Theory of macroscopic quantum tunneling with Josephson-Leggett collective excitations in multiband superconducting Josephson junctions

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Collective excitations reveal fundamental properties and potential applications of superconducting states. We theoretically study macroscopic quantum tunneling (MQT) in a Josephson junction composed of multiband superconductors, focusing on a phase mode induced by interband fluctuations: the Josephson-Leggett (JL) collective excitation mode. Using the imaginary-time path-integral method, we derive a formula for the MQT escape rate for low-temperature switching events. We clarify that the JL mode has two major effects on the MQT: (i) the zero-point fluctuations enhance the escape rate, and (ii) the quantum dissipation induced by the couplings to the gauge-invariant phase difference suppresses the quantum tunneling. We show that the enhancement exceeds the suppression for a wide range of junction parameters. This enhancement originates from the single-mode interaction between the tunneling variable and the interband fluctuations.

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I. INTRODUCTION

Josephson junctions show phenomena caused by macroscopic-scale quantum coherence and nonlinear dynamical properties. Their unique properties come from couplings between superconducting gauge-invariant phase differences and the electromagnetic field, leading to practical applications, as seen, e.g., in Ref. [1]. Macroscopic quantum tunneling (MQT) [2–5] is one of the characteristic phenomena of Josephson junctions. Superconducting-to-resistive switching events in current-biased Josephson junctions are related to MQT at low temperatures, where thermal excitations are negligible. A wide variety of junction systems (artificial niobium-based junctions [6,7], grain boundary junctions [8,9], and intrinsic Josephson-junction stacks in single-crystalline high-T_c cuprate superconductors [10–12]) show this tunneling phenomenon. The theoretical aspects have been studied well, depending on the types of Josephson junctions [13–21]. MQT in Josephson junctions plays an important role in Josephson phase qubits and the relevant superconducting quantum engineering [22–26]. Hence, the study of MQT in Josephson junctions attracts a great deal of attention theoretically and experimentally in the search to find quantum characteristics of superconducting devices.

The discovery of superconducting materials, including magnesium diboride [27] and iron-based compounds [28], triggered the studies of multiband superconductivity. These superconductors have intriguing properties, originating from the multiple superconducting gaps opening in different parts of the Fermi surfaces [29–33]. Notable Josephson effects are predicted in junctions with multiband superconductors [21,34–40]. The characteristic behaviors in these systems originate from the presence of multiple gauge-invariant phase differences coupled by interband Josephson coupling [39,41–45]. Specifically, a phase mode induced by interband fluctuations, which is referred to as a Josephson-Leggett (JL) mode [39], can lead to singular behaviors. However, the JL-mode excitations are not coupled directly to the electric field, owing to their neutral-superfluid feature [43,45]. Instead, they interact with the Josephson-plasma (JP) mode (i.e., in-phase motion of superfluids) [39]. Thus, a careful and systematic study of the interaction between the JP and JL modes is desirable for exploring characteristic phenomena in Josephson junctions composed of multiband superconductors.

In this paper, we construct a theory of MQT in a Josephson junction formed by a conventional single-band superconductor and a two-band superconductor (a hetero-Josephson junction) to clarify the effects of the JL mode on low-temperature switching events in Josephson junctions. From theoretical considerations of the dynamics of gauge-invariant phase differences, we choose a tunneling path along the center-of-mass motion of the phase differences, and consider the interband fluctuations to be the environment for the center-of-mass motion. We evaluate the MQT escape rates, varying different junction parameters. We show that the interband fluctuations have both positive and negative effects on the MQT. Zero-point fluctuations of the JL mode enhance the MQT escape rate, whereas the quantum dissipation induced by the JL mode suppresses the quantum tunneling. The former was found by two of the authors (Y.O. and M.M.) [21], focusing only on a specific junction parameter. Thus, the present approach successfully extends the previous results in Ref. [21] and reveals two distinct features of the JL mode, i.e., amplification and reduction. Moreover, we show that the zero-point-fluctuation enhancement exceeds the quantum-dissipation suppression. Therefore, we find that the escape rate is significantly enhanced by the JL mode. In these junctions, the dissipation effect is marginal because there is only one

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dissipation channel corresponding to a monochromatic JL mode. We also examine the dependence of the escape rate on the interband Josephson energy for junction parameters which are typical for $BaFe_2As_2$ and MgB_2. We find that the effects of interband fluctuations on MQT strongly depend on the nature of the JL mode characterized by the superconducting material parameters of the junction.

This paper is organized as follows. In Sec. II, we introduce a minimal model of multiband Josephson junctions. In Sec. III, we describe a theory of MQT in this Josephson junction and derive the MQT escape-rate formula. In Sec. IV, we evaluate the escape rate for various junction parameters. Section V presents a summary.

II. MODEL

We study a minimal model of a multiband Josephson junction, as seen in Fig. 1, to find an essential feature of the JP-JL coupling. The system is composed of a conventional single-gap superconductor and a two-gap superconductor. The superconducting electrodes are separated by an insulating layer with thickness D and area W. The dc current $I_{\rm ext}$ is applied to this junction. The right electrode has two gaps, $|\Delta_{\rm R}^{(1)}|e^{i\phi_{\rm R}^{(1)}}$ and $|\Delta_{\rm R}^{(2)}|e^{i\phi_{\rm R}^{(2)}}$, whereas the left electrode has a single gap, $|\Delta_{\rm L}|e^{i\phi_{\rm L}}$. These superconducting phases are coupled to each other via the Josephson couplings $E_{\rm J1}$, $E_{\rm J2}$, and $E_{\rm in}$. The standard Josephson energy associated with Cooper-pair tunneling between the superconducting electrodes is characterized by $E_{\rm J1}$ and $E_{\rm J2}$. The interband Josephson energy associated with the tunneling between the two bands is characterized by $E_{\rm in}$ [41].

Now we show two key variables in this paper, i.e., the center-of-mass phase and the relative phase. Using the gauge-invariant phase differences $\theta^{(1)}$ and $\theta^{(2)}$ between the electrodes (see Fig. 1), the center-of-mass phase is

$$\theta = \frac{\alpha_2}{\alpha_1+\alpha_2}\theta^{(1)} + \frac{\alpha_1}{\alpha_1+\alpha_2}\theta^{(2)}, \tag{1}$$

with a dimensionless constant α_i, related to the density of states of the ith-band electron near the interface between the

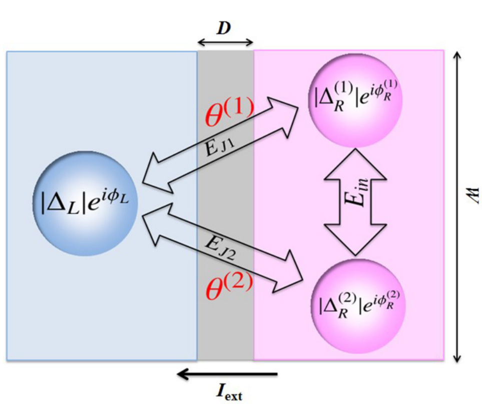

FIG. 1. (Color online) Schematic diagram of a Josephson junction composed of a single-gap superconductor (left electrode) and a two-gap superconductor (right electrode). Two gauge-invariant phase differences $\theta^{(1)}$ and $\theta^{(2)}$ are defined between the electrodes.

right electrode and the insulator [21]. The relative phase is

$$\psi = \theta^{(1)} - \theta^{(2)}. \tag{2}$$

When the voltage difference is V, the Josephson relation is [39]

$$\frac{\partial\theta}{\partial t} = \frac{2eD}{\hbar}V, \tag{3}$$

with the electric charge e and the Planck constant $\hbar$. The Josephson relation indicates that θ is directly coupled to the electric field, but ψ is not. In this paper, we focus on a short Josephson junction, that is, D is much smaller than the Josephson penetration depth. Hence, we ignore the spatial modulation of $\theta^{(1)}$ and $\theta^{(2)}$ and the influence of solitonic excitations shown in Refs. [17] and [18].

III. FORMULATION

We formulate the MQT escape rate based on the semiclassical approximation with the imaginary-time path-integral method [3]. Our discussion is divided into four steps. First, we show the Lagrange formalism of our junction, which is useful for the path-integral method. Second, from a physical point of view, we find a plausible tunneling path for low-temperature switching events in the present junction. Our approach is to choose a specific path and reduce the issue into an effective one-dimensional (1D) tunneling problem. Third, we show that the relevant Euclidean (imaginary-time) Lagrangian can be mapped into the Caldeira-Leggett model [2]. In this step, we mainly use a small ψ expansion. Finally, we obtain the MQT escape rate using a technique based on the influence-functional method [2,3,20].

A. Real-time Lagrangian

The real-time effective Lagrangian $\mathcal{L}$ for our junction is [39]

$$\mathcal{L}(\theta,\psi) = \frac{1}{2}m_{\rm cm}\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}m_{\rm rlt}\left(\frac{d\psi}{dt}\right)^2 + E_{\rm J1}\cos\theta^{(1)} + E_{\rm J2}\cos\theta^{(2)} + E_{\rm in}\cos\psi + E_{\rm J}\gamma\,\theta, \tag{4}$$

with $m_{\rm cm} = \hbar^2/2E_{\rm c}$ and $m_{\rm rlt} = \hbar^2/2(\alpha_1+\alpha_2)E_{\rm c}$. The charging energy is denoted by $E_{\rm c}$. The total Josephson-energy coupling is $E_{\rm J} = E_{\rm J1} + E_{\rm J2}$. The dimensionless bias current is $\gamma = I_{\rm ext}/I_{\rm c}$. The critical current $I_{\rm c}$ is related to $E_{\rm J}$.

In this paper, we take a positive value for the interband Josephson coupling $E_{\rm in}$ [39]. Thus, we focus on the case of a 0 phase shift in the two-band superconductor (i.e., $\phi_{\rm R}^{(1)} - \phi_{\rm R}^{(2)} \equiv 0 \mod 2\pi$). We can find that our results do not change qualitatively when a π phase shift (i.e., $\pm s$ wave) occurs.

B. Determining a tunneling path

We now seek a predominant tunneling path on (θ, ψ). In this paper, we choose an *in-phase tunneling path* along the θ axis. Here, we justify this choice based on the physical properties of θ and ψ. First, θ has a direct coupling to the electric field, whereas ψ does not. Thus, the switching event in this junction is caused by the tunneling of θ. This tunneling is strongly enhanced by the bias current γ because the potential barrier height along the θ axis decreases with increase of γ. Second, ψ tends to be fixed to 0 or π because the dynamics of the relative phase is subjected to a restoring force induced by the interband coupling (in this paper, ψ tends to be 0 because $E_{\text{in}} > 0$). Although ψ fluctuates around these fixed values, the amplitude of the fluctuations is relatively small compared to the oscillation of θ, as described below. Moreover, the fluctuations are not affected by the bias current γ, in contrast to the tunneling of θ. Therefore, the switching event in a high-bias current condition $\gamma \simeq 1$ may occur via the tunneling along the θ axis. Thus, we can reduce our issue to an effective 1D tunneling problem along this in-phase path.

Let us more closely examine the dynamical behaviors of θ and ψ around the in-phase tunneling path. The amplitudes of θ oscillations and ψ fluctuations are characterized by, respectively, m_{cm}^{-1} and m_{rlt}^{-1}. Since the dimensionless constants α_1 and α_2 are small ($\alpha_i < 1$), we have small fluctuations of ψ. Hence, we examine the switching event in this Josephson junction, using the in-phase tunneling path with small relative-phase fluctuations. Along this tunneling path, the dynamics of θ is expressed by a particle under the so-called washboard potential. Furthermore, the dynamics of ψ can be expressed by a simple harmonic oscillator, with angular frequency

$$\omega_{\text{L}} = \frac{1}{\hbar}\sqrt{2(\alpha_1+\alpha_2)E_{\text{c}}E_{\text{in}}}. \tag{5}$$

In other words, ψ is regarded as a bosonic *environment* for the center-of-mass phase. We will show these points via the derivation of the Euclidean Lagrangian with the expansion of $\mathcal{L}$ around the in-phase tunneling path.

C. Euclidean Lagrangian around an in-phase tunneling path

Now we derive the Euclidean Lagrangian with three steps. Throughout this paper, we denote the imaginary time as τ $(=it)$. First, we take a small ψ expansion, up to second order. We obtain the Euclidean Lagrangian $\mathcal{L}^{\text{E}} = \mathcal{L}^{\text{E}}_{\text{cm}} + \mathcal{L}^{\text{E}}_{\text{rlt}} + \mathcal{L}^{\text{E}}_{\text{int}}$, with

$$\mathcal{L}^{\text{E}}_{\text{cm}} = \frac{m_{\text{cm}}}{2}\left(\frac{d\theta}{d\tau}\right)^2 - E_{\text{J}}(\cos\theta + \gamma\theta), \tag{6}$$

$$\mathcal{L}^{\text{E}}_{\text{rlt}} = \frac{m_{\text{rlt}}}{2}\left(\frac{d\psi}{d\tau}\right)^2 + \frac{1}{2}m_{\text{rlt}}\omega_{\text{L}}^2\psi^2, \tag{7}$$

$$\mathcal{L}^{\text{E}}_{\text{int}} = g_+E_{\text{J}}\psi^2\cos\theta - g_-E_{\text{J}}\psi\sin\theta. \tag{8}$$

The first term ($\mathcal{L}^{\text{E}}_{\text{cm}}$) is the Lagrangian for the center-of-mass θ, the second term ($\mathcal{L}^{\text{E}}_{\text{rlt}}$) is for the relative phase ψ, and the third term describes the interaction between θ and ψ. The coupling constants g_+ and g_- are

$$g_+ = \frac{1}{2E_{\text{J}}}\left[\frac{\alpha_1^2}{(\alpha_1+\alpha_2)^2}E_{\text{J1}} + \frac{\alpha_2^2}{(\alpha_1+\alpha_2)^2}E_{\text{J2}}\right], \tag{9}$$

$$g_- = \frac{1}{E_{\text{J}}}\left[-\frac{\alpha_1}{(\alpha_1+\alpha_2)}E_{\text{J1}} + \frac{\alpha_2}{(\alpha_1+\alpha_2)}E_{\text{J2}}\right]. \tag{10}$$

We find that g_+ is positive, whereas g_- vanishes when the parameters of the respective gaps are equivalent: $\alpha_1 = \alpha_2$ and $E_{\text{J1}} = E_{\text{J2}}$.

Second, in order to remove the nonlinearity with respect to ψ in the interaction Lagrangian, we use the mean-field approximation [21]. The expectation values of ψ and ψ^2 for the ground state (i.e., zero-temperature limit) of $\mathcal{L}^{\text{E}}_{\text{rlt}}$ are

$$\langle\psi\rangle_\psi = 0,\ \ \langle\psi^2\rangle_\psi = \frac{\hbar}{2m_{\text{rlt}}\omega_{\text{L}}} \equiv \Omega^2, \tag{11}$$

where the symbol $\langle\cdot\rangle_\psi$ indicates the expectation value with respect to ψ. Using these values, we rewrite $\mathcal{L}^{\text{E}}_{\text{int}}$ as a summation of the expectation values and the deviation from them, $\mathcal{L}^{\text{E}}_{\text{int}} = \langle\mathcal{L}^{\text{E}}_{\text{int}}\rangle_\psi + \delta\mathcal{L}^{\text{E}}_{\text{int}}$. Omitting higher-order fluctuations, we obtain the linearized interaction Lagrangian,

$$\mathcal{L}^{\text{E}}_{\text{int}} = g_+\Omega^2\cos\theta - g_-\psi\sin\theta. \tag{12}$$

We then derive the effective Euclidean Lagrangian with the mean-field approximation,

$$\mathcal{L}^{\text{E}}(\theta,\psi) = \frac{m_{\text{cm}}}{2}\left(\frac{d\theta}{d\tau}\right)^2 + V_{\text{eff}}(\theta) + \mathcal{L}^{\text{E}}_{\text{rlt}} - g_-\psi\sin\theta, \tag{13}$$

with

$$V_{\text{eff}}(\theta) = -E_{\text{J}}[(1-\varepsilon)\cos\theta + \gamma\theta], \tag{14}$$

where $\varepsilon = g_+\Omega^2$.

Third, we expand Eq. (13) around the local minimum of V_{eff}, denoted by $\theta_0 = \arcsin[\gamma/(1-\varepsilon)]$. In this paper, we focus on the case $\gamma \simeq 1$; this is typical for MQT experiments. After performing a constant phase-shift transformation, which does not change the path-integral measure, we obtain

$$\mathcal{L}^{\text{E}}(\theta,\psi) \approx \frac{m_{\text{cm}}}{2}\left(\frac{d\theta}{d\tau}\right)^2 + \widetilde{V}_{\text{eff}}(\theta) + \mathcal{L}^{\text{E}}_{\text{rlt}} - C_{\text{int}}\theta\psi + \delta V, \tag{15}$$

with

$$\widetilde{V}_{\text{eff}}(\theta) = \frac{\hbar^2\omega_{\text{eff}}^2(\gamma)}{4E_{\text{c}}}\left(\theta^2 - \frac{\theta^3}{\theta_1}\right), \tag{16}$$

where $\theta_1 = \cot\theta_0$ and $C_{\text{int}} = E_{\text{J}}g_-\cos\theta_0$. We have dropped constants irrelevant to θ and ψ. The current-dependent Josephson-plasma frequency is

$$\omega_{\text{eff}}(\gamma) = \frac{\sqrt{2E_{\text{c}}E_{\text{J}}(1-\varepsilon)}}{\hbar}\left[1 - \left(\frac{\gamma}{1-\varepsilon}\right)^2\right]^{1/4}. \tag{17}$$

We stress that the effect of the JP-JL coupling explicitly appears in this formula. Furthermore, we find that the interaction term $C_{\text{int}}\theta\psi$ is essentially the same as the system-bath interaction in the Caldeira-Leggett model [2]. The last term in Eq. (15) is the counterterm [3] $\delta V = (C_{\text{int}}^2/2m_{\text{rlt}}\omega_{\text{L}}^2)\theta^2$, which is added for reproducing Hooke's law between θ and ψ [i.e., $(\theta-\psi)^2$].

D. Escape-rate formula

We now show the formula for the MQT escape rate Γ. At the low-temperature limit, the escape rate [3] is $\Gamma = (2/\hbar\beta)\text{Im}\,K(\beta)$, with the inverse temperature β and

$$K(\beta) = \int_{-\infty}^{\infty}d\psi\int_{\theta(0)=0,\psi(0)=\psi}^{\theta(\beta)=0,\psi(\beta)=\psi}\mathcal{D}\theta(\tau)\mathcal{D}\psi(\tau) \times \exp\left\{-\frac{1}{\hbar}\int_0^{\hbar\beta}d\tau\,\widetilde{\mathcal{L}}^{\text{E}}[\theta(\tau),\psi(\tau)]\right\}. \tag{18}$$

One of the authors (S.K.) [20] developed a method to evaluate the MQT escape rate for this class of Lagrangian. This approach is essentially the same as the influence-functional method [2,3]. Thus, we find that when $\beta \to \infty$ (i.e., zero-temperature limit) for the case $\gamma \simeq 1$,

$$\Gamma = \omega_{\rm eff}\sqrt{\frac{30S_{\rm B}}{\pi\hbar}}\left(1+\frac{S_{\rm D}}{2S_{\rm B}}\right)\exp\left[-\frac{1}{\hbar}(S_{\rm B}+S_{\rm D})\right], \tag{19}$$

with

$$S_{\rm B} = \frac{8}{15}\frac{\hbar^2}{2E_{\rm c}}\omega_{\rm eff}\,\theta_1^2, \tag{20}$$

$$S_{\rm D} = \frac{8\pi C_{\rm int}^2\theta_1^2}{m_{\rm rlt}\omega_{\rm L}^2\omega_{\rm eff}}\int_0^\infty g(z)\,dz, \tag{21}$$

where $g(z)$ means the effects of the memory kernel in terms of the influence functional method,

$$g(z) = \frac{z^4}{[(\omega_{\rm L}/\omega_{\rm eff})^2+z^2]\sinh^2(\pi z)}. \tag{22}$$

Here, $S_{\rm B}$ is the bounce action of the tunneling particle θ along the extremal path on the potential $\widetilde{V}_{\rm eff}(\theta)$, whereas $S_{\rm D}$ is the dissipative action, which corresponds to the energy dissipation from θ to the environment ψ.

Before closing this section, let us summarize the role of the JL mode on MQT based on our theory described above. On the one hand, the zero-point fluctuations give a positive nonzero ε. This quantity effectively reduces the tunneling barrier height, as seen in Eq. (14). As a result, the zero-point fluctuations enhance the MQT escape rate. On the other hand, the linear interaction $C_{\rm int}\theta\psi$ in Eq. (15) causes energy dissipation from θ to the environment ψ. The effect of this *quantum dissipation* appears as the dissipative action $S_{\rm D}$ in Eq. (19), then it suppresses the MQT escape rate. In Figs. 2(a) and 2(b), we show the schematics of these two major roles of the JL mode.

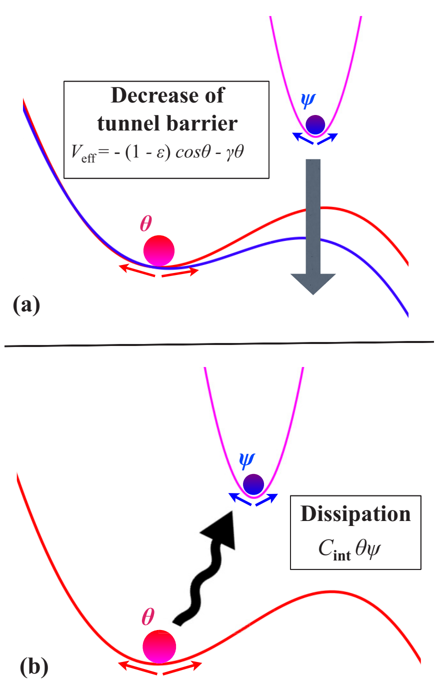

FIG. 2. (Color online) Schematic diagrams of two effects of the Josephson-Leggett (JL) mode on a macroscopic quantum tunneling (MQT) process. (a) MQT enhancement by the decrease of the barrier height. The curves indicate the potential energy with (blue) and without (red) JL mode. (b) MQT suppression by dissipation (quantum dissipation). The wiggling arrow schematically shows the energy dissipation from θ to ψ via linear coupling, $C_{\rm int}\theta\psi$.

IV. ESCAPE RATES WITH DIFFERENT JUNCTION PARAMETERS

We now numerically evaluate the MQT escape rate (19). In order to discuss the MQT in general two-band Josephson junctions, we perform the calculations for different junction parameter sets $(\alpha_i, E_{\rm J\it i})$, with fixed $E_{\rm c}/E_{\rm J}$ (=0.002), γ (=0.9), and $\alpha_1+\alpha_2$ (=0.1). We use two parameters characterizing the differences between the two tunneling channels,

$$\delta\alpha = \frac{\alpha_1-\alpha_2}{\alpha_1+\alpha_2}, \quad \delta E_{\rm J} = \frac{E_{\rm J1}-E_{\rm J2}}{E_{\rm J}}. \tag{23}$$

The former characterizes the density-of-states difference in the vicinity of the interface, while the latter is the normalized Josephson-energy difference. We examine Γ in the parameter space $(\delta\alpha, \delta E_{\rm J})$.

Figure 3 shows the MQT escape rate as a function of $\delta\alpha$ and δE_J, for $E_{\rm in}/E_{\rm J} = 0.1$. $\omega_p = \sqrt{2E_{\rm C}E_{\rm J}}/\hbar$ is the Josephson-plasma frequency. The black mesh indicates the escape rate with the JL mode, while the purple surface indicates the *bare* escape rate Γ_0, namely, Γ without the JL mode (i.e., $C_{\rm int} = \varepsilon = 0$). At the origin of the $(\delta\alpha, \delta E_J)$ space in which all band parameters are equivalent, $C_{\rm int}$ becomes zero. Therefore, the MQT suppression by quantum dissipation does not appear at this point. The MQT in this ideal condition was studied by two of the authors (Y.O. and M.M.) [21]. Figure 3 indicates that the MQT escape rate is enhanced by the JL mode for various hetero-Josephson junctions with two-band superconductors, whereas the effect of MQT suppression is marginal.

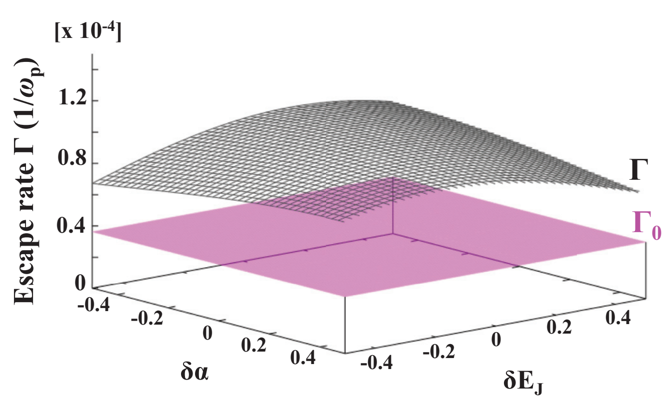

FIG. 3. (Color online) Macroscopic quantum tunneling escape rate vs the density-of-states difference $\delta\alpha$ and the Josephson-energy difference $\delta E_{\rm J}$ between the two tunneling channels shown in Fig. 1. $\omega_p = \sqrt{2E_{\rm C}E_{\rm J}}/\hbar$ is the Josephson-plasma frequency. The black mesh surface (Γ) shows the escape rate with the interband fluctuations. In contrast, the purple mesh surface (Γ_0) indicates the escape rate without the interband fluctuations.

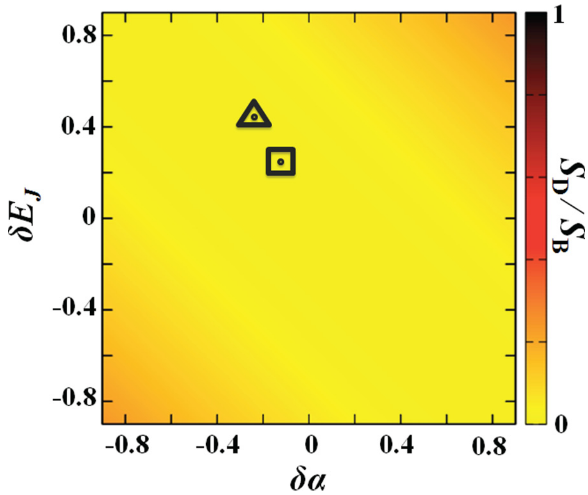

FIG. 4. (Color online) Ratio of the dissipative action to the bounce action, S_D/S_B. The open square and triangle correspond to the parameters for the iron-based superconductor $BaFe_2As_2$ and MgB_2, respectively.

The results in Fig. 3 indicates that the energy dissipation is relatively small, compared to the energy of the bounce motion of θ. To clarify this point, we calculate the ratio of the dissipative action to the bounce action. Figure 4 shows the contour map of S_D/S_B, with different junction parameters $(\delta\alpha, \delta E_J)$. The open square and the triangle in Fig. 4 indicate the parameter sets $(\delta\alpha, \delta E_J) = (-0.126, 0.251)$ and $(-0.236, 0.447)$, respectively. The former is evaluated by typical material parameters for $BaFe_2As_2$ [46,47], while the latter for MgB_2 [48]. We find that S_D is much smaller than S_B. It is noteworthy that the environment ψ oscillates with single angular frequency ω_L. Thus, there is only one dissipation channel in our system. This fact would lead to a small energy dissipation.

Finally, for clarifying the dependence of Γ on the interband Josephson energy, we calculate Γ, with different E_{in}/E_J. Figure 5(a) shows Γ as a function of E_{in}/E_J. In this calculation, we use again the typical material parameters for $BaFe_2As_2$ and MgB_2, as seen in Fig. 4. We find that Γ sharply increases with decreasing E_{in}/E_J. In order to understand this behavior, we plot ε and C_{int} as functions of E_{in}/E_J in Figs. 5(b) and 5(c). The magnitude of the zero-point fluctuations of the JL mode ε is large, with decreasing E_{in}/E_J. This behavior corresponds to the fact that the JL angular frequency ω_L decreases when E_{in} decreases. Thus, the zero-point fluctuations Ω^2 increase for small E_{in} [see Eq. (11) as well]. Therefore, the reduction of the tunneling barrier height is marked for small E_{in}. We also find that the coupling strength of the quantum dissipation C_{int} for the MgB_2 parameter set is larger than the $BaFe_2As_2$ parameter set, as shown in Fig. 5(c). In contrast, we find little difference in ε for these two parameter sets. Hence, the energy dissipation of MgB_2 is remarkable, compared to $BaFe_2As_2$. As a result, the enhancement of the MQT rate for MgB_2 is smaller than that of $BaFe_2As_2$.

Let us now summarize our results. The MQT in hetero-Josephson junctions with two-band superconductors is strongly affected by the presence of the JP-JL coupling.

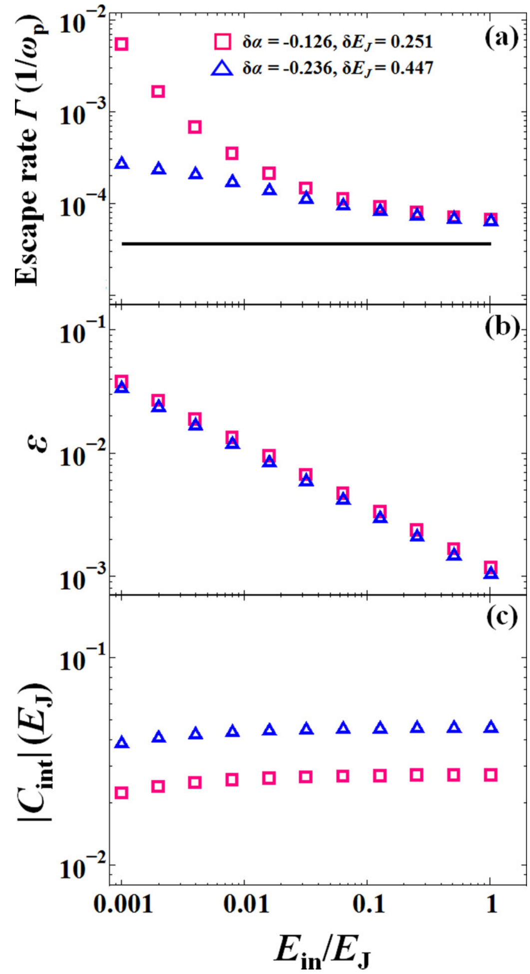

FIG. 5. (Color online) (a) Macroscopic quantum tunneling escape rate, (b) parameter $\varepsilon = g_+\Omega^2$ for the magnitude of the zero-point fluctuations of the Josephson-Leggett mode [see Eq. (14)], and (c) parameter $C_{int} = E_J g_- \cos\theta_0$ for the coupling strength between the tunneling particle θ and the environment ψ [see Eq. (15)], with different interband Josephson couplings, E_{in}/E_J. The ε and $|C_{int}|$ contribute to the enhancement and the suppression of the quantum tunneling, respectively. The horizontal solid line in (a) indicates the escape rate without interband fluctuations. The open square and the triangle indicate the results for $(\delta\alpha = -0.126, \delta E_J = 0.251)$ and $(\delta\alpha = -0.236, \delta E_J = 0.447)$, respectively.

In other words, the MQT escape rate reflects the nature of the JL mode characterized by the superconducting material parameters, i.e., the density of states near the interfaces and the interband Josephson energy.

V. CONCLUSION

We constructed a theory of macroscopic quantum tunneling (MQT) in Josephson junctions consisting of multiband superconductors, and clarified the effect of interband phase fluctuations, namely, the Josephson-Leggett (JL) mode on the MQT. In order to discuss the essential effect of the JL mode, we employed a minimal model of the multiband Josephson junction: a hetero-Josephson junction consisting of a conventional single-gap superconductor and a two-gap superconductor. We focused on the in-phase tunneling path along the center-of-mass motion of the phase differences, which is directly related to low-temperature switching events. In the tunneling process along the in-phase path, the effect of the JL mode is caused by the interaction between the JP mode and the JL mode. We derived a Lagrangian which explicitly includes the JL-JP coupling by using a mean-field approximation. The derived Lagrangian is similar to that of the Caldeira-Leggett model for dissipative quantum tunneling. Based on the imaginary-time path-integral method, we derived a formula for the escape rate from this Lagrangian.

In our junction, the JL mode plays two major roles which are opposite to each other: (i) the enhancement of quantum tunneling by lowering the tunneling barrier height and (ii) the suppression of quantum tunneling by quantum dissipation. We calculated the MQT escape rate, systematically varying the junction parameters. We clarified that the enhancement effect is dominant and that the MQT escape rate is significantly enhanced by the JL mode. The amount of the MQT enhancement depends on the properties of the JL mode characterized by the superconducting material parameters, such as the interband Josephson energy and the density of states near the interfaces of the junctions. Therefore, a precise analysis of the MQT would provide valuable information for the JL mode in multiband superconductors.

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[47] We can estimate $\delta\alpha$ and δE_{J}, with the density of states on the Fermi surface, $D_i(E_{\mathrm{F}})$ for the ith band. We obtain $\alpha_i \propto 1/\sqrt{D_i(E_{\mathrm{F}})}$ and $E_{\mathrm{J},i} \propto D_i(E_{\mathrm{F}})$. For $BaFe_2As_2$, we choose the contributions from the first and the second largest superconducting gaps of the five bands.

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