

Critical currents in quasiperiodic pinning arrays: One-dimensional chains and Penrose lattices

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Summary

We have studied the critical depinning current J_c versus the applied magnetic flux Φ , for quasiperiodic (QP) one-dimensional (1D) chains and 2D arrays of pinning centers placed on the nodes of a five-fold Penrose lattice. In 1D QP chains, the peaks in $J_c(\Phi)$ are determined by a sequence of harmonics of the long and short segments of the chain. The critical current $J_c(\Phi)$ has a remarkable self-similarity. In 2D QP pinning arrays, we predict analytically and numerically the main features of $J_c(\Phi)$, and demonstrate that the Penrose lattice of pinning sites (which has many built-in periods) provides an enormous enhancement of $J_c(\Phi)$, even compared to triangular and random pinning site arrays. This huge increase in $J_c(\Phi)$ could be useful for applications.

Model

We perform simulated annealing simulations of

$$\eta \mathbf{v}_i = \mathbf{f}_i = \mathbf{f}_i^{vv} + \mathbf{f}_i^{vp} + \mathbf{f}_i^T + \mathbf{f}_i^d.$$

- The force due to the vortex-vortex interaction is

$$\mathbf{f}_i^{vv} = \sum_j^{N_v} f_0 K_1(|\mathbf{r}_i - \mathbf{r}_j|/\lambda) \mathbf{r}_{ij}$$

N_v is the number of vortices, K_1 is a modified Bessel function, λ is the penetration depth, $\mathbf{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j) / |\mathbf{r}_i - \mathbf{r}_j|$, $f_0 = \Phi_0^2/8\pi^2\lambda^2$, and $\Phi_0 = hc/2e$.

- The pinning force is

$$\mathbf{f}_i^{vp} = \sum_k^{N_p} f_p \cdot (|\mathbf{r}_i - \mathbf{r}_k^{(p)}|/r_p) \Theta[(r_p - |\mathbf{r}_i - \mathbf{r}_k^{(p)}|)/\lambda] \mathbf{r}_{ik}^{(p)},$$

N_p is the number of pinning sites, f_p (expressed in f_0) is the maximum pinning force of each short-range parabolic potential well located at $\mathbf{r}_k^{(p)}$, r_p is the range of the pinning potential, Θ is the Heaviside step function.

- All the lengths (fields) are expressed in units of λ (Φ_0/λ^2).
- In the equation of motion, \mathbf{f}_i^T is the thermal stochastic force, and \mathbf{f}_i^d is the driving force; η is the viscosity.

1D Quasicrystal

Construction of a 1D QP chain (1D quasicrystal): Iteratively apply the Fibonacci rule $L \rightarrow LS, S \rightarrow L$

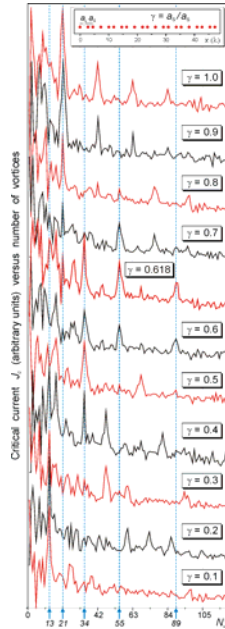
LSLLSLSLLSLSLLSLSLLS...

For an infinite QP sequence, the ratio of the numbers of L to S elements is the *golden mean* $\tau = (1 + \sqrt{5})/2$.

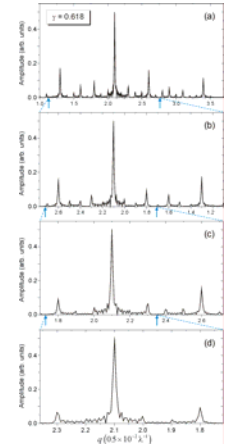
The n th point where a new segment (e.g., for $S = 1$ and $L = \tau$) begins is:

$$x_n = n + [n/\tau] / \tau,$$

where $[x]$ denotes the integer part of x . This sequence exhibits self-similarity and has a hierarchy of built-in periods.

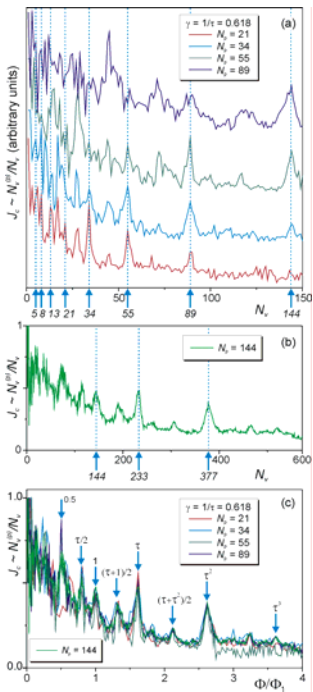


Self-similarity of J_c in k -space



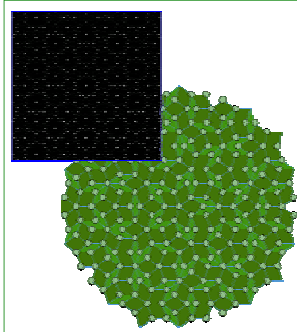
Critical current J_c for 1D QP chains

Self-similarity of J_c in real space



2D Quasicrystal: Penrose lattice

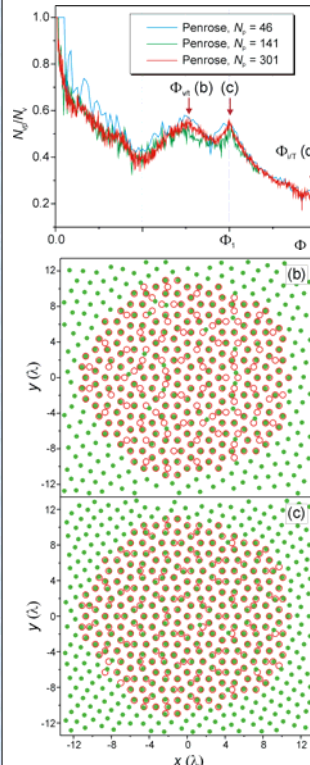
Five-fold Penrose lattice



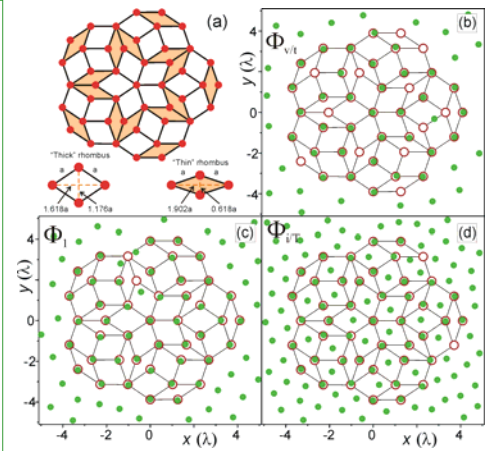
A Penrose lattice is a 2D quasiperiodic (QP) structure, or 2D quasicrystal.

- Has a local rotational (five- or ten-fold) symmetry, but does not have translational long-range order.
- Is constructed using certain simple shapes combined according to specific local rules, and can extend to infinity without any defects.
- Self-similar diffraction patterns of a Penrose lattice exhibit a dense set of "Bragg" peaks.

J_c for increasing sample size



Critical current J_c in a Penrose-lattice array of pinning sites

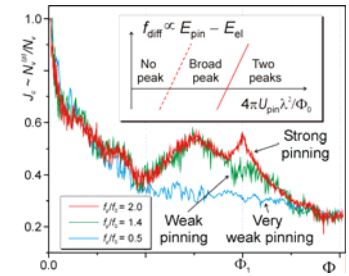


Stability of the maxima of J_c

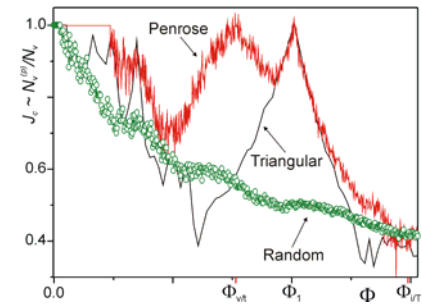
The dimensionless difference of the pinning and elastic energies is $E_{pin} - E_{el} = \beta f_{diff} n_{pin} \Phi_0^2 / (4\pi\lambda^2)$, where

$$f_{diff} = 4\pi\lambda^2 U_{pin} / \Phi_0^2 - \beta \cdot [1 - b(3^{1/2} \beta n_{pin} / 2)^{1/2}]^2.$$

Here $E_{pin} = U_{pin} \beta n_{pin}$; $E_{el} = C_{11} [(a_{eq} - b) / a_{eq}]^2$; n_{pin} is the density of pinning centers, $\beta (H \leq H_1) = H/H_1 = B/(\Phi_0 n_{pin})$, and $\beta (H > H_1) = 1$ is the fraction of occupied pinning sites, $a_{eq} = [2(3^{1/2} \beta n_{pin})^{1/2}]$ and b are the inter-vortex distances in triangular and distorted lattices ($b = a/\tau$ for $H = H_1$ and $b = a$ for $H = H_0$), and $C_{11} = B^2 / [4\pi(1 + \lambda^2 k^2)]$ is the compressibility modulus for local deformations. Near matching fields, J_c has a peak only if $f_{diff} > 0$.



Strong enhancement of the critical current J_c in a Penrose-lattice array of pinning sites



Quasiperiodic (Penrose) lattice provides an **unusually broad** critical current $J_c(\Phi)$, that could be useful for practical applications demanding **high J_c 's over a wide range of fields**