loop because the size of the loop is usually very small (~ 1 μm). The Hamiltonian of the system then becomes

$$H = \sum_{k=1}^{N} H_k + \frac{1}{2}LI^2\,, \tag{2}$$

with H_k given by

$$H_k = E_{ck}(n_k - n_{Xk})^2 - E_{Jk}(\Phi_{Xk})(\cos\phi_{kA} + \cos\phi_{kB}). \tag{3}$$

Here

$$E_{ck} = 2e^2/(C_k + 4C_{Jk}) \tag{4}$$

is the charging energy of the superconducting island and $I = \sum_{k=1}^{N} I_k$ is the total persistent current through the superconducting inductance, as contributed by all coupled Cooper-pair boxes. The offset charge $2en_{Xk} = C_kV_{Xk}$ is induced by the gate voltage V_{Xk}. The phase drops ϕ^L_{kA} and ϕ^L_{kB} are related to the total flux

$$\Phi = \Phi_e + LI \tag{5}$$

through the inductance L by the constraint

$$\phi^L_{kB} - \phi^L_{kA} = 2\pi\Phi/\Phi_0, \tag{6}$$

where Φ_e is the externally applied magnetic flux threading the inductance L. In order to obtain a simpler expression for the interbit coupling, and without loss of generality, the magnetic fluxes through the two SQUID loops of each Cooper-pair box are designed to have the *same* values but *opposite* directions. If this were not to be the case, the interbit coupling can still be realized, but the Hamiltonian of the qubit circuits would just take a more complicated form. Because this pair of fluxes cancel each other in any loop enclosing them, then

$$\phi^L_{kB} - \phi^L_{kA} = \phi^R_{kB} - \phi^R_{kA}. \tag{7}$$

This imposes the constraint

$$\phi_{kB} - \phi_{kA} = 2\pi\Phi/\Phi_0 \tag{8}$$

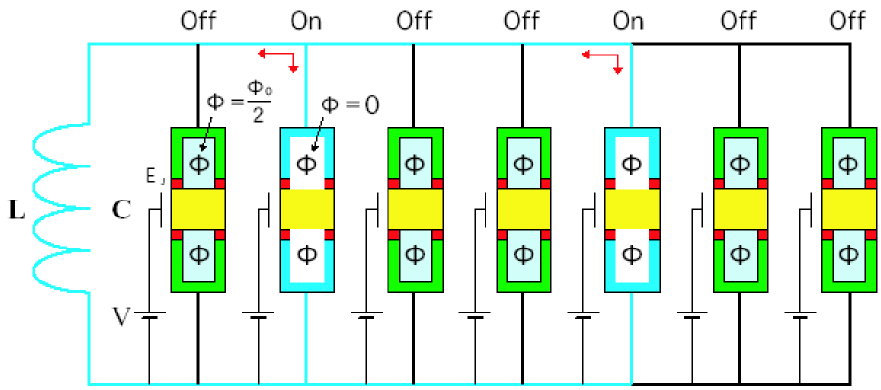

Figure 2. Simplified diagram of the circuit shown in Fig. 1. Here we explicitly show how two charge qubits (not necessarily neighbors) can be coupled by the inductance L, where the cyan SQUIDs are switched on by setting the fluxes through the cyan SQUID loops zero, and the green SQUIDs are turned off by choosing the fluxes through the green SQUID loops as $\Phi_0/2$. This applies to the case when any selected charge qubits are coupled by the common inductance [21].

for the average phase drops across the Josephson junctions in the SQUIDs. The common superconducting inductance L provides the coupling among Cooper-pair boxes. The coupling of selected Cooper-pair boxes can be implemented by switching "on" the SQUIDs connected to the chosen Cooper-pair boxes. In this case, the persistent currents through the inductance L have contributions from all the coupled Cooper-pair boxes. The essential features of our proposal can be best understood via the very simplified diagram shown in Fig. 2.

ONE-BIT CIRCUIT

As seen in Fig. 3(a), for any given Cooper-pair box, say i, when

$$\Phi_{Xk} = \frac{1}{2}\Phi_0, \quad V_{Xk} = (2n_k + 1)e/C_k$$

for all boxes except $k = i$, the inductance L only connects the ith Cooper-pair box to form a superconducting loop. The Hamiltonian of the system can be reduced to [11]

$$H = \varepsilon_i(V_{Xi})\,\sigma_z^{(i)} - \overline{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)\,\sigma_x^{(i)}, \tag{9}$$

where

$$\varepsilon_i(V_{Xi}) = \frac{1}{2}E_{ci}[C_i V_{Xi}/e - (2n_i + 1)] \tag{10}$$

is controllable via the gate voltage V_{Xi}, while the intrabit coupling $\overline{E}_{Ji}$ can be controlled by both the applied external flux Φ_e through the common inductance, and the local flux Φ_{Xi} through the two SQUID loops of the ith

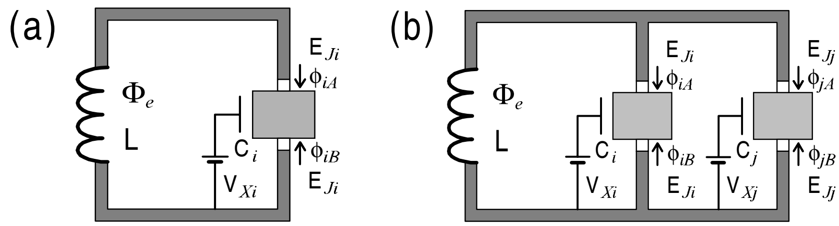

Figure 3. (a) One-bit circuit with a Cooper-pair box connected to the inductance. (b) Two-bit structure where two Cooper-pair boxes are commonly connected to the inductance. Here, each SQUID connecting the superconducting island is represented by an effective Josephson junction.

Cooper-pair box. Retained up to second-order terms in the expansion parameter

$$\eta_i = \pi L I_{ci}/\Phi_0, \tag{11}$$

where

$$I_{ci} = -\pi E_{Ji}(\Phi_{Xi})/\Phi_0, \tag{12}$$

we obtain

$$\overline{E}_{Ji}(\Phi_{Xi}, \Phi_e, L) \ = \ E_{Ji}(\Phi_{Xi}) \cos(\pi\Phi_e/\Phi_0)\,\xi, \tag{13}$$

with

$$\xi = 1 - \frac{1}{2}\eta_i^2 \sin^2(\pi\Phi_e/\Phi_0). \tag{14}$$

The intrabit coupling $\overline{E}_{Ji}$ in (9) is different from that in [1, 3] because a very different contribution by L is considered.

Two-bit circuit

To couple *any* two Cooper-pair boxes, say i and j, we choose

$$\Phi_{Xk} = \frac{1}{2}\Phi_0, \quad V_{Xk} = (2n_k + 1)e/C_k$$

for all boxes except $k = i$ and j. As shown in Fig. 3(b), the inductance L is shared by the Cooper-pair boxes i and j to form superconducting loops. The reduced Hamiltonian of the system is given by [11]

$$H = \sum_{k=i,j} [\varepsilon_k(V_{Xk})\,\sigma_z^{(k)} - \overline{E}_{Jk}\ \sigma_x^{(k)}] + \Pi_{ij}\,\sigma_x^{(i)}\sigma_x^{(j)}. \tag{15}$$

Up to second-order terms,

$$\overline{E}_{Ji}(\Phi_{Xi}, \Phi_e, L) \ = \ E_{Ji}(\Phi_{Xi}) \cos(\pi\Phi_e/\Phi_0)\,\xi, \tag{16}$$

with

$$\xi = 1 - \frac{1}{2}(\eta_i^2 + 3\eta_j^2)\sin^2(\pi\Phi_e/\Phi_0), \tag{17}$$

and

$$\Pi_{ij} = -LI_{ci}I_{cj}\sin^2(\pi\Phi_e/\Phi_0). \tag{18}$$

Here the interbit coupling Π_{ij} is controlled by both the external flux Φ_e through the inductance L, and the local fluxes, Φ_{Xi} and Φ_{Xj}, through the SQUID loops.
Using these two types of circuits, we can derive the required one- and two-bit operations for QC. Specifically, the conditional gates such as the controlled-phase-shift and controlled-NOT gates can be obtained using one-bit rotations and only one basic two-bit operation. For details, see Ref. [11]. A sequence of such conditional gates supplemented with one-bit rotations constitute a universal element for QC [22, 23]. Usually, a two-bit operation is much slower than a one-bit operation. Our designs for conditional gates $U_{\rm CPS}$ and $U_{\rm CNOT}$ are *efficient* since *only one* (instead of two or more) basic two-bit operation is used.

Persistent currents and entanglement

The one-bit circuit modeled by Hamiltonian (9) has two eigenvalues $E_{\pm}^{(i)} = \pm E_i$, with

$$E_i = [\varepsilon_i^2(V_{Xi}) + \overline{E}_{Ji}^2]^{1/2}. \tag{19}$$

The corresponding eigenstates are

$$\begin{aligned} |\psi_+^{(i)}\rangle &= \cos\xi_i|\uparrow\rangle_i - \sin\xi_i|\downarrow\rangle_i, \\ |\psi_-^{(i)}\rangle &= \sin\xi_i|\uparrow\rangle_i + \cos\xi_i|\downarrow\rangle_i, \end{aligned} \tag{20}$$

where

$$\xi_i = \frac{1}{2}\tan^{-1}(\overline{E}_{Ji}/\varepsilon_i). \tag{21}$$

At these two eigenstates, the persistent currents through the inductance L are given by

$$\langle\psi_\pm^{(i)}|I|\psi_\pm^{(i)}\rangle = \pm\left(\frac{\overline{E}_{Ji}I_{ci}}{E_i}\right)\sin\left(\frac{\pi\Phi_e}{\Phi_0}\right) + \left(\frac{\pi L I_{ci}^2}{2\Phi_0}\right)\sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right), \tag{22}$$

up to the linear term in η_i. In the case when a dc SQUID magnetometer is inductively coupled to the inductance L, these two supercurrents generate different fluxes through the SQUID loop of the magnetometer and the quantum-state information of the one-bit structure can be obtained from the measurements. In order to perform sensitive measurements with weak dephasing, one could use the underdamped dc SQUID magnetometer designed previously for the Josephson phase qubit [6].

For the two-bit circuit described by Eq. (15), the Hamiltonian has four eigenstates and the supercurrents through inductance L take different values for these four states. The fluxes produced by the supercurrents through L can also be detected by the dc SQUID magnetometer. For example, when $\varepsilon_k(V_{Xk}) = 0$ and $\overline{E}_{Jk} > 0$ for $k = i$ and j, the four eigenstates of the two-bit circuit are

$$\begin{aligned}
|1\rangle &= \frac{1}{2}\left(|\uparrow\rangle_i|\uparrow\rangle_j - |\uparrow\rangle_i|\downarrow\rangle_j - |\downarrow\rangle_i|\uparrow\rangle_j + |\downarrow\rangle_i|\downarrow\rangle_j\right), \\
|2\rangle &= \frac{1}{2}\left(|\uparrow\rangle_i|\uparrow\rangle_j + |\uparrow\rangle_i|\downarrow\rangle_j - |\downarrow\rangle_i|\uparrow\rangle_j - |\downarrow\rangle_i|\downarrow\rangle_j\right), \\
|3\rangle &= \frac{1}{2}\left(|\uparrow\rangle_i|\uparrow\rangle_j - |\uparrow\rangle_i|\downarrow\rangle_j + |\downarrow\rangle_i|\uparrow\rangle_j - |\downarrow\rangle_i|\downarrow\rangle_j\right), \\
|4\rangle &= \frac{1}{2}\left(|\uparrow\rangle_i|\uparrow\rangle_j + |\uparrow\rangle_i|\downarrow\rangle_j + |\downarrow\rangle_i|\uparrow\rangle_j + |\downarrow\rangle_i|\downarrow\rangle_j\right). \qquad (23)
\end{aligned}$$

Retained up to linear terms in η_i and η_j, the corresponding supercurrents through the inductance L are

$$\langle k|I|k\rangle = \mathcal{I}_k \sin\left(\frac{\pi\Phi_e}{\Phi_0}\right) + \frac{\pi L \mathcal{I}_k^2}{2\Phi_0}\sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right) \qquad (24)$$

for $k = 1$ to 4, where

$$\begin{aligned}
\mathcal{I}_1 &= -(I_{ci} + I_{cj}), \qquad \mathcal{I}_2 = I_{cj} - I_{ci}, \\
\mathcal{I}_3 &= I_{ci} - I_{cj}, \qquad \mathcal{I}_4 = I_{ci} + I_{cj}. \qquad (25)
\end{aligned}$$

These supercurrents produce different fluxes threading the SQUID loop of the magnetometer and can be distinguished by dc SQUID measurements. When the two-bit system is prepared at the maximally entangled Bell states

$$|\Psi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_i|\downarrow\rangle_j \pm |\downarrow\rangle_i|\uparrow\rangle_j), \qquad (26)$$

the supercurrents through L are given by

$$\langle\Psi^{(\pm)}|I|\Psi^{(\pm)}\rangle = \frac{\pi L}{2\Phi_0}(I_{ci} \pm I_{cj})^2 \sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right). \qquad (27)$$

These two states should be distinguishable by measuring the fluxes, generated by the supercurrents, through the SQUID loop of the magnetometer.

Estimates of the inductance for optimal coupling

The typical switching time $\tau^{(1)}$ during a one-bit operation is of the order of $\hbar/E_J^0$. Using the experimental value $E_J^0 \sim 100$ mK, then $\tau^{(1)} \sim 0.1$ ns. The switching time $\tau^{(2)}$ for the two-bit operation is typically of the order

$$\tau^{(2)} \sim (\hbar/L)(\Phi_0/\pi E_J^0)^2.$$

Choosing $E_J^0 \sim 100$ mK and $\tau^{(2)} \sim 10\tau^{(1)}$ (i.e., ten times slower than the one-bit rotation), we have

$$L \sim 30 \text{ nH}$$

in our proposal. This number for L is experimentally realizable. A small-size inductance with this value can be made with Josephson junctions. Our expansion parameter η is of the order

$$\eta \sim \pi^2 L E_J^0/\Phi_0^2 \sim 0.1.$$

Our inductance L is related with the inductance L' in [1, 3] by

$$L' = (C_J/C_{qb})^2 L. \tag{28}$$

Let us now consider the case when $\tau^{(2)} \sim 10\tau^{(1)}$. For the earlier design [3], $C_J \sim 11 C_{qb}$ since $C_g/C_J \sim 0.1$, which requires an inductance $L' \sim 3.6\ \mu$H. Such a large inductance is problematic to fabricate at nanometer scales. In the improved design [1], $C_J \sim 2C_{qb}$, greatly reducing the inductance to $L' \sim 120$ nH. This inductance is about four times larger than the one used in our scheme, making it somewhat more difficult to realize than our proposed L.

Conclusion

We propose a *scalable* quantum information processor with Josephson charge qubits. We use a common inductance to couple all charge qubits and design *switchable* interbit couplings using two dc SQUIDs to connect the island in each Cooper-pair box. The proposed circuits are scalable in the sense that any two charge qubits can be effectively coupled by an experimentally accessible inductance. In addition, we formulate [11] an efficient QC scheme in which only one two-bit operation is used in the conditional transformations, including controlled-phase-shift and controlled-NOT gates.

Acknowledgments

We thank Yu. Pashkin, B. Plourde and Xuedong Hu for useful discussions. This work is supported in part by ARDA, the AFOSR, and the US National Science Foundation grant No. EIA-0130383.

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