

Entanglement swapping and testing quantum steering into the past via collective decay

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We propose a scheme to realize entanglement swapping via superradiance, entangling two distant cavities without a direct interaction. The successful Bell-state-measurement outcomes are performed naturally by the electromagnetic reservoir, and we show how, using a quantum trajectory method, the nonlocal properties of the state obtained after the swapping procedure can be verified by the steering inequality. Furthermore, we discuss how the unsuccessful measurement outcomes can be used in an experiment of delayed-choice entanglement swapping. An extension of testing the quantum steering inequality with the observers at three different times is also considered.

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I. INTRODUCTION

Entanglement swapping [1] is a procedure to create entanglement between two qubits which have never directly interacted with each other and has been demonstrated experimentally by Pan *et al.* [2]. Together with quantum memories [3], one can, in principle, use entanglement swapping to build quantum repeaters [4] to overcome the decoherence problem in quantum communication [5]. In fact, entanglement swapping can also be viewed as a special example of quantum teleportation [6] if the unknown state is replaced by an entangled state [7]. Very recently, the notion of delayed-choice entanglement swapping was experimentally demonstrated by Ma *et al.* [8], an idea first considered in Peres's [9] and Cohen's [10] gedanken experiments. The intriguing feature of this mechanism is that the entanglement is made a posteriori, after the entangled qubits have been measured.

Another route to create entanglement between two qubits is to make use of the common environment [11], i.e., the one via which they can effectively interact. Taking this one step further, Chen *et al.* [12] combined the concept of teleportation and a common environment and proposed a method to teleport charge qubits via a common reservoir using the superradiance effect. The necessary Bell-state measurements are performed naturally by the collective decay, i.e., the sub- and superradiance channels [13]. An analysis of the fidelity and the success probability was further examined [14] using the quantum trajectory method.

In this work, we propose a scheme to accomplish entanglement swapping via collective decay, effectively swapping the entanglement between two atoms into entanglement between two distant cavities. The steering inequality [15] is utilized to verify the nonlocal properties of the final state obtained between the two distant cavities. We further point out that the unsuccessful outcomes, naturally occurring from the superradiance effect we employ here, may be useful in a

delayed-choice entanglement-swapping experiment. Finally, we consider the scenario of quantum steering into the past when the observers perform their measurements at three different times.

II. ENTANGLEMENT SWAPPING VIA SUPERRADIANCE

It is well known that highly entangled states can be “naturally” generated via collective spontaneous decay [13]. For two identical qubits interacting with a common photon reservoir, separated by a distance shorter than the emitted radiation wavelength, entanglement appears in the two intermediate states, $|S_0\rangle = (|+-\rangle - |-+\rangle)/\sqrt{2}$ and $|T_0\rangle = (|+-\rangle + |-+\rangle)/\sqrt{2}$, of the two decay channels [16] from the excited state $|T_1\rangle = |++\rangle$ to the ground state $|T_{-1}\rangle = |--\rangle$. Armed with this information, we now propose how to accomplish the entanglement-swapping protocol through such collective decay processes.

We first consider two identical two-level atoms, each passing through a separate cavity, as shown in Fig. 1. At this stage atom-cavity system 1 and atom-cavity system 2 are well separated from each other. In the strong atom-cavity coupling regime the interactions between the atoms and their respective cavities can be written as [17]

$$H' = \hbar g_j (\sigma_{j,+} b_j^- + \sigma_{j,-} b_j^+), \quad (1)$$

where $j = 1$ or 2 , g_j is the atom-cavity coupling strength, and b_j^\pm and the Pauli matrices $\sigma_{j,\pm}$ are the cavity photon and the atom operators, respectively. With the appropriate preparation of the initial states and control of the passage times through the cavities, the singlet entangled states

$$(|0\rangle_{c,j}|+\rangle_j - |1\rangle_{c,j}|-\rangle_j)/\sqrt{2} \quad (2)$$

are prepared between the j th atom and its corresponding cavity. Here, $|0\rangle_{c,j}$ ($|1\rangle_{c,j}$) and $|+\rangle_j$ ($|-\rangle_j$) represent the j th

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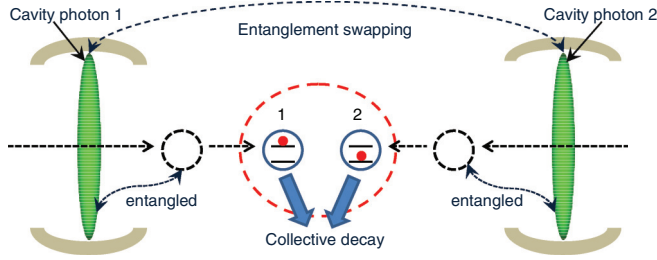


FIG. 1. (Color online) Schematic description of entanglement swapping via superradiance. First, the entanglement is generated between the j th atom and its corresponding cavity. The next step is to trap the two atoms, so that they decay collectively. If the measurement outcome is a single photon, the entanglement swapping can be achieved.

cavity with no (one) photon and the j th atom in its excited (ground) state, respectively.

Our goal is to use these entangled atom-cavity states and the phenomena of collective decay, as well as postselection, to perform entanglement swapping and entangle the photonic states of the two distant cavities. To achieve this, the next step is to remove the atoms from the cavities and trap the two atoms close together so that they experience superradiant collective decay processes in the common electromagnetic environment. This is a collective decay phenomenon which is enhanced when the interatom distance is much shorter than the wavelength of the emitted photon. One can also have a similar enhanced effect by placing the two atoms at the antinodes of a cavity [11]. Before the collective decay occurs, the total wave function of the combined atom-cavity systems can be written as

$$\begin{aligned}
 |\Psi\rangle &= \left[\frac{1}{\sqrt{2}}(|0\rangle_{c,1}|+\rangle_1 - |1\rangle_{c,1}|-\rangle_1) \right. \\
 &\quad \left. \otimes (|0\rangle_{c,2}|+\rangle_2 - |1\rangle_{c,2}|-\rangle_2) \right] \\
 &= \frac{1}{2} [|0\rangle_{c,1}|0\rangle_{c,2} \otimes |T_1\rangle_{12} + |1\rangle_{c,1}|1\rangle_{c,2} \otimes |T_{-1}\rangle_{12} \\
 &\quad + |0\rangle_{c,2}|1\rangle_{c,1} - |1\rangle_{c,2}|0\rangle_{c,1} \otimes |S_0\rangle_{12} \\
 &\quad - (|0\rangle_{c,2}|1\rangle_{c,1} + |1\rangle_{c,2}|0\rangle_{c,1}) \otimes |T_0\rangle_{12}]. \quad (3)
 \end{aligned}$$

Assuming that all photonic decay processes from the two atoms can be observed (inefficient detection is introduced below), there are four possible outcomes due to the collective decay: zero photons emitted ($|T_{-1}\rangle_{12}$), two photons emitted ($|T_1\rangle_{12}$), and, finally, just one photon via the subradiant channel ($|S_0\rangle_{12}$) or one photon via the superradiant channel ($|T_0\rangle_{12}$). If the measurement outcome is a single photon, i.e., via either $|S_0\rangle_{12}$ or $|T_0\rangle_{12}$, then entanglement swapping can be achieved, provided that the sub- and superradiant decays can be distinguished. As pointed out in Ref. [12], the momentum of the emitted photon \vec{k} depends on the separation of the two atoms \vec{r} , i.e., $\vec{k} \cdot \vec{r} = 0$, or π corresponds to the emission of a super- or subradiant photon. Therefore, to distinguish between the sub- and superradiant photons one can place the detectors at the appropriate angles.

One can use the recently proposed steering inequality [15] or Bell–Clauser–Horne–Shimony–Holt (CHSH) inequality [18] to verify the nonlocal properties of the state obtained after the swapping procedure. The density operator of the state obtained

after the postselection measurement can easily be calculated by using the quantum trajectory method [14], where it can be described as a probabilistic mixture of different measurement outcomes:

$$\rho(t) = \sum_i p_i |\psi_i(t)\rangle \langle \psi_i(t)|, \quad (4)$$

where i denotes the events, or measurements, of photodetection and $|\psi_i(t)\rangle$ is the pure state conditioned on this event. For example, the total master equation of the evolution of two two-level atoms and their collective decay phenomena, without postselection, can be written as

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H, \rho] + \sum_i^n (2\hat{J}_i\rho\hat{J}_i^\dagger - \hat{J}_i^\dagger\hat{J}_i\rho - \rho\hat{J}_i^\dagger\hat{J}_i). \quad (5)$$

H is any Hermitian Hamiltonian evolution (e.g., dipole-dipole interactions between the atoms or external magnetic fields). Here we neglect such terms and set $H = 0$. The photon-emission-event operators are

$$\hat{J}_1 = \sqrt{\frac{\gamma + \Gamma}{2}}(\sigma_{1,-} + \sigma_{2,-}), \quad (6)$$

$$\hat{J}_2 = \sqrt{\frac{\gamma - \Gamma}{2}}(\sigma_{1,-} - \sigma_{2,-}). \quad (7)$$

We assume the two atoms are separated by a distance d and that the wavelength of the emitted light is λ . Thus, γ is the spontaneous emission rate for a single atom and $\Gamma = \frac{\sin(2\pi d/\lambda)}{(2\pi d/\lambda)}\gamma$.

To obtain the postselected state, one can unravel the evolution (see Ref. [14] for a full description of a similar derivation for a teleportation scheme driven by superradiance), and then the un-normalized state of the two atoms given by

$$\begin{aligned}
 \overline{|\psi_i(t)\rangle} &= e^{-iH_B(t-t_n)} \hat{J}_{i_n} e^{-iH_B(t_n-t_{n-1})} \dots \hat{J}_{i_2} \\
 &\quad \times e^{-iH_B(t_2-t_1)} \hat{J}_{i_1} e^{-iH_B t_1} |\psi(0)\rangle, \quad (8)
 \end{aligned}$$

with the effective non-Hermitian Hamiltonian written as

$$H_B = -i\hbar(\hat{J}_1^\dagger\hat{J}_1 + \hat{J}_2^\dagger\hat{J}_2). \quad (9)$$

As pointed out in Ref. [14], the sub- and superradiant outcomes can also be distinguished in a statistical sense. In this case, one notes that state $|S_0\rangle_{12}$ ($|T_0\rangle_{12}$) favors the longer (shorter) emission times. Following the derivation in Ref. [14], one can define a crossover time t_1^* , such that the swapped state should be corrected (i.e., $|0\rangle_{c,2}|1\rangle_{c,1} + |1\rangle_{c,2}|0\rangle_{c,1} \rightarrow |0\rangle_{c,2}|1\rangle_{c,1} - |1\rangle_{c,2}|0\rangle_{c,1}$) for a single-photon emission at time $t_1 < t_1^*$. For $t_1 > t_1^*$, there is no need to correct the swapped state. We now assume that after the atoms are brought together one waits a time T and retains the state when exactly one photon, emitted by the atoms, is detected in the period T (with detection efficiency η). After averaging over all such detection events (distinguished by the statistical scheme mentioned above) and tracing out the atomic states [14], the remaining

two-cavity state is

$$\begin{aligned} \rho_c(\eta, T) = & \eta\rho_c(T) + \frac{\eta(1-\eta)}{4} |0\rangle_{c,1}|0\rangle_{c,2} \otimes {}_{c,2}\langle 0|_{c,1}\langle 0| \\ & \times \left\{ \frac{2(\gamma^2 + \Gamma^2)}{\gamma^2 - \Gamma^2} (1 + e^{-2\gamma T}) - \frac{2}{\kappa} e^{-(\gamma+\Gamma)T} \right. \\ & \left. - 2\kappa e^{-(\gamma-\Gamma)T} - \frac{4\Gamma^2}{\gamma^2 - \Gamma^2} (1 - e^{-2\gamma T}) \right\}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \rho_c(T) = & |0\rangle_{c,1}|0\rangle_{c,2} \otimes {}_{c,2}\langle 0|_{c,1}\langle 0| \left\{ \frac{1}{4\kappa} [e^{-(\gamma+\Gamma)T} - e^{-2\gamma T}] \right. \\ & \left. + \frac{\kappa}{4} [e^{-(\gamma-\Gamma)T} - e^{-2\gamma T}] \right\} \\ & + \frac{1}{4} |\psi^+\rangle \langle \psi^+| \{ 1 - \kappa^{(\gamma+\Gamma)/2\Gamma} \\ & + \kappa^{(\gamma-\Gamma)/2\Gamma} - e^{-(\gamma-\Gamma)T} \} \\ & + \frac{1}{4} |\psi^-\rangle \langle \psi^-| \{ 1 - \kappa^{(\gamma-\Gamma)/2\Gamma} \\ & + \kappa^{(\gamma+\Gamma)/2\Gamma} - e^{-(\gamma+\Gamma)T} \}. \end{aligned} \quad (11)$$

Here,

$$\kappa \equiv (\gamma - \Gamma)/(\gamma + \Gamma), \quad (12)$$

$$|\psi^-\rangle = (|0\rangle_{c,2}|1\rangle_{c,1} - |1\rangle_{c,2}|0\rangle_{c,1})/\sqrt{2}, \quad (13)$$

$$|\psi^+\rangle = (|0\rangle_{c,2}|1\rangle_{c,1} + |1\rangle_{c,2}|0\rangle_{c,1})/\sqrt{2}. \quad (14)$$

To test the nonlocal properties of the resultant cavity state $\rho_c(\eta, T)$, we use both the steering inequality [15] and the maximum value of a Bell inequality violation [18]. For the steering inequality, the correlated measurements observed by Alice and Bob on the two cavities are described by the probability distribution $P(B_i = b, A_i = a)$, with $b = \pm 1$ or 0 and $a = \pm 1$. If the two cavities are not entangled, the steering inequality is written as [15]

$$S_N \equiv \sum_{i=1}^N E[\langle \hat{A}_i \rangle_{B_i}^2] \leq 1, \quad (15)$$

where $N(= 2$ or $3)$ is the number of mutually unbiased measurements that Alice implements on her qubit (cavity) and

$$E[\langle \hat{A}_i \rangle_{B_i}^2] \equiv \sum_{b=\pm 1, 0} P(B_i = b) \langle \hat{A}_i \rangle_{B_i=b}^2, \quad (16)$$

with Alice's expectation value for a measurement (conditioned on Bob's result) defined as

$$\langle \hat{A}_i \rangle_{B_i=b} \equiv P(A_i = +1 | B_i = b) - P(A_i = -1 | B_i = b). \quad (17)$$

For the maximum value of a Bell inequality violation, the Bell operator associated with the CHSH inequality has the following form [18]:

$$\hat{B}_{CHSH} \equiv \hat{a} \cdot \hat{\sigma} \otimes (\hat{b} + \hat{b}') \cdot \hat{\sigma} + \hat{a}' \cdot \hat{\sigma} \otimes (\hat{b} - \hat{b}') \cdot \hat{\sigma}, \quad (18)$$

where \hat{a} , \hat{a}' , \hat{b} , \hat{b}' are unit vectors in \mathbf{R}^3 . Here, $\hat{a} \cdot \hat{\sigma} \equiv \sum_{i=1}^3 a_i \sigma_i$, where σ_i are the standard Pauli matrices. If the

two cavities are not entangled, the CHSH inequality of the two-cavity state ρ obeys

$$|\langle \hat{B}_{CHSH} \rangle_\rho| = |\text{Tr}(\rho \hat{B}_{CHSH})| \leq 2. \quad (19)$$

The maximum value of the CHSH inequality is given by

$$B_{\max} = \max_{\hat{a}, \hat{a}', \hat{b}, \hat{b}'} \text{Tr}(\rho \hat{B}_{CHSH}). \quad (20)$$

Thus, violations of the steering parameter S_N or the maximum value of the CHSH inequality B_{\max} can mean that the resultant cavity state is entangled.

In Fig. 2(a), we plot the steering parameter S_N and the maximum value of the CHSH inequality B_{\max} for various

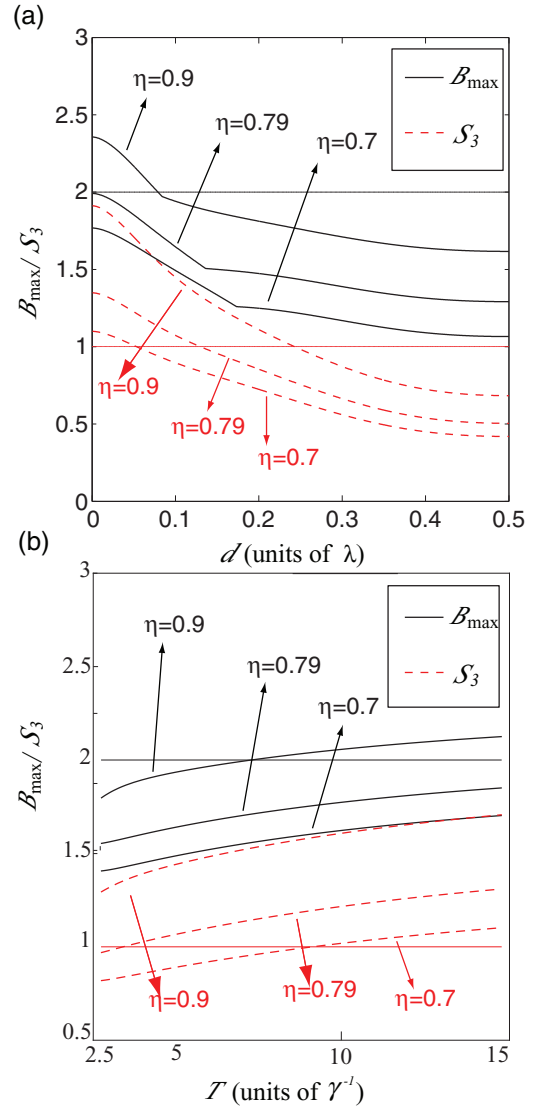


FIG. 2. (Color online) Testing the steering and Bell-CHSH inequalities for the two-cavity state $\rho_c(T)$ after entanglement swapping. The solid curves and dashed curves represent the results of the maximum value of Bell's inequality B_{\max} and the steering parameter S_3 , respectively. The horizontal black line is the Bell-CHSH inequality bound, and the horizontal red (gray) line is the S_3 bound. In plotting (a), we set the waiting time $T = (5/\gamma)$, and the interatomic distance d is in units of the wavelength λ of the emitted photon. In plotting (b), we set the interatomic distance $d = 0.1\lambda$.

photon-detection efficiencies. When decreasing the photon-detection efficiency η below 0.79, the steering inequality (dashed curves) is still violated, while the Bell-CHSH one (solid curves) is not. For the interatomic distance d , the range for the violation of the steering parameter is larger than that for the mean value of the Bell observable. In Fig. 2(b), we also plot S_N and B_{\max} as a function of the waiting time T . As seen, if the waiting time T is too short, it is possible that the inequalities are not violated. This coincides with the results obtained in Ref. [14]: The longer the waiting time T is, the higher the success probability is. In addition, the results in Figs. 2(a) and 2(b) all show that the steering inequality has better tolerance in examining the nonlocal properties of the entangled states [15].

III. DELAYED-CHOICE ENTANGLEMENT SWAPPING

From Eq. (3), naively, one knows there is a 50% chance that this protocol may fail, i.e., one may obtain the outcomes $|T_{-1}\rangle_{12}$ or $|T_1\rangle_{12}$. This drawback is actually useful if one wishes to implement delayed-choice entanglement swapping. To illustrate this, let us start with Peres's [9] original gedanken experiment. The joint state of a pair of singlets (particles a and b and particles c and d) takes the form

$$|\Psi\rangle_{abcd} = |\psi^-\rangle_{ab} \otimes |\psi^-\rangle_{cd}, \quad (21)$$

where $|\psi^-\rangle_{ab} = (|\uparrow\rangle_a|\downarrow\rangle_b - |\downarrow\rangle_a|\uparrow\rangle_b)/\sqrt{2}$ and likewise for $|\psi^-\rangle_{cd}$. Here, $|\uparrow\rangle_k$ and $|\downarrow\rangle_k$ are the two spin states of the particles $k = a, b, c, d$. Equation (21) can be rewritten in the basis of Bell states of particles a and d and b and c :

$$|\Psi\rangle_{abcd} = \frac{1}{2}(|\psi^+\rangle_{ad} \otimes |\psi^+\rangle_{bc} - |\psi^-\rangle_{ad} \otimes |\psi^-\rangle_{bc} - |\phi^+\rangle_{ad} \otimes |\phi^+\rangle_{bc} + |\phi^-\rangle_{ad} \otimes |\phi^-\rangle_{bc}), \quad (22)$$

where $|\psi^\pm\rangle_{ad} = (|\uparrow\rangle_a|\downarrow\rangle_d \pm |\downarrow\rangle_a|\uparrow\rangle_d)/\sqrt{2}$ and $|\phi^\pm\rangle_{ad} = (|\uparrow\rangle_a|\uparrow\rangle_d \pm |\downarrow\rangle_a|\downarrow\rangle_d)/\sqrt{2}$ are the symmetric Bell states (and likewise for particles b and c). In the normal scheme of delayed-choice entanglement swapping, particles a and d are sent to Alice and Bob. The third observer, Eve, performs the Bell-state measurement on particles b and c after Alice and Bob have measured the values of their spin components (randomly chosen along arbitrary directions). With Eq. (22), Alice and Bob can then sort their data into four subsets according to the measurement outcomes of Eve. If Alice and Bob test Bell's inequality with only the data in one of the subsets, they would find the inequality is readily violated.

Very recently, Ma *et al.* [8] experimentally demonstrated the delayed-choice entanglement-swapping protocol using photons. In their experiment, instead of choosing all four Bell-state measurements, they performed measurements which projected either onto entangled states or onto the separable states $(|\uparrow\rangle_a|\uparrow\rangle_d \pm |\downarrow\rangle_a|\downarrow\rangle_d)$. This allowed them to *a posteriori* decide whether Alice and Bob's states are entangled or separable.

Returning to our entanglement-swapping scheme in Eq. (3), although there is a 50% chance that the swapping may fail, the unsuccessful results ($|T_{-1}\rangle_{12}$ and $|T_1\rangle_{12}$) are just those that project the particles into the separable states [8] and therefore can be used in the experiment for delayed-choice entanglement swapping. An additional advantage here, over that of a scheme based on purely photonic degrees of freedom, is that we can

make use of other internal metastable states of the atoms to postpone the collective decay (Fig. 3). When atoms b and c are trapped, laser pulses can be applied to store the information in metastable states with a much longer dephasing or decay time. After Alice and Bob have performed their measurements on their own particles (which are now the photonic degrees of freedom in the cavities) at time t_1 , the reverse pulses are applied to the atoms b and c to continue the collective decay at a later time t_2 .

IV. TESTING QUANTUM STEERING INTO THE PAST

In our scheme, the Bell measurements are made ‘‘automatically’’ by the common photon reservoir. Therefore, the delayed choice is intrinsically random, i.e., in the stochastic sense of spontaneous decay. However, a loophole may arise when Alice and Bob both happen to choose their measurements along the z axis. If their measurement outcomes are the same ($|0\rangle_{c,1}|0\rangle_{c,2}$ or $|1\rangle_{c,1}|1\rangle_{c,2}$), from Eq. (3) they would know in advance that the later joint-measurement settings can be only the separable ones and could never be the entangled ones. To overcome this, the relative temporal order of the three observers should be changed: $t_1(\text{Alice}) < t_2(\text{Eve}) < t_3(\text{Bob})$, as shown in Fig. 4. In this case, even if Alice chooses her setting along the z axis, the wave function of Eq. 3 may, for example, collapse onto

$$|\Psi\rangle_{bcd} = \frac{1}{2}[|0\rangle_{c,2} \otimes |T_{-1}\rangle_{12} - |1\rangle_{c,2} \otimes |S_0\rangle_{12} + |1\rangle_{c,2} \otimes |T_0\rangle_{12}]. \quad (23)$$

The later joint atomic measurement driven by the common photon reservoir can still either project it into the separable state ($|T_{-1}\rangle_{12}$) or the entangled states ($|S_0\rangle_{12}, |T_0\rangle_{12}$). Actually, such an arrangement can be viewed as a special kind of quantum steering if the atomic regime and Bob's cavity are considered as a black box [15]. This means Bob can steer

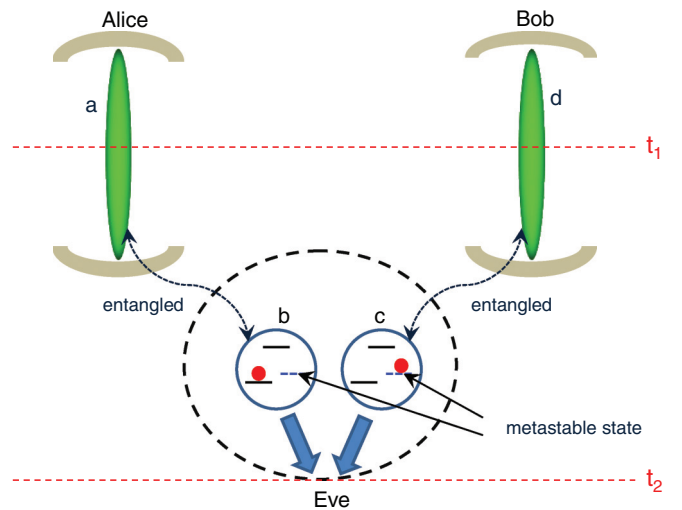


FIG. 3. (Color online) Delayed-choice entanglement swapping via superradiance. When atoms b and c are trapped, laser pulses are applied to store the information in metastable states with a much longer dephasing or decay time. After the measurements are performed on the cavities (a and d) at time t_1 , the reverse pulses are applied to atoms b and c to continue the collective decay at a later time t_2 .

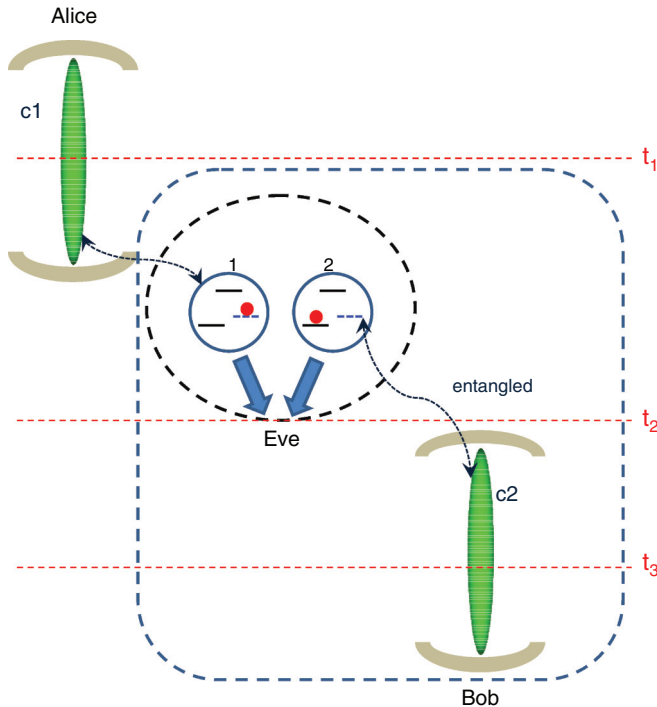


FIG. 4. (Color online) To test quantum steering into the past via superradiance, the relative temporal order of the three observers should be arranged as $t_1(\text{Alice}) < t_2(\text{Eve}) < t_3(\text{Bob})$. If the atomic regime and Bob's cavity are considered as a black box (blue dashed line), Bob can steer Alice's state by his measurement.

Alice's state by his measurement, which is conditioned on the random choice of the collective decay. Remember that Alice's measurement is performed before Bob's measurement. Similar to Bell's test in Ref. [8], one can therefore test "quantum steering into the past" by using the steering inequality [15].

The above proposal may also be further simplified by using only one cavity with two atoms. As shown in Fig. 5, one first lets atom 1 pass through the cavity, and as it does so, the cavity plays the role of cavity c_1 in the above scheme. After atom 1 has exited the cavity and is trapped and its state has been transferred to an internal metastable state, one then performs a measurement on the photonic state in cavity c_1 and records the data. The next step is to "reset" the cavity and let atom 2 pass through it, so that the cavity now plays the role of cavity c_2 . When atom 2 arrives at the trap, atom 1 is transferred back to its original state so that the collective decay can occur. Then, a measurement is again performed on cavity c_2 .

Following the above procedure, one can obtain the violation of the steering inequality and may have the illusion of "self-entanglement" since the measurement data are produced by the same cavity. However, this is not the case because when

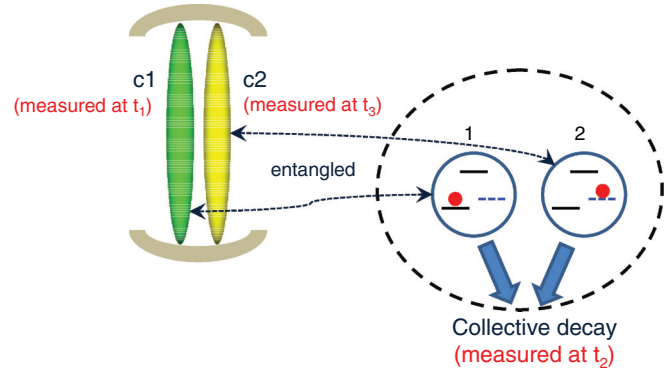


FIG. 5. (Color online) A simplified version of testing quantum steering into the past. After atom 1 is trapped and the information is stored in its metastable state, one then performs a measurement on cavity c_1 at time t_1 . The next step is to reset the cavity and let atom 2 pass through it. When atom 2 arrives at the trap, the reverse pulse is applied to atom 1, so that a collective decay can occur at time t_2 . Then, the measurement is performed on cavity c_2 at time t_3 .

we reset the cavity, it (of course) becomes a new system. The use of a single cavity simplifies a possible experimental implementation and emphasizes the fact that the relative temporal order of the three observers' events is irrelevant [8,9].

V. SUMMARY

In summary, we have proposed a scheme to accomplish entanglement swapping via superradiance. We outlined how the swapping protocol could be combined with collective decay to entangle two distant cavities. After postselection and averaging over all single-photon events, the nonlocal properties of the two-cavity state were analyzed by both the steering and Bell-CHSH inequalities. We found that the steering inequality has a better tolerance in verifying the nonlocal properties of the swapping state. Furthermore, we have also pointed out that the unsuccessful events in our scheme can be used in a delayed-choice entanglement-swapping protocol.

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