Magnetic and mechanical buckling: modified Landau theory approach to study phase transitions in micro-magnetic disks and compressed rods

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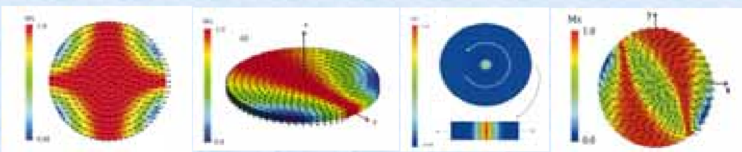
Summary: Using the rigid magnetic vortex model, we develop a substantially modified Landau theory approach for analytically studying phase transitions between different spin arrangements in circular sub-micron magnetic dots subject to an in-plane externally-applied magnetic field. We introduce a novel order parameter: the inverse distance between the center of the circular dot and the vortex core. This order parameter is suitable for describing closed spin configurations such as curved or bent-spin structures and magnetic vortices. Depending on the radius and thickness of the dot as well as the exchange coupling, there are five different regimes for the magnetization reversal process when decreasing the in-plane magnetic field. The magnetization-reversal regimes obtained here cover practically all possible magnetization reversal processes. Moreover, we have derived the change of the dynamical response of the spins near the phase transitions and obtained a "critical slowing down" at the second order phase transition from the high-field parallel-spin state to the curved (C-shaped) spin phase. We predict a transition between the vortex and the parallel-spin state by quickly changing the magnetic field --- providing the possibility to control the magnetic state of dots by changing either the value of the external magnetic field and/or its sweep rate. We study an illuminating mechanical analog (buckling instability) of the transition between the parallel-spin state and the curved spin state (i.e., a magnetic buckling transition). In analogy to the magnetic-disk case, we also develop a modified Landau theory for studying mechanical buckling instabilities of a compressed elastic rod embedded in an elastic medium. We show that the transition to a buckled state can be either first or second order depending on the ratio of the elasticity of the rod and the elasticity of the external medium. We derive the critical slowing down for the second-order mechanical buckling transition.

Novel spin arrangements: Micromagnetic simulations by J.K. Ha, R. Hertel, J. Kirschner, Phys. Rev. B 67, 224432 (2003); Phys. Rev. B 67, 064418 (2003).

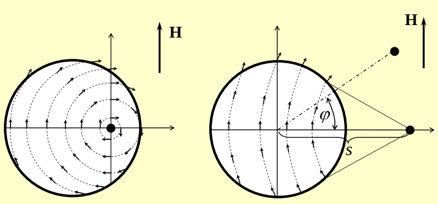


Single-domain (onion) state — Buckled C-state — In-plane vortex state — Buckled S-state

Model: rigid vortex model. K.Yu. Guslienko, V. Novosad, Y. Otani, H. Shima, and K. Fukamichi, Phys. Rev. B 65, 024414 (2001).

Magnetic vortex, C-, and single-domain phases within the rigid vortex model:

K.Yu. Guslienko, V. Novosad, Y. Otani, H. Shima, and K. Fukamichi, Phys. Rev. B 65, 024414 (2001).



Vortex inside the disk — Vortex outside the disk: C-phase

Single-domain state: $s \to \infty$

Exchange, Magnetostatic, and Zeeman contributions to the energy of a magnetic dot

$$W_{ex} = \frac{1}{2} C \int d^3\vec{r}\,\left[(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2\right] \quad (1)$$

where $\vec{m} = \vec{M}/M_s$ is the dimensionless magnetization normalized to the saturation magnetization M_s, C is the exchange constant. ← **Exchange**

$$W_m = \frac{1}{2}\int dS \int dS' \frac{\sigma(\vec{r})\sigma(\vec{r}')}{\vec{r}-\vec{r}'} \quad (2)$$

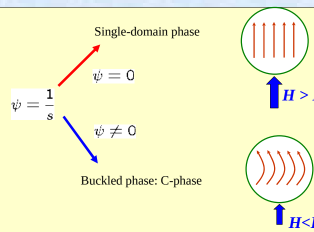
where $\sigma = \vec{M}\cdot\vec{n}$ is the magnetic charge density with unit vector $\vec{n}$ directed outside of the dot surface, while $\vec{r}$ and $\vec{r}'$ are the positions of the elements of the surface. ← **Magnetostatic**

$$W_H = -\int_V d^3\vec{r}\;\vec{M}(\vec{r})\cdot\vec{H} \quad (3)$$

← **Zeeman energy**

Novel order parameter

Inverse magnetic vortex distance as a novel order parameter

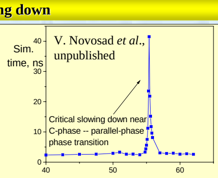


Critical slowing down

$$\frac{d(\delta\psi)}{dt} = -\frac{(\delta\psi)}{\tau}, \quad \text{for } h < h_c$$
$$\frac{d\psi}{dt} = -\frac{2A}{\gamma}\psi^5, \quad \text{for } h > h_c$$

Relaxation time diverges at phase transition :

$$\tau = -\frac{\eta B^2}{A^3} = \frac{512\eta B^2}{(h_c - h)^3}$$



Modified Landau theory approach

Energy expansion

$$w(\psi) = w(\psi = 0) + A(h)\,\psi^2 + B(h)\,\psi^4 + \mathcal{O}(\psi^6).$$

Analytically derived Landau-like coefficients

$$A(h) = w_m^{(2)}(\beta) + h/8 + R_0^2/2R^2.$$
$$B(h) = w_m^{(4)}(\beta) + h/64 + R_0^2/4R^2$$

where

$$w_m^{(2)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \int_{-\pi}^{\pi} d\phi'\, \mathcal{K}(\phi-\phi',\beta)\,\sin\phi\sin\phi'\left(\frac{1}{2}+\frac{3}{2}\cos 2\phi + \cos\phi\cos\phi'\right).$$
$$w_m^{(4)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \int_{-\pi}^{\pi} d\phi'\, \mathcal{K}(\phi-\phi',\beta)\,\sin\phi\sin\phi' \times \left(\frac{11}{32} + \cos 2\phi + \frac{35}{32}\cos 4\phi + \frac{3}{4}\cos\phi\cos\phi' + \frac{5}{4}\cos 3\phi\cos\phi' + \frac{9}{16}\cos 2\phi\cos 2\phi'\right).$$

A>0 and B>0 at any h>0:

(1) single-domain phase is stable or metastable up to h=0

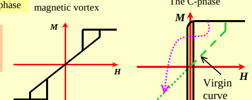
(2) Single-domain phase rotates as a whole at h=0, vortex does not contribute to remagnetization process

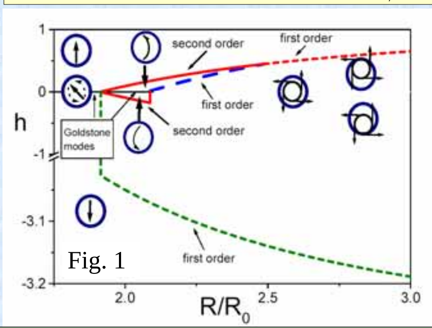
$A(h_c)=0$, $B(h_c)>0$: Second order transition to the C-phase

$A(h_c)=0$, $B(h_c)<0$: First order transition to the vortex or C-phase

Magnetization reversal via nucleation and annihilation of magnetic vortex

Magnetization reversal via Goldstone rotation of The C-phase





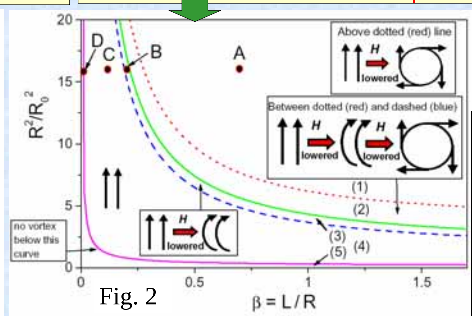


Fig. 2. Phase diagram plotted on the plane (magnetic field = (generalized force)$^{-1}$ = h= H/M_s reduced radius=R/R_0). This shows magnetization reversal processes for a fixed aspect ratio β=L/R=1 when lowering the applied magnetic field h from a high value $h>h_c$. Colored dashed (solid) lines correspond to first- (second-) order phase transitions. The short black segment at the top left corner corresponds to Goldstone modes, where the magnetization can be rotated with zero or little energy cost. For the case of increasing h from a high negative value $h<-h_c$, the diagram is inverted with respect to the h=0 axis.

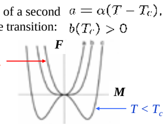
Fig. 1. Phase diagram showing the domains of parameters for the different sequences of transitions among three stable and metastable spin configurations inside a micromagnetic disk. The parameters are the square of the reduced radius R^2/R_0^2 versus the aspect ratio β=L/R. Above the top curve, the parallel spin state, which exists for high applied in-plane magnetic fields, discontinuously transforms to a magnetic vortex sitting inside the disk. This is because the large radius of the microdisk can easily accommodate a vortex inside. For parameter values between the red and green curves (e.g., for smaller disk radius at a fixed β), a second-order phase transition from the parallel spin configuration to the C-phase occurs first. Upon further lowering the in-plane field, the C-phase abruptly transforms to a vortex sitting inside the disk. Between the blue and green curves, the C-phase survives down to $h=H/M_s=0$, and rotates as a whole when the magnetic field changes its polarity. This rotation costs little or no energy and it is a Goldstone mode. Below the blue curve, the magnetization reversal process proceeds via a rotation of the parallel-spin state as a whole at h=0. Even though the vortex state does not contribute to the magnetization reversal process below the green curve, the magnetic vortex is stable or metastable at low magnetic fields above the magenta curve located at the bottom of the diagram. In practice, the stable or metastable vortex states below the green curve cannot be reached besides at high temperatures or when H changes suddenly. Below this magenta bottom curve, the magnetic vortex does not correspond to an energy minimum and it is unstable for any value of h.

The standard Landau approach for studying transitions in Ferro-magnetic materials

Landau free energy:

$$F = F_0 + aM^2 + bM^4 - HM$$

Conditions of a second order phase transition: $a = \alpha(T - T_c)$, $b(T_c) > 0$



	Standard Approach	Our Approach
Order parameter	Magnetization M	$\psi = 1/s$ = (disk-vortex-distance)$^{-1}$
High T or high H phase	$M = 0$ (high T)	$\psi = 0$ (parallel-spin phase) (High H)
Low T or low H phase	$M \neq 0$ (low T)	$\psi \neq 0$ (C or vortex phase) (Low H)
$T = 0$ or $H = 0$ phase	$M = M_s$ ($T = 0$)	For certain parameter values $\psi = \infty$ (vortex centered in the dot center at $H = 0$)
Energy[order parameter]	$F_L[M] = F_{L0} + a M^2 + b M^4 - HM$	$F[\psi] = F_0 + A\,\psi^2 + B\,\psi^4 + O(\psi^6)$
Energy expansion coefficients	T – dependent, phenomenological	H – dependent, analytically derived
Stability of the phases determined by	Truncated expansion of $F_L[M]$	Full expansion of $F[\psi]$ (at all orders in ψ)
Odd power are zero because of	Symmetry $F_L[M] = F_L[-M]$	Analytically derived; this already satisfies the symmetry restrictions $F[\psi] = F[-\psi]$
Coefficient of ψ^2 or M^2	$a \propto (T - T_c)$	$H \propto (h - h_c)$
Coefficient of ψ^4 or M^4	$b > 0$	$B(h,C,R,L) \gtrless 0$
1st order transition	Requires cubic term $\propto M^3$	A changes sign then B<0
2nd order transition	a changes sign then b > 0	A changes sign then B > 0
Order parameter near T_c or H_c	$M \propto (T - T_c)^{1/2}$	$\psi \propto (h - h_c)^{1/2}$
Susceptibility near T_c or H_c	$\frac{\partial M}{\partial H} = \lvert T - T_c\rvert^{-1}$ (near T_c)	$\frac{\partial\psi(h)}{\partial T} = \lvert h - h_c\rvert^{-p}$ (near h_c)
Critical slowing down near T_c or H_c	δM - deviation from equilibrium; $\delta M(t) \propto \exp(-t/\tau_m)$; $\tau_m = (T_c - T)^{-1}$, $T < T_c$; $\tau_m = (T - T_c)^{-1}$, $T > T_c$	$\delta\psi$ - deviation from equilibrium; $\delta\psi(t) \propto \exp(-t/\tau)$; $\tau = \eta\mu_0^{(2)} = \frac{512\eta B^2}{(h_c-h)^3}$, $h < h_c$; $\psi = (\frac{2At}{\eta} + \psi^{-4}(t=0))^{-\frac{1}{4}}$, $h > h_c$

Mechanical buckling

$$F_b = \int_0^{l_{max}} dl \left\{\frac{IE(y'')^2}{2\left(1-(y')^2\right)} + f\left(\sqrt{1-(y')^2} - 1\right) + \frac{\alpha}{2}y^2\right\}. \qquad y(l) = \psi_b \sin\left(\frac{n\pi l}{l_{max}}\right)$$

Energy expansion $\quad F_b = A_b(f)\,\psi_b^2 + B_b(f)\,\psi_b^4 + \mathcal{O}(\psi_b^6)$

Coefficients

$$A_b = \frac{\pi^2 n^2}{4 l_{max}}\left(f_c(n) - f\right).$$
$$B_b = \frac{\pi^4 n^4}{64 l^3}\left(\frac{4IEn^2\pi^2}{l_{max}^2} - 3f\right),$$
$$f_c(n) = \frac{IE\pi^2}{l_{max}^2}\left(n^2 + \frac{\alpha l_{max}^4}{IE\pi^4 n^2}\right).$$

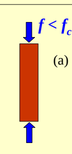
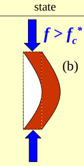
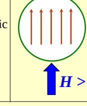
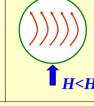
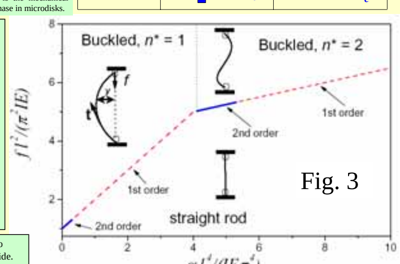
	Symmetric state	Buckled (broken-symmetry) state
compressed elastic rod	$f < f_c^*$ (a) 	$f > f_c^*$ (b) 
micromagnetic disk	(c)  $H > H_c$	(d)  $H<H_c$

Fig. 3. Phase diagram for the mechanical buckling instability. The parameters shown are the normalized force = $f\,l^2/(\pi^2 IE)$ versus the ratio of the medium/rod elasticities $\alpha\,l^4/(\pi^4 IE)$. Solid blue segments correspond to second-order phase transitions from the straight rod state to the buckled state, while the first-order phase transitions are indicated by the dashed red lines. The different buckling modes, with different undulation numbers n^*, are sketched in the corresponding phase domains. The black vertical dotted segment around $\alpha\,l^4/(\pi^4 IE) \approx 4$ corresponds to a first-order transition between the $n^*=1$ (C-phase-like) and the $n^*=2$ (S-phase-like). The transition from the straight rod phase to the $n^*=1$ buckled phase is the mechanical analog of the magnetic phase transition between the straight and bent spin-configuration phases: this diagram corresponds the one shown in Fig. 2. The transition to the $n^*=2$ buckled rod state is the mechanical analog of the magnetic transition to the S phase in microdisks.



References: [1] S. Savel'ev, F. Nori, Phys. Rev. B **70**, 214415 (2004);
[2] S. Savel'ev, X.Hu, F. Nori, New J, Phys. **8**, 105 (2006)